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THE MORDELL PROPERTY OF HYPERBOLIC FIBER SPACES WITH NONCOMPACT FIBERS

Dedicated to Professor Nobuyuki Suita on his sixtieth birthday

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Abstract. We show that under some boundary conditions a hyperbolic fiber space with noncompact complete hyperbolic fibers has at most finitely many essentially nontrivial sections.

Introduction. Let $\mathscr{X}$ and $\mathscr{B}$ be irreducible, reduced complex spaces and $p: \mathscr{X} \to \mathscr{B}$ a surjective holomorphic mapping with connected fibers. We call $(\mathscr{X}, p, \mathscr{B})$ a complex fiber space. We shall say that $(\mathscr{X}, p, \mathscr{B})$ is *Mordellic* or has the *Mordell property* if it has at most finitely many meromorphic sections (essentially nontrivial sections) except for the ones which come from trivial sections of meromorphically trivial fiber subspaces of $(\mathscr{X}, p, \mathscr{B})$ modulo base change. The term "meromorphically trivial" means that it is bimeromorphic to a trivial fiber subspace as fiber spaces. In 1974, Lang [11, p. 781] conjectured the following analog of Mordell's conjecture (cf. Manin [17] and Grauert [5]); an algebraic family of compact hyperbolic complex spaces is Mordellic. Noguchi [24, Theorem B] proved that if a hyperbolic fiber space $(\mathscr{X}, p, \mathscr{B})$ with compact fibers is hyperbolically imbedded into its compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ along $\partial\mathscr{B}$, then it is Mordellic. In this paper, we study a noncompact version of Noguchi's result. Let $\bar{\mathscr{X}}$ and $\bar{\mathscr{B}}$ be irreducible, reduced compact complex spaces and $\bar{p}: \bar{\mathscr{X}} \to \bar{\mathscr{B}}$ a surjective holomorphic mapping. Let $\mathscr{X}$ and $\mathscr{B}$ be Zariski open subsets of $\bar{\mathscr{X}}$ and $\bar{\mathscr{B}}$, respectively and $p=\bar{p}|\mathscr{X}$ the restriction over $\mathscr{X}$ which is a surjection from $\mathscr{X}$ to $\mathscr{B}$. We consider the fiber space $(\mathscr{X}, p, \mathscr{B})$ with hyperbolic fibers, which we call a hyperbolic fiber space. Let $\partial_h\mathscr{X}$ be the union of all irreducible components of $\partial\mathscr{X} = \bar{\mathscr{X}} \setminus \mathscr{X}$ which are not contained in $\bar{p}^{-1}(\partial\mathscr{B})$ where $\partial\mathscr{B} = \bar{\mathscr{B}} \setminus \mathscr{B}$.

MAIN THEOREM. *Let $(\mathscr{X}, p, \mathscr{B})$ and $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ be as above. Assume that $(\mathscr{X}, p, \mathscr{B})$ is hyperbolically imbedded in $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ along the fibers and that $\bar{\mathscr{X}} \setminus \partial_h\mathscr{X}$ is locally complete hyperbolic in $\bar{\mathscr{X}}$. Then $(\mathscr{X}, p, \mathscr{B})$ is Mordellic.*

See Definition 1.1 in §1 for the assumptions. This extend the result in the case of trivial fiber spaces obtained in [28].

Some higher dimensional cases were considered by Riebesehl [27], Noguchi [20],

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[21], [24], Parshin [26] and Maehara [16]. Noncompact variants for a family of curves were treated by Imayoshi and Shiga [6] in the case where fibers are Riemann surfaces of fixed finite type with genus $g \geq 2$, by Zaidenberg [31] in the case of compactifiable families with hyperbolic fibers and by Browder and Yamaguchi [3] in the case of topologically trivial and locally Stein fiber spaces over the complex plane. For more complete references see the above cited references and the surveys [11], [12], [32] and [19].

We prove the main theorem by making use of the method established by Noguchi [21], which was also used in Zaidenberg [31]. After some reductions (§1), we prove the relative compactness of the space of sections (§2). Then we show that it has a universal complex structure (§3). Constructing the fiber subspace by the evaluation mapping, we see that it turns out to be trivial (§4). Finally, we give a remark on the assumptions of the main theorem (§6).

In this paper, complex spaces are supposed to be paracompact and reduced. The word "hyperbolic" is taken in the sense of Kobayashi (cf. [9] see also [13] and [25] for general reference).

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1. Preliminaries and some reductions. Let $\mathscr{X}$ and $\mathscr{B}$ be irreducible complex spaces and $p: \mathscr{X} \to \mathscr{B}$ a surjective holomorphic mapping such that general fibers $\mathscr{X}_t = p^{-1}(t)$ with $t \in \mathscr{B}$ are irreducible complex subspaces of $\mathscr{X}$. We call the triple $(\mathscr{X}, p, \mathscr{B})$ a fiber space. If $\mathscr{X}_t$ is hyperbolic for all $t \in \mathscr{B}$, we call $(\mathscr{X}, p, \mathscr{B})$ a *hyperbolic fiber space*. We treat the case of a hyperbolic fiber space with noncompact fibers which has a "good" compactification in the following sense. Let $\bar{\mathscr{X}}$ and $\bar{\mathscr{B}}$ be irreducible compact complex spaces and $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ be a compact fiber space; that is, the total space is compact. We say that $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ is a compactification of the fiber space $(\mathscr{X}, p, \mathscr{B})$, if $\mathscr{X}$ and $\mathscr{B}$ are Zariski open subsets of $\bar{\mathscr{X}}$ and $\bar{\mathscr{B}}$, respectively, and $p = \bar{p} | \mathscr{X}$. Put $\partial \mathscr{X} = \bar{\mathscr{X}} \setminus \mathscr{X}$ and $\partial \mathscr{B} = \bar{\mathscr{B}} \setminus \mathscr{B}$. Let $\partial_h \mathscr{X}$ be the union of all irreducible components of $\partial \mathscr{X}$ which are not contained in $\bar{p}^{-1}(\partial \mathscr{B})$, which is called the *horizontal boundary* of $\mathscr{X}$.

