THE MINIMUM NUMBER OF FEEDBACK STATE VARIABLES FOR THE DECOUPLING CONTROL OF A BINARY DISTILLATION COLUMN

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In this paper, it is shown what influences remain in a controlled system if the decoupling controller is designed by using incomplete state variable feedback. A sufficient condition for the existence of a decoupling controller based on the incomplete state variable feedback is derived by using the special structure of the model.

It is also shown for a binary distillation column that only five state variables (1st, 2nd, 3rd, $(n-1)$-th, n-th) need be fed back for design of a decoupling controller.

Introduction

In many chemical processes, it is often required to install suitable control systems for maintaining the output variables at some preassigned constant values. But it is not easy to design such control systems, because chemical processes are inherently highly interconnected, multivariable systems.

The control system design for such a multivariable system is an important problem from both theoretical and practical viewpoints. Decoupling control has been intensively investigated as a promising approach and practical approaches have been proposed based on;

1) compensator[1,6]
2) state variable feedback controller[2-5,7,8].

Many theoretical results concerning these approaches have been published, but only a few applications to practical process control problems have been reported.

Buckley[1] applied the first approach to a distillation column and proposed the simple and intuitively appealing scheme of inserting two interaction compensators to cancel the effect of each manipulating variable on the compositions at the other ends of the column. Luyben[6] also took this approach and designed two types of decouplers, ideal and simplified, from a linear distillation column model. These decouplers are so designed as to make all the non-diagonal elements of the transfer function zero. With this approach there is a possibility of producing uncontrollable modes, and it cannot be applied to systems with implicitly diverging modes.

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As for the second method, Morgan[7] first proposed state space formulation of the decoupling problem for linear multivariable systems.

He dealt with the problem such that

$$\dot{x}=Ax+Bu \tag{1}$$

$$y=Cx \tag{2}$$

$$C=[I_m, 0] \tag{3}$$

where x = n-dimensional state variables
u = m-dimensional manipulating variables
y = m-dimensional output variables
A = $n \times n$ constant matrix
B = $n \times m$ constant matrix
C = $m \times n$ constant matrix
I_m = $m \times m$ unit matrix

and obtained the feedback control law

$$u=Fx+Gw \tag{4}$$

which allows the decoupling of the above system where w are m-dimensional external inputs and F and G are $m \times n$ and $m \times m$ constant matrices, respectively. By this method, decoupling can only be accomplished if the $m \times m$ matrix B_m, composed of the upper m rows of the matrix B, is non-singular. Rekasius[8] extended his results.

Falb and Wolovich[2,3] pointed out that Morgan and Rekasius's result is a sufficient condition and also derived for the first time the necessary and sufficient condition for decoupling general linear time-invariant multivariable systems.

Gilbert[4] and Pivnichny[5] reexamined these results and developed a computer program for synthesizing the decoupling controller. Even in their method, the feedback controller is designed so as to make the transfer function matrix diagonal.

In these methods it is presumed that all the state variables can be fed back, but this assumption is unacceptable for most chemical processes where the dynamical behavior is usually expressed by higher-order ordinary differential equations. It is very common in actual chemical processes that only some of the state variables can be measured and fed back. This means that we should try to decouple the system by utilizing incomplete state variable feedback.

In this paper we first ask whether it is possible to decouple the system by using incomplete state variable feedback. To do this we introduce a matrix H that indicates the state variables which will be measured and fed back. Then, a sufficient condition for the existence of such a controller is derived, and at the same time, the minimum number of state variables which need to be measured and fed back for composing the decoupling controller is set out. A method to determine these state variables is also proposed. As a practical example, a binary distillation column control problem is considered where it is shown how to control two output variables, the compositions of both the distillate and the bottom product, by an incomplete state variable feed back decoupling controller.

For a distillation column, it is shown that measurement and feedback of only five state variables is sufficient to compose a decoupling controller.

1. Decoupling Control Using Complete State Variable Feedback

It is first assumed that the control system is expressed by

$$\dot{x}=Ax+Bu \tag{1}$$

$$y=Cx \tag{2}$$

where C is an $m\times n$ arbitrary constant matrix free from the restriction of Eq. (3).

