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THE METHODOLOGY OF STOCHASTIC OPTIMIZATION OF PARAMETERS AND CONTROL LAWS FOR THE AIRCRAFT GAS-TURBINE ENGINES FLOW PASSAGE COMPONENTS

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ABSTRACT

This paper presents the main theses of stochastic approach to the multimeasure parameters and control laws optimization for the aircraft gas-turbine engines. The methodology allows us to optimize the engines taking into account the technological deflections which inevitably take place in the process of manufacturing of the engine's components as well as engine's control deflections. The stochastic optimization is able to find highly robust solutions, stable to inaccuracies in technological processes.

The effectiveness of the methodology is shown by example of optimization problem solution to find the control laws for the flow passage controllable elements of the 4-th generation aircraft mixed-flow turbofan engine. The use of information about the existing and advanced production technology levels during the optimization process, including some components manufacturing accuracy, allows us to considerably increase the probability of optimum solution implementation in practice. In real engine there are some components manufacturing deflections as well as control accuracy deflections. It results a certain engine's performance deviation. An engine optimization classic deterministic approach can not take into account this circumstance, so the probability of an optimum design implementation is too low.

NOMENCLATURE

GTE - gas turbine engine;
LPC - low pressure compressor;
HPC - high pressure compressor;
IGV - inlet guide vanes;
GV - guide vanes;
SFC - specific fuel consumption;
c_R - specific fuel consumption;
R - thrust;
σ - the middle square deflection;
$Z(\bar{\sigma})$ - the scale factor for middle square deflections vector;
P - probability;

Subscripts

Conf - confidence;
Cond - conditional;
Constr - observing of constraints;
Def - defined value;
t - technological.

INTRODUCTION

Till now a wide class of aircraft GTE optimization problems has been solved with the help of deterministic approach and it has been quite enough. GTE was considered as ideal system. Strictly speaking, such consideration of any engineering object is incorrect because under the real-life conditions an object is a stochastic system, which has certain indefinites. Therefore the probabilistic estimation of optimization problem results is required.

Let us consider from this point of view the GTE control laws optimization problem. As a control law we consider the dependence of position of a certain GTE controllable element upon a certain GTE mode operation parameter. As an example, for the case of GTE throttle performance optimization one can consider the position dependence upon an engine rotor rotation speed or upon an engine thrust. The optimum laws of control and therefore the optimization criterion extremum value obtained with the deterministic approach, are nothing but the maximum achievable effect, one ought to seek to. The question is being opened concerning the possibility of the maximum effect achievement. First, there is the problem of assurance of the stability of obtained solution under possible deflections of optimized parameters, which inevitably take place in practice while realizing any engineering project, even if the most perfect technology levels are used. Secondly, it is hard to predict if one can influence either the property or the extent of solution stability, optimizing the control laws of an object being researched.

The main theses of the stochastic approach to aircraft GTE and its components control laws optimization can be found in [Egorov, 1992b, Egorov, 1993]. The implementation of any technical solution is known to be directly connected with an object production technology level. In real-life conditions the dispersion of design parameters is inevitable. This is shown in Fig. 1, which demonstrates the dependence of the deterministic solution implementation probability upon the production technology level [Egorov, 1992b]. Meanwhile it is important not only to obtain an object high performance, but to assure its

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fulfillment according to the high probability level as well. Concerning development of aircraft GTE, it makes necessary the search of the optimum control laws taking into account the dispersion of components parameters due to technological deflections.

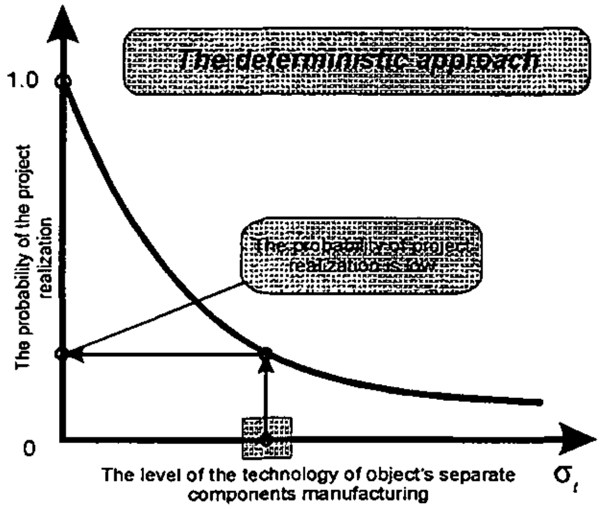

Figure 1. The influence of production technology level upon the probability of the project implementation.

The main idea is following. The effectiveness indicator can be presented as a certain functional, depending upon the vector of the optimized parameters $\bar{a}$, which defines the control laws, and upon the vector of accidental parameters $\bar{\xi}$, which influences the value of the functional. The latter vector defines the errors in object control. Note, that when the components of accidental vector $\bar{\xi}$ are fixed, the problem is nothing but the deterministic problem of control laws optimization.

