

The Methodology of Mathematics and the Emergence of a Proto-Discipline

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ABSTRACT

A key issue in the institutionalisation of Mathematics' Education and in the professionalisation of teachers educators consists in characterizing the type of knowledge involved in the formation of primary school teachers of mathematics (Valente, 2017). The aim of this article is to explain how “*knowledge to teach*” and “*knowledge for teaching*” have been historically present in the institutions devoted to the training of teachers in Spain since the formation of the *Escuelas Normales* (teacher training colleges), from the beginning of the second third of the 19th century until their incorporation into the university system. Likewise, this work looks into the institutional conditions and other restrictions that operated over the mathematics studied in such institutions.

Moreover, the disciplines' consolidation is related to the institutionalisation of knowledge. In this article it is explained how the subject *Metodología de las Matemáticas* (*Methodology of Mathematics*), which appeared in the curriculum for primary school teachers during the Second Spanish Republic (1931-1936), reached what could be described as a *proto-disciplinary* status. Finally, the historical background that made this possible is detailed together with the circumstances that led to the interruption of the process.

Keywords: teacher training colleges, methodology of mathematics, mathematics *to teach*, mathematics *for teaching*, proto-discipline

INTRODUCTION

The teaching profession distinguishes two types of knowledges. The institution in which the learning takes place defines the *knowledges to teach*, build up as objects of teaching and systematically organized in formal disciplines. Moreover, there are the *knowledges for teaching*, different from the first ones and considered as work tools for the teacher. These are knowledges *for the action* of the teacher (Hofstetter & Schneuwly, 2009).

Valente (2018) emphasises the difference between mathematics as a discipline and mathematics as a part of teachers professional knowledge:

Thus, mathematics that integrate a training for teaching, mathematics as a tool for the professional who teaches, have a character different from disciplinary mathematics, typical of mathematical science, not professionally committed to its teaching. There are mathematics for teaching; they constitute a professional knowledge.

Thus, characterizing the *mathematics to teach* and the *mathematics for teaching* is directly linked to the construction of the professional knowledge of the teacher who teaches mathematics (Valente, 2018, 384, italics in the original).

To understand the professionalisation process of the professor in charge of the academic training of teachers in mathematics and its didactics one must consider the role played by mathematics. In turn, in Spain this professionalisation process is linked to the constitution of a characteristic discipline for the profession. Determining to what extent the *mathematics for teaching* and the *mathematics to teach* were present in the training institutions is a key element when researching how the necessary professional teacher knowledges have become a discipline.

These knowledges of the profession, which integrate *knowledge for teaching* and *knowledge to teach*, come into place during several historical periods and in the context of specific social and political situation. These circumstances lead to aspirations and expectations of the protagonists, while determining what is possible to do at each moment. This work addresses the institutional and “ecological” dimension, in the sense of the Anthropological Theory of Didactics (ATD). Especially, the notion of *ecological* analysis (Chevallard, 2002) is used, in the sense of trying to understand the conditions or restrictions influencing teachers'

formation and, more specifically, in the mathematical training (knowledge *for teaching* and knowledge *to teach*) they received. At the same time, the evolution of such conditions shall favour or prevent certain changes.

This study begins with an overview of the legislation to determine the topics of mathematics that appeared in the different curricula of *Escuelas Normales* (teacher training colleges). It then proceeds with the identification of the references to the methodology of mathematics in the legislation, in the subjects' names.

Afterwards, teachers training is characterized in each of the historical periods in terms of mathematics *for teaching* and mathematics *to teach*. The restrictions that new curricula had to face are described together with the tensions between the law and its actual possibilities of application in real life. The impact of the creation of the subject *Metodología de las Matemáticas* (Methodology of Mathematics) is analysed. Finally, it is studied the extent to which the political situation of the moment combined with the renovation effort of the previous years resulted in mathematics *for teaching* taking on major importance.

From the above, and considering how expert voices characterize any process of disciplinarization, certain elements are identified confirming that such process was initiated when the 1931 curriculum came into effect, although it could not be consolidated.

MATHEMATICS IN THE CURRICULUM OF THE TEACHER TRAINING COLLEGES

To characterize which mathematics were taught in the teacher training colleges in Spain, I have first gathered mathematics subjects in the curricula since these centres were created, from the second third of the 19th century, until their definite incorporation into the universities at the beginning of the 1970s.

The mathematics subjects that were taught to future teachers in teacher training colleges during the nineteenth century refer to disciplinary content. The Regulations of 1837 and 1842 established by the Escuela Normal Central, created in Madrid to train professors for the *escuelas normales* in the provinces, establish the subject Arithmetic and Elements of Geometry. The *Ley de Instrucción Pública* (Public Instruction Law) of 1857, also known as the *Ley Moyano* (Moyano Law), has been the education law in force for the longest time in our country. It considered, in articles 68 and 69, the subjects Arithmetic, and Notions of Geometry, and Linear Drawing and Surveying in order to obtain the title of primary teacher. It included Notions of Algebra only for teachers of upper primary education. The following year's curriculum involves, in addition to this introduction to algebra, further studies of arithmetic and geometry for these teachers.