The necessary and sufficient condition for the existence of feedback control

$$u=Fx+Gw \tag{4}$$

which allows the decoupling of the input-output relations, was derived by Falb and Wolovich and is expressed as follows.

Let d_i be defined by

$$\begin{aligned} d_i &= \min\{j\,|\,C_iA^jB\neq 0;\ j=0,\cdots,n-1\} \\ &= n-1 \quad (\text{if } C_iA^jB=0 \text{ for all } j) \end{aligned} \tag{5}$$

where C_i is the i-th row of C.
The necessary and sufficient condition is

$$B^*=\begin{pmatrix} C_1A^{d_1}B \\ \cdot \\ \cdot \\ \cdot \\ C_mA^{d_m}B \end{pmatrix} \to C: \tag{6}$$

and is non-singular.

Hereafter, we say the system expressed by Eqs. (1) and (2) "can be decoupled" if and only if

$$\det B^*\neq 0 \tag{7}$$

When $\det B^*\neq 0$, there exists the inverse of B^*, and it is possible to choose F and G of the decoupling controller so that

$$F=-B^{*-1}A^* \tag{8}$$

$$G=B^{*-1} \tag{9}$$

where

$$A^*=\begin{pmatrix} C_1A^{d_1+1} \\ \cdot \\ \cdot \\ \cdot \\ C_mA^{d_m+1} \end{pmatrix} \to C: \tag{10}$$

Substituting Eq. (4) into Eqs. (1) and (2), and using the relation defined in Eq. (5), we find that the i-th output, y_i, is expressed by

$$y_i^{(d_i+1)}=C_i(A+BF)^{d_i+1}x+C_i(A+BF)^{d_i}BGw \tag{11}$$

Substituting Eqs. (8) and (9) into Eq. (11), we obtain

$$y_i^{(d_i+1)}=w_i. \quad i=1,2,\dots,m \tag{12}$$

It is clear that the i-th input w_i affects only the i-th output y_i. In other words, a one-to-one correspondence is accomplished between the external input variables $\boldsymbol{w}$ and the output variables $\boldsymbol{y}$.

The transfer function for such a decoupled system is given by

$$G(s)=\begin{pmatrix} s^{-(d_1+1)} & & 0 \\ & \ddots & \\ 0 & & s^{-(d_m+1)} \end{pmatrix} \tag{13}$$

and the decoupled system is called an "integrator decoupled system".

The block diagram for this decoupled system is shown in **Fig. 1**. If we define "the control loop i" by a single input and a single output, and the transfer function of the control system is given by the i-th element of the diagonal of the transfer function matrix $G(s)$, then the classical control theory is easily applied to this decoupled control loop i.

According to the suggestion of Falb and Wolovich, F is chosen instead of Eq. (8) as follows:

$$F=B^{*-1}\Big[\sum_{k=0}^{\delta} M_kCA^k-A^*\Big] \tag{14}$$

where

$$\delta=\max\{d_i|i=1,2,\cdots,m\} \tag{15}$$

and M_k is an arbitrary $m\times m$ diagonal matrix whose elements are given by $\mathscr{M}_k^i(i=1,2,\cdots,m)$ and, moreover, $\mathscr{M}_k^i=0$ if $k<d_i$. Then the following relations are obtained:

$$y_i^{(d_i+1)}=\sum_{k=0}^{\delta}\mathscr{M}_k^i y_i^{(k)}+w_i \quad (i=1,2,\cdots,m) \tag{16}$$

where $\mathscr{M}_k^i=0$ if $k<d_i$.

And the transfer function matrix $G(s)$ is expressed by

$$G(s)=\begin{pmatrix} \dfrac{1}{s^{d_1+1}-\sum_{k=0}^{\delta}\mathscr{M}_k^1 s^k} & & 0 \\ & \ddots & \\ 0 & & \dfrac{1}{s^{d_m+1}-\sum_{k=0}^{\delta}\mathscr{M}_k^m s^k} \end{pmatrix} \tag{17}$$

For the control loop i, the values of $\mathscr{M}_k^i(k=0,1,2,\ldots,d_i)$ can be chosen arbitrarily; in other words, some suitable values can be assigned to $\mathscr{M}_k^i(k=0,1,2,\cdots,d_i)$ in such a way that any desirable pole assignment is accomplished.