In real-life conditions there is the lack of specific information about vector $\bar{\xi}$ components. However there is just enough practical experience of GTE production, which allows us to estimate the law of distribution of $\bar{\xi}$ components with acceptable accuracy. Moreover, in most engineering cases the components of vector $\bar{\xi}$ have normal or close to normal distribution laws [Draper and Smith, 1981]. Hence, for immutable conditions of engines components manufacturing, in the frameworks of our approach we use the hypothesis that the distribution law of $\bar{\xi}$ is known.

It should be noted that as well as the components of vector $\bar{\xi}$, the effectiveness indicator $Y(\bar{a},\bar{\xi})$ has an accident character. Here the deterministic optimization can not be used. However, the stochastic optimization of some statistical indicators can be carried out.

Thus, when GTE control laws being optimized while using the stochastic approach, one ought to find the extremum of statistical criterion $Y(\bar{a},\bar{\xi})$, depending upon both deterministic component $\bar{a}$ and stochastic one $\bar{\xi}$. As the effectiveness indicators one can consider the same sense criteria, as for the deterministic case (the SFC, the thrust, the efficiency of elements etc.), but these criteria must be presented in probabilistic form. For example, one can maximize the probability of such event as $c_R < c_{R\,Def}$, where $c_{R\,Def}$ is some pre-defined admissible value of c_R:

$$P_{Cond}\left|Y(\bar{a}^*,\bar{\xi})\right| = \max P\{Y(\bar{a},\bar{\xi}) < Y_{Def}(\bar{a})\}$$

To calculate the probabilistic criterion we used following algorithm. In the process of extremum search for each current vector of optimized parameters $\bar{a}$, the value of the stochastic criterion is obtained using the generation of the set of vectors $\bar{\xi}$ according to given distribution law (for example, the normal law), using the computer's generator of random numbers. Because of different control parameters dispersion inequality in a common case, we use the vector of middle-square deflection $\bar{\sigma}_{Def}$. Thus, for each combination of deterministic vector and stochastic ones $(\bar{a},\bar{\xi}_j), j=\overline{1,m}$ the values of effectiveness indicators $Y(\bar{a},\bar{\xi}_j), j=\overline{1,m}$ are being determined. Here m is the number of experiments. On the base of the results for current vector $\bar{a}$, we define the sequence $e_1, e_2, \ldots, e_m$, where

$$e_j = \begin{cases} 0, & when\, Y(\bar{a},\bar{\xi}) > Y_{Def}(\bar{a}); \\ 1, & when\, Y(\bar{a},\bar{\xi}) \le Y_{Def}(\bar{a}); \end{cases}$$

This expression corresponds to the case of minimization of an effectiveness criterion. Then the estimation of probability is carried out:

$$\hat{P}_{Cond}(Y(\bar{a},\bar{\xi})) = \frac{1}{m}\sum_{j=1}^{m} e_j\,.$$

This probability converges to $P(Y(\bar{a},\bar{\xi}))$ if $m \to \infty$.

Besides the above mentioned probabilistic criterion, for stochastic optimization of control laws other statistical indicators of accidental value $Y(\bar{a},\bar{\xi})$ can be used too. For example, its mathematical expectation can be used as

$$M_x = M\left|Y(\bar{a},\bar{\xi})\right| = \frac{1}{m}\sum_{j=1}^{m} Y(\bar{a},\bar{\xi}_j)\,, \qquad \textbf{(A)}$$

and the complex criterion as

$$\mu_x = \sigma_x M_x\,, \qquad \textbf{(B)}$$

where σ_x is middle square deflection.

$$\sigma_x = \sqrt{\sum_{j=1}^{m} \frac{(M_x - Y(\bar{a},\bar{\xi}_j))^2}{m-1}}\,.$$

Moreover, the dispersion can be used as criterion

$$D_x = \sigma_x^{\,2} \qquad \textbf{(C)}$$

Some other probabilistic indicators can be also considered as criteria.

The obtained estimation $\hat{P}_{Cond}(Y(\bar{a},\bar{\xi}))$ (M_x, μ_x, D_x) is used just in the algorithm of optimization to determine the direction of further extremum search. The peculiarity of such approach is that it is not necessary to calculate highly accurate estimations of $\hat{P}_{Cond}(Y(\bar{a},\bar{\xi}))$ (M_x, μ_x, D_x). It is explained by noiseproof features of the method of indirect optimization on the base of self-organization (IOSO) [Egorov et al., 1989]. So, we can use low values of *m* (approximately 20) while solving stochastic optimization problem, in order to reduce time-consuming.

As we use the approximate estimations of probabilistic criteria during the optimization, after the solution has been obtained one can carry out special statistical research using the greater *m* values to improve the estimations.

As the drawbacks of criteria (A), (B) and (C) one can treat the fact that these criteria are connected directly with the probability of GTE control law implementation. The probabilistic characteristics are present here indirectly, through the statistical effectiveness indices. In this case, the solution obtained with these criteria is rough from the point of view of assurance of given probability level. However, when the distribution is normal or similar to it, such solutions can assure highly robust features [Egorov and Kretinin, 1993].

