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The *N* Plate Problem of Plate Tectonics

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Summary

Plate tectonics theory is used to develop a least squares method for finding all the relative motions of any n interacting plates simultaneously. The method ensures internal consistency of results and allows full use to be made of data on relative movement of plates. The least squares fitting criteria are chosen to avoid ambiguity. Relative motions of eight major plates of the world are calculated in preliminary form, and regional and global rates of creation and destruction of lithosphere are derived from these motions.

Introduction

In the theory of plate tectonics, one of our main concerns is how the various rigid parts of the Earth's surface move relative to one another. Until now, present-day plate movements have been studied by finding the relative motions for pairs of plates for which there is enough information, then using vector circuits to deduce motions between other pairs (Morgan 1968; Le Pichon 1968). If we have more than the minimum amount of data needed to solve such a system, there is no guarantee that motions deduced from vector circuits will agree with those found directly, since those data will contain errors of measurement which introduce uncertainty into the direct results and can compound in the vector circuits. If, however, all the relative motions of all the plates in our system are calculated simultaneously, self-consistency is ensured. Only two assumptions are needed: rigidity of the plates; and constant area of the Earth.

Method: the *n*-plate problem

The instantaneous relative motions of n rigid plates on the surface of the spherical earth can be completely described by $n-1$ relative angular velocity vectors, although a possible $n(n-1)/2$ such vectors exist. These vectors will be referred to here as poles of relative motion. The convention of sign used in quoting them is that seen from above the pole $\boldsymbol{\omega}(b, a)$, plate b moves counterclockwise with respect to plate a. Let us consider the case of three plates: a, b, and c. If we assume plate a to be fixed and we know the two poles $\boldsymbol{\omega}(b, a)$ and $\boldsymbol{\omega}(c, a)$, then the third pole $\boldsymbol{\omega}(c, b)$ is determined and is equal to $\boldsymbol{\omega}(c, a)-\boldsymbol{\omega}(b, a)$. This is because the relation

$$\boldsymbol{\omega}(a, b)+\boldsymbol{\omega}b, c)+\boldsymbol{\omega}(c, a) = 0 \qquad (1)$$

always holds (McKenzie & Parker 1967), and $\boldsymbol{\omega}(a, b) = -\boldsymbol{\omega}(b, a)$. The argument for extension to n plates is analogous. We can take the $n-1$ vectors together to form a $3(n-1)$-dimensional vector $\boldsymbol{\Omega}$.

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The problem then is to use all the information available on the motions of the set of n plates to determine a best fit $\mathbf{\Omega}$ in the least squares sense. This information is of two types, relative directions and relative rates of plate movement. The kth place on a plate boundary at which an observation is made will be represented by $\mathbf{r}(k)$, the vector from the Earth's centre to the observation point. The local directions of relative plate motion $\mathbf{s}$ are best found from transform fault trends and, less precisely, from the slip vectors of the mechanism solutions for shallow earthquakes. $\mathbf{s}(i, j, k)$ is the unit vector tangent to the Earth's surface at $\mathbf{r}(k)$ and pointing in the direction of motion of plate j relative to plate i.

The magnitudes of the rates of relative motion $\mathbf{v}$ are found by identifying young sea-floor spreading magnetic anomalies on the ridge crests and applying a geomagnetic reversal time scale such as that of Heirtzler *et al.* (1968). $\mathbf{v}(i, j, m)$ is the vector tangent to the Earth's surface at $\mathbf{r}(m)$, normal to the trend of the magnetic anomalies and ridge crest, in the direction consistent with the motion of plate j with respect to plate i, and with magnitude equal to twice the spreading half-rate in cm yr^{-1}.

The least squares criteria used follow from the definition of $\mathbf{s}$ and $\mathbf{v}$. On the i, j plate boundary a pole $\boldsymbol{\omega}(j, i)$, formed from the appropriate components of $\mathbf{\Omega}$, predicts for the value of $\mathbf{s}(i, j, k)$ a unit vector

$$\mathbf{x}(i, j, k) = [\boldsymbol{\omega}(j, i) \wedge \mathbf{r}(k)]/|\boldsymbol{\omega}(j, i) \wedge \mathbf{r}(k)| \tag{2}$$

and a vector error

$$\mathbf{e} = \mathbf{s} - \mathbf{x} \tag{3}$$

is defined (Fig. 1(a)). This criterion turns out to be equivalent to that expressed in equation (26) of Appendix A, McKenzie & Sclater (1971). It has two main advantages. One is that it is formulated in Cartesian vector form, which makes it easy to handle by computer. The other is that the sense of motion between the plates is unambiguously defined, so for a particular data item no secondary minimum in the error is introduced at $-\boldsymbol{\omega}(j, i)$.

