



The Monetary Dynamics of International Adjustment under Fixed and Flexible Exchange Rates

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THE MONETARY DYNAMICS OF INTERNATIONAL ADJUSTMENT UNDER FIXED AND FLEXIBLE EXCHANGE RATES*

By ROBERT A. MUNDELL

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I. INTRODUCTION

The basic criticism hitherto advanced against a system of fixed exchange rates, that the powerful instruments of monetary policy are tied to the goal of external balance thereby ruling out a domestic rate of interest compatible with full employment, has been the subject of continued debate over past decades. First advanced over a century ago during the bullionist controversy, and revived in this century by Fisher and Keynes, the argument poses the conflict between internal stability (stable employment and price levels) and external stability (balance-of-payments equilibrium at a fixed exchange parity). In the absence of trade restrictions, which cause inefficiency and invite retaliation, one of the targets implicit in the concepts of internal and external stability must be abandoned. But, so the argument runs, full employment is a prime goal of public policy, balance-of-payments equilibrium is a long-run necessity, and sufficient price flexibility simply does not exist in the modern world: the rate of exchange must therefore be freed.

The argument is based on money illusion: the community is unwilling to accept variations in real income through changes in money prices, but it will accept the same changes in real income through adjustments in the rate of exchange. A flexible-exchange system may then be interpreted as a device for providing a more acceptable means (than employment changes) of altering the real income of the community. But what if money illusion is absent? Then, it is argued, there is no reason for changing to a system of flexible exchange rates: "If internal prices were as flexible as exchange rates, it would make little economic difference whether adjustments

* I am grateful to colleagues at Stanford for helpful discussion of some of the points raised in this paper.

were brought about by changes in exchange rates or by equivalent changes in internal prices."¹

In this paper I shall demonstrate that while this view, under certain circumstances, may be valid in statics, it is entirely erroneous in dynamics. The dynamical differences between the two systems are based on an inversion of the roles, in the dynamic adjustment process, of the terms of trade and the rate of interest. In the fixed exchange system money income (the price level) moves to equilibrate the market for domestic goods and services, and monetary policy is directed at the requirements of the foreign balance; but in the flexible exchange system the rate of exchange moves to correct external disequilibrium, and monetary policy aims at the goal of internal stabilization. These dynamical dissimilarities have important implications for economic policy.

A dynamic analysis of the two systems is necessary to provide a description of the relative merits of fixed and flexible exchange rates — one system may work well (dynamically) under one set of static parameters and speeds of adjustment, but badly under another. Moreover, the type of model which such an analysis requires reveals a conspicuous gap in the international trade literature. Since the days of Hume's classic analysis of the price-specie-flow mechanism, the adjustment process has been rightly recognized as dynamic. Yet it is seldom stated, or even hinted, in expositions of the gold-standard mechanism that an explicit dynamic model is necessary for examining processes which occur simultaneously but at different speeds. The conclusions which follow from this rich field of analysis are essential to even a minimal understanding of the meaning of the adjustment mechanism.

This paper offers a simplified exposition of the dynamics of the international adjustment process, and provides preliminary answers to the following questions: (1) under what conditions will one system be stable while the other system is unstable; (2) how is the cyclicity or directness of the paths to equilibrium, in each system, affected by the extent to which capital is internationally mobile; (3) to what extent should the central bank be concerned, in the fixed exchange system, about the absolute level of its reserves, as opposed to the situation in the current balance of payments; and (4) to what extent can offsetting central bank action stabilize a system which is inherently unstable because of speculative capital movements?

1. Milton Friedman, "The Case for Flexible Exchange Rates," *Essays in Positive Economics* (University of Chicago Press, 1953). See also James E. Meade, *The Balance of Payments* (London: Oxford University Press, 1951), p. 190, for a similar statement. It is, of course, recognized that monetary differences between the two systems exist based on considerations of fixed debts.

The assumption is made throughout that money prices are flexible unless stabilized by the monetary authorities. The purpose of this assumption is to maintain the static and real equivalence of the two systems and thereby isolate most clearly the dynamical dissimilarities. It will nevertheless become clear that the main conclusions would hold in an underemployed economy or in one in which prices were rigid downward and flexible upward.

II. THE STATIC SYSTEM

The economic system to be investigated below is one which is dominated by the conditions of equilibrium in two markets: the market for domestic goods and services, and the market for foreign exchange. Equilibrium in the goods-and-services market is assumed to prevail when the current world demand for domestic goods and services is equal to the current supply of domestic goods and services, a condition which is equivalent to the equality of the excess of domestic saving over domestic investment, and the trade balance surplus. When the excess of domestic saving over domestic investment is greater than the trade balance surplus there is deflationary potential, and when it falls short of the trade balance surplus there is inflationary potential.

Equilibrium in the foreign exchange market requires equality of foreign exchange payments and receipts (excluding central bank transactions), or, equivalently, equality of the rate of lending (net capital exports) and the trade balance surplus. When this equality does not hold there is a balance-of-payments surplus or deficit depending on whether lending is greater or less than the trade balance surplus.

I assume that the goods-and-services market and the foreign exchange market are subject to two main influences: the domestic rate of interest, and the ratio of home and foreign prices (the terms of trade). The rate of interest is assumed to be determined by the monetary policy of the central bank — which means that the latter must always supply funds to the public (through, say, open market operations) to make any given interest rate compatible with equilibrium in the capital market.² I also assume that all foreign prices,

2. For the entire system to be in equilibrium the following conditions must prevail:

$$\text{Saving} - \text{Investment} = \text{Trade Balance Surplus}$$

$$\text{Securities Supplied} - \text{Securities Demanded} = \text{Net Capital Indebtedness}$$

$$\text{Trade Balance Surplus} = \text{Net Capital Exports.}$$

These equalities guarantee equilibrium in the goods-and-services market, the capital market, and the foreign exchange market, respectively.

incomes and interest rates are constant during the period under consideration: this means that changes in the terms of trade can result only from changes in the exchange rate or variations in the domestic price level.

Given these assumptions, it is possible to construct a simple geometric interpretation of the forces governing the rate of interest and the terms of trade. Consider first the foreign exchange market. This market can be divided into two components: the balance of trade governed primarily by the terms of trade, and the net flow of capital, influenced chiefly by the rate of interest. For every level of the terms of trade there will correspond a given balance of trade, and for every rate of interest there will be a specific rate of lending. At high levels of the terms of trade, other things equal, the balance of trade will be smaller than at low levels of the terms of trade.³ Similarly, at high rates of interest the net inflow of capital will be larger, or the net outflow will be smaller, than at low rates of interest. For any given rate of interest, therefore, an increase in the price level or an appreciation of the rate of exchange worsens the balance of payments; and for any given level of the terms of trade, an increase in the rate of interest improves the balance of payments.⁴

One can now conceive of a schedule indicating what the terms of trade would have to be, for different rates of interest, in order to make

The chief limitations of the assumption that the central bank determines the rate of interest are that allowances cannot be made for the cash-balance effect and varying speeds of response in the capital market. Naturally, the dynamic possibilities of the system are increased when these modifications are introduced. In an earlier version of this paper I analyzed the more complete system and found that, providing the cash-balance effect is not large, and the speed of response in the capital market is relatively fast, the main conclusions of the present paper hold.

Alternatively, one could suppose that the central bank adjusts only its own lending rate (Bank Rate) over which it does have complete control. But in this case Bank Rate may not be a true reflection of the general level of interest rates so the slopes of the *XX* and *FF* curves (see below) would be different.

3. This is necessarily the case if domestically-produced goods are not "Giffen goods" in world consumption, or if the sum of the elasticities of demand for imports is greater than unity.

