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The minimum mass ratio of W UMa-type binary systems

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ABSTRACT

When the total angular momentum of a binary system $J_{\rm tot} = J_{\rm orb} + J_{\rm spin}$ is at a certain critical (minimum) value, a tidal instability occurs which eventually forces the stars to merge into a single, rapidly rotating object. The instability occurs when $J_{\rm orb} = 3J_{\rm spin}$, which in the case of contact binaries corresponds to a minimum mass ratio $q_{\rm min} \approx 0.071$–$0.078$. The minimum mass ratio is obtained under the assumption that stellar radii are fixed and independent. This is not the case with contact binaries where, according to the Roche model, we have $R_2 = R_2(R_1, a, q)$. By finding a new criterion for contact binaries, which arises from $\mathrm{d}J_{\rm tot} = 0$, and assuming $k_1^2 \neq k_2^2$ for the component's dimensionless gyration radii, a theoretical lower limit $q_{\rm min} = 0.094$–0.109 for overcontact degree $f = 0$–1 is obtained.

Key words: instabilities – methods: analytical – binaries: close – blue stragglers.

1 INTRODUCTION

When the total angular momentum of a binary system $J_{\rm tot} = J_{\rm orb} + J_{\rm spin}$ is at a certain critical (minimum) value, a secular tidal instability occurs (Darwin's instability) which eventually forces the stars to merge into a single, rapidly rotating object. It is likely that at least some of the blue stragglers in star clusters are formed in this way (see e.g. Lombardi et al. 2002). The instability occurs when $J_{\rm orb} = 3J_{\rm spin}$, which in the case of contact binaries corresponds to a minimum mass ratio $q_{\rm min} \approx 0.071$–$0.076$ (Rasio 1995; Li & Zhang 2006), depending on dimensionless gyration radii of stars k^2 and on the overcontact degree f (Rasio & Shapiro 1995).

The minimum mass ratio is obtained under the assumption that stellar radii are fixed and independent. This is not the case with contact binaries where, in accordance with the Roche model, component's radii are correlated, $R_2 = R_2(R_1, a, q)$, where a is the orbital radius. More importantly, $k^2 \approx 0.06$ was adopted in previous studies in order to place the well-known AW UMa with $q = 0.075$ (Rucinski 1992) just at the stability boundary. Nevertheless, $n = 3$ polytrope (fully radiative star with $\Gamma_1 = 4/3$) has $k^2 \approx 0.075$, implying that AW UMa primary cannot have much of the convective envelope and must be slightly evolved (Rasio 1995). In most W UMa-type binaries the primary is a Sun-like main-sequence star (spectral types late F to K, see e.g. Hilditch 2001). For the Sun $k^2_\odot = 0.059 \approx 0.06$ (Allen 1973), and it is possible that primaries of W UMa systems also have k^2 slightly below the $n = 3$ value, but this k^2 may not be the same for W UMa-type secondaries.

In this paper we find a new criterion for the stability of contact binaries, which arises from $\mathrm{d}J_{\rm tot} = 0$ and the assumption that $R_2 = R_2(R_1, a, q)$. This taken into consideration and assuming $k_1^2 \neq k_2^2$, the minimum mass ratio of W UMa-type binary systems is derived. Finally, the obtained results are briefly discussed and compared with the observational data.

2 ANALYSIS

The orbital angular momentum of a binary can be written as

$$J_{\rm orb} = \mu a^2 \Omega = \frac{q\sqrt{GM^3 a}}{(1+q)^2}, \tag{1}$$

where $\mu = M_1 M_2/M$, $M = M_1 + M_2$, $q = M_2/M_1 < 1$, M_1 and M_2 are masses of the primary and secondary component, respectively, and Ω is the orbital angular velocity. Synchronization assumed, the spin angular momentum of a binary is

$$J_{\rm spin} = k_1^2 M_1 R_1^2 \Omega + k_2^2 M_2 R_2^2 \Omega, \tag{2}$$

where R_1 and R_2 are taken to be the volume radii.