If we choose $\mathscr{M}_k^i=0$ $(i=1,\cdots,m,\ k=0,\cdots,d_i)$, the system expressed by Eqs. (1) and (2) becomes an integrator decoupled system given by Eq. (13). Hereafter, we use Eq. (14) as a general type of F.

2. Incomplete State Variable Feedback

In the previous section, we explained the condition for accomplishing the decoupling by using complete state variable feedback. In many chemical processes, however, it is often the case that we cannot feedback all the state variables due to various restrictions. Even if all the state variables could be feedback, for economic reasons it is not unusual to utilize only some of them.

Thus, it may be possible to take the approach whereby an approximate model is made by using only the state variables which can be fed back, and then to try to design a control system based on this model.

Here we take a different approach. Without using any simplification or approximation of the model of Eqs. (1) (2), we determine whether it is possible to accomplish decoupling by using only incomplete state variable feedback, by using only some of the state variables.

For this purpose, we introduce a new matrix H for indicating which state variables are to be measured and fed back. This matrix H has the following properties.

1) H is an $n\times n$ diagonal matrix

2) each diagonal element of H has the value 1 if the corresponding state variable is measured, otherwise it is zero.

When H is used, the feedback control law of Eq. (4) becomes

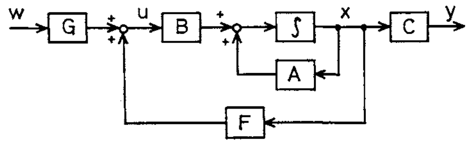

Fig. 1 Diagram of the decoupling control system

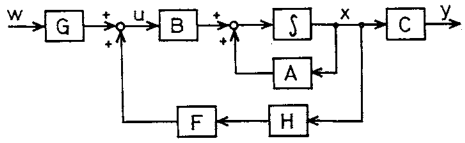

Fig. 2 Diagram of the control system by using incomplete state variable feedback

$$\boldsymbol{u}=FH\boldsymbol{x}+G\boldsymbol{w} \tag{18}$$

Substituting Eq. (18) into Eqs. (1) and (2) the following equation is derived with respect to the i-th output y_i.

$$y_i^{(d_i+1)}=C_i(A+BFH)^{d_i+1}\boldsymbol{x}+C_i(A+BFH)^{d_i}BG\boldsymbol{w} \tag{19}$$

Choosing F and G as given by Eqs. (14) and (9), respectively, Eq. (19) is further rewritten as

$$y_i^{(d_i+1)}=(C_iA^{d_i+1}(I-H)+\sum_{k=0}^{\delta}\mathscr{M}_k^i C_iA^kH)\boldsymbol{x}+w_i \tag{20}$$

and this equation shows the resulting behaviour of the system controlled by the incomplete state variable feedback control law of Eq. (18). The block diagram of this system is shown in **Fig. 2**. This system is not said to be completely decoupled because the influence of the unmeasured state variable exists in the input-output relation of "the control loop i" as expressed by Eq. (20). It is also evident that we cannot assign arbitrary desired values to the poles of the closed loop.

3. Minimum Number of State Variables Necessary to be Measured

As shown in Eq. (20) the system is not completely decoupled as the influence of the unmeasured state variables still remains in the input-output relations.

If we can choose H in Eq. (20) so as to satisfy

$$C_iA^k(I-H)=\boldsymbol{0} \text{ for } i=1,\cdots,m \quad k=0,\cdots,d_i+1 \tag{21}$$

the system expressed by Eqs. (1) and (2) is completely decoupled and Eq. (20) becomes exactly equal to Eq. (16).

For a system able to be decoupled, there exists at least one H which realizes the complete decoupling, because by choosing $H=I$, complete state variable feedback, the system able to be decoupled can be completely decoupled as shown in the previous section.