Since the trend of oceanic ridges is not necessarily perpendicular to the actual spreading direction the observed quantity $\mathbf{v}$ may not be parallel to the direction of plate separation. Thus the predicted velocity of $\boldsymbol{\omega}(j, i) \wedge \mathbf{r}(m)$ is projected onto the observed direction of $\mathbf{v}$, and

$$\mathbf{y}(i, j, m) = [\mathbf{v}(i, j, m) . \boldsymbol{\omega}(j, i) \wedge \mathbf{r}(m)]/|\mathbf{v}(i, j, m)| \tag{4}$$

is used to define a scalar error (Fig. 1(b))

$$f = [|\mathbf{v}| - |\mathbf{y}|]/|\mathbf{v}|. \tag{5}$$

The normalizing factor $1/|\mathbf{v}|$ prevents undue weight being given to fast spreading rates. Projection of the observed onto the predicted direction could have been used instead, but in absence of reasons to the contrary I chose the form above, which is easier to compute and results in a criterion linear in the components of $\mathbf{\Omega}$. As in the relative directions case, the sense of plate motion is unambiguous.

It has been the practice in previous studies of plate motions (Morgan 1968; Le Pichon 1968; McKenzie & Sclater 1971, and others) to separate the spreading rates and the directions of motion and either find best-fit poles for each kind of information separately, or use the direction information to find the pole direction and then apply the spreading rates to assign it a magnitude. In this simultaneous n-plate approach, both kinds of information must be used together, since the vector sum rules of the form of equation (1) depend upon both magnitude and direction of the poles involved. No particular difficulty is encountered in combining the information since both kinds of error are given vector definitions. It does become desirable,

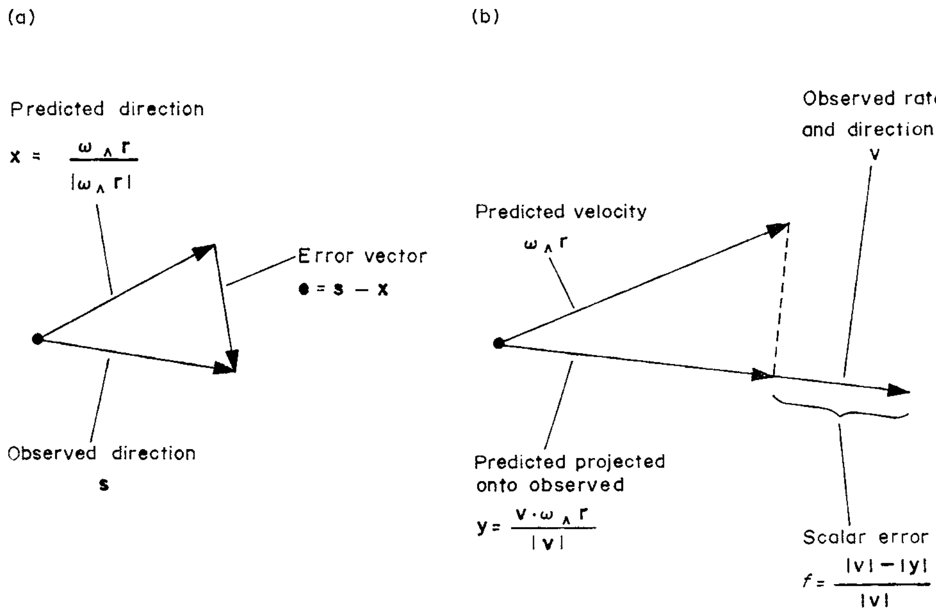

FIG. 1. Errors used in least squares fitting criteria, looking down on observation point on given plate boundary. (a) Vector error for directions of motion found from transform fault trends or earthquake slip vectors. (b) Scalar error for relative rates of motion from sea-floor spreading magnetic anomalies.

however, to be able to assign the various data items individual weighting factors $w(k)$. Ideally, these $w(k)$ should be inversely proportional to the variance of the data involved.