DEFINITION 1.1. (i) Let $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ be a compactification of a fiber space $(\mathscr{X}, p, \mathscr{B})$. We say that the fiber space $(\mathscr{X}, p, \mathscr{B})$ is *hyperbolically imbedded in* $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ *along the fibers*, if for any $b \in \bar{\mathscr{B}}$, there is a neighborhood N of $\bar{p}^{-1}(b) \cap \partial \mathscr{X}$ such that $N \setminus \partial \mathscr{X}$ is hyperbolically imbedded into $\bar{\mathscr{X}}$.

(ii) We say that a relatively compact subspace $\mathscr{Y}$ of a complex space $\mathscr{W}$ is *locally complete hyperbolic in* $\mathscr{W}$ (cf. Kiernan [8, p. 205] or Lang [12, p. 35]), if for any y in the closure of $\mathscr{Y}$ in $\mathscr{W}$, there is a neighborhood U of y in $\mathscr{W}$ such that $U \cap \mathscr{Y}$ is complete hyperbolic.

If all generic fibers are compact hyperbolic, the notion of hyperbolic imbeddedness along the fibers is nothing but the one of hyperbolic imbeddedness along $\partial\mathscr{B}$, which was introduced by Noguchi [21, Definition 1.3 and Corollary 1.8].

We now consider a hyperbolic fiber space $(\mathscr{X}, p, \mathscr{B})$ which has a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ satisfying the following two conditions:

(LC) $\bar{\mathscr{X}} \setminus \partial_h \mathscr{X}$ is locally complete hyperbolic in $\bar{\mathscr{X}}$,

(HI) $(\mathscr{X}, p, \mathscr{B})$ is hyperbolically imbedded in $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ along the fibers.

PROPOSITION 1.2. *Let $(\mathscr{X}, p, \mathscr{B})$ be a hyperbolic fiber space which has a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ satisfying the conditions* (LC) *and* (HI). *Then for any $t \in \mathscr{B}$ there is a neighborhood U of t such that $\mathscr{X}_U := p^{-1}(U)$ is complete hyperbolic and that $\mathscr{X}_U$ is hyperbolically imbedded into $\bar{\mathscr{X}}_U = \bar{p}^{-1}(U)$. In particular, $\mathscr{X}_t$ is complete hyperbolic and is hyperbolically imbedded into $\bar{\mathscr{X}}_t$.*

PROOF. In view of the condition (HI), take a neighborhood N of $\bar{p}^{-1}(t) \cap \partial\mathscr{X}$ such that $N \setminus \partial\mathscr{X}$ is hyperbolically imbedded into N. Let U be a neighborhood of t which is relatively compact in $\bar{p}(N)$ and complete hyperbolic. Then by the hyperbolicity of U, we see that $\mathscr{X}_U = p^{-1}(U)$ is Brody hyperbolic (see the definition after Remark 1.3 below). By the condition (LC), $\mathscr{X}_U$ is locally complete hyperbolic in $\bar{\mathscr{X}}_U = \bar{p}^{-1}(U)$. Moreover, $\partial\mathscr{X}_U := \bar{\mathscr{X}}_U - \mathscr{X}_U$ contains no limiting entire holomorphic curve with respect to $\mathscr{X}_U$ since the restriction of $\bar{p}$ mapping to U exists. The result follows from the theorem of Zaidenberg [30, Theorem 2.1]. ∎

REMARK 1.3. Note that even if each fiber $\mathscr{X}_t$ is complete hyperbolic and is hyperbolically imbedded into $\bar{\mathscr{X}}_t$, the result of Proposition 1.2 need not hold. This is related with the problem of stability of these properties (cf. §6).

In general, it is difficult to know whether a given complex space is hyperbolic or not. Under the conditions (LC) and (HI), we can replace the hyperbolicity condition of the fibers by the following condition, which seems to be more practical. A reduced complex space $\mathscr{Y}$ is said to be *Brody hyperbolic*, if $\mathscr{Y}$ contains no nonconstant entire holomorphic curves, i.e., there are no nonconstant holomorphic mappings from $\boldsymbol{C}$ to $\mathscr{Y}$.

PROPOSITION 1.4. *Let $(\mathscr{X}, p, \mathscr{B})$ be a fiber space whose fibers are Brody hyperbolic. Suppose that $(\mathscr{X}, p, \mathscr{B})$ has a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ satisfying the conditions* (LC) *and* (HI). *Then the same conclusion as in Proposition* 1.2 *holds.*

The proof is the same as that of Proposition 1.2.

In our situation $\mathscr{B}$ may have singularities. Then put

$$\mathscr{B}' := \bar{\mathscr{B}} - (\operatorname{Sing} \bar{\mathscr{B}} \cup \partial\mathscr{B}), \quad \partial\mathscr{B}' := \bar{\mathscr{B}} - \mathscr{B}' = \partial\mathscr{B} \cup \operatorname{Sing} \bar{\mathscr{B}}, \quad \mathscr{X}' := p^{-1}(\mathscr{B}'),$$

and $p' := \bar{p} | \mathscr{X}'$, where $\operatorname{Sing} \bar{\mathscr{B}}$ denotes the singularity of $\bar{\mathscr{B}}$. The fiber space $(\mathscr{X}', p', \mathscr{B}')$ is a hyperbolic fiber space which has a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ satisfying the conditions (LC) and (HI). Moreover, take a resolution $\bar{\mathscr{R}}$ of $\bar{\mathscr{B}}$, $\alpha : \bar{\mathscr{R}} \to \bar{\mathscr{B}}$ with $\mathscr{R} := \alpha^{-1}(\mathscr{B}')$ such that $\partial\mathscr{R} := \bar{\mathscr{R}} \setminus \mathscr{R}$ is a hypersurface with only normal crossings. Put

$$\bar{\mathscr{W}} := \bar{\mathscr{R}} \times_{\bar{\mathscr{B}}} \bar{\mathscr{X}}, \quad \mathscr{W} := \mathscr{R} \times_{\mathscr{B}'} \mathscr{X}', \quad \partial\mathscr{W} := \bar{\mathscr{W}} \setminus \mathscr{W}.$$