The overcontact degree for a contact binary (or, by some authors, the degree of contact) is defined as

$$f = \frac{\Phi - \Phi_{\rm IL}}{\Phi_{\rm OL} - \Phi_{\rm IL}} \approx \frac{R - R_{\rm IL}}{R_{\rm OL} - R_{\rm IL}}, \tag{3}$$

where we have adopted the linear dependence of f on the volume radius, which is a quite good approximation in the narrow range $0 \leqslant f \leqslant 1$ involved (see tables 6 and 7 in Mochnacki 1984). Volume radii for the inner Roche lobe are, following Eggleton (1983),

$$\frac{R_{{\rm IL}i}}{a} = \begin{cases} \frac{0.49q^{-2/3}}{0.6q^{-2/3}+\ln(1+q^{-1/3})}, & i = 1, \\ \frac{0.49q^{2/3}}{0.6q^{2/3}+\ln(1+q^{1/3})}, & i = 2, \end{cases} \tag{4}$$

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while volume radii for the outer Roche lobe (Yakut & Eggleton 2005) are

$$\frac{R_{\mathrm{OL}i}}{a} = \begin{cases} \frac{0.49q^{-2/3}+0.15}{0.6q^{-2/3}+\ln(1+q^{-1/3})}, & i=1, \\ \frac{0.49q^{2/3}+0.27q-0.12q^{4/3}}{0.6q^{2/3}+\ln(1+q^{1/3})}, & i=2. \end{cases} \quad (5)$$

The latter are defined as the radii of the spheres, each being of the same volume as the volume of the respective figure obtained by cutting the equipotential surface passing through the L2 point by a plane through the L1 point which is perpendicular to the line of centres.

As the component's surfaces in the contact system are at the same potential (the same *f*), by combining equations (3)–(5) one obtains

$$R_2 = P(q)a + Q(q)R_1, \quad (6)$$

where

$$Q(q) = \frac{R_{\mathrm{OL2}} - R_{\mathrm{IL2}}}{R_{\mathrm{OL1}} - R_{\mathrm{IL1}}}, \quad (7)$$

and

$$P(q) = \frac{R_{\mathrm{IL2}}}{a} - Q(q)\frac{R_{\mathrm{IL1}}}{a}. \quad (8)$$

Total angular momentum of a contact binary system $J_{\mathrm{tot}} = J_{\mathrm{orb}} + J_{\mathrm{spin}}$, with the help of equations (1), (2) and (6), can then be expressed as

$$J_{\mathrm{tot}} = \frac{q\sqrt{GM^3R_1}}{(1+q)^2}\left(\frac{a}{R_1}\right)^{1/2}\left\{1 + \frac{k_1^2(1+q)}{q}\left[(1+q\tilde{Q}^2)\left(\frac{R_1}{a}\right)^2 + 2q\tilde{P}\tilde{Q}\left(\frac{R_1}{a}\right) + q\tilde{P}^2\right]\right\}, \quad (9)$$

where $\tilde{Q} = \frac{k_2}{k_1}Q$ and $\tilde{P} = \frac{k_2}{k_1}P$. From the condition $\frac{\mathrm{d}J_{\mathrm{tot}}}{\mathrm{d}(a/R_1)} = 0$ one finds the critical separation

$$\frac{a_{\mathrm{inst}}}{R_1} = \frac{q\tilde{P}\tilde{Q} + \sqrt{(q\tilde{P}\tilde{Q})^2 + 3(1+q\tilde{Q}^2)\left\{q\tilde{P}^2 + q/\left[(1+q)k_1^2\right]\right\}}}{q\tilde{P}^2 + q/\left[(1+q)k_1^2\right]}. \quad (10)$$

Let us first examine two special cases:

(i) $\tilde{P} = \tilde{Q} = 0$

This is the situation when the secondary (i.e. its angular momentum) has been neglected. Equation (10) is then reduced to

$$\frac{a_{\mathrm{inst}}}{R_1} = k_1\sqrt{\frac{3(1+q)}{q}}, \quad (11)$$

which is a result obtained by Rasio (1995). Inserting equations (4) and (5), setting $k_1^2 = 0.06$, and solving equation (11) numerically results in $q_{\mathrm{min}} = 0.071$–0.077, for the inner and the outer Roche radius ($f = 0$–1).

