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The Modulo Network Simplex with Integrated Passenger Routing[§]

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Abstract

Periodic timetabling is an important strategic planning problem in public transport. The task is to determine periodic arrival and departure times of the lines in a given network, minimizing the travel time of the passengers. We extend the modulo network simplex method [6], a well-established heuristic for the periodic timetabling problem, by integrating a passenger (re)routing step into the pivot operations. Computations on real-world networks show that we can indeed find timetables with much shorter total travel time, when we take the passengers' travel paths into consideration.

1 Introduction

Classical optimization approaches to periodic timetabling are based on formulations in terms of the period event scheduling problem (PESP) [7], see, e.g., Liebchen [4] and the references therein. A powerful heuristic for the PESP is the *modulo network simplex method*, which has been proposed by Nachtigall and Opitz [6]. It iteratively improves a given feasible solution by pivot operations. This algorithm has been improved by Goerigk and Schöbel [3] who introduced pivot selection rules and cuts to escape local optima.

Standard PESP models work with fixed travel paths. The passengers, however, choose their routes depending on the timetable. Approaches to integrate passenger routing in periodic timetabling have been presented recently, see, e.g., [1, 2]. In this paper, we propose to apply the modulo network simplex method to a periodic timetabling model with variable passenger routing, i.e., to the integrated periodic timetabling and passenger routing problem. We show that a pivot selection that considers updated passenger routes allows to find better timetables in terms of total travel time.

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2 Periodic Timetabling with Fixed Passenger Routes

Consider a directed graph $N = (V,A)$, the *event-activity network*. The nodes V are called *events* and represent arrivals and departures of lines at their stations. The arcs $A \subseteq V \times V$ model *activities* of lines (driving between stations, waiting at stations) and possible transfers between lines at stations. Further, we are given lower and upper time bounds $\ell_a, u_a \in \mathbb{Q}_{\geq 0}$, respectively, for the duration of activity $a \in A$. Activity weights $w \in \mathbb{R}^A_{\geq 0}$ represent the number of the passengers traveling on arc $a \in A$.

A *periodic timetable* $\pi : V \to \mathbb{Q}$ determines for each line periodic arrival and departure times at its stations. We call a timetable *feasible* if π satisfies the *periodic interval constraints* $\ell_a \leq [\pi_j - \pi_i]_T \leq u_a$ for each activity $a = (ij) \in A$; here, we define $[y]_T := y \bmod T$ for $y \in \mathbb{R}$. We may assume without loss of generality that $0 \leq \ell_a \leq u_a$, $u_a - \ell_a < T$, and $\ell_a < T$ holds for all $a \in A$, see [4]. By Serafini and Ukovich [7], π satisfies the periodic interval constraints if and only if there exist *modulo parameters* $z \in \mathbb{Z}^A$ such that $\ell_a \leq \pi_j - \pi_i + T z_a \leq u_a \forall a = (ij) \in A$. For a feasible timetable π with modulo parameters z, the resulting duration of activity $a = (ij) \in A$ is given by $x_a := \pi_j - \pi_i + T z_a$, and is called *periodic tension*. The *periodic slack* is defined by $y_a := x_a - \ell_a$; it measures how much the lower bound is exceeded. The goal is to find a feasible timetable such that the resulting weighted total travel time of all passengers is minimized.

Periodic tensions and slacks can be characterized by means of cycles in N, see [5, 4]. Let $\mathcal{T} \subseteq A$ be a spanning tree of N. For a co-tree arc $\bar{a} \in A \setminus \mathcal{T}$, denote by $C_{\bar{a}}$ the *fundamental cycle of* $\bar{a}$, i.e., the unique oriented cycle $C_{\bar{a}}$ induced by adding $\bar{a}$ to the tree. Arcs in $C_{\bar{a}}$ with the same orientation as $\bar{a}$ are called forward arcs $C^+_{\bar{a}}$, arcs with opposite orientation are called backward arcs $C^-_{\bar{a}}$. The *fundamental cycle matrix* $\Gamma \in \{-1,0,1\}^{A \setminus \mathcal{T} \times A}$ of $\mathcal{T}$ is defined by $\Gamma_{\bar{a}a} = 1$ if $a \in C^+_{\bar{a}}$, $\Gamma_{\bar{a}a} = -1$ if $a \in C^-_{\bar{a}}$, and $\Gamma_{\bar{a}a} = 0$ if $a \notin C_{\bar{a}}$ for all $\bar{a} \in A \setminus \mathcal{T}$ and $a \in A$.