Let $\bar{\Pi} : \bar{\mathscr{W}} \to \bar{\mathscr{R}}$ and $\Pi := \bar{\Pi}|_{\mathscr{W}}$ be the natural projections. Let $\partial_h\mathscr{W}$ be the union of all irreducible components of $\partial\mathscr{W}$ not contained in $\bar{\Pi}^{-1}(\partial\mathscr{R})$. Let α^* be the holomorphic bimeromorphic mapping such that the following diagram commutes:

$$\begin{array}{ccccccccc} \mathscr{W} & \subset & \bar{\mathscr{W}} & \xrightarrow{\alpha^*} & \bar{\mathscr{X}} & \supset & \mathscr{X} & \supset & \mathscr{X}' \\ {\scriptstyle \Pi}\downarrow & & {\scriptstyle \bar{\Pi}}\downarrow & & \downarrow{\scriptstyle \bar{p}} & & \downarrow{\scriptstyle p} & & \downarrow{\scriptstyle p'} \\ \mathscr{R} & \subset & \bar{\mathscr{R}} & \xrightarrow{\alpha} & \bar{\mathscr{B}} & \supset & \mathscr{B} & \supset & \mathscr{B}' \end{array} .$$

Then the compact fiber space $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$ is a compactification of the fiber space $(\mathscr{W}, \Pi, \mathscr{R})$.

PROPOSITION 1.5. *The fiber spaces* $(\mathscr{W}, \Pi, \mathscr{R})$ *and* $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$ *satisfy the conditions* (LC) *and* (HI).

PROOF. They satisfy the condition (LC) as a result of the following, which is a direct consequence of the distance decreasing principle of the Kobayashi distance: Let $\mathscr{M}$ and $\mathscr{N}$ be Zariski open subsets of compact irreducible complex varieties $\bar{\mathscr{M}}$ and $\bar{\mathscr{N}}$, respectively. Suppose that there is a surjective holomorphic mapping $F: \bar{\mathscr{M}} \to \bar{\mathscr{N}}$ with $\partial\mathscr{M} = F^{-1}(\partial\mathscr{N})$. Then if $\mathscr{N}$ is locally complete hyperbolic, then so is $\mathscr{M}$.

Note that the condition (HI) is equivalent to the following condition: for any point $t \in \bar{\mathscr{R}}$, there is a neighborhood U of t in $\bar{\mathscr{R}}$ such that for any distinct points w_1 and w_2 in $\bar{\mathscr{W}}_t = \bar{\Pi}^{-1}(t)$ there are neighborhoods $V_i \subset \bar{\mathscr{W}}_U = \bar{\Pi}^{-1}(U)$ of w_i, $i=1, 2$ such that $d_{\mathscr{W}_{U-\partial\mathscr{R}}}(V_1 \cap \mathscr{W}, V_2 \cap \mathscr{W}) > 0$, where

$$d_{\mathscr{W}_{U-\partial\mathscr{R}}}(V_1 \cap \mathscr{W}, V_2 \cap \mathscr{W}) := \inf\{d_{\mathscr{W}_{U-\partial\mathscr{R}}}(x_1, x_2) : x_i \in V_i \cap \mathscr{W}, i=1, 2\}$$

and $d_{\mathscr{W}_{U-\partial\mathscr{R}}}(x_1, x_2)$ denotes the Kobayashi distance on $\mathscr{W}_{U-\partial\mathscr{R}}$. Then the conclusion also follows from the distance decreasing principle of the Kobayashi distance. ■

2. Topological structure of the spaces of sections. We use the same notation as in the previous section. We consider the relation between the topologies of the spaces of sections of the fiber spaces. We denote by $\Gamma(\circ, \bullet)$ the space of all holomorphic sections of a fiber space ($\bullet$, projection, $\circ$) endowed with the relative topology of the compact open topology of the space of all holomorphic mappings from $\circ$ to $\bullet$.

LEMMA 2.1. *A holomorphic section of* $(\mathscr{W}, \Pi, \mathscr{R})$ *is extended to a holomorphic section of* $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$.

PROOF. We may localize the situation at points of $\partial\mathscr{R}$. By the condition (HI), we take a neighborhood U of $t \in \partial\mathscr{R}$ such that $\mathscr{W}_U$ is hyperbolically imbedded into $\bar{\mathscr{W}}_U$. Then it follows from the well-known Picard-type extension theorem (cf. Kobayashi [9], Kiernan [7] and Noguchi [21, Lemma 2.1]). ■

By the above lemma, $\Gamma(\mathscr{R}, \mathscr{W})$ is regarded as a subset of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. We denote the holomorphic extension of $s\in\Gamma(\mathscr{R}, \mathscr{W})$ by $\bar{s}$.