Thus, in the frameworks of given approach we optimize GTE control laws in stochastic statement and search the extremum solution taking into account its implementation in accordance with existing level of production technology of concrete engines-manufacturing firm.

THE ESTIMATION OF MIXED FLOW TURBOFAN ENGINE CONTROLLABLE ELEMENTS OPTIMUM CONTROL CONCERNING THE PROBABILISTIC CRITERIA

The first stage of research was to determine how control inaccuracies influence engine's performance when implementing the obtained optimum control laws.

As an example, Fig. 2 and Fig. 3 present the probabilistic performance of engine, obtained considering the noise only by the low pressure compressor IGV control law implementation. In this case the variable parameter (IGV position) is a random parameter, distributed (in accordance with the assumed conditions of statistical testing) according to normal law, with the mathematical expectation equal to the given IGV position defined by control law, with dispersion σ defined by admissible dispersions of modern engines LPC IGV angles setting.

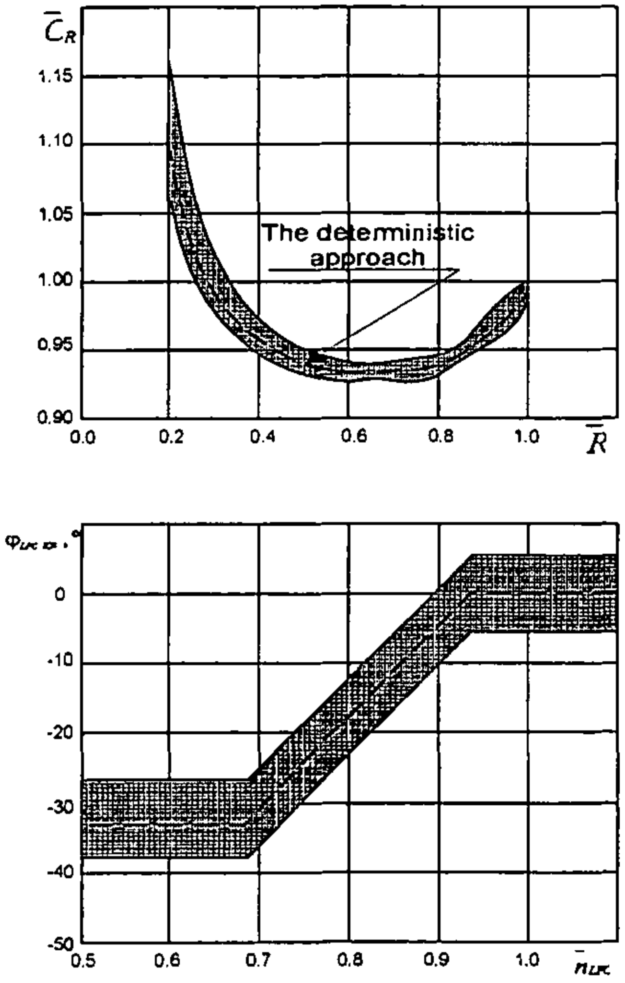

Figure 2. The influence of IGV control dispersion upon engine's integral performance.

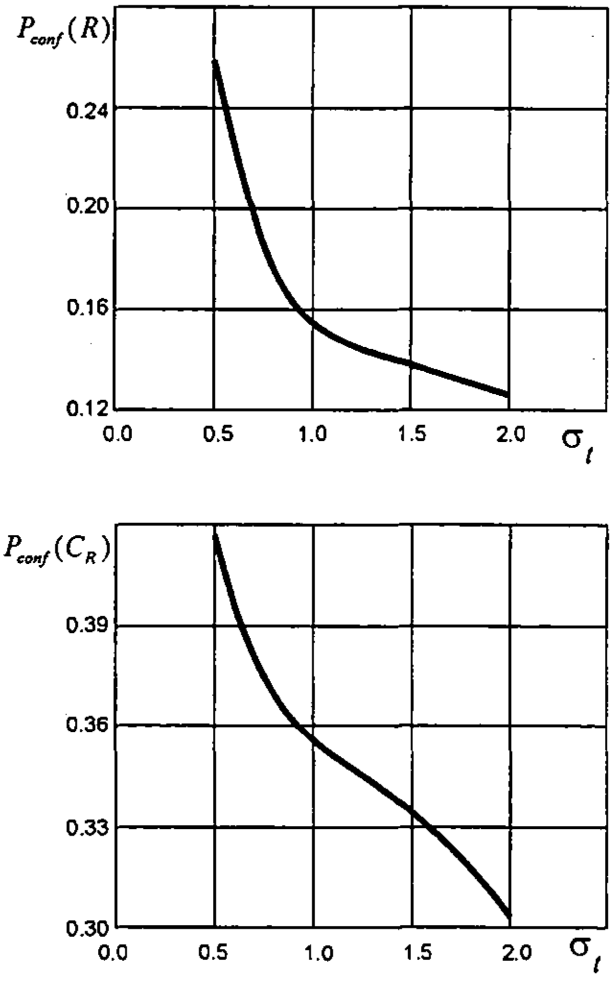

Figure 3. The influence of IGV control dispersion upon engine's probabilistic characteristics.