If the given system has any special structure, that is, the matrices A, B and C of the given system have any special structure, it is possible to choose H in such a way that Eqs. (21) are satisfied and at the same time

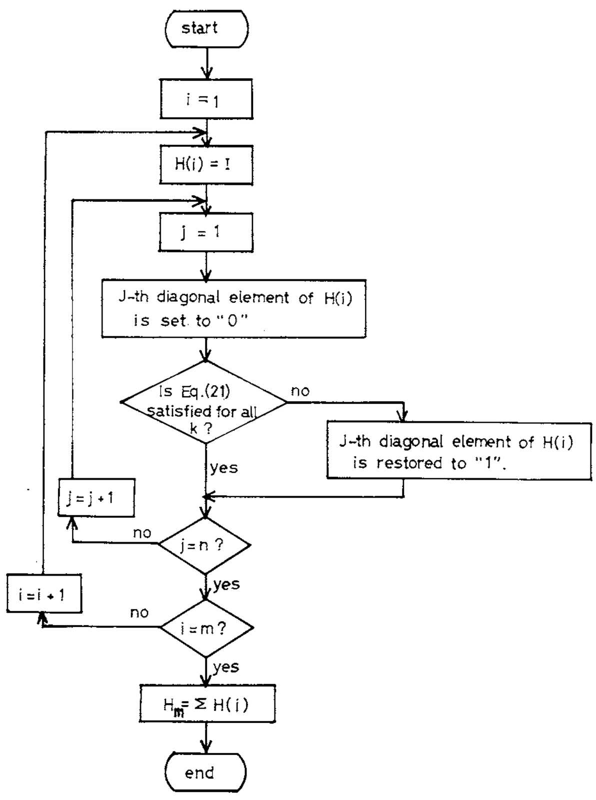

Fig. 3 Flowchart for finding the minimum measuring points

some of the diagonal elements of H are equal to zero. This means that it is possible in some cases to accomplish the complete decoupling and pole assignment of the system by measuring only some of the state variables and feeding them back.

The number of non-zero diagonal elements of H, means the number of state variables which have to be measured. Here, we define "the minimum measuring points" as the fewest state variables which have to be measured to accomplish complete decoupling, and we express the matrix comprising the fewest measuring points as H_m.

To decouple completely the i-th input and output variables, the following relations have to be satisfied for all $k(k=0, \cdots, d_i+1)$.

$$C_i A^k (I-H)=0$$

This means that the only diagonal elements of $(I-H)$ which have to be zero are those which are multiplied by the non-zero elements of the vector $C_i A^k$. The other diagonal elements of the matrix H can take on arbitrary values. Thus we make all the other elements of the matrix H zero except those which are assigned "1", so that the diagonal elements of $(I-H)$ will be zero. Hereafter we represent the matrix H so obtained by H_i. The matrix H_i can be uniquely obtained for each i and it shows which state variables have to be measured so as to decouple the i-th input-output relation. Thus the matrix H_m is obtained by taking the logical sum of H_i for all $i(i=1, \cdots, m)$.

The matrix H_m shows the minimum number of state variables which have to be measured and fed back to realize the complete decoupling among the all input and output variables. The uniqueness of the matrix H_m is understood from the fact that H_i is uniquely determined for all i $(i=1, 2, \cdots, m)$.

A way of finding the fewest measuring points

First, let H be I and change the "1" elements on the diagonal to zero one by one. At each stage check whether Eq. (21) is still satisfied or not. If Eq. (21) is still satisfied, continue; but when Eq. (21) ceases to be satisfied, change the last altered element back to "1" and continue with the next step.

Repeat this procedure for all i $(i=1, \cdots, m)$. The matrix H_m finally obtained contains the minimum number of measuring points.

4. Application to a Binary Distillation Column

In this section, the decoupling control problem of a binary distillation process is taken up as a practical example and the fewest measuring points are obtained by using the procedure explained in the previous section.

