

The Method of Inter-Transformation between Horizontal and Space Coordinate

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Abstract. Realizing coordinates transformation between WGS-84 datum and local datum is the key problems of GPS surveying. It is difficult to obtain the precise geodetic height in the national or local coordinate system which affects the accuracy of 7 parameter models. This paper puts forward a way of transformation which can achieve the WGS-84 to BJ-54 and WGS-84 to Xi'an 80 coordinate transformation combining with an "adjustment" method. It only needs to know the horizontal coordinate of each datum. The results of experiment show that in a certain longitude-latitude grid it is able to achieve the high accuracy horizontal coordinate transformation to meet the demand of the actual work in the case of lacking geodetic height.

Introduction

Usually the coordinates of GPS positioning is WGS-84 coordinates[1], and it should be transformed to national, local or engineering coordinates to meet the requirement of actual produced operation[2-3]. The methods of space coordinate transformation in the engineering commonly used bursa model[4-7], while the bursa model could be used only in spatial right-angle coordination system, as a consequence, we need to get geodetic height in the national or local coordinate system to transform the geodetic coordinates into spatial right-angle coordinates[8-12]. Although high precision WGS-84 coordinates are easier to obtain, it is difficult to get geodetic height in the national or local coordinate system by reason of its definition[13,14]. The overall precision of 7-parameter models is effected which would be used inconveniently.

There is little difference between national or local ellipsoid and WGS-84 ellipsoid in the process of coordinate transformation. Therefore, in a certain longitude-latitude grid, the shape of WGS-84 ellipsoidal surface is the same as regional ellipsoidal surface, and the main difference is the "relative position" of them, which includes the height difference, nonparallelism, and so on.

The coordinate transformation of surface can be understood as a problem of an "adjustment" of the two known surface. This paper presents the method of 7-parameter coordinate transformation based on "adjustment" method. It simulates, calculates and analysis the error characteristics and application scope of this method which needs not geodetic height and would be used conveniently.

An "Adjustment" Method

Take the WGS-84 to BJ-54 as an example, we assumed the ellipsoidal height H of calculating points be zero in both WGS-84 and BJ-54 coordinate, which means that all of the calculation points were constrained on their ellipsoid surface, then, the seven parameters were solved with the method of the curved surface approximation. In this method, the ellipsoidal height has no practical significance, and

was used as the standard to evaluating the approximation level of the two ellipsoids. This algorithm only adjusted the relative position of two ellipsoids but not change the curvature of the two ellipsoids. And the key of calculation is still the problem of 7-parameter coordinate transformation. Bursa model is in Eq. 1[15-17]:

$$\begin{bmatrix} \mathbf{X}_b \\ \mathbf{Y}_b \\ \mathbf{Z}_b \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{X} \\ \Delta\mathbf{Y} \\ \Delta\mathbf{Z} \end{bmatrix} + (1 + k) \begin{bmatrix} \mathbf{X}_a \\ \mathbf{Y}_a \\ \mathbf{Z}_a \end{bmatrix} + \begin{bmatrix} 0 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 0 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 0 \end{bmatrix} \begin{bmatrix} \mathbf{X}_a \\ \mathbf{Y}_a \\ \mathbf{Z}_a \end{bmatrix} \quad (1)$$

Where $\mathbf{X}_a = (X_a, Y_a, Z_a)^T$, which is considered to be the spatial right-angle coordinates in the original coordinate system, $\mathbf{X}_b = (X_b, Y_b, Z_b)^T$, which is considered to be the spatial right-angle coordinates in the objective coordinate system, $\Delta\mathbf{X} = (\Delta X, \Delta Y, \Delta Z)^T$, which is the translation parameters. $\varepsilon_x, \varepsilon_y, \varepsilon_z$ respectively represent the parameters of rotation matrix, k is scale parameter.