(ii) $\tilde{P} = 0$, $\tilde{Q} = \frac{k_2 R_2}{k_1 R_1}$

In this case, the component's radii are treated as independent. Setting $k_1^2 = k_2^2 = 0.06$ and solving the equation numerically, one finds $q_{\mathrm{min}} = 0.071$–0.078 (Li & Zhang 2006).

If $k_1^2 = k_2^2 = 0.06$, by inserting equations (4) and (5) into (10) and solving the equation numerically, one finds $q_{\mathrm{min}} = 0.072$–0.080. Thus, rigorous derivation does not basically change the result, that is, since q is small all of the above assumptions and simplifications are justified. For the radiative main-sequence primary, however, it would be more appropriate to set $k_1^2 \approx 0.075$. The secondary in all low-q systems is a very low-mass star (see fig. 2 in Gazeas & Niarchos 2006), $M_2 \sim 0.1\,\mathrm{M}_{\odot}$.[1] For a low-mass main-sequence secondary $n = 1.5$ polytrope would be appropriate (fully convective star with $\Gamma_1 = 5/3$), for which $k^2 \approx 0.205$. This gives the theoretical lower limit $q_{\mathrm{min}} = 0.094$–0.109. The exact value probably depends on the overcontact degree *f* (Rasio 1995; Rasio & Shapiro 1995).

If k_1^2 is slightly lower than $n = 3$ value, for example, $k_1^2 = k_{\odot}^2 = 0.059$, and $k_2^2 \approx 0.205$, by solving equation (10) for the inner and the outer Roche radius we obtain $q_{\mathrm{min}} = 0.076$–0.087 (see also Fig. 1). All these values are higher than the values obtained previously. This makes the systems with low mass ratio like AW UMa even more difficult to understand.

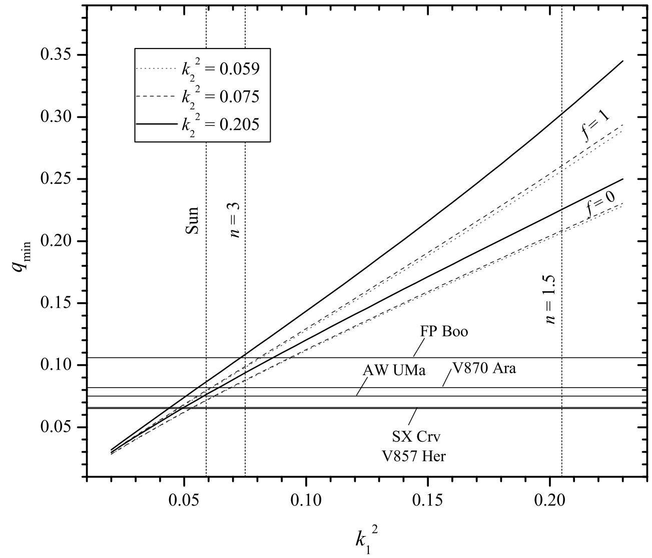

Figure 1. Dependence of minimum mass ratio q_{min} on dimensionless gyration radius k_1^2, for critical Roche lobes and different values for k_2^2. Vertical lines are k_1^2 values for the Sun, $n = 3$ polytrope and $n = 1.5$ polytrope, while horizontal lines are the empirical mass ratios for FP Boo, V870 Ara, AW UMa, SX Crv and V857 Her.

Table 1. Contact systems with the lowest published q values.