The mathematical model of a binary distillation column is obtained by linearizing the material balance equation of each plate at the steadystate condition as follows (see **Appendix**):

$$\dot{\boldsymbol{x}}=A\boldsymbol{x}+B\boldsymbol{u} \qquad (22)$$

where

$\boldsymbol{x}^T=[\Delta x_D, \Delta x_1, \cdots, \Delta x_B]$ (state variables)

$\boldsymbol{u}^T=[\Delta L_n, \Delta V_m]$ (manipulating variables)

The output equation is expressed by

$$\boldsymbol{y}=C\boldsymbol{x} \qquad (23)$$

where

$\boldsymbol{y}^T=[\Delta x_D, \Delta x_B]$ (output variables)

The matrices A, B and C of Eq. (22) are given by

$$A=\begin{pmatrix} a_{11}, a_{12} & & & 0 \\ a_{21}, a_{22}, a_{23} & & & \\ & \cdot\ \cdot\ \cdot & & \\ & & \cdot\ \cdot\ \cdot & \\ & & & \cdot\ \cdot\ \cdot \\ 0 & & & a_{n,n-1}, a_{nn} \end{pmatrix} \qquad (24)$$

$$B=\begin{pmatrix} 0 & 0 \\ b_{21} & b_{22} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ b_{n1} & b_{n2} \end{pmatrix} \qquad (25)$$

$$C=\begin{pmatrix} 1 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 0 \\ 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 & 1 \end{pmatrix} \qquad (26)$$

Here, our objective is to control the compositions of both the distillate and the bottom products.

To check whether the system expressed by Eqs. (22) and (23) can be decoupled or not, we examine the det B^*.

For this system, d_i and B^* as defined by Eqs. (5) and (6) are

$$d_1=1 \tag{27}$$

$$d_2=0 \tag{28}$$

$$B^*=\begin{pmatrix} C_1AB \\ C_2B \end{pmatrix}=\begin{pmatrix} a_{12}b_{21} & a_{12}b_{22} \\ b_{n1} & b_{n2} \end{pmatrix} \tag{29}$$

For the usual binary distillation column, $(b_{21}b_{n2}-b_{22}b_{n1})$ is not zero, and therefore the det B^* is not zero. This means that the system expressed by Eqs. (22) and (23) is "able to be decoupled".

By using the procedure explained in the previous section, the matrix composing the minimum number of measuring points is uniquely determined as follows:

$$H_m=\begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & 0 & & & & & \\ & & & & \cdot & & & & \\ & & & & & \cdot & & & \\ & & & & & & 0 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{pmatrix} \tag{30}$$

H_m shows that to realize decoupling control it is sufficient to measure three state variables (1st, 2nd, 3rd) in the enriching section, two ((n-1) th, n-th) in the stripping section and to feed back these five state variables.

Matrices F and G of the decoupling controller defined by Eq. (18) are obtained as

$$F=1/\gamma$$
$$\begin{pmatrix} b_{n2}f_1, & b_{n2}f_2, & -b_{n2}a_{23}, & 0, & \cdots, & 0, & b_{22}a_{nn-1}, & -b_{22}f_3 \\ -b_{n1}f_1, & -b_{n1}f_1, & b_{n1}a_{23}, & 0, & \cdots, & 0, & -b_{21}a_{nn-1}, & b_{21}f_3 \end{pmatrix} \tag{31}$$

$$G=\frac{1}{a_{12}\gamma}\begin{pmatrix} b_{n2}, & -a_{12}b_{22} \\ -b_{n1}, & a_{12}b_{21} \end{pmatrix} \tag{32}$$

where

$$\gamma=(b_{21}b_{n2}-b_{22}b_{n1}) \tag{33}$$

$$f_1=(1/a_{12})(\mathscr{M}_0^1+\mathscr{M}_1^1a_{11}-a_{11}^2-a_{12}a_{21}) \tag{34}$$

$$f_2=(\mathscr{M}_1^1-a_{11}-a_{22}) \tag{35}$$

$$f_3=(\mathscr{M}_0^2-a_{nn}) \tag{36}$$

The values of f_1, f_2 and f_3 depend on the chosen values of $\mathscr{M}_0^1$, $\mathscr{M}_1^1$, and $\mathscr{M}_0^2$. It so happens that all the elements of matrix F become zero except for the 3rd and $(n-1)$-th columns when suitable values of $\mathscr{M}_0^1$ $\mathscr{M}_1^1$, and $\mathscr{M}_0^2$ are chosen. In this case, we cannot arbitrarily assign the poles of the controlles system.