PROPOSITION 2.2. *The set* $\{\bar{s} : s\in\Gamma(\mathscr{R}, \mathscr{W})\}\subset\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ *forms a normal family.*

This follows from the following lemma, which is a direct consequence of the extension and convergence theorem of Noguchi [22, Theorem 1.19]:

LEMMA 2.3. *If a sequence* $\{s_\nu\}_{\nu=1}^{\infty}$ *in* $\Gamma(\mathscr{R}, \mathscr{W})$ *converges uniformly to a holomorphic sections* $s\in\Gamma(\mathscr{R}, \bar{\mathscr{W}}_{\mathscr{R}})$ *on compact subsets of* $\mathscr{R}$, *then the sequence* $\{\bar{s}_\nu\}_{\nu=1}^{\infty}$ *of their holomorphic extensions converges uniformly to the holomorphic extension* $\bar{s}\in\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ *on* $\bar{\mathscr{R}}$.

THEOREM 2.4. $\Gamma(\mathscr{R}, \mathscr{W})$ *is a relatively compact topological subspace of* $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$.

PROOF. By Lemmas 2.1 and 2.3, $\Gamma(\mathscr{R}, \mathscr{W})$ is a topological subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. Let h be a Hermitian metric on $\bar{\mathscr{W}}$. Let $\{s_\nu\}$ be a sequence in $\Gamma(\mathscr{R}, \mathscr{W})$. By Proposition 1.2, for any $t\in\mathscr{R}$ there is a neighborhood U of t in $\mathscr{R}$ such that $\mathscr{W}_U$ is complete hyperbolic and is hyperbolically imbedded into $\bar{\mathscr{W}}_U$. There is a positive constant C_U such that $\sqrt{h}\le C_U F_{\mathscr{W}_U}$ on $\mathscr{W}_U$, where $F_\bullet$ denotes the Kobayashi differential metric on the space $\bullet$. Since $s_\nu^* F_{\mathscr{W}_U}\le F_U$, it follows that

$$\sqrt{s_\nu^* h}\le C_U F_U\,.$$

From this we see that $\{s_\nu\}$ are equicontinuous on U. By the standard argument, we obtain a subsequence of $\{s_\nu\}$ which converges uniformly to a holomorphic section in $\Gamma(\mathscr{R}, \bar{\mathscr{W}}_{\mathscr{R}})$ on compact subsets of $\mathscr{R}$. Thus relative compactness follows from Lemma 2.3. ■

3. The universal complex structure of the space of holomorphic sections. We denote by $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$ the space of all holomorphic mappings from a complex space $\mathscr{M}$ to a complex space $\mathscr{N}$ equipped with compact open topology. If the domain $\mathscr{M}$ is compact, by Douady's theory [4] $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$ has a universal complex structure whose topology coincides with the underlying compact open topology. Let $(\mathscr{W}, \Pi, \mathscr{R})$ and $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$ be as in §1. Since the mapping

$$\bar{\Pi}^* : f\in\mathrm{Hol}(\bar{\mathscr{R}}, \bar{\mathscr{W}})\to\bar{\Pi}\circ f\in\mathrm{Hol}(\bar{\mathscr{R}}, \bar{\mathscr{R}})$$

is holomorphic, $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})=\bar{\Pi}^{*-1}(\mathrm{id}_{\bar{\mathscr{R}}})$ is a complex subspace of $\mathrm{Hol}(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. By the universality of the complex structure of $\mathrm{Hol}(\bar{\Pi}(\partial\mathscr{W}), \bar{\mathscr{W}})$, $\mathrm{Hol}(\bar{\Pi}(\partial\mathscr{W}), \partial\mathscr{W})$ has a universal complex structure as a complex subspace of $\mathrm{Hol}(\bar{\Pi}(\partial\mathscr{W}), \bar{\mathscr{W}})$. Let $\partial\mathscr{W}=\bigcup_j\bar{\mathscr{V}}_j$ be the irreducible decomposition. Put

$$\mathscr{E}:=\partial_h\mathscr{W}\cap\bar{\mathscr{W}}_{\partial\mathscr{R}}=\bigcup_j\{\bar{\mathscr{V}}_j\cap\bar{\mathscr{W}}_{\partial\mathscr{R}} : \mathscr{V}_j\cap\bar{\mathscr{W}}_t\ne\emptyset \text{ for some } t\in\mathscr{R}\}$$

and $\partial\mathscr{R}=\bigcup_i\mathscr{A}_i$, where $\mathscr{A}_i$ is an irreducible hypersurface of $\bar{\mathscr{R}}$ and $\bar{\mathscr{W}}_{\partial\mathscr{R}}=\bar{\Pi}^{-1}(\partial\mathscr{R})$. By the universality of the induced complex structure, $\Gamma(\mathscr{A}_i, \mathscr{E} \cap \bar{\Pi}^{-1}(\mathscr{A}_i))$ is a complex subspace of $\Gamma(\mathscr{A}_i, \bar{\mathscr{W}}_{\mathscr{A}_i})$. The restriction mapping $\rho_i : \bar{s} \in \Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}}) \to \bar{s}|_{\mathscr{A}_i} \in \Gamma(\mathscr{A}_i, \bar{\mathscr{W}}_{\mathscr{A}_i})$ is holomorphic for each i. Thus if $\mathscr{A}_i \subset \bar{\Pi}(\mathscr{E})$, then $\rho_i^{-1}(\Gamma(\mathscr{A}_i, \mathscr{E} \cap \bar{\Pi}^{-1}(\mathscr{A}_i)))$ is a complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. Put

$$\Xi := \bigcup \{\rho_i^{-1}(\Gamma(\mathscr{A}_i, \mathscr{E} \cap \bar{\Pi}^{-1}(\mathscr{A}_i))) : \mathscr{A}_i \subset \bar{\Pi}(\mathscr{E})\} ,$$

which is a complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. Put $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})' := \Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}}) \setminus \Xi$ and take any connected component Z of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})'$. Then Z is a Zariski open subset of its closure in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$. Put $Z' := Z \cap \Gamma(\mathscr{R}, \mathscr{W})$.

