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The Naive Quartet Quark Model and the ψ's

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The naive quartet quark model approach to the ψ-particle is developed. Narrow decay widths of the ψ's are explained on the basis of the Okubo-Zweig-Iizuka rule. $SU(4)$ mass formula results in a scaling relation $\delta_{\phi\phi}=f(\delta_{\psi\omega})$. Small $\delta_{\psi\omega}$ required by the narrow width of $\psi(3.1)$ is consistent with the mass formula. The relatively large $\Gamma(\psi(3.7)\to\psi(3.1)+2\pi)$ is a result of large $\bar{Q}Q\epsilon$-coupling which also explains the dominance of $\rho+2\pi$ decay mode of $\rho'(1.6)$ decay. The breaking of the O-Z-I rule through ϵ is about 8% in amplitude. No sensible difference between two assignments for $\psi(3.7)$ (orbital and radial excitations) is found.

§ 1. Introduction

The lepton-baryon symmetry[1),**)] has suggested the fourth Sakaton (c)[2)] and $SU(4)$ quark model.[3)] In $SU(4)$ quark model, there are two ways of quantum number assignment for ψ.[4)] Case I: $\psi(3.1)$ is assigned to ${}^3S_1$ multiplet together with ρ, K^*, ω and ϕ, and $\psi(3.7)$ to ${}^3D_1$ together with $\rho'(1.6)$. Case II: $\psi(3.1)$ is the same as above and $\psi(3.7)$ is assigned to be $L=0$ radial excitation state of $\psi(3.1)$.

In this model, the narrowness of the decay widths of $\psi(3.1)$ and $\psi(3.7)$ is explained on the basis of the Okubo-Zweig-Iizuka rule[5)] (O-Z-I rule). ψ can decay into non-charmed hadrons only through the breaking of the O-Z-I rule at the vertex of quark and emitted particle and/or deviation of the ψ's from pure $\bar{c}c$ and the $\bar{c}c$ components of the ordinary hadrons.

We will examine whether these pictures are reasonable or not: especially, whether the state mixing parameters estimated by the decay widths are consistent or not with the Okubo type $SU(4)$ mass formula,[6)] and whether small Γ ($\psi(3.1)\to$hadrons) is consistent with relatively large Γ $(\psi(3.7)\to\psi(3.1)+2\pi)$ or not.

§ 2. Mixing parameters determined by mass formula

Okubo type $SU(4)$ mass formula

$$\mathcal{M}^2=T_0+\bar{\beta}(T_8+\bar{\alpha}T_{15})\tag{2·1}$$

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**) The proposal of the full symmetry among p, n, λ was encouraged by the lepton-baryon symmetry and the μ-e universality.

See S. Ogawa, Prog. Theor. Phys. **21** (1959), 209.

has five parameters for each 16-plet. For the vector multiplet in question, these parameters are determined by the masses of ρ, K^*, ω, ϕ and $\psi(3.1)$. At the same time, the mixing parameters of ω, ϕ and ψ are determined.

We parametrize the state mixing as

$$\begin{pmatrix}\omega\\ \phi\\ \psi\end{pmatrix}=\begin{pmatrix}\delta_{\omega\omega} & \delta_{\omega\phi} & \delta_{\omega\psi}\\ \delta_{\phi\omega} & \delta_{\phi\phi} & \delta_{\phi\psi}\\ \delta_{\psi\omega} & \delta_{\psi\phi} & \delta_{\psi\psi}\end{pmatrix}\begin{pmatrix}\omega_I\\ \phi_I\\ \psi_I\end{pmatrix}, \tag{2·2}$$

where ω_I, ϕ_I and ψ_I are the ideal mixing states,

$$\omega_I=\frac{1}{\sqrt{2}}(\bar{p}p+\bar{n}n)=\frac{1}{\sqrt{2}}\phi_0+\frac{1}{\sqrt{3}}\phi_8+\frac{1}{\sqrt{6}}\phi_{15},$$

$$\phi_I=\quad -\bar{\lambda}\lambda \quad =-\frac{1}{2}\phi_0+\sqrt{\frac{2}{3}}\phi_8-\frac{1}{\sqrt{12}}\phi_{15},$$

$$\psi_I=\quad \bar{c}c \quad =\frac{1}{2}\phi_0 \qquad -\frac{\sqrt{3}}{2}\phi_{15}.$$

Since the physical masses have some errors as

$$m_\rho=770\pm10(\mathrm{MeV}),\qquad m_{K^*}=892.2\pm0.5,$$

$$m_\omega=782.7\pm0.6,\qquad m_\phi=1019.7\pm0.3,\qquad m_\psi=3105\pm3,$$

we obtain an allowed region for the mixing parameters δ_i, which is shown in Figs. 1, 2 and 3. The values $\delta_{\psi\omega}$ and $\delta_{\psi\phi}$ depend sensitively on m_ρ (more exactly on $(m_{K^*}^2-m_\rho^2)$) which has a large error. $\delta_{\psi\omega}$ is restricted by m_ρ as is shown in