At this early stage of the teacher training colleges existence, it is not like the curriculum does not include any aspect relative to the mathematics methodology. But the truth is that there was no subject as such dedicated to it. In the first decades of the existence of teacher training colleges, arithmetic methodology was included among the general teaching methods and was taught as part -usually a chapter or lesson- of one of the pedagogical subjects. On the other hand, the student was supposed to complete that training during his internship, by observing a teacher giving arithmetic lessons in a primary school (Carrillo, 2005).

During the last third of the century and the first decade of the following century, teacher training colleges suffered a decline (Escolano, 1982). The following curricula from 1898, 1901 and 1903 included similar subjects to those of the previous period. Future teachers studied arithmetic and geometry, and primary school teachers of the higher grade had some training in algebra-and in some cases trigonometry-as well. But before the Moyano Law geometry as such was only taught in the *Escuelas Normales Superiores* (teacher colleges for upper primary teachers). The reality is that, during most of the reign of Isabel II, in these *Escuelas Normales Superiores*, teachers-to-be studied together with children who took the so-called *upper primary education*, since these centres were the only ones that functioned, in practice, as upper primary schools (Sánchez-Jiménez, 2006).

Teacher colleges were reformed towards the end of the 19th century. However, the articles of the Royal Decree of 1898 do not mention methodological aspects. Only the article 32 refers to "academies, school trips and excursions and other practices of educational and didactic value that will be organized and arranged by the Board of Teachers of each School" (Real Decreto de 23 de septiembre de 1898, p. 1254). The sole mention of special methodologies is found in the additional provisions. In the first of them, concerning programmes and textbooks, it is stated that the *Consejo de Instrucción Pública* (Board of Public Instruction) shall ensure that "programmes, as well as textbooks, contain the necessary lessons in methodology and teaching procedures related to the subject in question" (p. 1256).

With the creation of the *Escuela Superior del Magisterio* (Higher School of Teaching), in 1909, teachers in teacher training colleges and inspectors of primary education began being trained there. And with the Royal Decree of 1914, which reorganized the teacher training colleges, the training of teachers is boosted and comes out from its previous lethargy. Despite the incorporation of important improvements such as the creation of a teacher's diploma and new internship schools attached to each *escuela normal*, the curriculum is once again based on mathematical topics. The subjects of mathematics are: Notions and exercises of Arithmetic and Geometry, Arithmetic and Geometry, and Algebra. In other words, the emphasis is once again on essentially disciplinary training. However, the 1914 curriculum does mention the methodological orientation in the articles of the Royal Decree. Thus, article 19 states: "All professors shall teach their students the methodology of their respective subjects as applied to primary school" (Real Decreto de 30 de agosto de 1914, p. 564). Nonetheless, the subject titles do not mention this orientation.

In 1931 the new political context allowed for essential changes. With the establishment of a new system of government in Spain, the Second Republic, a true revolution in the formation of the teaching professionals took place. During the years in which the previous plan had been in force, the teacher trainers had been participating in an important methodological renewal, in line with the spirit of renewal that permeated all education, in a period that has come to be known as the "Silver Age" of Spanish culture.

Those who formed the most innovative group had been trained, for the most part, at the Escuela Superior del Magisterio and had visited foreign primary schools and institutions for the training of teachers. They were grant-holders of the *Junta para Ampliación de Estudios e Investigaciones Científicas* (Board for the Extension of Studies and Scientific Research). A group with strong pedagogical concerns, they had shown great interest in the didactic aspects of primary school mathematics and also in those aspects that future teachers should study. Their books and contributions in pedagogical journals, as well as the training activities in which they participated are proof of this (Sánchez-Jiménez, 2015).

The so-called “professional plan” abolishes disciplinary subjects and instead includes subjects of a didactic nature; thus, for the first time, a subject called Methodology of Mathematics appears, which concentrates all the mathematical training that teachers receive during this period.

The Spanish Civil War (1936-1939) and the advent of the Franco dictatorship (1939-1975) were a major setback to the teaching profession. In 1940, a decree “establishing in all its force the Royal Decree of August 30, 1914 and corresponding provisions” was issued (Decreto de 10 de febrero de 1940, 1200). Many teachers and teacher trainers were considered dissidents by the new political regime and therefore sanctioned and removed from their posts. Not only was the entire renewal movement aborted, but every effort was made to erase the memory of these teachers and their publications.

The 1946 curriculum for teacher colleges-what were then called Escuelas de Magisterio-prescribed that the disciplines that students had studied in the early years of the baccalaureate should be expanded. There was just one subject in mathematics called Extension and Methodology of Sciences: Mathematics (Orden de 14 de octubre de 1946). The reference to methodology, in a generic way, in the titles of each group of science or arts subjects was preceded by the term “Extension”, which left no room for doubt as to the intended orientation of the subjects, or their relationship with those of the baccalaureate. In fact, those who had studied the baccalaureate were awarded the title of teacher with only a few subjects-mainly pedagogical and religious ones-, in addition to the practice in primary schools. The methodology of mathematics, or of the other subjects, was not a concern of the legislators.