The method of the “adjustment” was to make the coordinate in BJ-54 ellipsoid denoted by β closing the corresponding coordinate in WGS-84 ellipsoid denoted by α in the study area, and the more visual demonstration was shown in Fig.1. We assumed the calculation points’ ellipsoid height in both ellipsoid be zero. Actually, as shown in Fig.2, 3, the ellipsoid height of common points are different in the two coordinate system, and this difference value was calculated and used to measure the approximation level of the two ellipsoid surface during the iteration process, and this algorithm does not change the shape or the curvature of the ellipsoid surface, which means that the common points in two ellipsoid cannot totally overlap and there are errors in the common points, and these errors can be attributed as the model error. The model errors change by the different ellipsoid parameters, and can be calculated based on these ellipsoid parameters.

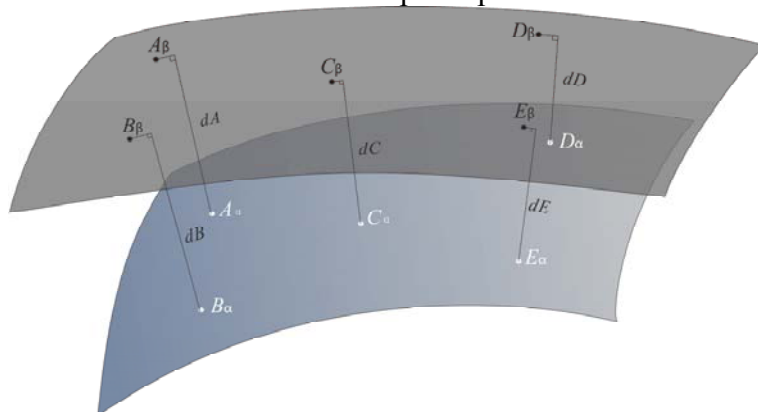


Figure 1.

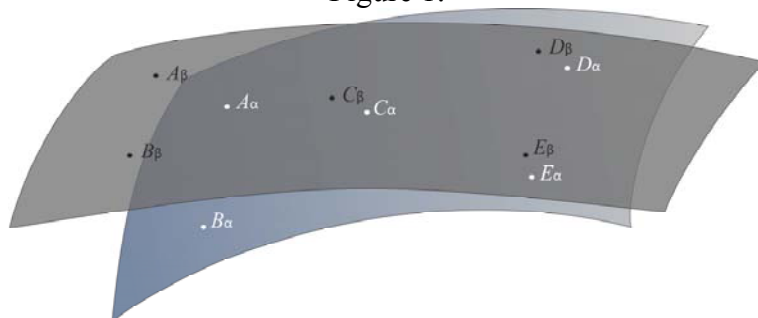


Figure 2.



Figure 3.

And the flowchart of the iterative computations was shown in Fig. 4:

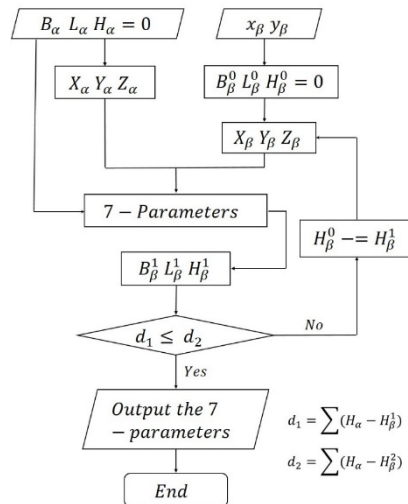


Figure 4.

Given the inconsistent of the curvature of two ellipsoids, the common points in two ellipsoid cannot overlap perfectly, as Fig.3 shown, which lead to the oscillation of ellipsoid height correction values in small range that can be calculated by the ellipsoid parameter. The horizontal errors caused by the oscillation of ellipsoid height was very small in middle and small scale of zone considering the error of triangular control points of BJ-54. The iteration stopped when $d_1 < d_2$, that is, the stop condition of the iteration is the beginning of the oscillation. The approaching of the two ellipsoid surface in the calculating process can be attributed as that the minimization of common points on the two ellipsoid β - α surface. When the iteration stopped, the two ellipsoid surface can be attributed as approaching by the distance minimization of common points and these distances approached average vertical distance

Simulation Experiment

In order to compare the applicability and stability of this method in different areas, select two areas to calculate, and choose two groups of different points in each area.