Lemma 3.1. *The closure $\bar{Z}'$ of Z' in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ is a compact complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$, and Z' is a Zariski open subset of $\bar{Z}'$.*

Proof. Suppose that $Z' \neq \varnothing$. Let s be in Z'. By the condition (LC) in §1, for each $w \in \bar{\mathscr{W}}_{\partial\mathscr{R}} \cap \partial_h \mathscr{W}$ there exist a neighborhood N of w in $\bar{\mathscr{W}}$ and a neighborhood U of $\bar{\Pi}(w)$ such that $\bar{\mathscr{W}}_U \cap N$ is complete hyperbolic. By the extension theorem (cf. Kobayashi [9, Ch. VII, Theorem 3.4]), the set $s^{-1}(\partial_h \mathscr{W})$ is empty or a hypersurface of $\bar{\mathscr{R}}$, which is contained in $\partial\mathscr{R}$. If not empty, then $s(\mathscr{A}_i) \subset \mathscr{E}$ for some i. Thus $s \in \rho_i^{-1}(\Gamma(\mathscr{A}_i, \mathscr{E} \cap \bar{\Pi}^{-1}(\mathscr{A}_i)))$. This is absurd since s is in Z. Hence $s^{-1}(\partial_h \mathscr{W}) = \varnothing$. We have

$$s \in Z'' := \{s \in Z' \,;\, s(\partial\mathscr{R}) \subset \bar{\mathscr{W}}_{\partial\mathscr{R}} \setminus \mathscr{E}\} ,$$

i.e., $Z' = Z''$.

Now, it is clear that Z'' is open in Z. Moreover, we claim that Z'' is closed in Z. Indeed, let $\{s_\nu\}$ be a sequence in Z'' such that s_ν converges to some $s \in Z$. Take a point t_0 in $\mathscr{R}$ such that $s(t_0) \in \mathscr{W}$. According to Proposition 1.2, take a neighborhood U of t_0 such that $\mathscr{W}_U$ is complete hyperbolic. Then we have

$$\begin{aligned} d_{\mathscr{W}_U}(s_\nu(t), s(t_0)) &\leq d_{\mathscr{W}_U}(s_\nu(t), s_\nu(t_0)) + d_{\mathscr{W}_U}(s_\nu(t_0), s(t_0)) \\ &\leq d_U(t, t_0) + d_{\mathscr{W}_{t_0}}(s_\nu(t_0), s(t_0)) \end{aligned}$$

for $t \in U$, where each $d_\bullet$ denotes the Kobayashi distance function of the complex space $\bullet$. By the complete hyperbolicity of $\mathscr{W}_U$ we have $s(t) \in \mathscr{W}$. Since $\mathscr{R}$ is connected, we have $s \in \Gamma(\mathscr{R}, \mathscr{W})$, thus $s \in Z''$. Therefore, $Z' = Z'' = Z$ and the compactness of $\bar{Z}$ follows from Theorem 2.4. ■

Next we consider the remaining part

$$\Gamma(\mathscr{R}, \mathscr{W}) \cap \Xi = \{s \in \Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}}) \,;\, \operatorname{Supp} s^{-1}(\partial_h \mathscr{W}) \subset \partial\mathscr{R}\} .$$

Put $\Xi' := \Xi \setminus \Gamma(\bar{\mathscr{R}}, \partial\mathscr{W})$. Let Z be a connected component of Ξ'. Then the closure $\bar{Z}$ of Z in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ is a complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ and Z is a Zariski open subset of $\bar{Z}$. Put $Z' := Z \cap \Gamma(\mathscr{R}, \mathscr{W})$.

Lemma 3.2. *The closure $\bar{Z}'$ of Z' in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ is a compact complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ and Z' is a Zariski open subset of $\bar{Z}'$.*

The proof is a slight modification of the proof of the structure theorem of Noguchi [22, Theorem 2.8]. We restate the theorem in the form suitable for our purpose. Let $\mathscr{M}$ be a connected, compact complex manifold, $\mathscr{N}$ a compact irreducible complex space, V a hypersurface in $\mathscr{M}$ and K a proper subvariety of $\mathscr{N}$. Then $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$ has a universal complex structure and $\mathrm{Hol}(\mathscr{M}, K)$ is a complex subspace of $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$. Let Σ be a closed complex subvariety of $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$. Put $\Sigma' := \Sigma - \mathrm{Hol}(\mathscr{M}, K)$. Take any connected component Z of Σ'. Z is a Zariski open subset of the closure $\bar{Z}$ of Z in $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$, which is a complex subvariety.

THEOREM 3.3. *Suppose that* $\mathscr{N} \setminus K$ *is locally complete in* $\overline{\mathscr{N} \setminus K} = \mathscr{N}$. *If* $Z \cap \{f \in \mathrm{Hol}(\mathscr{M}, \mathscr{N}) ; f^{-1}(K) \subset V\}$ *is relatively compact in* $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$, *then it is a Zariski open subset of its closure in* $\mathrm{Hol}(\mathscr{M}, \mathscr{N})$, *which is a compact complex subspace.*

Note that under the assumption above, for any $f \in \mathrm{Hol}(\mathscr{M}, \mathscr{N})$, $f^{-1}(K)$ is an empty set, a hypersurface in $\mathscr{M}$ or $\mathscr{M}$ itself by the extension theorem (cf. Kobayashi [9, Ch. VII, Theorem 3.4]). Lemma 3.2 follows from Theorem 3.3, since the relative compactness is guaranteed by Theorem 2.4.

4. A triviality theorem. We now state a triviality theorem in somewhat general setting and give a sketch of a proof for the sake of convenience.