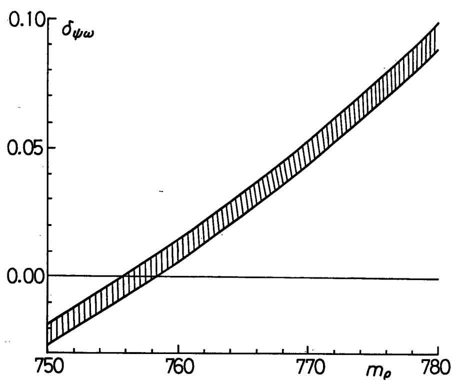

Fig. 1. The dependence of mixing parameter $\delta_{\psi\omega}$ on the ρ mass with 750 MeV$\leq m_\rho \leq$780 MeV. Errors indicated by shaded region come mainly from experimental errors in masses of K^*, ω, ϕ and ψ. This is the same for Figs. 2 and 3.

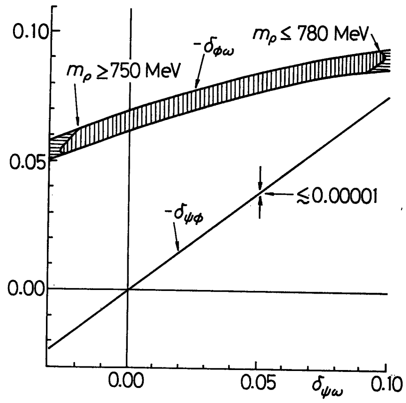

Fig. 2. Showing the relations of mixing parameters $\delta_{\phi\omega}$ and $-\delta_{\psi\phi}$ to mixing parameter $\delta_{\psi\omega}$. The relation of $-\delta_{\psi\phi}$ and $\delta_{\psi\omega}$ scales.

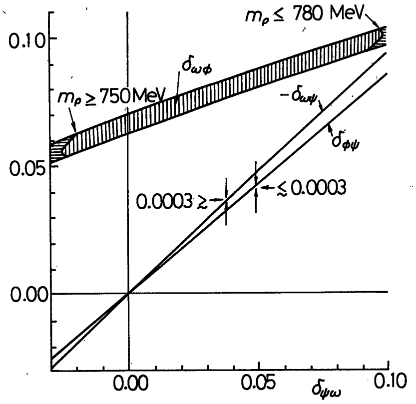

Fig. 3. The relations of $\delta_{\omega\phi}$, $-\delta_{\omega\psi}$ and $\delta_{\phi\psi}$ to $\delta_{\psi\omega}$. The relations of $\delta_{\omega\psi}$ and $\delta_{\phi\psi}$ to $\delta_{\psi\omega}$ are also scales.

Fig 1. However if we choose a value for $\delta_{\psi\omega}$, $\delta_{\psi\phi}(\simeq -\delta_{\phi\psi})$ and $\delta_{\omega\psi}(\simeq -\delta_{\psi\omega})$ are sharply determined as is shown in Figs. 2 and 3. In other words, $\delta_{\psi\phi,\omega\psi} = f_{\psi\phi,\omega\psi}(\delta_{\psi\omega}, m_i^2)$ scales very sharply.

Especially, the formula

$$-\delta_{\psi\phi} = -0.014\delta_{\psi\omega}^2 + 0.7650\delta_{\psi\omega} - 0.00007 \tag{2·3}$$

holds within the accuracy of 0.00001. It is to be noted that $|\delta_{\psi\omega}|$ and $|\delta_{\psi\phi}|$ can be simultaneously smaller than even 0.00005 if

$$755.6 \leq m_\rho \leq 758.2 \text{ MeV}.$$

§ 3. Decay of ψ

We study the decay of ψ on the basis of the single quark transition assumption.

ψ can decay into non-charmed hadrons only through the breaking of the O-Z-I rule at the vertex of the quark transition and/or deviation of ψ from the pure $\bar{c}c$- and $\bar{c}c$-components of the ordinary hadrons. In principle, the breaking of the O-Z-I rule at the vertex and the deviation from ideal mixing should be related dynamically, but the dynamics is not yet known. Therefore, we take them as independent phenomenological parameters.

In the single quark transition, the hadron decay of $\psi(3.1)$ occurs only through the deviation from the ideal mixing. On the other hand, the process $\psi(3.7) \to \psi(3.1) + h$ (ordinary hadron) occurs dominantly through the breaking of the O-Z-I rule at the $\bar{Q}Qh$-vertex and through the $\bar{c}c$-component of h.