4. I assume that expenditure on goods and services is affected by a capital inflow only insofar as the latter affects the domestic rate of interest. This assumption can usually be justified — especially in a short-run analysis — for some types of capital movements but not for others. The analysis traditionally associated with the transfer problem can easily be incorporated into the present analysis, but it would complicate the exposition without affecting the conclusions fundamentally. A brief summary of the transfer analysis is included in my review of classical theory, "The Pure Theory of International Trade," *American Economic Review*, L (March 1960). The assumption that the trade balance is not directly affected by the rate of interest is made for simplicity, and would have to be modified if imports consisted of durable goods.

the country's foreign exchange payments equal to its foreign exchange receipts (or its net capital exports equal to the balance-of-trade surplus). This curve, which I shall refer to as the "foreign-balance schedule," is plotted as the *FF* line in Figure I. At any point on this line the balance of payments is in equilibrium, although its composition changes in favor of higher rates of lending and higher balance-of-

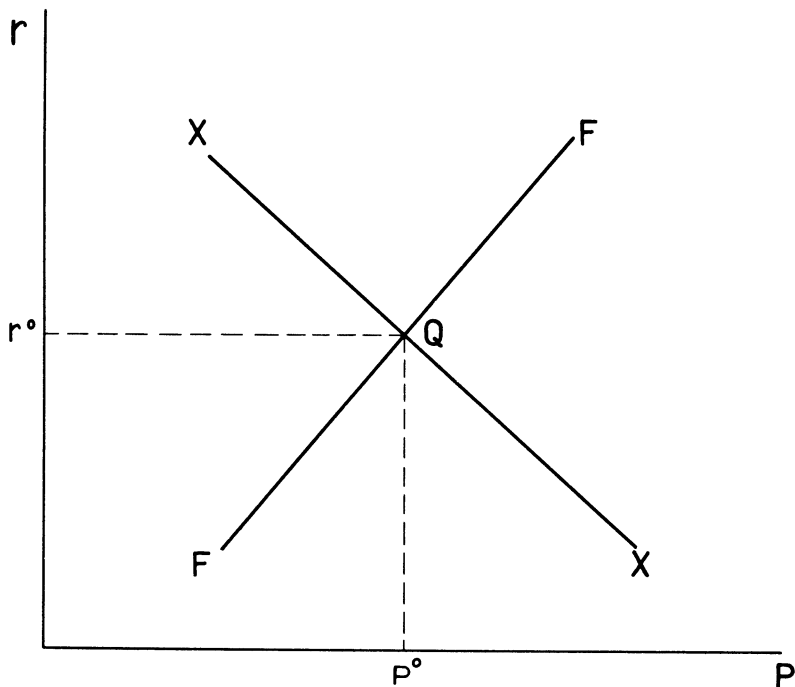


FIGURE I

trade surpluses as we move upward and to the right along it. Any point below or to the right of *FF* represents a point of balance-of-payments deficit: the rate of interest is too low, or the price level or exchange rate is too high, for equilibrium. Similarly, any point above or to the left of *FF* indicates a point of surplus in the balance of payments: the rate of interest is too high, or the relative price of domestic goods is too low, for equilibrium. The economic system can be in equilibrium only on this line.

It may be seen that there is a whole series of combinations of rates of interest and terms of trade at which the balance of payments is in equilibrium. But the entire economic system cannot be in

equilibrium unless there is also balance in the market for goods and services. For this market to be in equilibrium, the balance of trade must equal the excess of saving over investment. Now the balance of trade, as we have seen, is affected primarily by the terms of trade: a rise in the price level or exchange rate worsens the balance of trade so higher levels of the terms of trade tend to cause an excess supply of goods and services. We must add to this, however, the deflationary effect of an increase in the price level or the exchange rate on the rate of saving. Since at high levels of the terms of trade real income is higher than at low levels of the terms of trade, the level of saving also tends to be higher.⁵ An increase in the terms of trade, therefore, is deflationary for two reasons: it lowers the balance of trade and it increases saving. On the other hand, changes in the rate of interest influence primarily the rate of investment spending. At high rates of interest the rate of investment is lower than at low rates of interest so an increase in the interest rate is deflationary. Both increases in the price level or the exchange rate, and increases in the rate of interest, are deflationary.

One can now construct an "internal balance schedule" for the goods-and-services market analogous to the foreign balance schedule developed for the foreign exchange market. From any point on this schedule an increase in the rate of interest causes deflationary pressure and a fall in the price level or the exchange rate causes inflationary pressure. A hypothetical rise in the rate of interest, starting from a position of balance, must therefore be compensated by a fall in the price level or the exchange rate in order to retain balance. This means that the internal balance schedule, plotted as the XX line in Figure I, must have a negative slope. At any point above and to the right of this line there is deflationary potential, and at any point below and to the left of this line there is inflationary potential. Only along XX is the goods-and-services market in equilibrium.

We have now described the conditions necessary for equilibrium in each of two markets. At any point on FF the balance of payments is in equilibrium, and at any point on XX the goods-and-services market is in equilibrium. The entire economic system can be in equilibrium only at the point common to both schedules, Q . The equilibrium interest rate is therefore r_o , and the equilibrium terms of trade is p_o .

5. An excellent survey of the extensive literature on the effects of a change in the terms of trade on the rate of saving has been provided by Harry G. Johnson in "The Transfer Problem and Exchange Stability," *Journal of Political Economy*, LXIV (June 1956).

III. THE DYNAMIC SYSTEMS

The static system described by the foreign balance schedule and the internal balance schedule provides a convenient framework for analyzing the dynamic responses appropriate to systems of fixed and flexible exchange rates. These responses are determined in part by free market reactions and in part by the stabilization policy of the central bank. In the absence of stabilization there is a tendency for the price level to rise or fall depending on whether there is excess demand (inflationary potential) or excess supply (deflationary potential) in the goods-and-services market, and a tendency for the exchange rate to rise or fall depending on whether there is a surplus or deficit in the balance of payments. But if the monetary authorities stabilize the exchange rate they must be prepared to buy and sell foreign exchange reserves at a fixed price, and if they stabilize the price level they must buy and sell goods and services at a fixed price. In order to protect their reserve position (of foreign exchange in the one system, or of goods in the other system) they will pursue a monetary policy which tends to relieve the disequilibrium.

Consider first the case where the central bank pegs the exchange rate. It will raise the interest rate when there is a balance-of-payments deficit, and lower the interest rate when there is a balance-of-payments surplus.⁶ In this case the price level is free to respond to disequilibrium in the market for goods and services. Thus, the interest rate will be rising at any point below and to the right of the foreign balance schedule, and falling at any point above and to the left of the foreign balance schedule. Similarly, the price level will be rising at any point below and to the left of the internal balance schedule, and falling at any point above and to the right of this schedule. These dynamic responses are described by the arrows in Figure II.

The points *A*, *B*, *C* and *D* represent four points not on either of the two schedules. From the point *A*, the interest rate is rising because of the balance-of-payments deficit, and the price level is falling because of the deflationary gap. Similarly, at the point *B* both the interest rate and the price level fall; at the point *C* the price level rises and the rate of interest falls; and at the point *D* both the price level and the interest rate rise.

One of the arrows in each quadrant points in the direction of equilibrium, while the other arrow points in a direction which suggests

6. The central bank adjusts interest rates to prevent *changes* in its reserve position. Later I consider the effect of a monetary policy which attempts to maintain a given *level* of reserves.

a cyclical motion around equilibrium. This means that the equilibrium Q is a stable equilibrium, and that it may be approached cyclically. Consider, for example, the point Z . This point is in the same quadrant as the point D so, from Z , the price level rises because of inflationary pressure and the interest rate rises because of the deficit in the balance of payments. These changes work in opposite

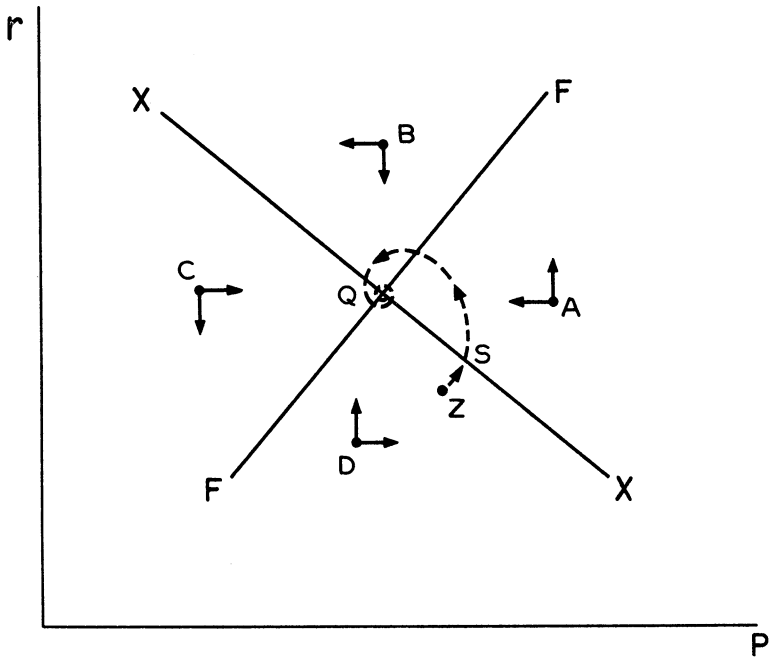


FIGURE II

directions on the foreign balance — capital is attracted while the balance of trade is worsened — but in the same direction on the goods-and-services market — both the interest-rate and price-level changes relieve excess demand. The goods-and-services market is therefore equilibrated before the foreign balance is brought into equilibrium. Now, at the point S on the XX line, the internal market is in equilibrium but there is a deficit in the balance of payments; the interest rate thus continues to rise and this works to produce deflationary pressure in the goods-and-services market. In quadrant A the interest rate rises and the price level falls. Both these forces now operate to relieve the external disequilibrium, but they operate in opposite directions on the internal market; this time the foreign exchange

market is cleared before the goods-and-services market. Now the path of the interest rate and the price level moves into quadrant *B* in which the interest rate is lowered and the price level falls: and so the cycle continues in a counter-clockwise direction.⁷