Star	q	Reference
V857 Her[a]	0.065 ± 0.001	Qian et al. (2005)
SX Crv	0.066 ± 0.003	Rucinski et al. (2001)
AW UMa	0.075 ± 0.005	Rucinski (1992)
V870 Ara	0.082 ± 0.030	Szalai et al. (2007)
FP Boo	0.106 ± 0.005	Rucinski et al. (2005)
CK Boo	0.111 ± 0.052	Rucinski & Lu (1999)
FG Hya	0.112 ± 0.004	Lu & Rucinski (1999)
GR Vir	0.122 ± 0.044	Rucinski & Lu (1999)
V776 Cas	0.130 ± 0.004	Rucinski et al. (2001)
TZ Boo	0.130 ± 0.030	McLean & Hilditch (1983)

[a] The q s.e. estimated for V857 Her is actually 0.0002. Nevertheless, this is the only system included for which the mass ratio is determined photometrically.

3 DISCUSSION AND CONCLUSIONS

Contact systems with the lowest mass ratio, that the author is aware of, are given in Table 1. The first four, at least, seem to have q

[1] Only V776 Cas secondary is slightly more massive, $M_2 \approx 0.2\,\mathrm{M}_{\odot}$ (Djurašević et al. 2004).

values below the theoretical limit for stability, although the errors in some cases may be too optimistic. A spectroscopic mass ratio is determined from the semi-amplitudes of the sine fitted radial velocity curves, and a formal error usually given arises from the least-squares method used. Radial velocity data were previously obtained from the spectra by using techniques such as *cross-correlation* (see Hilditch 2001), or *broadening function* approach of Rucinski (1999), each with their own uncertainties. Qian et al. (2005), for example, quote $q = 0.72$ for SX Crv. Nevertheless, a spectroscopic mass ratio is by far more reliable that the one obtained photometrically through q-search (e.g. $q = 0.65$ for V857 Her), due to a number of parameters involved in the light-curve modelling.

If we want to explain the existence of low-q systems such as AW UMa, SX Crv or V857 Her, in the framework of current theory, the only solution is in setting k_1^2 (and k_2^2) to match the observations. Essentially, this requires $k_1^2 < 0.075$. This can be understood if the mainly radiative primary is slightly evolved, that is, more centrally condensed than $n = 3$ polytrope, while keeping its main-sequence mass and radius.

Another possible solution to the problem is to consider differential rotation of the primary. Differential rotation was recently proposed by Yakut & Eggleton (2005) as a possible mechanism for thermal energy transfer from the primary to the secondary component in contact binaries, which leads to the equalization of temperatures in the common envelope. In this case, k_1^2 in equation (2) could be provisionally replaced with χk_1^2, where $\chi = 1$ for solid body rotation, and $\chi < 1$ for differential rotation (see Hilditch 2001). It is also possible that other phenomena occurring in close binaries, such as mass transfer or mass-loss, would make parameters in equations (1) and (2) (e.g. components' masses) depend on each other and on binary separation a. This would ultimately lead to a new criterion for the critical separation, and possibly to the explanation of the existence of low-q systems.

In both cases the situation may not be that simple and the stability analysis itself may have to be modified. The condition $J_{\rm orb} = 3J_{\rm spin}$ at the onset of instability is likely to be no longer valid, and even the more general condition that $dJ_{\rm tot} = 0$, that is, $J_{\rm tot}$ is minimum, needs to be reconsidered. What should be kept constant along the equilibrium sequence defining the $J_{\rm tot}$ curve? For a detail discussion of this in the case of compressible Riemann ellipsoids, see Lai, Rasio & Shapiro (1993, 1994a,b). Since the viscous forces conserve angular momentum, the binary evolution driven by viscosity proceeds along sequences of constant $J_{\rm tot}$. If the system loses angular momentum, for example, through gravitational radiation (Lai et al. 1994a), then fluid circulation C is conserved and the evolution proceeds along sequences of constant C. In the late-type binaries system would probably lose angular momentum due to magnetized stellar wind (Stępień 2006). The onset of instability will depend on what is driving the evolution, for example, angular momentum loss or viscous dissipation. There are many other possibilities in contact binaries including the mass-loss and mass transfer.

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