With definitions (2), (3), (4), and (5), the total square error becomes

$$F = \sum_{i,\,j,\,k,\,m} [w(k)\,\mathbf{e}(i,j,k).\mathbf{e}(i,j,k)+w(m)f^2(i,j,m)] \quad (6)$$

with $j > i$ to avoid duplication. The desired choice of $\mathbf{\Omega}$ is that for which F is a minimum. A reasonable initial $\mathbf{\Omega}_0$ is chosen, and F is then minimized by a $3(n-1)$-dimensional search from $\mathbf{\Omega}_0$, descending along the analytically found gradient of F. The contribution to F from the f's is linear in $\mathbf{\Omega}$, but that of the e's is not, so secondary minima may be introduced. Care must be taken to avoid these, but in practice all reasonable $\mathbf{\Omega}_0$'s have given the same final $\mathbf{\Omega}$ from the same set of data.

Preliminary results

The relative motions of eight major plates of the world have been calculated by the method discussed above. The plates and their boundaries are plotted in Fig. 2. Seven of them are large, and together cover much of the Earth's surface. They are: AME—North and South America; PAC—Pacific; NAZ—Nazca plate; EUA—Europe and Asia; AFR—African plate; IND—plate containing India and Australia; ANT—Antarctica. The eighth plate, COC for Cocos, is smaller, but is included because of the wealth of data concerning its movement relative to the larger plates surrounding it.

For all the plates, 176 directions of relative motion and 59 rates of plate separation by sea-floor spreading were selected from published and unpublished sources. The

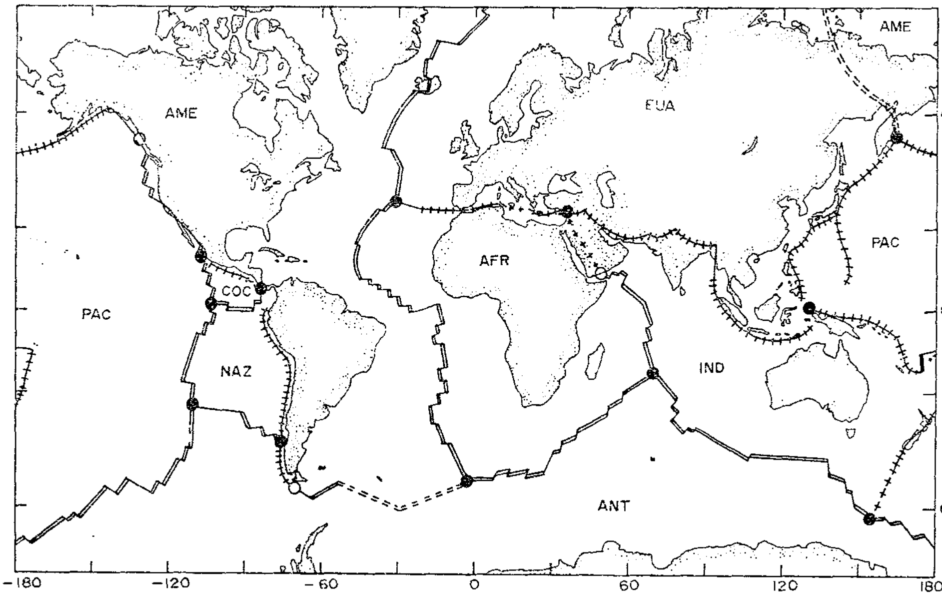

FIG. 2. Major plates considered in this study, on Mercator projection. Plate boundaries are marked by heavy lines: double for spreading centres, single for transform faults, cross-hatched for subduction zones. Solid circles are triple junctions used in calculating crustal creation and destruction rates listed in Table 2, and open circles mark places at which plate boundaries change from extensional to compressional nature.

information used by Morgan (1968) and Le Pichon (1968) is included, together with many measurements that have become available since. The data themselves and their sources will be published later, in conjunction with a statistical treatment of the problem.

Table 1 displays the best-fit pole set found for these eight plates using the data at hand. The label S005ALL will serve to distinguish these results from future pole sets to be calculated with more complete and carefully screened data. The program prints out all the relative motions of all the plates, but only those that correspond to plates in actual contact are shown in Table 1. In general, these results are comparable to those of Le Pichon (1968) but are more complete, include additional data, and have shown greater predictive accuracy in detail.

In particular, McKenzie & Parker (1967) and Morgan (1968) found quite different positions for the pole of relative motion of AME with respect to PAC. Le Pichon (1968) modified Morgan's pole only slightly. None of these poles, however, agreed with the trend mapped by Larson (1971) for the Rivera fracture zone, a transform fault separating the American and Pacific plates. The AME/PAC pole of Table 1, which lies between the previous pole locations, predicts the Rivera trend within 1°. A pole calculated by the same method, but considering only the two plates PAC and AME, was somewhat less compatible with the Rivera trend. It seems fair to conclude that both the fitting criteria and inclusion of the other plates contributed to the accuracy of the AME/PAC pole presented here.