Now consider the flexible exchange system in which the central

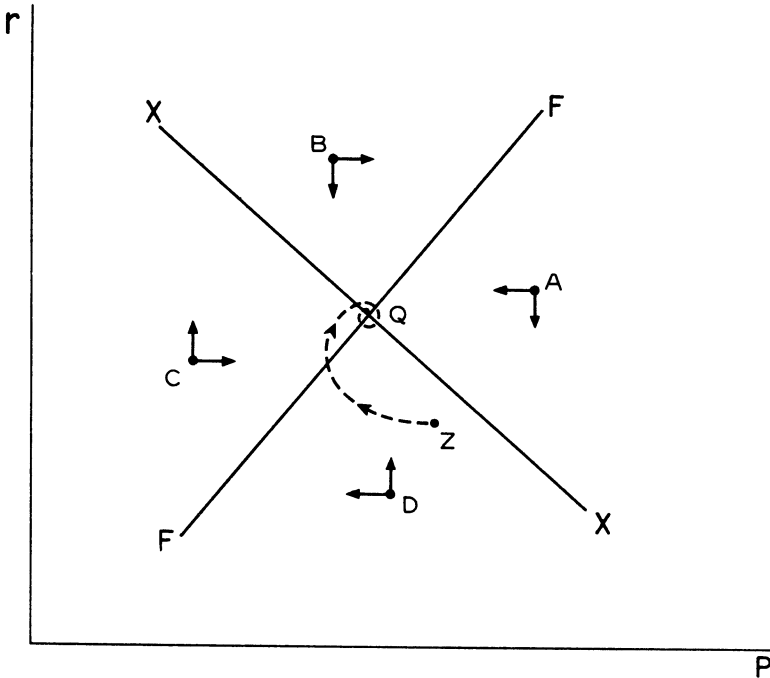


FIGURE III

bank stabilizes the domestic price level. When there is an inflationary gap in the market for goods and services the central bank will tighten credit, raising the interest rate; and when there is a deflationary gap

7. A formal proof of stability, based on a linear system, is given in the Appendix. The assumption of linearity is an important one. It must be expected that central bankers — not governed by the restrictions of mathematical laws — would anticipate future positions and modify their reactions accordingly. But the linear system must be stable if the nonlinear system is to be stable.

The diagrammatic method used here, which provides an intuitive proof of stability, was, I think, first used in economics by Marshall in his 1879 manuscript, *The Pure Theory of Foreign and Domestic Trade*. See also L. A. Metzler, "Wealth, Saving and the Rate of Interest," *Journal of Political Economy*, LIX (Apr. 1951) and D. Patinkin, *Money, Interest and Prices* (Evanston, Ill.: Row, Peterson, 1956), where this technique is ingeniously applied to general equilibrium analysis of a closed economy.

it will ease credit conditions, lowering the interest rate.⁸ On the other hand, the rate of exchange is, in this system, free to move to preserve external balance. Under these conditions the interest rate will be rising at any point below and to the left of the *XX* line in Figure III, and falling at any point above and to the right of this schedule. Similarly, the rate of exchange will be falling at any point below and to the right of the *FF* line because of the payments deficit, and rising at any point above and to the left of this line. The points *A*, *B*, *C*, and *D* represent four typical points in the four quadrants, and the directions of the arrows describe the paths of the interest rate and the exchange rate.

As in the fixed exchange case one of the arrows always points in the direction of equilibrium, while the other arrow tends to impart a cyclical motion to the system: the system is stable but equilibrium may be reached cyclically. Consider the point *Z*, referring to the same point of real disequilibrium as that analyzed in the fixed exchange case. From the point *Z* the exchange rate falls because of the external deficit, and the interest rate rises because of the inflationary pressure. Both of these changes work to correct the foreign balance, but they work in opposite directions on the internal market; the foreign balance is therefore equilibrated before the inflationary gap is relieved. The cycle may continue, as in the fixed-exchange system, but it moves in an opposite direction. The path to equilibrium under flexible exchange rates is *clockwise* but under fixed exchange rates it is *counter-clockwise*.

While the cycles around the real equilibrium *Q* may occur, this result is by no means a necessary one. From any of the quadrants the approach to equilibrium may be direct, depending, in part, on the relative speeds with which the interest rate and the terms of trade move in response to disequilibrium. I have described the cycles which may occur only to emphasize the difference in the nature of the approach to equilibrium. This difference does not, in itself, demonstrate that one system is superior to the other. But the difference in the paths that the interest rate and the terms of trade follow assumes more importance as we consider different values of the static parameters, and the importance to the central bank of a given level of foreign exchange reserves.

8. To stabilize the price level the central bank (or the government) must be willing to buy and sell goods at a fixed price in terms of local currency. The monetary policy may then be interpreted as an attempt to prevent further changes in the stocks of goods held by the government. This often has no practical institutional counterpart in the real world, so the central bank may not be completely successful in its stabilization policy; the abscissa must then be taken to reflect changes in the price level and the exchange rate.

IV. THE CRUCIAL ROLE OF CAPITAL MOVEMENTS

In order to show how the nature of the path to equilibrium is affected by the static parameters and the speeds with which the central bank responds in each system, I shall demonstrate that one system may work well with capital internationally mobile, but badly if capital is internationally immobile; and that the opposite applies to the other system. It should be recalled that the slope of the foreign balance schedule depends upon the values of two parameters: the responsiveness of capital flows to the interest rate, and the responsiveness of the trade balance to the terms of trade. The higher is the degree of capital mobility, relative to the responsiveness of the trade balance, the smaller is the slope of the FF curve. Two extreme cases can be identified: the foreign balance schedule is flat if capital is perfectly mobile and vertical if capital is completely immobile. Figures IV and V are drawn on the assumption that capital is almost completely mobile.

In the fixed exchange system (Figure IV) the path to equilibrium cannot be cyclical. This may be demonstrated by considering the path of the interest rate and the price level in either quadrant A or quadrant C . In these quadrants both the rate of interest and the price level move in directions which work to equilibrate both the internal and the external market. Once the path moves into either of these quadrants it becomes trapped and moves directly to equilibrium. But, as our previous analysis made clear, from quadrant D the path moves either directly to equilibrium or into quadrant A , and from quadrant B it moves to equilibrium or into quadrant C . In both cases it cannot cross the FF line; the path to equilibrium is therefore noncyclical. The economic interpretation of this result is clear: The central bank has little power to affect the domestic rate of interest, and any attempt of the central bank to do so only results in changes in the rate of capital imports. The actual interest rate remains near its equilibrium level, so there can be little interaction of the interest rate with the goods-and-services market. A disequilibrium in the latter market can therefore be eliminated directly by changes in the price level; and, while this will affect the balance of payments, very small changes in the interest rate are sufficient to restore equilibrium.