A few years later, the 1950 School Regulations (Decreto de 7 de julio de 1950) referred to the methodology of the subjects; the two subjects in mathematics were called in the new plan: Mathematics: Arithmetic and its Methodology, Algebra in the first year, and Mathematics: Geometry, Extension and Methodology, Trigonometry in the second year.

The methodological aspects took on once again special importance in 1967, at least in the official precepts, with the two mathematics subjects being renamed Didactics of Mathematics (Orden de 1 de junio de 1967).

The 1967 plan, which gave a boost to teacher training, is considered the closest to the one of the Republican era for several reasons. Among the most important ones are the entrance requirements. It was necessary to have completed the baccalaureate in order to be admitted to the *escuela normal*. In addition, the last year-from a total of three-was dedicated to paid internships in a primary school. However, we will see that the ‘ecological’ conditions (in the sense of the ATD) were not the same as in 1931.

Finally, in 1970 the *Ley General de Educación* (General Education Law) dictated that the Teacher Training Schools had to be integrated into the universities, and the 1972 Decree (Decreto 1381/1972) converted them into *Escuelas Universitarias del Profesorado de Educación General Básica* (University Schools for the Training of Teachers of Basic General Education). After an experimental study plan, the definitive plan was published in 1977 (Orden de 13 de junio de 1977). With the integration of teacher training into the university system, the emphasis was placed on bringing their training into line with that of other university qualifications, rather than on offering specific mathematics for the teaching profession. There was Matemáticas (Mathematics), a common subject for all the students. Furthermore, the students of the Physical-Mathematical Sciences speciality took two other Mathematics subjects and only one subject of a methodological nature, Didáctica de las Matemáticas (Didactics of Mathematics), with very little weight (one hour per week) compared to the Mathematics subjects themselves. As for the teachers of the Preschool Education speciality, the mathematical subject they were studying, The Logical-Mathematical Area at Preschool Age, did suggest a didactic orientation, although it should be borne in mind that this speciality was not created until 1981.

It was not until 1983 that, with a democratic system of government, the issue of university reform was addressed. The *Ley de Reforma Universitaria* (University Reform Law, Ley Orgánica 11/1983) created the legal framework to establish a catalogue of knowledge areas the following year, including Didactics of Mathematics. This subject is thus officially configured as an academic discipline, although during the first years it was, in practice, a budding discipline in this country.

MATHEMATICS TO TEACH AND MATHEMATICS FOR TEACHING IN TEACHER TRAINING COLLEGES

As explained in the previous section, the mathematics that was taught in the teachers colleges before the 1931 plan was what Cirade (2006) calls the *mathematics to teach*, that is, “the mathematics that a teacher must know in order to take responsibility for teaching what the curriculum prescribes” (p. 133). Yet if we take into account what was really taught, it could hardly be considered that during the 19th century training covered sufficient mathematical content needed for the profession. The contents of arithmetic that future teachers studied were the same as those studied in elementary or upper primary schools. The attempt to introduce algebraic contents-which could have had a validating function with respect to arithmetic techniques-was more driven by the intention of raising the prestige of teacher training institutions, bringing them closer to professional schools or to secondary schools, which did not happen either (Carrillo, 2005). And something similar happened, as has already been said, with geometry (Sánchez-Jiménez, 2006).

Despite the fact that since the last quarter of the century important reforms in teacher training were demanded in professional journals and pedagogical conferences, until the reform of 1914 the mathematical preparation of teachers was rather scarce. In 1915, the *Sociedad Matemática Española* (Spanish Mathematical Society) complained that the level of mathematical knowledge required in the competitive examinations for teachers was even below that of other subjects, which had repercussions on the teaching of mathematics in primary schools and, as a consequence, on the mathematical backwardness of the country:

We Spaniards, in general, and with few honourable exceptions, are in a very bad state of mathematical knowledge; a bad state that has its roots in primary school itself; it is enough to compare, in the competitive examinations for teachers, the exercises that deal with mathematics with all the other exercises, in which a great superiority will be noted with respect to the former; roots that then extend throughout our entire social organism (A nuestros lectores, 2012/1915, p. 61).

For a long time, teacher training colleges taught their students, essentially, the mathematics *to teach*, plus an added pedagogical training that was supposed to provide the future teachers with professional training. In other words, the contents of mathematics being studied were the same as those to be taught later by the future teachers in the primary schools.

The Royal Decree of 1914 raised the level of mathematical knowledge of teachers. Nevertheless, as happened with curricula before and after 1931, in the mathematical training provided by teacher colleges-despite the references in the laws to the teaching of the methodology of each subject- mathematics *to teach* continued to predominate in most of teacher colleges. In the next subsection the reasons for that will be commented.