We introduce slack variables $y \in \mathbb{Q}^A$ for the arcs and modulo parameter variables $z \in \mathbb{Z}^{A \setminus \mathcal{T}}$ for the co-tree arcs of $\mathcal{T}$. As suggested by Nachtigall [5], the periodic timetabling problem can be formulated as the following integer program:

$$
\begin{aligned}
(\mathrm{PTT}_w) \quad \min \quad & \sum_{a \in A} w_a (y_a + \ell_a) & & \\
\text{s.t.} \quad & \Gamma y - T z = -\Gamma \ell & & (1)\\
& 0 \leq y_a \leq u_a - \ell_a & \forall a \in A \quad & (2)\\
& y_a \in \mathbb{Q} & \forall a \in A \quad & (3)\\
& z_a \in \mathbb{Z} & \forall a \in A \setminus \mathcal{T}. \quad & (4)
\end{aligned}
$$

The model (PTT_w) minimizes the total passenger travel time for a fixed passenger routing given by the arc weights $w \in \mathbb{R}^A_{\geq 0}$. A timetable given by tensions is feasible if and only if the tensions sum up to a multiple of the period time along every fundamental cycle. This is expressed by Equations (1) in terms of slack variables.

3 The Modulo Network Simplex Method

In this section, we recall the modulo network simplex method as proposed by Nachtigall and Opitz [6].

A point $(y,z) \in \mathbb{R}^A \times \mathbb{Z}^{A\setminus\mathcal{T}}$ is called a *spanning tree solution* for (PTT_w), if there exists a spanning tree structure $\mathcal{S} = \mathcal{S}_\ell \cup \mathcal{S}_u$, where $\mathcal{S}$ is a spanning tree of N, the periodic slack y_a is zero for all $a \in \mathcal{S}_\ell$, and at its upper bound $u_a - \ell_a$ for all $a \in \mathcal{S}_u$. The values for all non-tree arcs and the modulo parameters are uniquely determined by equation (1). The spanning tree solution is called feasible if $0 \le y_a \le u_a - \ell_a$ for all $a \in A$, i.e., (y,z) is a feasible solution of (PTT_w).

Theorem 1 (Nachtigall [5])**.** *Define the* periodic slack polyhedron *by*

$$\mathcal{Y} := \mathrm{conv}\left\{(y,z) \in \mathbb{R}^A \times \mathbb{Z}^{A\setminus\mathcal{T}} : 0 \le y \le u - \ell, \Gamma y - T z = -\Gamma \ell\right\}.$$

Then, $(y,z) \in \mathcal{Y}$ *is an extremal point of* $\mathcal{Y}$ *if and only if it is a spanning tree solution.*

The idea of the modulo network simplex is as follows: starting with a feasible spanning tree solution (y,z) for a spanning tree structure $\mathcal{S} = \mathcal{S}_\ell \cup \mathcal{S}_u$, the current solution is iteratively improved by exchanging a co-tree arc $\bar{a} \in A \setminus \mathcal{S}$ with a tree arc $\hat{a} \in \mathcal{S}$ in its fundamental cycle. This is done by shifting the slack from the co-tree arc $\bar{a}$ to the other arcs in the fundamental cut of $\hat{a}$. For every tree arc $\hat{a} \in \mathcal{S}$, the *fundamental cut induced by* $\hat{a}$ is defined by the unique minimal oriented cut $\mathcal{X}_{\hat{a}} \subseteq A$ of N such that $\mathcal{X}_{\hat{a}} \cap \mathcal{S} = \hat{a}$. As commonly known, $\bar{a}$ is contained in the fundamental cut induced by $\hat{a}$ if and only if $\hat{a}$ is contained in the fundamental cycle induced by $\bar{a}$.

Let $\tilde{\Gamma}$ be the fundamental cycle matrix of $\mathcal{S}$ and let $\delta \in \{y_{\bar{a}}, y_{\bar{a}} - u_{\bar{a}} + \ell_{\bar{a}}\}$. Then

$$y'_a = \begin{cases} [y_a + \tilde{\Gamma}_{\bar{a}\hat{a}}\, \delta]_T & \text{if } a \in \mathcal{X}^+_{\hat{a}}, \\ [y_a - \tilde{\Gamma}_{\bar{a}\hat{a}}\, \delta]_T & \text{if } a \in \mathcal{X}^-_{\hat{a}}, \\ y_a & \text{else,} \end{cases} \qquad \forall a \in A,$$

induces a feasible spanning tree solution if $y'_a \le u_a - \ell_a$ for all $a \in A$. That is, if $y'_a \le u_a - \ell_a$ for all $a \in A$, then there exists $z' \in \mathbb{Z}^{A\setminus\mathcal{T}}$ such that (y',z') is a feasible spanning tree solution of (PTT_w) with respect to $\mathcal{S}' = \mathcal{S} \cup \{\bar{a}\} \setminus \{a\}$. If $\delta = y_{\bar{a}}$, then we are pivoting the co-tree arc $\bar{a}$ into $\mathcal{S}'_\ell$, i.e., $y'_{\bar{a}} = 0$. On the other hand, if $\delta = y_{\bar{a}} - u_{\bar{a}} + \ell_{\bar{a}}$, then we are pivoting the co-tree arc $\bar{a}$ into $\mathcal{S}'_u$, i.e., $y'_{\bar{a}} = u_{\bar{a}} - \ell_{\bar{a}}$.