Consider now the flexible exchange system (Figure V). In this chart, one arrow in each quadrant points toward equilibrium while the other arrow suggests a circular motion around equilibrium. Consider quadrants A and C . If the interest rate and exchange rate combination passes into one of these quadrants it is at once clear that the equilibrium cannot be approached directly. This is in direct

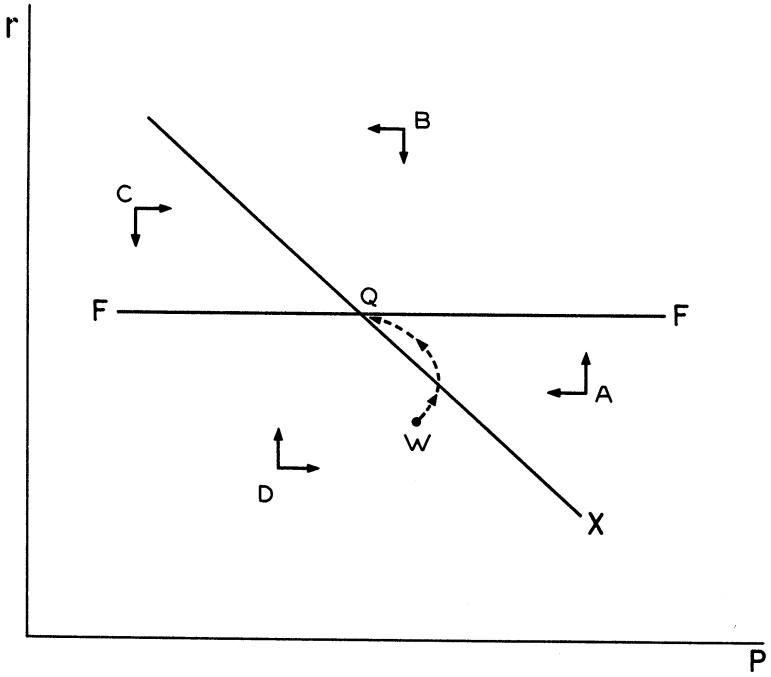


FIGURE IV

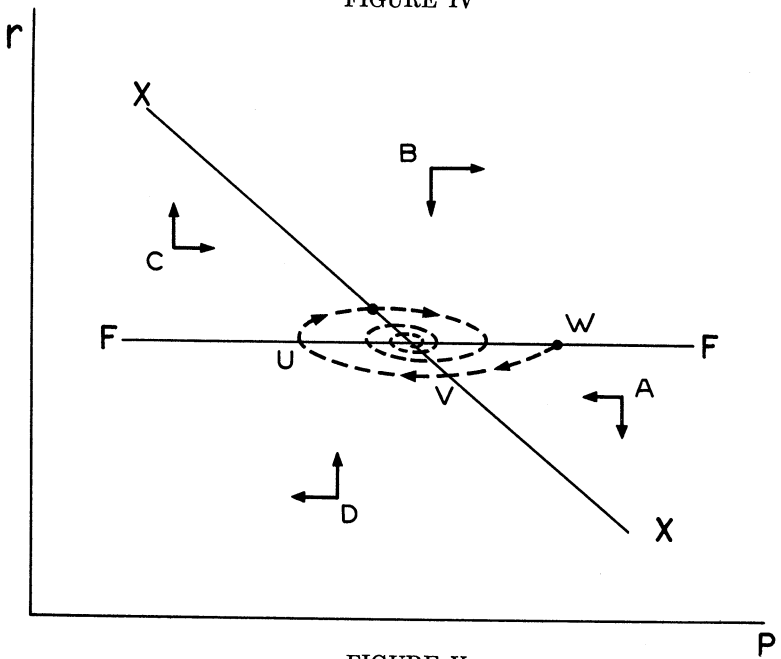


FIGURE V

contrast with the previous case studied where equilibrium had to be reached directly from these quadrants. The system must oscillate around equilibrium in a flat convergent cycle.

To interpret this economically consider the position W , where there is balance-of-payments equilibrium, but a deflationary gap. To prevent deflation or unemployment, the central bank eases credit conditions; but instead of this lowering, significantly, the rate of interest it causes a capital outflow. The exchange rate depreciates and this works to stimulate effective demand; eventually the deflationary gap is removed. But at V , the interest rate is slightly below equilibrium so the depreciation of the exchange rate continues, now causing an inflationary gap and a reversal of central bank policy. Tighter credit conditions now correct the balance of payments, and introduce a surplus, reversing, at U , the direction in which the exchange rate is moving. Eventually the cycle draws closer to equilibrium. But notice that internal stability is not achieved by the *direct* effect of changed credit conditions on effective demand: it is achieved instead by the *indirect* effect of changes in the exchange rate.⁹ To achieve internal stability the central bank changes credit conditions, and this affects the balance of payments and the exchange rate, which in turn produces the desired effect on effective demand. Because the central bank is not master of the domestic rate of interest it can stabilize the domestic market only through the effect of changed credit conditions on the rate of exchange. Because its action is indirect the approach to equilibrium is cyclical.

Let us now consider the other extreme case: capital immobility. The vertical slope of the FF curves in Figures VI and VII reflect the fact that a change in the rate of interest has no direct effect on the balance of payments. Figure VI demonstrates that, in the fixed exchange system, capital immobility may lead to a cyclical approach to equilibrium. The interest rate moves in response to the foreign balance, yet can affect the foreign balance only *indirectly* through interaction with the goods-and-services market. Consider the point S where there is equilibrium in the foreign-exchange market, but a deflationary gap. As the price level falls the balance of trade improves, inducing the central bank to lower interest rates. Both the change in the price level and the fall in the interest rate work to remove the deflationary gap. But at T there remains a surplus in the balance of

9. It is reassuring to find that my theoretical analysis of this situation conforms closely to the empirical analysis in R. R. Romberg's unpublished dissertation: *Fluctuating Exchange Rates in Canada* (Yale University, 1959), although I have not considered the sensitivity of capital flows to the exchange rate, a factor which Romberg concludes is important.

payments so the interest rate continues to fall, introducing inflationary pressure and an increase in prices. Now, the increase in the price level tends to correct the balance of payments surplus and the inflationary gap; but the fall in the rate of interest has no effect on the balance of payments and it works to aggravate the inflationary gap. The balance of payments is first equilibrated and then, because of the continued rise in prices, becomes a deficit. The cycle may therefore continue in a spiral motion around equilibrium.

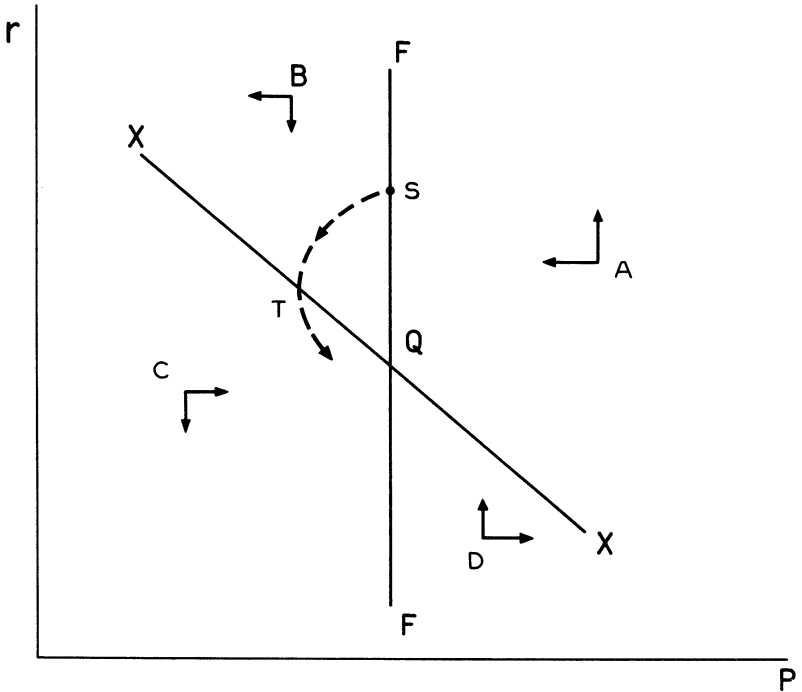


FIGURE VI

But a cycle is not inevitable. Indeed, it can be seen from the graph that the existence of a cycle depends on the speed with which the central bank acts relative to the speed with which the price level is changing. The more slowly does the central bank adjust the interest rate in response to a surplus or deficit the less likely it is that equilibrium will be reached cyclically. From the point *T*, for example, it is clear that very small decreases in the interest rate, per unit of time, would result in a path to equilibrium along the *XX* line. But if the central bank does respond slowly it must be willing to accumulate a larger quantity of reserves than if it had acted quickly. The

disadvantage of this policy may be reflected in both the expense of accumulating reserves, and in the problems this accumulation would create for deficit countries. On the other hand, the speed of central bank action during a period of deficit would be conditioned by the existing level of reserves.