The calculation procedures are as follows:

At first: Suppose the 7 parameters which are from WGS-84 to BJ54 are: $dx = 10$ $dy = -20$ $dz = 15$ $\varepsilon_x = 5$ $\varepsilon_y = 2$ $\varepsilon_z = 1$ $s = 20$. Where the basic unit of translation parameter is m, the rotation parameter is second, the zooming parameter is ppm. According the value of B84、L84、H84 of the points in each area to calculate the value of B54、L54、H54, then, the real value x_{54} 、 y_{54} of horizontal coordinate can be computed by Gauss projection *the longitude of central meridian is 120°*.

Secondly, supposing the values of the points' geodetic heights in WGS-84 coordinate system are zero, choose points to form coordinate groups to calculate its geodetic coordinate- B_{54}' 、 L_{54}' . Compare the x_{54}' 、 y_{54}' in the BJ-54 coordinate system which are calculated by Gauss projection with the real value of the horizontal coordinate x_{54} 、 y_{54} , and analysis the calculation results.

The difference between x_{54} 、 y_{54} and x_{54}' 、 y_{54}' of the four test data in each area is as shown in Table 1, where $\Delta x = x_{54} - x_{54}'$ $\Delta y = y_{54} - y_{54}'$.

Table 1

		117°-123°E 26°-34°N		117°-123°E 48°-56°N	
group	point	Δx	Δy	Δx	Δy
1	A	0.044	-0.055	0.037	0.052
	B	0.042	0.057	0.032	0.045
	C	-0.016	-0.043	0.012	-0.038
	D	0.024	0.020	-0.041	-0.052
	E	-0.039	0.052	0.025	0.055
	F	0.021	-0.018	-0.013	0.021
2	G	-0.031	0.023	0.019	0.021
	H	0.013	-0.007	-0.039	-0.050
	I	0.015	0.026	-0.025	0.024
	J	-0.022	0.026	0.013	-0.049

Similarly, experiment on four groups data from WGS-84 to Xi'an-80 to obtain the difference between the real value and the calculated value of Xi'an-80 horizontal coordinate. And the result was shown in Table 2:

Table 2

		117°~123°E 26°~34°N		117°~123°E 48°~56°N	
group	point	Δx	Δy	Δx	Δy
1	A	0.047	-0.043	0.036	-0.050
	B	-0.019	0.055	-0.024	0.042
	C	0.025	0.010	0.022	0.005
	D	-0.043	0.045	-0.045	0.031
	E	0.014	0.057	0.016	0.055
	F	-0.035	-0.047	-0.023	-0.020
2	G	0.015	-0.028	0.015	0.021
	H	-0.003	0.030	-0.019	-0.000
	I	-0.036	-0.016	0.017	0.018
	J	0.010	0.022	0.010	0.019

The results show that the inner average precision of this model has high accuracy whose error is less than 6cm. The far from the center, the higher error is. Furthermore, error will decrease gradually with the increase of latitude and is associated with the curvature variation of meridian.

Taking the area as an example whose longitude ranges from 117° to 123° and latitude ranges from 26° to 56°, use this method to carry out two groups of experiment. By the experiment data we can see all the maximum error is less than 5cm and all the average error is less than 3cm. Different ways of choosing points will have different effect on coordinate transformation in this model. Furthermore, high accuracy horizontal coordinate can be obtained by this method in which longitude is the same and the latitude varies.

Conclusions

The results of study show that high accuracy horizontal coordinate transformation can be completed by using the idea of "adjustment" method in different areas. Because there is not accurate geodetic data in the astro-geodetic network of China, it avoids the effect of geodetic height error on coordinate transformation by using this method. As well as it is able to complete the high accuracy horizontal coordinate transformation to meet the demand of the actual work in the case of geodetic height unknown or inaccurate.

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