Fig. 2 shows the excerpt, consisting of 50 possible realizations of IGV control laws, and corresponding to it the dependence of c_R upon the thrust. One can see that with the given level of IGV control disturbances the dispersion of c_R can exceed 2%. In this case the estimation of IGV control accuracy influence upon engine's performance is reasonable to be carried out using such probabilistic indicators as confidence probability of assurance of the given values of integral engine parameters (for example, the parameters has been obtained in deterministic statement),

$$P_{Conf} = P(c_R \le c_{R\,Def}),\ P_{Conf} = P(R \ge R_{Def}),$$

as well as the confidence interval, within the frameworks of which one, the integral parameters will be insured with the given probability, as

$$P(c_R \le c_{R\,Conf}) = 90\%,\quad P(R \ge R_{Conf}) = 90\%.$$

The reason of dispersion of integral engine performance is obvious to be the dispersion of internal engine parameters due to changes of LPC characteristic.

The results of estimation of influence of disturbance level upon the engine performance is presented in Fig. 3, where the dependence of confident probability of insurance of the given integral engine parameters upon the level of disturbance σ_t is shown. These results show that increasing of disturbances level leads to worsening of probabilistic characteristics of engine.

Thus, the problems of optimum control laws search under the conditions of random character of control implementation are of great interest.

THE STATEMENT OF THE OPTIMIZATION PROBLEM

This paper presents some results dealing with the search of optimum control laws of mixed flow turbofan engine for throttle operating modes. The researches have been carried out using the mathematical model of the engine with 2-D axial symmetrical simulation of LPC [Egorov, 1992a]. The dispersions in the operation of actuators have been taken into account, i.e. we have treated the probabilistic character of obtained control laws.

For the simpler analysis let us consider the problem of search of optimum control laws of LPC IGV, HPC GV and LPC rotation rate. The problem is to determine such optimum control laws, which would ensure the best engine economy within the wide range of operating modes under the required level of their implementation probability. In other words, it is the problem of search of stable (robust) control laws of controllable elements from the point of view of ensuring of high levels of fuel economy indices as well as observing of defined constraints.

Using the stochastic approach, we defined the accuracy $Z(\bar{\sigma}_{Def})$, which could be ensured while implementing the control laws of one or another controllable element. For example, when Z=3, for fan IGV and HPC GV setting angles $\sigma = 1°$, and for the rotation rate of LPC $\sigma = 0.25\%$. While varying the value of current disturbance $Z(\bar{\sigma}_{Def})$ the value σ of each controllable element is being changed proportionally. The given levels of control accuracy correspond to the levels, inherent to the engine under consideration.

As the components of the vector of control we consider the control laws of regular controllable elements of the engine:

$$\bar{u}(n_{HPC}) = (n_{LPC}(n_{HPC}),\ \varphi_{IGV}(n_{HPC}),\ \varphi_{GV}(n_{HPC}))\,.$$

These control laws have been defined using different basic functions, such as piece-wise linear and piece-wise parabolic.

To the engine and control vector parameters the constraints have been applied to keep the main engine restrictions, such as maximal values of temperature before the turbine and rotors rotation rates, minimal surge margins etc.

Solving the optimization problem in stochastic statement, we've used the mathematical expectation M_x (A) and the complex criterion μ_x (B) as the probabilistic criteria of effectiveness indicator Y, which displays engine fuel economy in integral form

$$Y(\bar{a}^*) = \int_{R_{MIN}}^{R_{MAX}} c_R(R)\,dR$$

The estimation of probabilistic criteria was carried out by the analysis of random excerpt, consisting of 20 experiments.

As the result of solving of the series of optimization problems, we determined the optimum control laws in deterministic and stochastic statements. For comparison Fig. 4 shows the dependence $c_R(R)$ for different control laws, which were obtained for different effectiveness indices.