Consider now the flexible exchange mechanism (Figure VII). In quadrants *B* and *D* both arrows point directly to equilibrium so the approach to equilibrium is direct (asymptotic): these two quadrants

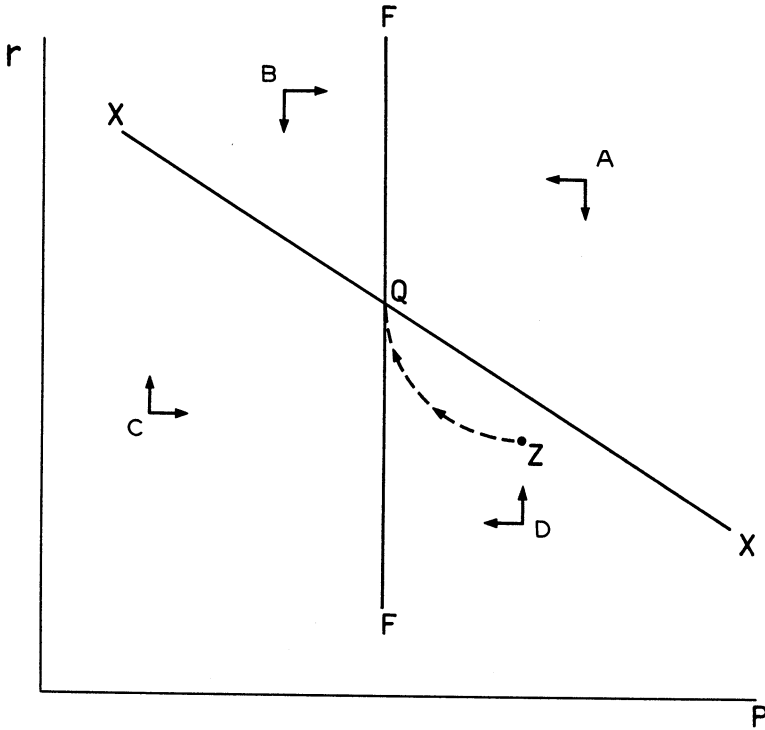


FIGURE VII

are traps out of which the actual interest rate and exchange rate position cannot emerge. From the point *Z*, for example, inflationary pressure induces the central bank to tighten credit, and the foreign deficit causes depreciation of the exchange rate. As the foreign exchange market approaches equilibrium, the movement of the exchange rate slows so that the inflationary gap can be removed by a direct upward movement of the interest rate. No cycles develop because this change in the interest rate has no effect on the foreign balance.

I have now demonstrated that the ease of correcting a disequilibrium under systems of fixed and flexible exchange rates depends partly on the extent to which capital is internationally mobile. If capital is highly mobile the fixed exchange type of response leads directly to equilibrium while the flexible exchange mechanism generally leads to cycles around equilibrium. On the other hand, if capital flows are insensitive to changes in the rate of interest the fixed exchange system leads to cycles if the central bank reacts too quickly to a foreign deficit, while the flexible exchange system leads directly to equilibrium regardless of the speeds of adjustment.¹

V. FOREIGN EXCHANGE RESERVES

As a description of central bank policy in the modern world, the above analysis of the fixed exchange system is subject to one serious qualification. I have assumed that the central bank eases or tightens credit conditions depending on whether *current* foreign exchange receipts exceed, or fall short of, *current* foreign exchange payments. This leaves out of account the lingering effects of past deficits and surpluses on central bank policy. But past deficits and surpluses affect the level of foreign exchange reserves, so this implicitly assumes that the central bank is concerned only with *changes* in the level of reserves, and not at all with the absolute level, at any point in time. It is unrealistic, however, to suppose that the central bank would react to disequilibrium in the balance of payments in the same way at substantially different levels of reserves. If reserves are excessively high the authorities are more likely to allow a deficit in the current balance to continue before remedial action is taken; and if reserves are too low, imperiling confidence in convertibility, the central bank is likely to keep tighter monetary conditions until reserves are again built up.

1. My conclusions conflict with the analysis of Professor Meade (*op. cit.*, pp. 255-58) who writes: "The reader is left to himself . . . to establish the fact that the mobility of labour and capital upon the ease of adjustment is similar for both the gold-standard and the variable-exchange-rate mechanisms of price adjustment." He then speaks of different spontaneous disturbances and how, for each case, the process of adjustment will be affected by the extent to which capital is mobile.

Meade defines the ease of the adjustment process in static terms: in the extent to which the terms of trade must adjust to correct a disequilibrium caused by a spontaneous disturbance. His method can be described in my model by shifting the two curves and comparing the terms of trade at the new equilibrium with the original level.

As suggested in the introduction, I believe that one should discuss the "process" or "ease" of adjustment in dynamic terms. Meade's textual exposition (unlike the *Mathematical Supplement*) hints at a dynamic process but without the use of an explicit model all the implications cannot be derived.

In this section I shall consider the extreme case where the central bank is concerned only with the level of stocks, in order to isolate the effects of this response. I assume that there is a normal or "optimum" level of reserves and consider what happens if the central bank aims at maintaining this level by raising or lowering the interest rate in proportion to the discrepancy between the desired and actual level. As before, I assume that the price level responds to any disequilibrium in the goods-and-services market.

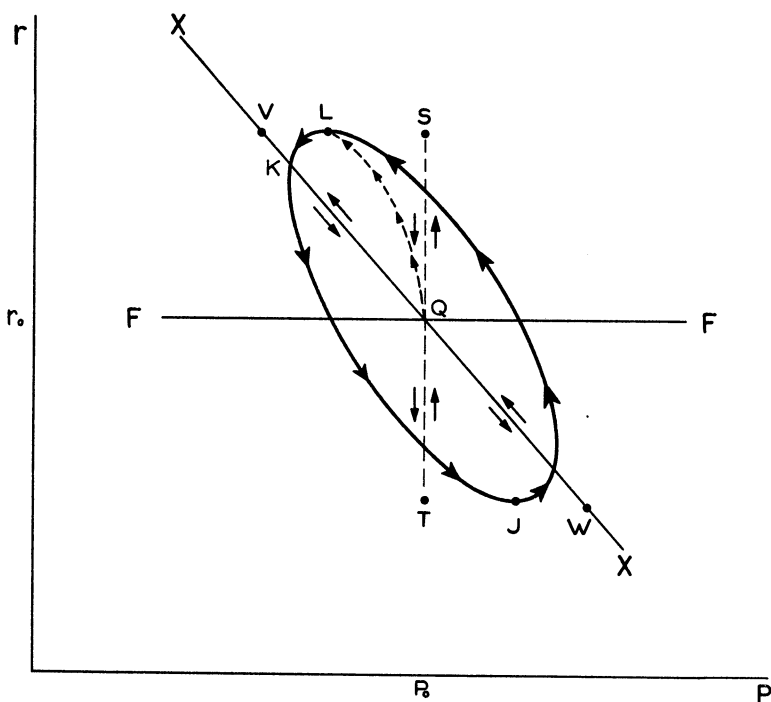


FIGURE VIII

For purposes of exposition it will be convenient first to drop some of the assumptions and then gradually reintroduce them. Suppose first that the price level does not exert any influence on the balance of payments; this means that there can be only one rate of interest at which there is external balance. The system is represented in Figure VIII.

Let the interest rate be initially r_0 and the price level, p_0 so that there is equilibrium in the balance of payments and the goods-and-services market. This does not now imply equilibrium in the entire

system because the level of central bank reserves may not be at the desired level. Suppose that reserves are below the desired level.

Consider first the movement of the system if the price level is fixed. From Q the rate of interest rises (because reserves are below the desired level) and capital is attracted from abroad. As the foreign balance improves reserves accumulate, reducing the discrepancy between the desired and actual level and slowing down the change in the rate of interest. But the rate of interest nevertheless continues to rise until stocks reach the required level. At a point such as S the rise in the rate of interest stops: reserves are at the desired level. But the point S cannot be an equilibrium point because there is a surplus in the foreign balance. Reserves begin to accumulate and the rate of interest is moved downward. Throughout the return path from S to Q the foreign balance is in surplus so reserves increase until the point Q is reached. At Q the discrepancy between actual and desired reserves reaches a maximum, and so does the speed with which the rate of interest is changing. The rate of interest now falls below Q and the excess reserves are gradually depleted until another point T is reached at which reserves are in equilibrium. The point T is below Q by as much as the point S is above Q . The cycle therefore continues in pure undamped harmonic motion with an amplitude of the interest rate fluctuation equal to ST . The motion exactly resembles that of a frictionless pendulum pivoted above Q .

This cycle applies only when the price level does not respond to disequilibrium in the market for goods and services. In other words I have thus far assumed a speed of response in the goods-and-services market equal to zero. But consider now the opposite case where the price level responds instantaneously (i.e., an infinite speed of response). In this case the initial rise in the rate of interest from Q causes deflationary pressure which induces an immediate fall in the price level. But above I have tentatively assumed that the price level does not affect the balance of payments (FF is horizontal) so the cycle of the interest rate, central bank stocks and the foreign balance goes on undisturbed. The only difference from the previous case studied is that a price cycle is added. The interest rate cycle induces the price cycle, but the price cycle does not, in turn, affect the interest rate cycle. The cycle is described in the graph by movements back and forth along the segment VW of the XX line. The amplitude of the interest rate fluctuation is the same as before, and the amplitude of the price level fluctuation is given by the horizontal distance between V and W .