The reform of the structure and the curricula of teacher training colleges undertaken by the government of the Second Republic (1931-1936) had an explicit professionalizing intention. It represented a transcendental change with respect to everything that existed in previous times (and what was to exist afterwards for several decades). In fact, it would be necessary to wait until 1967 to observe these professionalizing traits in the legislation on teacher training. Actually, if there was one thing the two curricula shared it was their short period of validity, but for different reasons, since the circumstances and the underlying political intentions were not the same in 1967 as those at the beginning of the Franco dictatorship.

Before commenting on the training in mathematics while the emblematic Republican plan was in force, we will see that the training that the teacher colleges could provide, at each historical moment, was subject to restrictions of an 'ecological' type. They made it difficult, in practice, for the training of teachers to cover all the elements of authentic mathematics *for teaching*.

'Ecological' Conditions and Constraints concerning Mathematics for Teaching

I have been examined the legislation on teacher training and pointed out in which periods it explicitly included aspects of mathematics teaching in the subject titles or, instead, only added some references to the treatment of these aspects. However, the 'praxeological' equipment that aspiring teachers can obtain depends on several factors (Cirade, 2006). Along with laws and curricula, other related elements must also be considered.

One of these are the requirements for access to teacher training institutions. It is not by chance that more weight was given to the didactic training of teachers-not only in the case of mathematics-precisely when high school studies were a requirement to enter those institutions, that is, in the 1931 and 1967 plans. The differences in the conditions in which both plans were designed and implemented influenced the mathematics actually taught in teacher training.

Furthermore, throughout the 19th century, and especially in the last quarter century, the necessary resources to teacher training were not allocated, either financially or in terms of teachers, greatly due to the political situations that Spain went through (Carrillo, 2005; Escolano, 1982). As for the programs, a teacher trainer wrote in the journal *La Escuela Moderna*: "The programs, instead of responding to a special plan, have been inspired by those of secondary education, without addressing the methodology of the disciplines" (Solís, 1894, in Escolano, 1982, p. 65). Nevertheless, when the requirements for access to courses in teacher colleges are quite low (Carrillo, 2005, sections 5.7 & 7.5), and the mathematical training with which one begins to study at them is very scarce, efforts are directed toward disciplinary training, in order to provide the teacher with the mathematical knowledge that he or she will have to teach his or her students. That is to say, those of primary education, despite what is in the programs. The effort that has to be dedicated to mathematics *to teach* leaves no room for mathematics *for teaching*, "and so, ultimately, professional suitability is diminished" (Herrainz, 1896, in Carrillo, 2005, p. 273).

The 1914 plan, or Bergamin Plan, deserves special mention. It was conceived with the idea of increasing the preparation of teachers, but this purpose was only partially fulfilled. Francisco Romero, a teacher of mathematics of the *Escuela Normal* of Ciudad Real, who was fully committed to pedagogical renovation, points out that the students who entered the teacher colleges had only a basic literacy and numeracy that, in mathematics, did not go beyond the algorithms of elementary operations, something that he describes as a "pseudoculture" (Romero, 1930). This explains why the efforts of teacher trainers were directed towards mathematics itself as a discipline. In the first years, they had to focus on teaching the same mathematics that their students would have to teach (in the primary school). Notwithstanding, this does not justify that mathematics in this plan, with respect to those in the previous plan, did not change but rather just expanded and continued to have a formalist and antiquated character, as some teachers pointed out at the time (Eyaralar, 1919). However, we will see that there are exceptions (Sánchez-Jiménez, 2015).

Another restriction that conditioned training that could be offered to teachers was the mathematical and pedagogical training of teacher trainers, which was related to the professional origin of these teachers. The *Escuela Superior del Magisterio* raised the level of knowledge of teacher trainers, particularly in mathematics, bringing it closer to university mathematics; but, unlike what happened with other disciplines-such as science or geography-, pedagogical training was absent. The students of the teacher colleges acquired their methodological training through the psycho-pedagogical subjects, and through the didactic training received in other subjects, extracurricular activities, seminars, etc. Thus, in 1919, Eyaralar wrote that "even in the mathematical

training of teachers of teacher colleges there is a great deal of disorientation with regard to the content, plan and method of teaching” (Eyaralar, 1919, p. 4). As will be seen, it was through other means that they acquired this training, throughout the years in which this curriculum was in force.

The teaching staff responsible for lecturing the subjects in a curriculum may be one of the main restrictions to which compliance with the legislation may be subject in practice. This is also the basis of the criticism made by the Inspector General de Escuelas Normales (Inspector General of Teacher Training Colleges) himself of the 1967 curriculum:

The turn it imposes on the old cultural or special disciplines by forcing them to give only their Methodologies is too great and exposed to failure or hypocrisy when there is a lack of tradition and adequate bibliographical information and little interest in these subjects (García Yagüe, 1970, in Holgado, 2000, p. 165).

Despite the social changes promoted by economic development in Spain in the 1960s, the renewing desire of many sectors of the teaching profession would have to coexist with the still-present Francoist ideology.