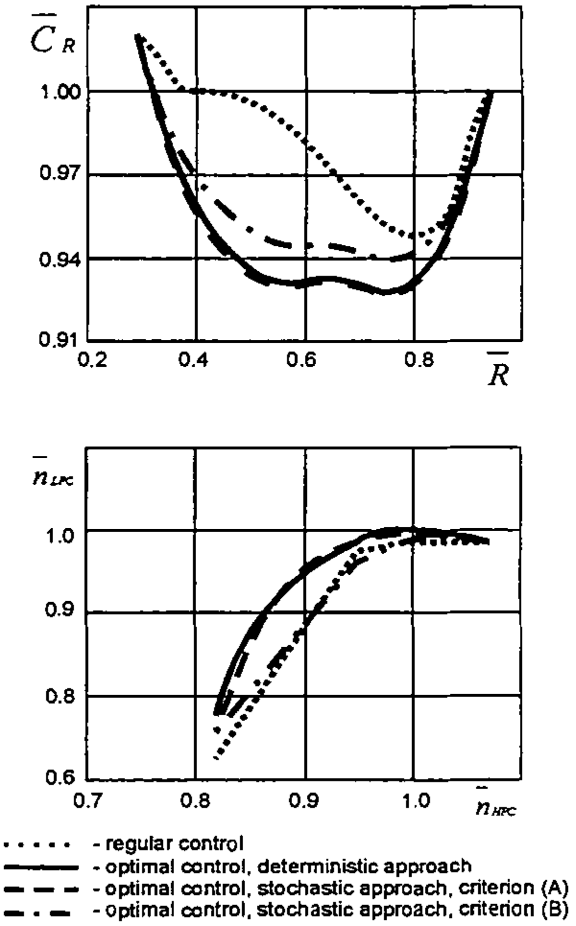

Figure 4. Comparison of different optimization approaches effectiveness.

The data for Fig. 4 have been obtained without the disturbances upon the parameters of control laws, i.e. with $Z(\bar{\sigma}_{Def}) = 0$. The results are shown for 4 cases: regular control laws; optimum control, obtained with use of deterministic statement $Z(\bar{\sigma}_{Def})=0$; two cases of stochastic statement of the optimization problems with $Z(\bar{\sigma}_{Def})=3$.

One can conclude from the analysis of Fig.4, that the best values of c_R are provided when solving the problems of optimum control in deterministic statement and in stochastic statement using the mathematical expectation as the effectiveness criterion (criterion A). It should be noted that the optimum solutions have the differences due to the parameters of control laws, i.e. the control laws themselves are different. The best advantage is ensured within the range of the thrust of about 40-80% of R_{Max} as well as for the deterministic case. A little bit worse result (about 2%) was reached when the effectiveness index was the complex criterion (B). Basing upon these results, one can make the important conclusion that the stochastic statement of the problem of optimum control of mixed-flow turbofan engine provides the fuel economy indicators extremely close to the best ones, having been obtained using the deterministic approach. Moreover, both deterministic and stochastic results are considerably better than the regular control performance. In other words, the solution has been obtained using the stochastic statement is quite acceptable for its use under deterministic conditions.

Of course, the question is stated concerning the truth of the following opposite assertion - «At what degree is the deterministic solution effective under stochastic conditions?». To answer this question one is to conduct the research in two directions. The first one is to determine how the constraints of optimization problem are stable under the dispersion of control parameters. The second direction is aimed at the estimation of the stability of optimum solution by means of the effectiveness criterion.

The analysis of the results presented in Fig. 5, shows the deterministic solution ($Z(\bar{\sigma}_{Def})=0$) under the conditions of noise of parameters. It is considerably worse than the optimum solution obtained using the stochastic statement. In this example as the robustness indicator we've used P_{Constr} - the probability of engine constraints observing while using the optimum control. Particularly, Fig. 5 shows the probability of defined engine constraints observing depending upon the value $Z(\bar{\sigma}_t)$ of normally-distributed noise with zero mathematical expectation.

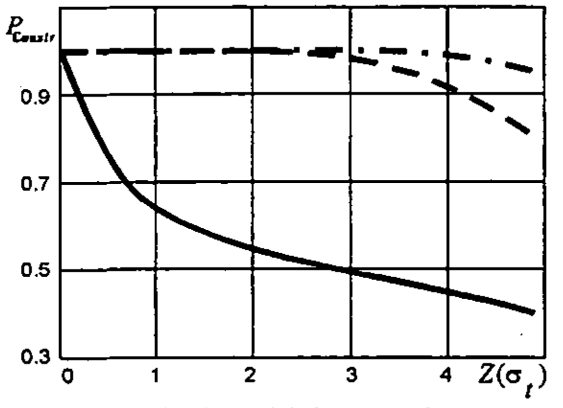

Figure 5. The probability of constraints observing for different optimization approaches.

To determine the value of the probability P_{Constr} the noise has been simulated by the generation of the noise vector $\bar{\xi}$ random values. Then there has been done an excerpt of 100 effectiveness indicators. One can see that for the optimum control laws obtained using the deterministic approach even the low level of the noise $Z(\bar{\sigma}_t)=1$ reduces the probability of constraints observing down to $P_{Constr}=0.65$, while the increase of $Z(\bar{\sigma}_t)$ up to 4 reduces P_{Constr} down to 0.45. Thus, only a half of implementations of the deterministic optimum control ensures the required conditions of engine operation under the given level of parameters noise. In other words, the benefits of the deterministic optimum solution by fuel economy indicators, which can be about 2% (see Fig.4), are practically realizable with the probability about 0.5. Hence it follows that the optimum solution obtained with deterministic approach is, first, the top achievable effect which could be possibly implemented, and secondly its practical implementation is possible only by low probability level.