I have now examined two extreme cases: one in which the price

level is unresponsive to excess demand in the goods-and-services market, and one in which it is instantaneously responsive. In the normal intermediate case the path of the initial departure from Q will fall somewhere between QS and QV . Eventually a point such as L is reached at which reserves are equal to the desired level. The slope of the path at L must be horizontal since the interest rate reaches a maximum at that point. From L the price level continues to fall (if L is to the right of XX); and, because of the balance-of-payments surplus and the gradual accumulation of excess reserves, the interest rate begins to fall. When the point K is reached the decline in the price level is reversed, but the interest rate continues to fall until the balance of payments has been in deficit long enough to bring reserves down to the desired level. At the point J reserves are again in equilibrium but, because the balance of payments is in deficit, the cycle continues. The path of the interest rate and the price level is indicated by the elliptical orbit; it is an undamped cycle with the same amplitude of the interest rate as before.

All the above systems have resulted in pure cycles of the interest rate, the level of foreign exchange reserves, the balance of payments, and, in the last two cases, the price level: these cycles were neither damped nor undamped. But once we relax the assumption that the price level has no effect on the foreign balance, the conservative motion of the system turns into unstable motion. When the foreign-balance has a positive tilt price changes react on the balance-of-payments schedule and subject the rate at which reserves are being accumulated or run down to sustained "shocks" which continually increase the amplitude of the fluctuations. The system therefore leads to ever-increasing cyclical movements of reserves, and also to the somewhat paradoxical conclusion that the central bank needs an *infinite* quantity of reserves to follow this policy designed to maintain *any* level of reserves!

I have not presented this system as an actual description of the policy followed by any central bank, nor even to suggest that the central bank should never be concerned with the *level* of foreign exchange reserves. It is more reasonable to suppose that the central bank governs its action according to both the level of reserves and the state of the current balance. If reserves are too low an interest rate high enough to develop a surplus in the current balance can be maintained, allowing reserves to build up gradually. Such a system is not necessarily unstable, as I have shown in the Appendix. What is important to notice, however, is that too great a concern for foreign exchange stocks *may* lead to instability. And because central banks

are prompted to act more vigorously when reserves are too low than when reserves are too high,² this means that an effective system of international payments based on fixed exchange rates must be one which provides a reasonably high degree of international liquidity. The social cost of these reserves — which is necessarily positive only when gold (or other commodities) are used as international currency — must then be weighed in the balance in considering the relative merits of fixed and flexible exchange rate systems.

VI. SPECULATION

The preceding analysis provides a useful introduction to an important kind of speculation inherent in the fixed exchange systems of today. The confidence that once prevailed in the permanence of the existing exchange parity no longer exists today. Indeed, permanence of this parity is not even an *espoused* policy of most major governments although it has become, in many cases, a *practical* policy. The decline of confidence means that the safety of capital values becomes a prime factor determining the international location of short-term capital. Fear of inconvertibility or devaluation often swamps the effects of small differences in rates of interest between money markets, and encourages capital outflows. But confidence is generally linked to the level of exchange reserves. Other things the same, confidence is higher the larger are the central bank holdings of foreign exchange: an increase in reserves makes a speculator more bullish with regard to the exchange value (or degree of convertibility) of a currency. The balance of payments therefore becomes a function of the level of exchange reserves with an improvement in the latter stimulating a capital inflow or restraining a capital outflow.

But is a system based on this type of speculative response stable? Intuition leads one to suspect that it may be stable or unstable depending on the strength of opposing forces. Speculators and the central bank engage in a fight for reserves. On the one hand, the central bank acts, through interest rate changes, to correct any foreign imbalance; on the other hand, speculators operate to exaggerate any imbalance because of the change in reserves and confidence.

To show that this view is substantially correct consider Figure IX and the initial disequilibrium *W*. The deficit in the balance of payments and the inflationary gap cause the interest rate and the price level to rise. But the deficit also lowers the level of exchange reserves and causes the foreign balance schedule itself to shift in a north-

2. Compare, for example, the British reaction to a deficit in the summer and fall of 1957, with the German reaction to a surplus in recent years.

westerly direction. There exists a different position of this schedule for every level of foreign exchange reserves. Now it may easily be seen that the stability of the system depends partly on how quickly the central bank reacts to a foreign deficit compared with the extent to which the FF curve is shifted because of speculation. For a new equilibrium to be established, the interest rate must move sufficiently fast to arrest (or reverse) the shift in the foreign balance schedule. A new equilibrium may, for example, be established at the point V .

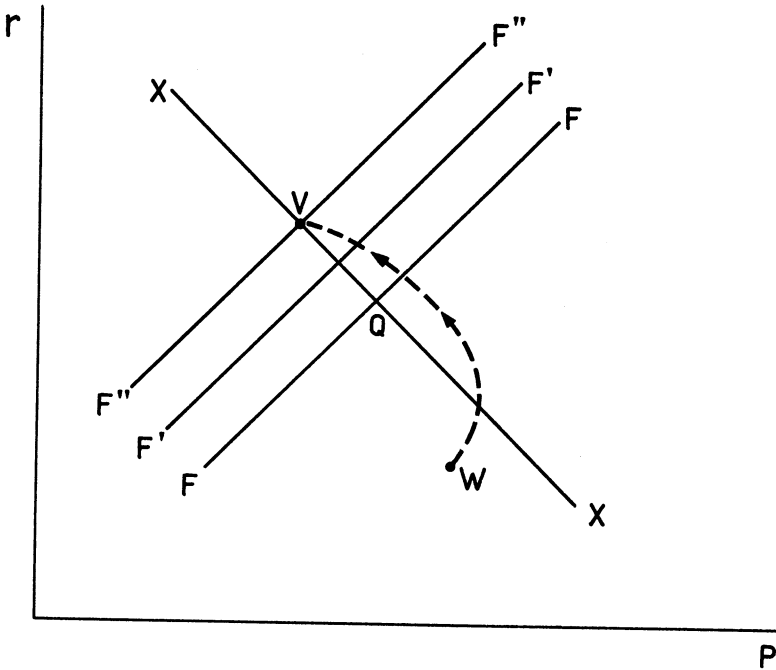


FIGURE IX

On the other hand, the interest rate may move too fast, turn the deficit into a surplus, and begin a downward path "chasing" the foreign balance schedule in the other direction. But in any case the system may be stable or unstable, cyclical or asymptotic, depending on the particular static parameters and speeds of response. It should be clear that a low sensitivity of the foreign balance to changes in reserves, a high sensitivity to the rate of interest, and relatively rapid responses of the central bank are stabilizing.³

3. The destabilizing nature of this type of speculation has been described by Friedman and Meade in the works already cited.

Among the more important factors influencing speculation under the fixed-

The precarious nature of the stability conditions of the above system — its sensitivity to speculative capital movements — is based on the reactions of speculators to changes in central bank stocks. It has no practical counterpart in a system of flexible exchange rates because central banks, in that system, do not need to hold foreign exchange reserves; and if they do hold reserves, the central banks may use them merely to even out what they consider “sporadic” fluctuations in the exchange rate.⁴

There may, however, be a type of speculation which applies to flexible, but not to fixed exchange systems. The greater ease, and lower storage costs, of buying and holding units of currency in the flexible exchange system, than of purchasing collections of goods in a fixed exchange system may induce more direct speculation in the former system. Speculators may interpret a current change in the exchange rate as a signal indicating further changes in the same direction, or as a signal indicating that the direction in which the rate is changing will be reversed. Proponents of flexible exchange rates have argued persuasively that, when the exchange rate appreciates or depreciates because of a change in import or export demand-and-supply schedules, speculators are likely to consider this a signal for an eventual movement of the exchange rate in the opposite direction, basing their expectations on the well-known tendency for elasticities to be greater in the long run than in the short run. The more conventional view, however, which was widespread after the war, urges that a change in the exchange rate is likely to be interpreted as a signal for further changes in the same direction because speculators may extrapolate past trends. This view is certainly confirmed during periods of monetary instability — during inflation the exchange rate would, in a free market, depreciate at the same rate as other prices are rising — but it is also irrelevant in this case: the main advantage of a flexible exchange system is that it permits central bank authorities to control price level fluctuations.⁵

exchange-rate system that I have not considered, may be included: (1) the extent of short-term liabilities and the so-called “flight-capital ratio”; (2) international credit facilities such as I. M. F. drawing rights; (3) the “lead-and-lag” effect on the trade balance; (4) the relation between spot and forward markets and the rate of interest; (5) international speculation in spot and forward commodity markets; and (6) the fact that much international speculation is of a once-for-all character. In spite of these obvious omissions the present model, outlined in the Appendix, is relatively complicated.