In any case, we cannot forget a third factor, the epistemology of the teachers themselves and the pedagogical model with which they sympathized. This factor gave rise to the existence of different sensibilities in the way they assumed the spirit of any reform. An example of this can be found in the teacher trainer Manuel Xiberta, student at the Escuela Superior del Magisterio between 1912 and 1915, who remained impervious to any legislative or ideological change. His books, which were written to serve as textbooks in the teacher colleges, were only modified in a structural way to adapt them to the different curricula. For the author, mathematics *for teaching* were identified with mathematics *to teach*. He identified the knowledge for the professionalization of future teachers with a methodology of mathematical science itself, and only if he is “forced” to consider them, because of the curriculum. This is what the prologue of his book states:

Under the pressure of the *Cuestionario* [syllabus for the subject Methodology of Mathematics] currently in force, we deal with questions that are alien to the spirit of the work; but we do not devote more space to the imposed subjects than we consider fair and acceptable, in order to expand on those that constitute the basis of a solid mathematical culture (Xiberta, 1934, p. 5).

In fact, when in the 1950 curriculum, texts for teacher training colleges had to include contents of methodology-which normally consisted of a chapter or appendix at the end of the book-Xiberta in his work *Aritmética y su Metodología* (Arithmetic and its Methodology), written with her daughter as co-author) dedicated only twelve pages to the methodology of arithmetic, of which four were dedicated to the methodology of science itself. This appendix did not even appear in the book’s index. In a sort of preamble entitled “Instrucciones pedagógicas”, they only referred to how to learn arithmetic using the book, nothing about teaching in primary school. The legend on the cover and under the title of the book constitutes an indicator of the similarities between the textbooks for future teachers and those of the high school:

Adaptation to the *Cuestionario de las Escuelas del Magisterio* [Teacher Training Schools Syllabus] of our work Elements of Arithmetic, chosen to be the only text of the National Secondary Education Institutes by virtue of the Competition and of the book Arithmetic which won first prize in the National Competition of books for Labour Institutes (Xiberta & Xiberta, 1961, p. 5).

On the contrary, the second part of the book *Geometría y ampliación. Didáctica. Trigonometría* (Geometry and Enlargement; Didactics. Trigonometria), which Sáiz Salvat published at the beginning of the 1950s (Sáiz Salvat, 1957), was reserved for methodological issues. He included in it part of the contents of his 1931 book, although it is considerably less extensive and makes no reference to works from the Republican period. It should be noted that Saiz Salvat’s book was an exception, as its author was one of the few teachers among those who had led the methodological renewal in the pre-war period who were allowed to continue teaching in a teacher training college.

The last restriction I will consider relates to the conditions for access to the profession of primary school teacher. We have an example in the first third of the 20th century. During the Bergamín Plan, some professional journals, such as the *Revista de Pedagogía* and the *Revista de Escuelas Normales*, criticized the competitive examination for access to a teaching post. The exam’s syllabus was based on mathematical knowledge that was already considered to be of little relevance at that time, both from a utilitarian point of view and from the point of view of intellectual training. Moreover, they did not even cover the teaching that was already being given in many teacher training colleges, at least by the sector of the most informed and most concerned with renewing mathematics teachers. Margarita Comas (1925) complained about the importance given to “alligation rules” (general procedure for solving problems related to mixtures of several substances of different quality and price) and other disused content, while symmetries, functions, graphs, coordinates and others were absent. She agrees in this with Eyaralar (1923), who describes the list of topics of the competitive examination as “pitiful”, since “it should collect the best of the new in our Teacher Training Colleges, and on the contrary marks a setback” (p. 221). Exam’s syllabus that were to influence, in practice, the mathematics that was studied in the centers where the aspiring teachers were trained.

The Preamble to a New Era

During the time when the 1914 curriculum was in force it is worth a progressive process of renewal took place in the field of mathematical training for future teachers in teacher training colleges. This process has been extensively researched (Sánchez-Jiménez, 2015) and here are some of the factors that contributed to this renewal-and were at the same time a sign of the desire for renewal, which also extended to primary schools. In the first place, the previous creation of institutions, among which the Junta

para Ampliación de Estudios and the Escuela Superior del Magisterio, in 1907 and 1909, respectively, played an essential role. As well, the foundation of the *Asociación del Profesorado Numerario de Escuelas Normales* (Association of the Numerary Members of Teacher Schools). Another factor was the appearance of new journals, such as the *Revista de Escuelas Normales*, in 1923, which was the expression journal of the aforementioned Association, or the *Revista de Pedagogía*, in 1922, although this one was much less concerned with the teaching of mathematics in teacher training colleges (Dólera & Sánchez-Jiménez, in press). In addition, there was the impetus given to culture and education in Spain in the first third of the 20th century and the dissemination of the principles of the international New Education movement.

An important sign of the interest in mathematical issues that teachers needed in order to meet the demands of their profession are the publications of these teacher trainers throughout this period. There were books for the mathematical subjects of the 1914 plan, and articles in journals, which in some cases already showed concern, not only for mathematical issues, but for methodological aspects and for issues that are part of what is considered mathematics *for teaching*. This advance facilitated what was to come during the next stage, and made it possible for the new reform to become a reality beyond the laws.