For Table 2, I have calculated the rates of crustal flux on the plate boundaries as shown in Fig. 2. The method I used is similar to that of Deffeyes (1970). The close correspondence of total rates of crustal generation (area of crust created annually by sea-floor spreading) and destruction (area of lithosphere subduced annually) is probably a happy coincidence, since the angular length of plate boundaries were

Table 1

Pole set S005ALL. quoted with the convention that seen from above pole A/B, plate A is moving counterclockwise with respect to plate B. North latitudes and east longitudes are positive. The number of observations of direction and rate of relative movement used for each plate boundary are given, together with the root mean square error associated with them in the fit.

Plates	Latitude (deg)	Longitude (deg)	Rate ($\times 10^{-7}$ deg/yr)	Relative directions #	Relative directions rms error	Relative rates #	Relative rates rms error
AME/PAC	+52	−73	7·86	65	0·178	2	0·037
NAZ/AME	+60	−129	7·35	4	0·103	—	—
EUA/AME	+48	+155	2·36	7	0·244	4	0·081
AFR/AME	+66	−29	3·44	29	0·092	12	0·104
AME/ANT	−64	−73	3·30	1	0·033	—	—
COC/AME	+27	−119	17·3	11	0·129	—	—
NAZ/PAC	+59	−97	14·6	10	0·061	4	0·029
EUA/PAC	+63	−90	8·85	—	—	—	—
PAC/IND	−58	+174	12·9	3	0·029	—	—
PAC/ANT	−70	+107	9·76	8	0·231	10	0·163
COC/PAC	+36	−108	23·7	1	0·089	8	0·035
NAZ/ANT	+36	−114	5·76	—	—	2	0·093
COC/NAZ	+8	−116	11·9	8	0·076	6	0·049
AFR/EUA	+25	−27	3·28	5	0·381	—	—
IND/EUA	+22	+25	8·04	—	—	—	—
IND/AFR	+15	+48	6·33	8	0·068	4	0·075
AFR/ANT	+4	−51	2·62	2	0·240	—	—
IND/ANT	+16	+24	6·50	14	0·098	7	0·078
Overall				176	0·164	59	0·095

Table 2

Balance of lithospheric generation and consumption based on pole set S005ALL Principal spreading centres and subduction zones indicated in parentheses.

Plates	Area of crust created ($km^2\ yr^{-1}$)	Area of crust destroyed ($km^2\ yr^{-1}$)
PAC/AME	0·056 (East Pacific Rise)	0·197 (Aleutian Trench)
NAZ/AME		0·454 (Peru–Chile Trench)
EUA/AME	0·167 (N. Mid-Atlantic Ridge)	
AFR/AME	0·358 (S. Mid-Atlantic Ridge)	
ANT/AME	0·038 (S.W. Atlantic Ridge)	0·008 (S. Chile Trench)
COC/AME		0·260 (Middle America Trench)
NAZ/PAC	0·678 (East Pacific Rise)	
EUA/PAC		0·609 (Kurile, Japan, Marianas, Phillipine Trench
IND/PAC		0·590 (Tonga–Kermadec, New Guinea Trench)
ANT/PAC	0·364 (Pacific–Antarctic Ridge)	
COC/PAC	0·240 (East Pacific Rise)	
ANT/NAZ	0·142 (Chile Rise)	
COC/NAZ	0·112 (Galapagos Rise)	
AFR/EUA		0·087 (Azores–Gibraltar Ridge, Mediterranean)
IND/EUA		0·697 (Caucasus, Himalayas, Java Trench)
IND/AFR	0·133 (Carlsberg Ridge)	0·040 fictitious—Arabian plate intervenes
ANT/AFR	0·148 (S.W. Indian Ridge)	
ANT/IND	0·489 (S.E Indian Ridge)	
Total	2·925	2·942

measured graphically on a globe. Of course, if done with complete precision, the assumption that the Earth's radius is not changing requires that the two quantities be exactly equal. The net rate of flux, almost 3 $km^2\ yr^{-1}$, is similar to the 2·65 $km^2\ yr^{-1}$ rate for sea-floor spreading found by Deffeyes (1970). For some of the plate boundaries listed in Table 2, complicated areas have been lumped together. For instance, under IND/PAC spreading on the Fiji Plateau (Chase 1971) is not included, and for EUA/PAC only the total subduction in both the Phillipine and Marianas trenches combined is listed, ignoring the intermediate Phillipine plate.

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