4. There is, however, an analytical parallel. The central bank or the government (through fiscal policy) must buy and sell goods at a fixed price in order to be completely successful in stabilizing the price level. There would be a direct analogy if speculation in commodity markets were conditioned by the level of commodity reserves of the government.

5. The relation between speculation and the stability of the foreign exchange

Nevertheless, it is easily proved that a system of flexible exchange rates may be stable even if speculators interpret a given variation in the exchange rate as a signal of further changes in the same direction: this is demonstrated in the Appendix. Furthermore, stability depends both on the behavior of speculators, and the speed with which the exchange rate responds to external imbalance. If, therefore, the system is unstable because of speculative capital movements, central bank intervention in the foreign exchange market can slow down the speed of the fluctuations and make the system stable. But this use of exchange equalization funds would not be necessary if speculators expected exchange rate changes to be reversed in direction, or if the extent of "destabilizing" speculation were not large.

VII. CONCLUDING REMARKS

Before concluding it may be useful to express in more general terms the problem initially posed. Society has two goals: full employment and balance-of-payments equilibrium. To achieve these goals it has two free variables: the terms of trade (the price level or the exchange rate) and the rate of interest (financial policy). The monetary authorities may stabilize the exchange rate, using financial policy to maintain external equilibrium and allowing the price level to maintain internal equilibrium; or it may stabilize the price level, using financial policy to maintain internal balance and allowing the exchange rate to preserve external balance. Which of these policies should be followed?

The answer was seen to depend on the values of the parameters and the speeds of adjustment. In the simple systems first studied, for example, it was demonstrated that the fixed-exchange-rate system operates most effectively if capital is highly mobile, while the flexible-exchange-rate system works best if capital is immobile. ("Best" is judged in terms of the directness of the approach toward equilibrium.) These conclusions have a useful application to economic policy. Equally interesting, however, is the general principle they illustrate.

The reason a high mobility of capital improves the effectiveness of the fixed-exchange-rate system is that the adjusting variable (the rate of interest) has a *direct* effect on the market to which it responds (the balance of payments); and the reason a low mobility of capital

market is discussed in M. Friedman (*op. cit.*); J. E. Meade (*op. cit.*), and "The Case for Variable Exchange Rates," *The Three Banks Review* (Sept. 1955); F. A. Lutz, "The Case for Flexible Exchange Rates," *Banca Nazionale del Lavoro* (Dec. 1954); W. J. Baumol, "Speculation, Profitability and Stability," *Review of Economics and Statistics*, XXXIX (Aug. 1957); S. C. Tsiang, "A Theory of Foreign Exchange Speculation Under a Floating Exchange System," *Journal of Political Economy*, LXVI (Oct. 1958).

may lead to cycles in this system is that the rate of interest can affect the balance of payments only through interaction with the goods-and-services market and the price level. On the other hand, the flexible exchange system works badly if capital is mobile because the rate of interest has a more direct effect on the balance of payments than on the market to which it responds (the goods-and-services market); and it works effectively if capital is immobile because this indirect repercussion is small or nil. In both these cases, it should be noticed, *a system works best if variables respond to the markets on which they exert the most direct influence.* It may be seen that this principle has a wider application to general problems in the theory of economic policy.⁶

In extreme cases this principle provides an unambiguous guide to "effective market classification": in addition to the example already cited one can consider varying responsivenesses of the goods-and-services market to changes in the rate of interest or the terms of trade and verify, for example, that a high responsiveness of this market to the rate of interest is conducive to the effective operation of the flexible-exchange-rate system, while a low responsiveness hinders its operation. In the less extreme cases a mixed system — where equilibrium conditions in both markets are considered before allowing any variable to adjust — may be theoretically preferable.⁷

6. The mathematical counterpart of this principle, which has obvious applications to a planned economy in which the "Ministry of Production" sets prices, is that variables should respond to markets in such a way as to make the diagonal elements of the characteristic matrix dominate. This, of course, may not always be possible.

7. Professor Nurkse has argued in "The Relation Between Home Investment and External Balance in the Light of British Experience, 1945-1955," *Review of Economics and Statistics*, XXXVIII (May 1956) that it is invalid to identify one instrument with a specific target (employing Tinbergen's terminology) since each instrument affects both markets: devaluation stimulates employment and improves the balance of trade, and changes in the interest rate affect the balance of payments in addition to the level of employment. In the mathematical system discussed in section (5) of the Appendix he is implicitly arguing that both the k 's and the h 's should be positive. This is the mixed system referred to above.

With perfect information, and international collaboration to prevent "beggary-neighbor" policies, this recommendation would be justified. In the absence of these conditions simpler rules would seem to be preferable.

The reader should notice that I have not examined the traditional argument for flexible exchange rates, which is based on the assumption of a rigid price level. This argument is formulated briefly in section (10) of the Appendix.

APPENDIX

1. Some of the propositions advanced in the text require more rigorous proof than was provided by the geometrical analysis. The purpose of this analytical appendix is to furnish these proofs.

2. Let X represent the excess demand for goods and services so that

$$X = \text{Investment} - \text{Saving} + \text{Trade Balance},$$

and let F be the balance-of-payments surplus so that

$$F = \text{Trade Balance} - \text{Capital Exports}.$$

Both X and F are assumed to depend on the domestic rate of interest, r , and the ratio of home and foreign price levels, p . The equilibrium conditions are therefore:

$$(1) \quad X(p, r) = 0 \quad (\text{goods-and-services market})$$

$$\text{and} \quad (2) \quad F(p, r) = 0 \quad (\text{foreign exchange market}).$$

By differentiation of (1) we can obtain

$$(3) \quad \left(\frac{dr}{dp} \right)_{X=0} = - \frac{X_p}{X_r} \quad (= \text{slope of the internal balance schedule})$$

and by differentiation of (2) we have

$$(4) \quad \left(\frac{dr}{dp} \right)_{F=0} = - \frac{F_p}{F_r} \quad (= \text{slope of the foreign balance schedule}).$$

The subscripts of X and F denote differentiation with respect to p and r .

It can normally be assumed that: $X_p < 0$ (appreciation of the exchange rate or an increase in the price level are deflationary in the sense that they lower the excess demand for goods); $X_r < 0$ (an increase in the interest rate is deflationary); $F_p < 0$ (appreciation or an increase in the price level worsens the balance of payments); and $F_r > 0$ (an increase in the interest rate improves the balance of payments). In the text I have made the simplifying assumption that the price level or the exchange rate affects only the balance of trade, and that the rate of interest affects only the rate of capital imports or exports.

3. The dynamic postulates¹ of the fixed-exchange system can be approximated by the following equations:

$$(5) \quad \frac{dp}{dt} = k_1 X(p, r)$$

1. The methods used in this appendix are based on the work of P. A. Samuelson, *Foundations of Economic Analysis*, Chaps. 9, 10, and Appendix B.

which states that the price level rises in proportion to the excess demand for goods and services; and

$$(6) \quad \frac{dr}{dt} = -k_2 F(p, r)$$

which states that the rate of interest rises and falls in proportion to the discrepancy between foreign exchange payments and receipts. The k 's indicate the speed of response in each market.

Expanding equations (5) and (6) in a Taylor series, and retaining only linear terms, we obtain:

$$(7) \quad \frac{dp}{dt} = k_1 X_p (p - p^\circ) + k_1 X_r (r - r^\circ)$$

$$(8) \quad \frac{dr}{dt} = k_2 F_p (p - p^\circ) + k_2 F_r (r - r^\circ)$$

where p° and r° refer to the price level and the rate of interest at equilibrium. This system has the characteristic equation

$$(9) \quad \begin{vmatrix} m - k_1 X_p & -k_1 X_r \\ k_2 F_p & m + k_2 F_r \end{vmatrix} \\ = m^2 - (k_1 X_p - k_2 F_r)m - k_1 k_2 (X_p F_r - X_r F_p) = 0$$

with latent roots

$$(10) \quad m_1, m_2 = \frac{k_1 X_p - k_2 F_r \pm \sqrt{(k_1 X_p + k_2 F_r)^2 - 4k_1 k_2 X_r F_p}}{2}$$

that, under the assumed signs, are negative if real and that have negative real parts if complex.