The Subject Methodology of Mathematics, Mathematics To Be Taught or For Teaching?

The subject Methodology of Mathematics, introduced in teachers' studies in the so-called professional plan in 1931, was located in the first year of those studies and was assigned three hours of teaching per week. No official programme was published for the so-called 'Special Methodologies'. However, the *Dirección General de Primera Enseñanza* (General Management of Primary Education), on which the teacher training colleges depended, called teacher trainers out to a meeting known as the *Cursillo de Información Metodológica* (Course on Methodological Information), so that the teacher trainers themselves could draw up the syllabuses ('Cuestionarios') for each of the disciplines. The need to integrate knowledge *to teach* and *for teaching* was already manifested in the will of the legislator, as shown by the request made to teacher training colleges:

The teacher needs to remind his students of the generic concept of the discipline he professes, to define it by its major current problems, to deduce its didactics from the concept of science and child psychology, by determining the methods, procedures, materials and aids best suited to its teaching. (...) This application [to the Primary School] must be preceded by a suitable discussion in terms of content, methods and material, as it must be followed by emulating criticism. (Dirección General de Enseñanza Primaria, 1932, p. 631).

To discover what knowledge was built up during the few years the curriculum was in force it is necessary to look at the textbooks for the new subject. The consideration of these sources is a constant in research about the history of the disciplines:

The reading and analysis of magazines, manuals, etc. would seek to capture methods, didactics, pedagogical guidelines that could be read as part of the movement of constitution of knowledge for teaching and knowledge to teach (Valente, Bertini, & Morais, 2017, p. 232).

These books, together with the works published by teachers dedicated to teacher training published-most of them-in the *Revista de Escuelas Normales*, have been analysed in Sánchez-Jiménez (2015) to determine what knowledge characterized the training of primary school teachers during that period.

This was a new subject and there were no previous textbooks that could serve as a reference. Therefore, from the beginning of the decade until the end of the Republican period appeared books that, although trying to cover what had been agreed by the teachers and assumed by the General Directorate, at the Course on Methodological Information, presented differences in terms of content and structure. There were treatises that focused on the methodology of the mathematical science itself (Rey Pastor & Puig Adam, 1933; Xiberta, 1934), books written with teachers in mind-with proposals for primary school (Comas, 1932)-, and some that focused on the methodology of a specific subject-such as problem solving or calculus (Charentón, 1930; Romero, 1933). Finally, books appeared that were adapted to the agreed *Cuestionario* and included the methodology of mathematics as a science and a perspective of the applications of mathematics that were considered current, as well as psychological, pedagogical and didactic knowledge applied to the teaching of mathematics and its organization in primary school. The books of Sáiz Salvat (1931), Eyaralar (1933) and Paunero (1935) correspond to this model.

Most innovative teacher trainers did not organize their teaching according to what was contained in published manuals. On the contrary, content of the books that were written for this subject during the Republican period reflected the teaching given by their authors during the previous years: "This book reflects the lessons explained in my course of the subject Metodología de la Matemática. I wanted to include some coursework here" (Paunero, 1935, p. 5). Proof of this is the inclusion in these books of shared reflections, proposals and examples, which had been published earlier in professional journals.

Nevertheless, in order to characterize the kind of training of future primary school teachers we have to wonder what kind of training was really given in the teacher colleges under the 1931 plan.

As will be explained in the following section, the arrival of the new curriculum boosted and accelerated the process of building the mathematics *for teaching* in teacher colleges-which had begun to build in previous years. However, I have reason to believe that the training that they could offer their students during the years that the plan was in force no longer consisted only of mathematics *to teach*, although neither could it be considered true mathematics *for teaching*, since a major restriction weighed on them. While the time assigned to the rest of the specific methodologies was at least six hours per week during a course (if the subjects are grouped according to the current areas of knowledge, in Didactics of Social Sciences, Didactics of Experimental Sciences, etc.), the Methodology of Mathematics had assigned only three hours per week in the curriculum. This was a major institutional constraint for setting up mathematics *for teaching* in the primary school classroom. Although, at first, it may seem

that the methodological training provided was sufficient to overcome the teaching based exclusively on the mathematics *to teach*, we cannot consider that all the elements of a true mathematics *for teaching* were there.

Cirade (2006) specified that going deeper into mathematics *for teaching* means asking about the reasons for the existence of a certain mathematical object, which is directly related to the possibility of motivating its study in the school.

Chevallard and Cirade (2010) used the notion of 'praxeology' (Chevallard, 1999) to emphasise that 'praxeologies *for teaching*' included-along with each of the 'mathematical praxeologies *to teach*' and the associated 'didactic praxeologies'-the corresponding mathematical praxeologies that allowed the didactic praxeologies to be constructed and understood. Therefore, within the framework of this theory, teacher training must take into account the role of mathematics itself as a teaching tool. Other researchers also value this role:

Synthetically, we are referring to that mathematics for teaching being characterized by means of [1] a more general knowledge about teaching, coming from pedagogy, psychology, among others [2] by the mathematics to teach, [3] by the higher knowledge that will support the knowledge to be taught and [4] by the didactics of mathematics that will allow to transform knowledge into objects of teaching (Rocha, 2019, p. 88).