As distinct from the deterministic solution, the stochastic optimum solution has the probability of ensuring of defined engine operating conditions about 1.0. It should be noted that the stochastic solutions with $Z(\bar{\sigma}_{Def})=3$ allow us to have the probability of constraints observing about 0.91 for criterion (A) by the current level of parameters noise $Z(\bar{\sigma}_t)=4$, and for criterion (B) under the same conditions the probability can reach ≈0.98. This fact characterizes the stability of the optimization problem solution, where as the measure of the stability we use the probability of insurance of defined engine operating conditions. Thus, we can conclude that in this case the use of the deterministic optimum result under the conditions of parameters noise is too problematical from the point of view of observing of the defined constraints.

The analysis of Fig. 6 allows us to understand the physical nature of improved stability (robustness) of the stochastic solution in comparison with the deterministic one. This figure shows the changes in effectiveness criterion contour lines for deterministic and stochastic solutions, depending upon two components of variable parameters vector, which are nothing but the coordinates of node points of control laws basic functions. Note, that as the effectiveness criterion both upper and lower maps use the value of the integral

$$Y(\bar{a})=\int_{R_{MIN}}^{R_{MAX}} c_R(R)dR ,$$

calculated for the region nearby the extremum under the deterministic conditions, i.e. with $Z(\bar{\sigma}_t)=0$. As the results of the deterministic optimization problem and the stochastic one are different, the components of optimum vectors are different as well. So, despite of the proximity of two variable parameters, shown in Fig. 6, other components of variable parameters vector of deterministic and stochastic optimum solutions are reciprocally different.

One can see that the deterministic solution has the sharp extremum and is situated near the boundary of admissible region of control (the dotted line shows the active constraint of HPC surge margin). This leads to the fact that a slight deflection of control law parameters from optimum ones inevitably worsens the effectiveness of the engine. It is just the reason of low robustness of the deterministic solution.

For the probabilistic optimization criterion (A) the contour of objective function has more gently sloping character, and the optimum solution if far sufficiently from the boundary of the admissible region. This reduces the influence of changes in the components of optimized parameters vector upon the effectiveness criterion, so for the stochastic solution there is the less dependence of effectiveness indicator upon the dispersion of control laws parameters while their implementation. The result is not unexpected, because just this goal was pursued using the stochastic statement of the problem of mixed-flow turbofan engine control laws optimization.

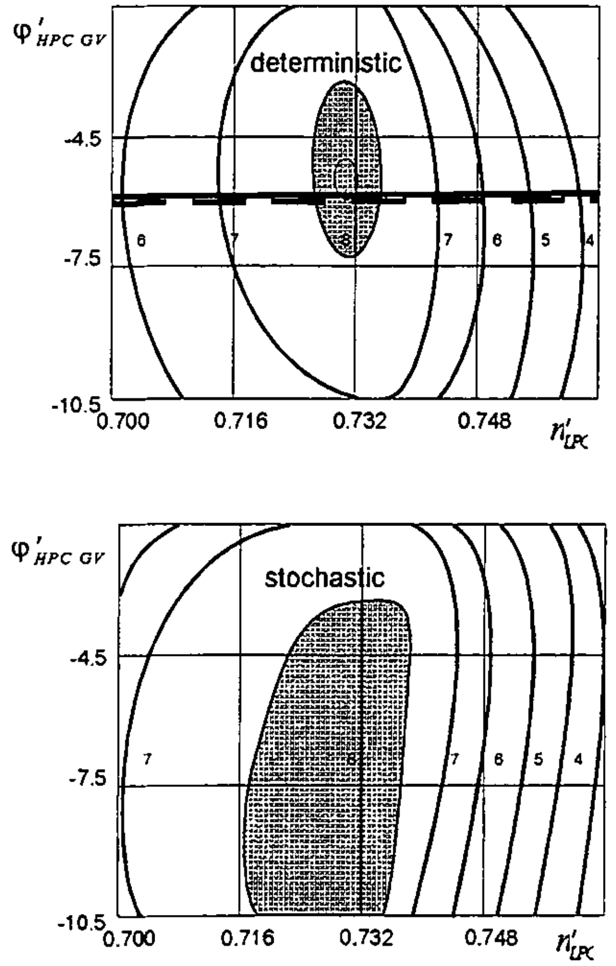

Figure 6. The topology of object functions for different optimization approaches.

No less important indicator of solution stability is the estimation of engine's performance (the effectiveness criterion), which has the probabilistic character under stochastic statement of the problem. For such analysis it should be necessary to know the distribution functions of researched random parameters. The components of the control vector are random parameters, distributed (according to the conditions of statistical testing) according to normal law. However, this doesn't mean that the other random parameters, such as effectiveness criterion or internal engine parameters, must be distributed by the same law. This laws depend upon the topology of object function. The estimation of these distribution laws can be carried out only by means of the experimental methods.

As the example, Fig. 7 presents the histograms of distribution laws of effectiveness criteria, corresponding to the statistical tests, carried out for engine optimum control when $Z(\bar{\sigma}_i)=3$. The dimension of statistical tests excerpt was 100 experiments. One can see that the distribution laws of effectiveness criteria random values are not normal. The reason of this is connected with the non-linearity of engine mathematical model. Fig. 8 shows that the stochastic solution obtained using the criterion (B) allows us to ensure the significantly less dispersion of specific fuel consumption values under parameters noise.