THEOREM 4.1 (cf. [21, Proof of Main Theorem (3.2)] and [28, Theorem 4.1]). *Let* $(\mathscr{W}', \Pi', \mathscr{R})$ *be a fiber space with a compactification* $(\bar{\mathscr{W}}', \bar{\Pi}', \bar{\mathscr{R}})$ *such that all fibers* $\mathscr{W}'_t = (\Pi')^{-1}(t)$ *are complete hyperbolic and hyperbolically imbedded into* $\bar{\mathscr{W}}'_t = (\bar{\Pi}')^{-1}(t)$ *for* $t \in \mathscr{R}$. *Let* $\bar{Z}'$ *be an irreducible compact complex subspace of* $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}}')$ *and* Z' *a Zariski open subset of* $\bar{Z}'$. *Supoose that the restriction* $\bar{\Psi} : \bar{Z}' \times \bar{\mathscr{R}} \to \bar{\mathscr{W}}'$ *of the evaluation mapping from* $\mathrm{Hol}(\bar{\mathscr{R}}, \bar{\mathscr{W}}') \times \bar{\mathscr{R}}$ *to* $\bar{\mathscr{W}}'$ *is surjective holomorphic and that the restriction* Ψ *of* $\bar{\Psi}$ *to* $Z' \times \mathscr{R}$ *is a surjection to* $\mathscr{W}'$. *Then* $(\mathscr{W}', \Pi', \mathscr{R})$ *and* $(\bar{\mathscr{W}}', \bar{\Pi}', \bar{\mathscr{R}})$ *are meromorphically trivial, and in fact, their normalizations are holomorphically trivial.*

PROOF. Let $\beta_1 : \bar{\mathscr{W}}'^{\natural} \to \bar{\mathscr{W}}'$ and $\beta_2 : \bar{Z}'^{\natural} \to \bar{Z}'$ be the normalizations. Let $\bar{\Psi}^{\natural}$ be the lifting of $\bar{\Psi}$ to the normalizations; i.e., the following diagram comutes:

$$\begin{array}{ccc} \bar{Z}'^{\natural} \times \bar{\mathscr{R}} & \xrightarrow{\bar{\Psi}^{\natural}} & \bar{\mathscr{W}}'^{\natural} \\ {\scriptstyle \beta_2}\downarrow & & \downarrow{\scriptstyle \beta_1} \\ \bar{Z}' \times \bar{\mathscr{R}} & \xrightarrow{\bar{\Psi}} & \bar{\mathscr{W}}'. \end{array}$$

Put

$$\mathscr{W}'^{\natural} := \beta_1^{-1}(\mathscr{W}'), \quad \bar{\Pi}'^{\natural} := \bar{\Pi}' \circ \beta_1, \quad \Pi'^{\natural} := \bar{\Pi}'^{\natural}|_{\mathscr{W}'^{\natural}}$$

and $Z'^{\natural} := \beta_2^{-1}(Z')$. Then we have $\mathscr{W}'^{\natural} = \bar{\Psi}^{\natural}(Z'^{\natural} \times \mathscr{R})$. It follows from the distance decreasing principle of the Kobayashi distance that $(\mathscr{W}'^{\natural}, \Pi'^{\natural}, \mathscr{R})$ is a hyperbolic fiber space with a compactification $(\bar{\mathscr{W}}'^{\natural}, \bar{\Pi}'^{\natural}, \bar{\mathscr{R}})$ such that all the fibers $\mathscr{W}_t'^{\natural} := \Pi'^{\natural -1}(t)$ are complete hyperbolic and hyperbolically imbedded into $\bar{\mathscr{W}}_t'^{\natural} := \bar{\Pi}'^{\natural -1}(t)$ for $t \in \mathscr{R}$. Note that since $\bar{Z}'^{\natural}$ is irreducible, $\bar{\mathscr{W}}_t'^{\natural}$ is irreducible. Put

$$\hat{\mathscr{W}}'^{\natural} := \bar{\Psi}^{\natural}(Z'^{\natural} \times \bar{\mathscr{R}}) \quad \text{and} \quad \hat{\Pi}'^{\natural} := \bar{\Pi}'^{\natural}|_{\hat{\mathscr{W}}'^{\natural}} .$$

Then we can construct a holomorphic horizontal direction field in the fiber space $(\bar{\mathscr{W}}'^{\natural}, \bar{\Pi}'^{\natural}, \bar{\mathscr{R}})$ as in the proof of Main Theorem 3.2 of Noguchi (cf. Noguchi [21, p. 37]) since $\bar{\mathscr{W}}'^{\natural}$ is normal. From the way of the construction of the field, we see that the restriction of the field to $\hat{\mathscr{W}}'^{\natural}$ gives a holomorphic horizontal direction field in the fiber space $(\hat{\mathscr{W}}'^{\natural}, \hat{\Pi}'^{\natural}, \bar{\mathscr{R}})$. Thus the fiber space $(\bar{\mathscr{W}}'^{\natural}, \bar{\Pi}'^{\natural}, \bar{\mathscr{R}})$ is in fact a holomorphic fiber bundle with typical fiber $\bar{\mathscr{W}}_0'^{\natural}$ and $(\hat{\mathscr{W}}'^{\natural}, \hat{\Pi}'^{\natural}, \bar{\mathscr{R}})$ is a fiber subbundle with typical fiber $\hat{\mathscr{W}}_0'^{\natural}$. Note also that $\hat{\mathscr{W}}_0'^{\natural}$ is complete hyperbolic and hyperbolically imbedded into $\bar{\mathscr{W}}_0'^{\natural}$. For a sufficiently small neighborhood U of $t \in \bar{\mathscr{R}}$, there are local trivializations

$$\bar{\mathscr{W}}_U'^{\natural} \cong \bar{\mathscr{W}}_0'^{\natural} \times U \quad \text{and} \quad \hat{\mathscr{W}}_U'^{\natural} \cong \hat{\mathscr{W}}_0'^{\natural} \times U .$$