But the agreed set of contents for the subject Methodology of Mathematics hardly included any issues related to the analysis of mathematical knowledge. Moreover, the analysis made of the content of the works written for this subject reveals, in general, this same scarcity (Sánchez-Jiménez, 2015). There can be found only some elements of the mathematical and didactic praxeologies related to the mathematical knowledge mentioned (for example, a comparison of techniques to deal with proportionality tasks, etc.), and not even for all of them. In the book *Nuevo Tratado de Aritmética* (New Treatise of Arithmetic, Eyaralar, 1922), written for the subjects of the 1914 curriculum, José María Eyaralar wonders about the reasons for the notions, properties and arithmetic techniques that teachers have to teach in primary school (Sánchez-Jiménez & Carrillo, 2018). However, this type of analysis of mathematical content is not planned in the subject Methodology of Mathematics. This might be the reason why the author considered necessary-despite the fact that the baccalaureate was required for access to teacher colleges-the publication of a new version of his arithmetic book, *Aritmética Intuitiva* (Intuitive Arithmetic, Eyaralar, 1932a), in which he revised the content of the previous one. Thus, although the Professional Plan has been, for many reasons, so well considered in our country by primary school teachers and by educational historians, the truth is that this plan could not allow future teachers to elaborate true Mathematics *for teaching* either.

THE GERM OF A NEW DISCIPLINE AND THE PROFESSIONALIZATION OF TEACHERS

The disciplines are a social and historical product, the result of a process of construction. In reality, they are a process of human construction, closely related to the process of the professionalization, specifically, of teachers. The key element associated with the existence of a discipline is its "disciplinary code". It includes a body of content-embodied in textbooks and other materials-, a reasoned presentation of the educational value and usefulness of this knowledge, and professional practices regarding the teaching of this content. Other basic elements are its position in the educational system and in relation to other disciplines, as well as the teachers who teach it (Viñao, 2013).

Hofstetter and Schneuwly (2017) characterize the constitution of a discipline by four factors. First, an institutional base that allows for the professionalization of research and generates a body of specialists. Then, the constitution of communication networks among those who deal with shared problems, such as teachers' associations, scientific meetings, or professional journals. The third is the elaboration of legitimate research methodologies, consequence of the institutional and communication infrastructure. And, finally, a social recognition of the discipline, which authorizes it to establish its own criteria of legitimacy to judge scientific production and decide who does or does not belong to that disciplinary field.

In the text, the aim is to know to what extent the subject of Methodology of Mathematics could have been a first step in the construction of the Didactics of Mathematics as a discipline. The research carried out in Sánchez-Jiménez (2015) shows the existence of features of the previous elements in the period of the Second Republic. I highlight some of them below.

At the institutional level, we have, first of all, the Decree of 29th September 1931 (Decreto de 29 de septiembre), which reformed teacher training studies and established the Methodology of Mathematics as a subject in the new curriculum. In previous years, a decree allowed high school graduates to obtain the title of teacher simply by taking pedagogical subjects and practicing in primary schools. The repeal of this decree placed special methodologies-including mathematics-as constituting the knowledge necessary to be able to practice the profession, and as a characteristic subject of the training given in *escuelas normales*.

The professional association was manifested with the creation of the Asociación del Profesorado Numerario de Escuelas Normales. Although teachers of all subjects were represented in it, the Association allowed contact among teachers of each subject. And in 1932, during the Cursillo de Información Metodológica, which brought together teachers according to their subject, those of mathematics-like those of the other subjects-reached a consensus on the contents of the Methodology of Mathematics. One of those teachers, the main representative of the renewal in the teaching of mathematics in teacher training, José María Eyaralar (1932b), underlines the unanimous agreement on the general objectives of the new subject. Such agreement was nothing more than the joint synthesis of what many of them had been publishing in previous years in the *Revista de Escuelas Normales* about what should constitute the mathematical and didactic training of teachers. The very existence of the journal, as well as the publications in other professional journals of the time by teacher trainers, is another of the features mentioned above.

Moreover, that group of teachers trainers acknowledged the need to build what was to be the basis of the mathematical methodology, looking for common elements in the proposals so as to “make possible the advance of the national didactics that must be made up of personal didactics” (Sáiz Salvat, 1933, p. 5). The New Education movement provided a theoretical framework for all the methodologies. Despite sharing a part of the psycho-pedagogical knowledge with the rest of the special methodologies, there was an attempt to define a (didactic) knowledge of mathematics in its own right, beyond the knowledge or techniques imported from other disciplines. Its scientific nature is also stressed. The teachers assembled at the request of the General Management of Primary Education to develop the programme for the new subject agreed that “the Methodology of Mathematics must not be a simple ‘recipe book’, but on the contrary, it must provide a solid scientific basis for the guidelines of the method” (Eyaralar, 1932b, p. 5).