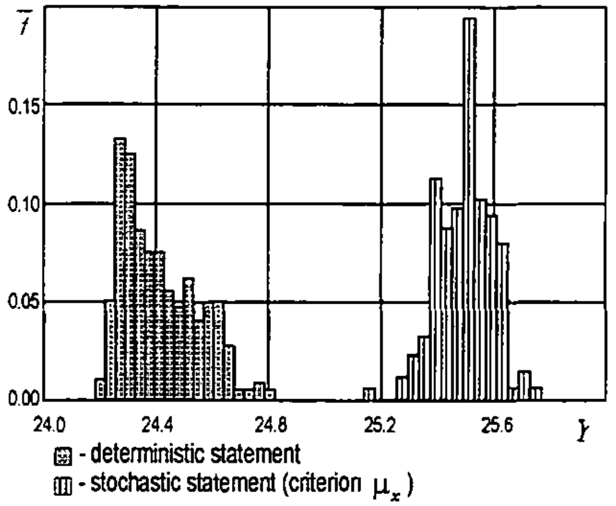

Figure 7. The frequency diagram of effectiveness criteria

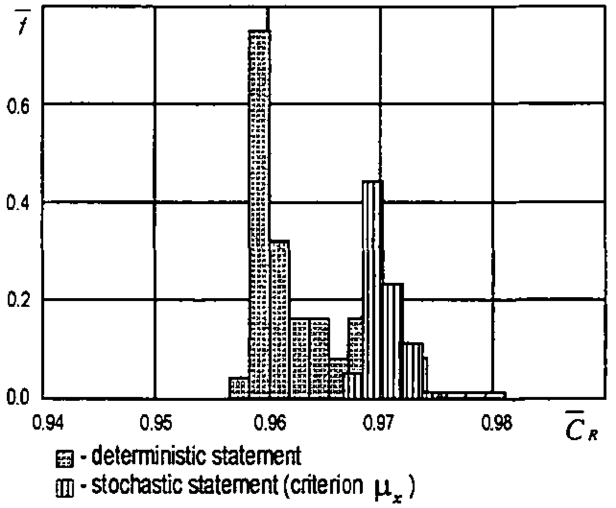

Figure 8. The frequency diagram of SFC value for $R=0.75R_{Max}$

It should be noted that as the alternative indicator of the optimum control stability one can consider the total probability of ensuring of defined effectiveness criterion - P_{Σ}. Let us determine the probability of ensuring of some pre-defined value of effectiveness indicator for the range of varying of parameter $Z(\bar{\sigma}_i)$, which can be presented by

$$P_\Sigma = P_{Constr} * P_{Cond},$$

where P_{Constr} is the probability of ensuring of defined operating conditions (observing of the constraints), $P_{Cond} = P(Y \leq Y_{Def})$ is the probability of ensuring of the effectiveness criterion which is not worse than some defined value under the condition of the constraints observing. Note that in the optimization problems which were considered in this paper there was necessary to minimize the object function.

Figs. 9-11 show the resuming results, which demonstrate the advantages of the stochastic approach as compared with the deterministic one. Fig. 9 demonstrates the changes in the value of conditional probability of ensuring of defined effectiveness criterion value (which corresponds to 101% of minimal possible effectiveness criterion), for both deterministic and stochastic approaches. One can see that for $0 < Z(\bar{\sigma}_t) < 1$ the probabilities of ensuring of pre-defined value of effectiveness criteria are nearly the same, while for the cases of $Z(\bar{\sigma}_t) > 1$ the deterministic solution is worse than the stochastic one from this point of view. Moreover, there is an interesting fact that the values of the probabilities of defined constraints observing P_{Constr} for the deterministic solution are significantly lower than that of the stochastic solution within the whole range of $Z(\bar{\sigma}_t)$, and when $Z(\bar{\sigma}_t) = 1$, these values are correspondingly 0.63 against 0.99. As the result, the total probability of ensuring of pre-defined effectiveness indicator P_Σ for the case of implementation of the stochastic solution is considerably higher than that of the deterministic solution, for all values of $Z(\bar{\sigma}_t) > 0$.

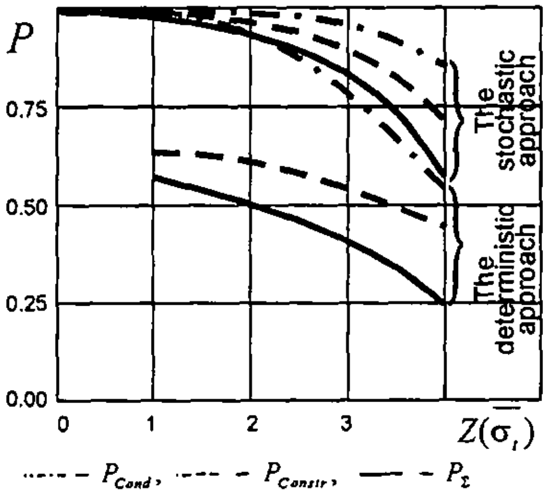

Figure 9. The probabilistic characteristics for the different approaches solutions.