We call y' a *feasible pivot operation* if y' is a feasible solution. If the difference in the objective value is negative, i.e., $\sum_{a\in A} w_a\,(y'_a + \ell_a) < \sum_{a\in A} w_a\,(y_{a+\ell_a)})$, then we call this an *improving pivot operation*.

The modulo network simplex iteratively applies improving pivot operations to the current tree solution until it terminates with a solution, which cannot be improved further by exchanging a co-tree arc with a tree arc.

4 Integrating Passenger Routing

In order to integrate passenger routing into the modulo network simplex method, we replace the fixed arc weights w by a variable passenger routing along paths in the network N.

The passenger demand is given in terms of an *origin-destination matrix* (OD-matrix) $(d_{st}) \in \mathbb{Q}_{\geq 0}$ specifying for each pair $(s,t) \in V \times V$ the number of passengers that want to travel from s to t. Let $D = \{(s,t) \in V \times V : d_{st} > 0\}$ be the set of all *OD-pairs* and for an OD-pair (s,t) let $\mathcal{P}_{st}$ be the set of (s,t)-paths in N and $\mathcal{P} := \bigcup_{(s,t)\in D} \mathcal{P}_{st}$ be the set of all passenger paths.

We extend the model (PTT_w) to a version $(\text{PTT}_{\mathcal{P}})$ with integrated passenger routing. We introduce passenger variables $f_p \geq 0$ for the fraction of passengers that travel on path $p \in \mathcal{P}$ and enforce the passenger flow by constraints $\sum_{p\in\mathcal{P}_{st}} f_p = 1$ for all $(s,t) \in D$. We include constraints (1)–(4) and change the objective as follows:

$$\min c(y,z,f) := \sum_{a\in A} \sum_{(s,t)\in D} \sum_{\substack{p\in\mathcal{P}_{st} \\ a\in p}} d_{st}\, f_p\, (y_a + \ell_a).$$

The resulting model $(\text{PTT}_{\mathcal{P}})$ is a mixed-integer non-linear program that minimizes the total passenger travel time among all feasible timetables.

Theorem 2. *There exists an optimal solution $(y^{\mathcal{S}}, z^{\mathcal{S}}, f^{\mathcal{S}})$ of $(\text{PTT}_{\mathcal{P}})$ such that $(y^{\mathcal{S}}, z^{\mathcal{S}})$ is a spanning tree solution, i.e., there exists a spanning tree structure $\mathcal{S} = \mathcal{S}_\ell \cup \mathcal{S}_u$ such that $y_a^{\mathcal{S}} = 0$ for all $a \in \mathcal{S}_\ell$ and $y_a^{\mathcal{S}} = u_a - \ell_a$ for all $a \in \mathcal{S}_u$.*

Proof. Let (y^*, z^*, f^*) be an optimal solution of $(\text{PTT}_{\mathcal{P}})$. Define arc weights $w_a^* := \sum_{(s,t)\in D} \sum_{p\in\mathcal{P}_{st}:a\in p} d_{st} f_p^*$, $a \in A$. Let $(y^{\mathcal{S}}, z^{\mathcal{S}})$ be an optimal spanning tree solution of (PTT_{w^*}) for the arc weights w^*. Since $(y^{\mathcal{S}}, z^{\mathcal{S}})$ is optimal and (y^*, z^*) is feasible for (PTT_{w^*}), we have:

$$c(y^{\mathcal{S}}, z^{\mathcal{S}}, f^*) = \sum_{a\in A} w_a^*(y_a^{\mathcal{S}} + \ell_a) \leq \sum_{a\in A} w_a^*(y_a^* + \ell_a) = c(y^*, z^*, f^*). \tag{5}$$

This inequality implies that $(y^{\mathcal{S}}, z^{\mathcal{S}}, f^*)$ is also an optimal solution of $(\text{PTT}_{\mathcal{P}})$. $\square$

Theorem 2 shows that it suffices to investigate spanning tree solutions as well when we integrate passenger variables. In the integrated case we have to consider the passenger flow in order to compute the difference in the objective value between two solutions. Let (y,z,f) be a feasible spanning tree structure solution of $(\text{PTT}_{\mathcal{P}})$ and let y' be a feasible pivot operation. The passenger flow that minimizes the travel time with respect to the modified timetable y' is given by

$$f' := \operatorname{argmin}\left\{ c(y',z',f) : \sum_{p\in\mathcal{P}_{st}} f_p = 1\, \forall\, (s,t) \in D, f \in [0,1]^{\mathcal{P}} \right\}.$$

Hence, y' is an improving pivot operation in the integrated case if $c(y',z',f') < c(y,z,f)$.