Since $\bar{\mathscr{W}}_U'^{\natural}$ is normal, so are $\bar{\mathscr{W}}_0'^{\natural}$ and $\hat{\mathscr{W}}_0'^{\natural}$. Through the above local trivializations, we obtain the surjective holomorphic mappings

$$\bar{Z}'^{\natural} \times U \ni (z, t) \to (\bar{\psi}(z, t), t) \in \bar{\mathscr{W}}_0'^{\natural} \times U$$

and

$$Z'^{\natural} \times U \ni (z, t) \to (\psi(z, t), t) \in \hat{\mathscr{W}}_0'^{\natural} \times U ,$$

where ψ is the restriction of $\bar{\psi}$ to $Z'^{\natural} \times U$. Since the mapping $\psi(\bullet, t) : Z'^{\natural} \to \hat{\mathscr{W}}_0'^{\natural}$ is surjective for each $t \in U$, it follows from the finiteness theorem of surjective holomorphic mappings (cf. [28, Theorem 2.3]) that ψ is independent of $t \in U$ and then so is $\bar{\psi}$. Let V be the nonnormal locus of $\bar{Z}'$. Then we see that for any two distinct elements z_1 and z_2 in $\bar{Z}'^{\natural} - \beta_2^{-1}(V)$, we have $z_1(t) \neq z_2(t)$ for any $t \in \bar{\mathscr{R}}$ since $\bar{Z}'^{\natural} - \beta_2^{-1}(V)$ is the space of sections. Hence the restriction of the holomorphic mapping $\bar{\psi}(\bullet, t)$ to $\bar{Z}'^{\natural} - \beta_2^{-1}(V)$ is bijective onto its image in $\bar{\mathscr{W}}_0'^{\natural}$. Since $\bar{\mathscr{W}}_0'^{\natural}$ is normal, it is biholomorphic by the Zariski main theorem. This implies that $\bar{\Psi}^{\natural}$ is bijective, hence biholomorphic. ∎

REMARK 4.2. By the same argument as above we see that if $(\mathscr{W}', \Pi', \mathscr{R})$ is a complex fiber space whose fibers are compact, irreducible and of general type, then under the same assumptions for the spaces of sections as above, $(\mathscr{W}', \Pi', \mathscr{R})$ is meromorphically trivial. In this case, Maehara [15] essentially proved that the conclusion also holds without assuming that $\bar{Z}'$ is a subset of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}}')$ (cf. Lang [13, Theorem 3.17] and [12, p. 202]). It seems to be true in the case of the hyperbolic fiber space in Theorem 4.1 unless $\bar{Z}'$ is bimeromorphic to $\bar{\mathscr{W}}'$.

5. Proof of the Main Theorem. We use the same notation as in §1. First note that every meromorphic mapping from a complex manifold into a hyperbolic complex space is in fact holomorphic (cf. [10, Theorem 1]). Thus the space of all meromorphic sections of $(\mathscr{X}, p, \mathscr{B})$ is imbedded into $\Gamma(\mathscr{B}', \mathscr{X}')$ as a subset. The space $\Gamma(\mathscr{B}', \mathscr{X}')$ is homeomorphic to $\Gamma(\mathscr{R}, \mathscr{W})$, which is relatively compact in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ as shown in Theorem 2.4. Then by Lemmas 3.1 and 3.2, the closure $\overline{\Gamma(\mathscr{R}, \mathscr{W})}$ of $\Gamma(\mathscr{R}, \mathscr{W})$ in $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ is a compact complex subspace of $\Gamma(\bar{\mathscr{R}}, \bar{\mathscr{W}})$ and $\Gamma(\mathscr{R}, \mathscr{W})$ is its Zariski open subset. Thus $\overline{\Gamma(\mathscr{R}, \mathscr{W})}$ is decomposed into only finitely many irreducible compact components $\overline{\Gamma(\mathscr{R}, \mathscr{W})} = \bigcup_{j=1}^{l} \bar{Z}_j$. Let

$$\bar{\Psi} : (\gamma, t) \in \overline{\Gamma(\mathscr{R}, \mathscr{W})} \times \bar{\mathscr{R}} \to \gamma(t) \in \bar{\mathscr{W}}$$

be the evaluation mapping which is the restriction of the one from $\mathrm{Hol}(\bar{\mathscr{R}}, \bar{\mathscr{W}}) \times \bar{\mathscr{R}}$ to $\bar{\mathscr{W}}$. Let Ψ be the restriction of $\bar{\Psi}$ to $\Gamma(\mathscr{R}, \mathscr{W}) \times \mathscr{R}$. Put

$$\bar{\mathscr{W}}_j := \bar{\Psi}(\bar{Z}_j \times \bar{\mathscr{R}}) , \quad \mathscr{W}_j := \Psi(Z_j \times \mathscr{R}) , \quad \bar{\Pi}_j := \bar{\Pi}|_{\bar{\mathscr{W}}_j}$$

and $\Pi_j := \bar{\Pi} | \mathscr{W}_j$. Then $(\mathscr{W}_j, \Pi_j, \mathscr{R})$ and $(\bar{\mathscr{W}}_j, \bar{\Pi}_j, \bar{\mathscr{R}})$ are fiber subspaces of $(\mathscr{W}, \Pi, \mathscr{R})$ and $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$, respectively. It follows from the distance decreasing principle of the Kobayashi distance that $(\mathscr{W}_j, \Pi_j, \mathscr{R})$ is a hyperbolic fiber space with a compactification $(\bar{\mathscr{W}}_j, \bar{\Pi}_j, \bar{\mathscr{R}})$ which satisfies the conditions (LC) and (HI). Then each irreducible component with positive dimension is a space of trivial sections of a meromorphically trivial fiber subspace of $(\bar{\mathscr{W}}, \bar{\Pi}, \bar{\mathscr{R}})$ by Theorem 4.1.