Fig. 10 shows the dispersion of effectiveness criteria values depending upon $Z(\bar{\sigma}_t)$ for different solutions of the optimization problems, where the probability of ensuring of effectiveness criteria values is defined by

$$P(Y - \frac{\Delta Y}{2} \leq Y \leq Y + \frac{\Delta Y}{2}) = 90\%.$$

One can see that the stochastic solution using the criterion (B) provides twice as less dispersion of the effectiveness criterion as compared with the deterministic solution and the stochastic solution using criterion (A).

Fig. 11 shows that the optimum solutions obtained using both the deterministic approach and the stochastic one by the criterion (A) can ensure the maximal improving of the effectiveness criteria $\Delta Y(c_R)$ equal to $\approx 9\%$ under $Z(\bar{\sigma}_t) = 1$, but the probabilities P_Σ of these improving are 0.63 and 0.92 correspondingly. The 6% improving of the effectiveness criterion can be assured with $P_\Sigma = 1$ when using the complex probabilistic criterion (B). This result seems to us to be very important, because even for the high enough level of manufacturing technology ($Z(\bar{\sigma}_t) = 1$) it is very difficult to ensure the value of the effectiveness criterion equal to the value obtained using the deterministic approach.

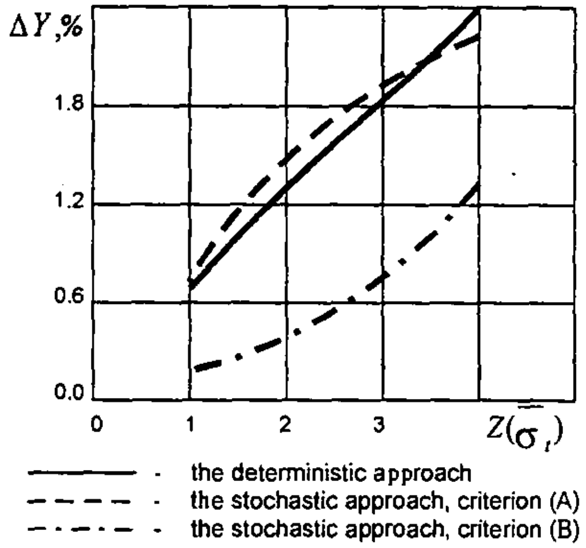

Figure 10. The dispersion of effectiveness criteria depending upon manufacturing technology level.

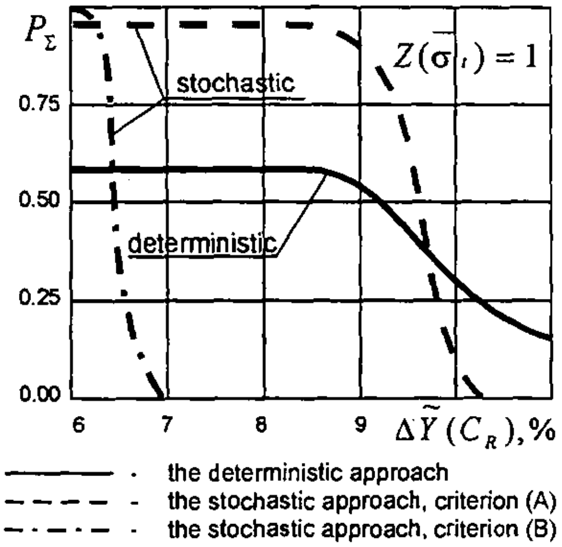

Figure 11. The probabilities of ensuring of effectiveness criteria for different optimization approaches.

Basing upon this result, we can make the important conclusion, which is seemed to be common for many branches of engineering. If one has the task to create some non-serial object with best performance and the expenditures to do it are not of paramount importance, it is reasonable to determine optimum parameters of the object using the deterministic approach. In such case the choice of one or several best specimens among the large enough series of the specimens, manufactured using such optimum solution, can ensure the record values of the object effectiveness indicator. But if one has the task to determine optimum parameters of some object designed for serial production, the stochastic approach will allow us to create the highly robust object, with well enough performances and low sensitivity to technological deflections of manufacturing.

CONCLUSIONS

The most important result of this research seems to be in substantiation of the reasonability to search the optimum control laws of aircraft GTE using the stochastic approach. The realization of optimum solution obtained using the deterministic optimization can be problematical due to the dispersion of object parameters. The use of the stochastic approach allows us to ensure the stability of the engine performance under the conditions of technological deflections and inaccuracies. This improving of the solution stability is reached through the slight worsening of average value of effectiveness criterion as compared with the value of the deterministic solution criterion. The regions of optimum control laws parameters can be significantly different for the deterministic approach and for the stochastic one, because the stochastic solution is placed far enough from the active constraints and from the regions of sharp worsening of the effectiveness indicator.

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