The system approaches equilibrium asymptotically or cyclically depending on whether the discriminant D is positive or negative. But $D > 0$ if F_r is very large (capital mobility); and $D \geq 0$ depending on whether k_2/k_1 (relative speeds of adjustment) is small or large, if $F_r = 0$ (capital immobility).²

2. Incidentally, this example illustrates a minor misconception which has developed in the theoretical literature on the stability of equilibrium. Lloyd Metzler in "The Stability of Multiple Markets," *Econometrica*, Vol. 13 (Oct. 1945) proved that the "Hicks conditions" are necessary, but not sufficient, if a system is to be stable for all possible speeds of adjustment: this theorem has been interpreted by Samuelson (*op. cit.*), p. 273, and by Arrow and Hurwicz, "On the Stability of the Competitive Equilibrium, I," *Econometrica*, Vol. 26 (Oct. 1958), to apply for *positive* speeds of adjustments. But any matrix with a pattern of signs

$$\begin{bmatrix} - & + \\ - & 0 \end{bmatrix} \quad (\text{the fixed exchange system with capital immobile})$$

is stable — yet the Hicks conditions are not satisfied. Metzler's theorem applies, as his proof indicates, to all possible *non-negative* speeds of adjustment.

4. The assumed dynamic postulates of the flexible exchange system are as follows:

$$(11) \quad \frac{dp}{dt} = h_1 F(p, r)$$

which states that the exchange rate rises and falls in proportion to the surplus or deficit in the balance of payments; and

$$(12) \quad \frac{dr}{dt} = h_2 X(p, r)$$

which states that the central bank raises or lowers the interest rate in proportion to the inflationary or deflationary gap in the goods-and-services market.

After linearizing equations (11) and (12) we find the latent roots of the characteristic equation:

$$(13) \quad m_1, m_2 = \frac{h_1 F_p + h_2 X_r \pm \sqrt{(h_1 F_p - h_2 X_r)^2 + 4h_1 h_2 F_r X_p}}{2}.$$

These roots are negative if real, and have negative real parts if complex, so the system is stable.

Note that if capital is immobile ($F_r = 0$) the discriminant is necessarily positive so the roots are *real*; the system leads directly (asymptotically) to equilibrium. On the other hand, if capital is highly mobile the roots are complex and the approach to equilibrium is oscillatory.

5. One can now consider a generalization of the above systems based on the following system of equations:

$$(14) \quad \frac{dp}{dt} = k_1 X(p, r) + h_1 F(p, r)$$

$$(15) \quad \frac{dr}{dt} = h_2 X(p, r) - k_2 F(p, r).$$

This is the fixed exchange rate system with h_1 and h_2 zero, and the flexible exchange rate case with k_1 and k_2 zero. For stability of the following linear system:

$$(16) \quad \frac{dp}{dt} = (k_1 X_p + h_1 F_p) (p - p^o) + k_1 X_r + k_1 F_r) (r - r^o)$$

$$(17) \quad \frac{dr}{dt} = (h_2 X_p - k_2 F_p) (p - p^o) + (h_2 X_r - k_2 F_r) (r - r^o)$$

the "trace" (sum of the diagonal coefficients) must be negative, and the basic determinant positive. Roughly, the larger are the absolute values of the diagonal elements, relative to the absolute values of the off-diagonal elements, the more direct will be the approach to equilibrium. This explains, in more general terms, the conclusion of the text that a high degree of capital mobility causes cycles in the flexible exchange system but not in the fixed exchange system; and that a low degree of capital mobility interferes with a direct approach to equilibrium under fixed exchange rates, and conduces to a direct approach under flexible exchange rates. The reader can easily demonstrate for himself other propositions relating to differences in the values of other parameters. It also helps to explain the "principle of effective market classification": instruments (i.e., variables) should be directed at those targets (i.e., markets) on which they have the most direct influence.

6. Consider now a system of fixed exchange rates in which the central bank bases its monetary policy on a discrepancy between the *desired* reserves of foreign exchange, Q^o , and *actual* reserves, $\int_0^t F(p, r) dt$ (past accumulations) according to the new system:

$$(18) \quad \frac{dp}{dt} = k_1 X(p, r)$$

$$(19) \quad \frac{dr}{dt} = b [Q^o - \int_0^t F(p, r) dt].$$

Equation (19) states that the interest rate moves in proportion to the difference between desired and actual reserves, the constant b indicating the speed of the reaction.

This system, linearized, has the following characteristic equation:

$$(20) \quad \begin{vmatrix} m - k_1 X_p & -k_1 X_r \\ bF_p & m^2 + bF_r \end{vmatrix} = 0.$$

By Descartes Rule, it has no positive real roots. It does, however, have complex roots with positive real parts so the system is unstable: the system moves further from equilibrium in an expanding spiral.

Notice that if the price level is fixed, the path of the interest rate is

$$(21) \quad r(t) = r^o + c e^{\sqrt{-F_r} t} + d e^{-\sqrt{-F_r} t} \\ = r^o + j \cos \sqrt{F_r} t + k \sin \sqrt{F_r} t,$$

an undamped harmonic. This also holds if F_p is zero. These results verify the step-by-step exposition in the text.

7. Suppose now that the central bank takes account of both the current balance and the level of stocks in the fixed exchange system. Then we have the following system:

$$(22) \quad \frac{dp}{dt} = k_1 X(p, r)$$

$$(23) \quad \frac{dr}{dt} = -k_2 F(p, r) + b [Q^0 - \int_0^t F(p, r) dt]$$

which has the characteristic equation

$$(24) \quad \begin{vmatrix} m - k_1 X_p & -k_1 X_r \\ k_2 F_p m + b F_p & m^2 + k_2 F_r m + b F_r \end{vmatrix} = 0$$

with no positive real roots. But it will have complex roots with positive real parts unless

$$k_1(F_p X_r - X_p F_r) [k_2/b (k_2 F_r - k_1 X_p) - 1] + F_r(k_2 F_r - k_1 X_p) > 0.$$

Notice that the lower is the weight attached to stocks (b small) and the higher is the degree of capital mobility (F_r large) the more likely it is that the system is stable.

8. Next, suppose that speculators react to a change in the level of central bank reserves (q), and that the central bank responds to the current balance. Then we have the system:

$$(25) \quad \frac{dp}{dt} = k_1 X(p, r)$$

$$(26) \quad \frac{dr}{dt} = -k_2 F(p, r, q) \text{ where } q = \int_0^t F(p, r, q) dt$$

which has a characteristic equation

$$(27) \quad m^3 + (k_2 F_r - F_q - k_1 X_p) m^2 + k_1 [k_2 (X_r F_p - X_p F_r) + X_p F_q] m = 0$$

where F_q is the change in the foreign balance which results from a change in the level of reserves, assumed to be positive. From (27) it is clear that high values of F_q and low values of F_r and k_2 are destabilizing.

9. Consider now speculation under flexible exchange rates. Suppose that speculative purchases of foreign exchange depend not

only on the exchange rate but also on its rate of change. Then we have the following system:

$$(28) \quad \frac{dp}{dt} = h_1 F\left(p, r, \frac{dp}{dt}\right)$$

$$(29) \quad \frac{dr}{dt} = h_2 X(p, r)$$

with the characteristic equation

$$(30) \quad m^2 - (aF_p + h_2 X_r)m + ah_2(F_p X_r - F_r X_p) = 0$$

in which $a = \frac{h_1}{1 - h_1 \eta}$; and η is the speculative outflow due to the

changing exchange rate (it is a variant of the coefficient of expectations). A necessary and sufficient condition for stability, given the assumed signs, is that $h_1 \eta < 1$. Note that $\eta < 0$ is not a necessary condition of stability: even if speculators believe the exchange rate will continue to move in the same direction as it is currently moving, the system may be stable.

10. The traditional argument for flexible exchange rates may be expressed briefly as follows:

- Let p = the domestic price level;
- y = the rate of domestic output;
- e = the price of a unit of domestic currency in terms of gold; and
- r = the domestic rate of interest.

The demand for goods and services and the balance of payments depends on all four variables. The equilibrium conditions are

$$X(p, e, y, r) = 0$$

$$F(p, e, y, r) = 0.$$

In this system there are two equations and four variables, so there are two degrees of freedom. But note that if a change in the exchange rate has the same effect on each market as an equivalent change in the price level it is not sufficient to allow both e and p to vary.

For the system to be consistent there must be two free variables (but only one of p, e , the terms of trade, can be counted). If the price level is rigid, because of institutional conditions, then either output or the exchange rate (besides the rate of interest) must be flexible. If the government stabilizes output (because of its full employment policy) then the exchange rate must be allowed to fluctuate. On the other hand, if the government stabilizes output *and* the exchange

rate one of the equilibrium conditions will be violated — either the goods-and-services market or the balance of payments will be in disequilibrium.

Trade controls or other policies could be introduced and this, it is true, would allow the authorities to stabilize both output and the exchange rate without conflicting with the equilibrium conditions. But these policies should either be directed at other goals or ruled out on grounds of inefficiency.

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