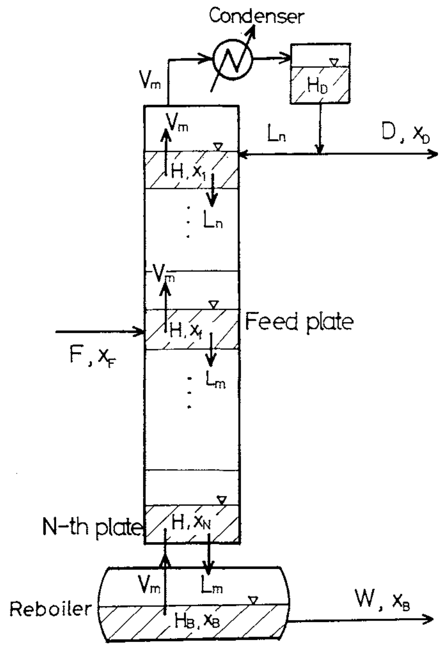

Fig. A-1 Schematic diagram of a binary distillation column

To accomplish decoupling of the binary distillation column expressed by Eqs. (22) and (23), and to be able to freely assign the poles, it is necessary to measure and feed back five state variables as shown in Eq. (31).

The fact that only five state variables have to be measured and fed back is a function of the structure of the matrices A and B and not of the number of plates in the distillation column. This is because the structural properties of matrices A and B, A being tri-diagonal and the first column of B being a zero vector, are preserved even when the number of plates in a distillation column is increased.

Conclusion

In applying the decoupling control techniques to chemical processes, it has often been problematic whether it is possible to realize decoupling controllers based on many state variables. But as shown in this paper there does exist for some special processes a high possibility of realizing complete decoupling by measuring and feeding back only a few state variables.

For a binary distillation column, it has been made clear that a complete decoupling controller can be achieved by feeding back only five state variables.

Appendix

1. Mathematical model of a binary distillation column

A binary distillation column is schematically expressed as shown in **Fig. A-1**. To obtain the mathematical model of this system, the following assumptions are introduced.

1) The binary system has constant volatility throughout the

column and each tray is a perfect, 100 percent efficient (theoretical) tray. Therefore a simple vapor-liquid equilibrium relationship can be used:

$$y_n = \frac{\alpha x_n}{1+(\alpha-1)x_n} \tag{A-1}$$

x_n: liquid composition on n-th tray (mol fraction of more volatile component)

y_n: vapor composition on n-th tray

α: relative volatility

2) A single feed stream is fed as a saturated liquid (at its bubble point) onto the feed tray.

3) The overhead vapor is totally condensed.

4) The liquid hold-up on each tray is constant and perfectly mixed.

5) The hold-up of vapor is negligible throughout the system.

6) The molal flow rates of the vapor and liquid through the stripping and rectifying sections respectively are constant.

Under these assumptions, the unsteady state of the system is expressed by the following equations.

i) condenser:

$$\frac{dx_D}{dt} = \frac{V_m}{H_D}(y_1 - x_D) \tag{A-2}$$

ii) n-th tray of enriching section $(1 \leqq n \leqq (f-1))$:

$$\frac{dx_n}{dt} = \frac{V_m}{H}(y_{n+1} - y_n) + \frac{L_n}{H}(x_{n-1} - x_n) \tag{A-3}$$

iii) feed tray:

$$\frac{dx_f}{dt} = \frac{F}{H}(x_F - x_f) + \frac{V_m}{H}(y_{f+1} - y_f) + \frac{L_n}{H}(x_{f-1} - x_f) \tag{A-4}$$

iv) m-th tray of stripping section $((f+1) \leqq m \leqq N)$:

$$\frac{dx_m}{dt} = \frac{V_m}{H}(y_{m+1} - y_m) + \frac{L_m}{H}(x_{m-1} - x_m) \tag{A-5}$$

v) bottom:

$$\frac{dx_B}{dt} = \frac{V_m}{H_B}(x_B - y_B) + \frac{L_m}{H_B}(x_N - x_B) \tag{A-6}$$