Table 1: The columns list the instances, the number of stations, the number of directed lines, the number of OD-pairs, the period time, the number of events, the number of activities, a lower bound on the optimal objective value for model $(\mathrm{PTT}_{\mathcal{P}})$, and the objective value of the starting solution.

instance	$\lvert S\rvert$	$\lvert\mathcal{L}\rvert$	$\lvert D\rvert$	T	$\lvert V\rvert$	$\lvert A\rvert$	lower bound	starting sol.
Wuppertal 98	123	98	32 857	20	1 370	10 994	2 043 083.52	2 239 330.56
Wuppertal 154	148	154	45 159	60	4 313	75 768	2 257 792.97	2 517 657.17
Wuppertal	1 582	311	196 158	60	13 202	78 090	5 016 813.33	5 625 657.98
Dutch	23	40	158	60	447	3 626	868 074.00	871 964.00

5 Computational Experiments

We implemented four variants of the modulo network simplex method in C++11 to assess the improvement potential of our integrated approach. We call the standard modulo network simplex method with fixed arc weights *static*. The variant with fully integrated passenger routing, which compares the objective values with updated passenger flows when searching for improving pivot operations, is called *integrated*. Since the integrated variant takes a toll on the runtime compared to the classic static variant, we also implemented an *iterative* version that applies the static modulo network simplex method and, at its end, updates the arc weights by passenger flow computations; this process is iterated until it cannot improve the solution any further. We finally tested a *hybrid* mode that updates the passenger flow induced arc weights after each pivot operation.

Instead of selecting the most improving pivot operation in each modulo network simplex iteration we used a faster "Quality First" rule as proposed in [3], which selects the first pivot with a satisfying improvement on the objective value. We used an improvement threshold of 0.1% and a scaling factor of 0.2 in all computations. A run was terminated after at most two hours plus finishing the incumbent iteration. All computations were done on an Intel(R) Xeon(R) CPU E3-1290 V2, 3.7 GHz computer (in 64 bit mode, 15 GB system memory), running Linux.

Statistics on four test instances are given in Table 1. The instance Wuppertal is based on the real multi-modal public transportation network of the city of Wuppertal for 2013. The remaining two Wuppertal-instances are obtained by selecting a subset of lines of this instance. The Dutch instance is based on a network that was introduced by Bussieck in the context of line planning. In all instances the lines are operated at different frequencies; their period times are 10, 15, 20, 30, or 60 minutes.

Statistics on the computations are given in Table 2. The integrated variant apparently outperforms the others in terms of quality but at the cost of a strong increase in the computation time. The computations confirm the existence of substantial optimization potentials of integrating passenger routing into periodic timetable computations.

Table 2: Computational results. The columns list the instances, the variant of the algorithm, the computation time, the number of pivot iterations, the average time per pivot operation, the final objective value, the optimality gap compared to the lower bound, and the improvement compared to the starting solution. For the iterative method, the number of (outer) iterations is given in parentheses.

instance	method	time [s]	pivot iter.	time/iter.	final obj.	gap in %	impr. in %
Wuppertal 98	static	16	3	5.34	2 237 571.31	8.69	0.08
	iterative (3)	25	6	4.18	2 233 814.57	8.54	0.25
	integrated	7 232	48	150.66	2 161 064.73	5.46	3.50
	hybrid	16	4	4.05	2 233 814.57	8.54	0.25
Wuppertal 154	static	6 496	6	1 082.74	2 516 268.31	10.27	0.06
	iterative (2)	7 115	7	1 016.41	2 515 421.58	10.24	0.09
	integrated	7 479	14	534.21	2 457 124.12	8.11	2.40
	hybrid	6 468	6	1 078.01	2 515 421.58	10.24	0.09
Wuppertal	static	7 479	19	393.62	5 622 157.01	10.77	0.06
	iterative (1)	7 490	19	394.20	5 622 157.01	10.77	0.06
	integrated	8 206	7	1 172.34	5 553 853.73	9.67	1.28
	hybrid	7 379	14	527.06	5 618 800.66	10.71	0.12
Dutch	static	4	6	< 1	871 697.00	0.42	0.03
	iterative (2)	4	7	< 1	871 697.00	0.42	0.03
	integrated	147	18	8.18	868 320.00	0.03	0.42
	hybrid	1	2	< 1	871 772.00	0.42	0.02

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