In order to treat the 2π-emission process like $\psi(3.7) \to \psi(3.1) + 2\pi$ in the single quark transition scheme, we assume that such a 2π-emission occurs through the virtual ϵ emission.

For the composite system, we use the oscillator wave function as before.[7] The oscillator parameter and the Ps-meson emission coupling constant are taken from our previous analysis.[8] $\alpha = 0.43$ GeV, $G_p^2/4\pi = 35.2$,*) $\mathcal{M}_q = 1.92$ GeV. The decay widths of ordinary meson calculated by using these values are shown in the Table.

3-1. $\psi(3.7) \to \psi(3.1) + 2\pi$ etc.

We use "ϵ-pole model" for $\psi(3.7) \to \psi(3.1) + 2\pi$, that is, we assume that this

*) G_p of this note is twice G_p in Ref. 8), because we added two kinds of amplitudes in 8), but here we adopt only one amplitude. See 8) for more details.

process occurs though the virtual ϵ[9] emission.*)

We parametrize the scalar emission interaction as

$$H_{\rm int}=g_s\{\sum_{i=0}^{15}\bar{Q}\lambda_i Q\phi_i+a\bar{Q}\lambda_0 Q\phi_0\}, \tag{3·1}$$

where the 16-plet scheme (which satisfies the O-Z-I rule) is broken by the second term within $SU(4)$ invariance. As to the $SU(4)$ structure of ϵ,

$$\begin{aligned}\epsilon&=\frac{1}{\sqrt{2}}(\bar{p}p+\bar{n}n)\cos\theta+(-\bar{\lambda}\lambda)\sin\theta+\delta_\epsilon\bar{c}c\\&=\frac{1}{2}(\sqrt{2}\cos\theta-\sin\theta+\delta_\epsilon)\phi_0+\frac{1}{\sqrt{3}}(\cos\theta+\sqrt{2}\sin\theta)\phi_8\\&\quad+\frac{1}{\sqrt{12}}(\sqrt{2}\cos\theta-\sin\theta-3\delta_\epsilon)\phi_{15},\end{aligned} \tag{3·2}$$

since δ_ϵ is expected to be small.

We describe the $\epsilon\pi\pi$-vertex completely phenomenologically by

$$H_{\rm int}(\epsilon\pi\pi)=\frac{g_\epsilon}{2}\phi_\pi\phi_\pi\phi_\epsilon \tag{3·3}$$

and then

$$\Gamma(\epsilon\to 2\pi)=\frac{g_\epsilon^2}{48\pi}\cdot\frac{\sqrt{(m_\epsilon/2)^2-\mu^2}}{m_\epsilon^2}, \tag{3·4}$$

where μ is the pion mass.

Now, we can write the amplitude of $\psi(3.7)\to\psi(3.1)+2\pi$ as

$$T=F(W)\frac{1}{q^\mu q_\mu-(m_\epsilon-i\Gamma_\epsilon/2)^2}\cdot\frac{g_\epsilon}{\sqrt{3}}, \tag{3·5}$$

where q^μ is four momentum of ϵ and $W\equiv q^0$ is its energy at the rest frame of $\psi(3.7)$. In the oscillator model, $\sum|F(W)|^2$ is given by

$$\begin{aligned}\sum_{\rm helicity}|F(W)|^2&=\frac{4}{5}M'(M'-W)(g_s\beta)^2\left(\frac{q}{2\alpha}\right)^4 e^{-q^2/4\alpha^2}\times\begin{pmatrix}1\\5/2\end{pmatrix},\\ \beta&=\left(\cos\theta-\frac{\sin\theta}{\sqrt{2}}\right)a+2\sqrt{2}\delta_\epsilon,\end{aligned} \tag{3·6}$$

where $q^2=(M'-W)^2-M^2$ is the 3-dimensional square of q^μ and the upper (lower) value of the last factor is to be read for $^3D_1$ (radial excitation) assignment of $\psi(3.7)$. M' and M are the masses of $\psi(3.7)$ and $\psi(3.1)$ respectively.

Finally

*) This assumption seems to be supported by recent experiments (G. S. Abrams et al., LBL-3669, SLAC-PUB-1556).

Table. Partial decay width of meson resonance including ψ.

$A=(1/2\pi)(G_P/2\mathcal{M}_q)^2\cdot(qE_1/M_J)\cdot q^2\cdot\exp(-q^2/4\alpha^2)$, $(G_P^2/4\pi=35.2,\ \alpha=0.43\ \mathrm{GeV},\ \mathcal{M}_q=1.92\ \mathrm{GeV},\ E_1$: the energy of non quantum decay product).

*: (3.7) is referred to the 2π-emission processes.