REMARK 5.1. We see that the number of components of $\Gamma(\mathscr{B}', \mathscr{X}')$ is finite, but do not know the number of components of $\Gamma(\mathscr{B}, \mathscr{X})$.

6. Some remarks. Let $(\mathscr{X}, p, \mathscr{B})$ be a fiber space with a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$. If $(\mathscr{X}, p, \mathscr{B})$ and $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$ satisfy the conditions (LC) and (HI), then each general fiber $\mathscr{X}_t$ is complete hyperbolic and is hyperbolically imbedded into $\bar{\mathscr{X}}_t$ (cf. Proposition 1.2). The converse is not true as we see in the following example.

EXAMPLE 6.1. We consider the well-known example of Brody-Green [2], a family of smooth hypersurfaces in $\boldsymbol{CP}^3$,

$$\mathscr{A} := \{(t, z) : z_0^d + z_1^d + z_2^d + z_3^d + t(z_0 z_1)^{d/2} + t(z_0 z_2)^{d/2} = 0\} \subset \boldsymbol{CP}^1 \times \boldsymbol{CP}^3 ,$$

where (z_0, z_1, z_2, z_3) is the homogeneous coordinate of $\boldsymbol{CP}^3$, t is the inhomogeneous coordinate of $\boldsymbol{CP}^1$ and d is even and ≥ 50. Let $\bar{\Pi} : \boldsymbol{CP}^1 \times \boldsymbol{CP}^3 \to \boldsymbol{CP}^1$ be the first projection. $(\mathscr{A}, \bar{\Pi}|_{\mathscr{A}}, \boldsymbol{CP}^1)$ is a compact complex fiber space. Brody and Green showed that the fibers with finite exceptions are hyperbolic hypersurfaces in $\boldsymbol{CP}^3$. Recently, Noguchi proved that if d is even and ≥ 62, there is a finite set $E \subset \boldsymbol{CP}^1$ such that for $t \in \boldsymbol{CP}^1 - E$, $\mathscr{A}_t = (\bar{\Pi}|_{\mathscr{A}})^{-1}(t)$ is hyperbolic and that $\boldsymbol{CP}^3 \setminus \mathscr{A}_t$ is complete hyperbolic and is hyperbolically imbedded into $\boldsymbol{CP}^3$ (cf. [18, Theorem 4.4]). Take a compact hyperbolic hypersurface V in $\boldsymbol{CP}^3$ and put

$$\mathscr{A}' := \bigcup_{t \in E} \{(t, w) : w \in V\} \cup \mathscr{A}$$

and

$$\mathscr{W} := (\boldsymbol{CP}^1 \times \boldsymbol{CP}^3) \setminus \mathscr{A}', \ \Pi = \bar{\Pi}|_{\mathscr{W}} .$$

Then by the distance decreasing principle, $(\mathscr{W}, \Pi, \boldsymbol{CP}^1)$ is a hyperbolic fiber space whose fibers are complete hyperbolic and are hyperbolically imbedded into $\bar{\mathscr{W}}_t \cong \boldsymbol{CP}^3$. Take a small neighborhood U of 0 in $\boldsymbol{CP}^1$ so that $\mathscr{A}_U$ is smooth and $\mathscr{A}_t$ is hyperbolic for $t \in U \setminus \{0\}$. $\mathscr{A}_0$ is the Fermat surface, which is nonsingular. By arguments similar to those in the proof of Noguchi [23, Lemma 2.1], we see that $(U \setminus \{0\}) \times \boldsymbol{CP}^3 \setminus \mathscr{A}_U$ is not hyperbolically imbedded into $U \times \boldsymbol{CP}^3$. It follows from the distance decreasing principle that $\mathscr{W}_U = (U \times \boldsymbol{CP}^3) \setminus \mathscr{A}'$ is not hyperbolically imbedded into $U \times \boldsymbol{CP}^3$. Therefore the hyperbolic fiber space $(\mathscr{W}, \Pi, \boldsymbol{CP}^1)$ does not satisfy the condition (HI). Clearly it does not satisfy the condition (LC), since $\mathscr{W}_U$ is not complete hyperbolic.

REMARK 6.2. We do not know whether $(\mathscr{W}, \Pi, \boldsymbol{CP}^1)$ is Mordellic or not.

PROPOSITION 6.3. *Let $(\mathscr{X}, p, \mathscr{B})$ be a complex fiber space with a compactification $(\bar{\mathscr{X}}, \bar{p}, \bar{\mathscr{B}})$. Suppose that $\mathscr{X}_t$ and $(\partial_h \mathscr{X})_t := \bar{p}^{-1}(t) \cap \partial_h \mathscr{X}$ are Brody hyperbolic for $t \in \mathscr{B}$ and that for $t \in \partial \mathscr{B}$ there is a neighborhood of t in $\bar{\mathscr{B}}$ such that $\mathscr{X}_U$ is complete hyperbolic and is hyperbolically imbedded into $\bar{\mathscr{X}}_U$. Then $(\mathscr{X}, p, \mathscr{B})$ is Mordellic.*

This follows from the stability theorem of hyperbolic imbeddedness of Zaidenberg [30, Corollary 2.1] and the Main Theorem. We remark that if $\bar{\mathscr{X}}_t$ and $(\partial_h \mathscr{X})_t$ are smooth and $\mathscr{X}_t$ is hyperbolically imbedded into $\bar{\mathscr{X}}_t$, then $(\partial_h \mathscr{X})_t$ is, in fact, hyperbolic. This also follows from the same argument as in the proof of Lemma 2.1 of Noguchi [23]. Noguchi pointed this out to the author.

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