Another sign of the emergence of a process of building a new discipline has to do with the importance awarded to research. Back in the previous decade, Eyaralar (1924a) pointed out the need of an experimental study to compare teaching proposals in terms of the organization of the study of geometry. Furthermore, he considered that research needed not be the exclusive preserve of university professors. He himself presented some results on a mathematical topic, the Pythagorean numbers, applicable in teacher colleges (Eyaralar, 1924b). Precisely in the *Bases para la Reforma de las Escuelas Normales* (Basis for the Reform of Teacher Training Schools), submitted to the assembly of teacher training colleges in 1931, the Association director proposed that “two research and reform schools with two or three grades each [be created], an indispensable body for the experience and practice of the new methods” (Gil Muñiz, 1931, p. 116).

As has been previously said, Spain is one of the countries in which the process of professionalization of teacher trainers has been related to that of the creation of a discipline itself. Associations and professional journals take part in this process, since it involves the communication of teaching experiences and research results. The organization of conferences or seminars is part of the process as well. In addition, most of those leading the renewal movement had visited foreign educational institutions granted by the Junta para Ampliación de Estudios.

On the one hand, the professionalization of a discipline calls for the existence of a group of teachers representing it. On the other hand, it requires identifying who is truly linked to such discipline (Viñao, 2006). In this sense, we notice that teacher trainers—at least the protagonists of methodological renovation—were somewhat aware that the field of special methodologies was reserved for teacher trainers. Most of them were trained in the same institution, the Escuela Superior del Magisterio. And this does not apply exclusively to mathematics. A well-known geography teacher expressed the following towards the *Cursillos de Información Metodológica* organised by the Dirección General de Primera Enseñanza:

These courses cannot be courses on methodological information, because *what has been done in Spain in terms of methodology, with a certain amount of systematisation, has been done by members of the teacher training colleges, so instead of being informed, they can be informers* (Chico, 1932, p. 6; italics added).

In addition, in the case of mathematics all the contributions came almost exclusively from teacher trainers. In fact, the proposals presented about mathematics and methodology of mathematics in the publications were produced by this same collective.

FINAL CONSIDERATIONS

There was a stage during the Second Republic, between the establishment of a national system of teacher training in teacher training colleges and the integration of such colleges into the university, when the mathematics didactics reached a development that could be considered “something that is not yet fully a discipline, according to the model or the ideal type, that is on the way to becoming one and that can be described as a proto-discipline or, at best, as a discipline project” (Viñao, 2013, p. 376).

This was supported by the ‘ecological’ conditions of the time, fostered by the social and cultural renewal movement initiated in Spain a few decades earlier. It received special support thanks to the revitalisation in the field of education and the interest in primary education and teacher training, remarkably backed by the Republican government.

Although this impulse boosted the development of all the so-called special methodologies, when it comes to the subject of Methodology of Mathematics, the prominence of the teachers from the *escuelas normales* had more impact.

Viñao considers that teachers in *escuelas normales* together with inspectors of primary education and many primary school directors represented the ‘low pedagogy’, that is, pedagogy as a professional art. This was opposed to pedagogy as science, or ‘high pedagogy’, represented by teachers from the Escuela Superior del Magisterio and other institutions linked to the Junta para Ampliación de Estudios. However, and although this author is right when it comes to other Methodologies, such as those related to the Sciences or the Social Sciences, I disagree with that assertion in the case of the methodology of mathematics:

In the case of mathematics, unlike other disciplines such as pedagogy and some specific didactics (...), we found no evidence that the so-called ‘high pedagogy’ came from the teaching staff of the Escuela Superior del Magisterio. They were not the main characters in the origin of Didactics of Mathematics, as shows the detailed and critical analysis in the contributions of some teacher trainers, graduated from the said institution (...). Although the role of the Escuela Superior del Magisterio—in which they were trained—as well as that of the Junta para Ampliación de Estudios and that of other institutions was fundamental (Sánchez-Jiménez, 2015, p. 152).

The presence of the elements inherent to the constitution of a discipline, in relation to the didactics of mathematics in the years of the Second Spanish Republic, which I analysed in this article, has made it possible to discover, firstly, that steps were taken in the direction of constituting a discipline; secondly, the role played by the mathematics teachers of the teacher training colleges.

The republican regime being so short, it did not allow further progress in the process of generating this new discipline. Thus, the methodology of mathematics did not developed beyond a *proto-disciplinary* phase since the efforts of the new political regime in the following period were aimed at reversing the path taken and erasing any trace of modernization.

It was not until 1967 that there was again in Spain a plan for training teachers more like the one of 1931. Notwithstanding, in spite of the opportunity that this new reform represented, the initial situation-different from that of 1931-and again the short period of validity of the plan did not allow for the consolidation process of the didactics of mathematics to be completed this time either.

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