$$\begin{pmatrix} \frac{d}{dt}(\Delta x_D) \\ \frac{d}{dt}(\Delta x_1) \\ \frac{d}{dt}(\Delta x_2) \\ \vdots \\ \frac{d}{dt}(\Delta x_f) \\ \vdots \\ \vdots \\ \frac{d}{dt}(\Delta x_N) \\ \frac{d}{dt}(\Delta x_B) \end{pmatrix} = \begin{pmatrix} \frac{-\bar{V}_m}{H_D}, & \frac{\bar{V}_m\bar{M}_1}{H_D} & & & & & & & \\ \frac{\bar{L}_n}{H}, & \frac{-\bar{V}_m\bar{M}_1-\bar{L}_n}{H}, & \frac{\bar{V}_m\bar{M}_2}{H} & & & & & & \\ & \frac{\bar{L}_n}{H}, & \frac{-\bar{V}_m\bar{M}_2-\bar{L}_n}{H}, & \frac{\bar{V}_m\bar{M}_3}{H} & & & & & \\ & & & & & & & & \\ & & & \frac{\bar{L}_n}{H}, & \frac{-\bar{V}_m\bar{M}_f-\bar{L}_m}{H}, & \frac{\bar{V}_m\bar{M}_{f+1}}{H} & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & \frac{\bar{L}_m}{H}, & \frac{-\bar{V}_m\bar{M}_N-\bar{L}_m}{H}, & \frac{\bar{V}_m M_B}{H} \\ & & & & & & & \frac{\bar{L}_m}{H_B}, & \frac{-\bar{V}_m\bar{M}_B-\bar{W}}{H_B} \end{pmatrix} \begin{pmatrix} \Delta x_D \\ \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_f \\ \vdots \\ \vdots \\ \Delta x_N \\ \Delta x_B \end{pmatrix} + \begin{pmatrix} 0, & 0 \\ \frac{\bar{x}_D-\bar{x}_1}{H}, & \frac{\bar{y}_2-\bar{y}_1}{H} \\ \frac{\bar{x}_1-\bar{x}_2}{H}, & \frac{\bar{y}_3-\bar{y}_2}{H} \\ \cdot & \cdot \\ \frac{\bar{x}_{f-1}-\bar{x}_f}{H}, & \frac{\bar{y}_{f+1}-\bar{y}_f}{H} \\ \cdot & \cdot \\ \cdot & \cdot \\ \frac{\bar{x}_{N-1}-\bar{x}_N}{H}, & \frac{\bar{y}_B-\bar{y}_N}{H} \\ \frac{\bar{x}_N-\bar{x}_B}{H_B}, & \frac{\bar{x}_B-\bar{y}_B}{H_B} \end{pmatrix} \begin{pmatrix} \Delta L_n \\ \Delta V_m \end{pmatrix} + \begin{pmatrix} 0, & 0 \\ 0, & 0 \\ 0, & 0 \\ 0, & 0 \\ \frac{\bar{F}}{H}, & \frac{\bar{x}_F-\bar{x}_f}{H} \\ 0, & \frac{\bar{x}_f-\bar{x}_{f+1}}{H} \\ 0, & \cdot \\ 0, & \cdot \\ 0, & \frac{\bar{x}_N-\bar{x}_B}{H_B} \end{pmatrix} \begin{pmatrix} \Delta x_F \\ \Delta F \end{pmatrix} \tag{A-7}$$

where $L_m = L_n + F$.

By linearizing the above equations at steady-state condition, the following linear model is derived.

where Δ is the deviation from the steady state and the superscript $^{-}$ indicates the value at the steady state. M_i is defined by

$$M_i = \frac{\alpha}{\{1+(\alpha-1)\bar{x}_i\}^2} \tag{A-8}$$

2. State equations of the binary distillation column

We define the state, manipulating and output variables as follows:

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_{n-1} \\ x_n \end{pmatrix} = \begin{pmatrix} \Delta x_D \\ \Delta x_1 \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_N \\ \Delta x_B \end{pmatrix} \quad \boldsymbol{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \Delta L_n \\ \Delta V_m \end{pmatrix}, \quad \boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \Delta x_D \\ \Delta x_B \end{pmatrix}$$

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