†: From these imput, we get $g_s^2/4\pi\cdot\cos^2\theta\sim\begin{pmatrix}200\\80\end{pmatrix}$ for $\begin{pmatrix}\text{Case I}\\\text{Case II}\end{pmatrix}$, $\beta\sim0.08$.

Case I: $\psi_{3.7}$, $\rho'_{1.6}$ are orbital excitation.

Case II: $\psi_{3.7}$, $\rho'_{1.6}$ are radial excitation.

The ψ processes contained in this table will be sufficient for estimation of ψ-decay width: Our harmonic oscillator form factor has the form $(q/2\alpha)^{2L}\exp(-(q/2\alpha)^2)$. The Q-value dependence of the exponential factor is mild for our α-value. The factor $(q/2\alpha)^{2L}$ has strong suppression effect for small Q-value processes. The smallness of $\psi_{3.7}\to\psi_{3.1}+\eta$ has possibly the same origin as $\delta\to\eta\pi$.[8] θ' is defined as $\eta=\eta_8\cos\theta'+\eta_1\sin\theta'$.

	Decay Mode	Γ cal	Case I (MeV) orbital	Case II (MeV) radial	Γ exp (MeV)
$^3S_1$	$\rho\to\pi\pi$	$4/3A$	120		140～160
	$\phi\to\bar{K}K$	$4/3A$	6		4.0～4.4
	$\to\rho\pi$	$8A\delta_{\phi\omega}^2$	$180\delta_{\phi\omega}^2$		<0.66
	$K^*\to K\pi$	A	50		48.8～50.9
	$\psi_{3.1}\to\omega\pi\pi$	* *	$27000\delta_{\psi\omega}^2\cos^2\theta$	$27000\delta_{\psi\omega}^2\cos^2\theta$	
	$\to\phi\pi\pi$	* *	$69000\delta_{\psi\phi}^2\sin^2\theta$	$69000\delta_{\psi\phi}^2\sin^2\theta$	
	$\to\rho\pi$	$8A\delta_{\psi\omega}^2$	$3500\delta_{\psi\omega}^2$		
	$\to\omega\eta$	$8/9A\delta_{\psi\omega}^2$	$430\delta_{\psi\omega}^2$		0.08
	$\to\omega\eta'$	$16/9A\delta_{\psi\omega}^2$	$940\delta_{\psi\omega}^2$		
	$\to\phi\eta$	$32/9A\delta_{\psi\phi}^2$	$2000\delta_{\psi\phi}^2$		
	$\to\phi\eta'$	$16/9A\delta_{\psi\phi}^2$	$1100\delta_{\psi\phi}^2$		
	……	……	……		
	$A_2\to\rho\pi$	$8/5\ (q/2\alpha)^2A$	66		62.7～80.6
	$\to\bar{K}K$	$4/15(q/2\alpha)^2A$	9.6		3.7～5.8
	$f\to\pi\pi$	$4/15(q/2\alpha)^2A\ (\sin\theta_v+\sqrt{2}\cos\theta_v)^2$	140		115～176
	$\to\bar{K}K$	$4/45(q/2\alpha)^2A(\sin\theta_v-2\sqrt{2}\cos\theta_v)^2$	4.4		1.4～14.0

$^3P_2$	$f' \to \bar{K}K$	$4/45(q/2\alpha)^2 A(\cos\theta_v + 2\sqrt{2}\sin\theta_v)^2$		68		30∼50
	$K_N \to K\pi$	$2/5\ (q/2\alpha)^2 A$		61		47∼63
	$\to K^*\pi$	$3/5\ (q/2\alpha)^2 A$		12		24.8∼35.2
	$\to \rho K$	$3/5\ (q/2\alpha)^2 A$		4.7		6.1∼12.8
	$\to \omega K$	$1/5\ (q/2\alpha)^2 A$		1.2		2.4∼6.7
	$\to K\eta$	$2/27(q/2\alpha)^2 A$		4.7		0.0∼4.4
	$g \to \pi\pi$	$4/35(q/2\alpha)^4 A$		52		31.5∼65.1
	$\to \bar{K}K$	$2/35(q/2\alpha)^4 A$		8.7		3.0∼4.2
$^3D_1$	$\to \rho\pi\pi$	*		$220\cos^2\theta$		<147
	$\omega'_{1.67} \to \rho\pi$	$12/35(q/2\alpha)^4 A$		46		<162
	$\to \omega\pi\pi$	*		80.6		
	$\rho'_{1.6} \to \rho\pi\pi$	*	*	$(2(g_s^2/4\pi)\cos^2\theta)$ †)	$(5(g_s^2/4\pi)\cos^2\theta)$ †)	400 (input) †)
	$\to \pi\pi$	$8/45\ (q/2\alpha)^4 A$	$2/9\ (q/2\alpha)^4 A$	62.4	78.1	
	$\to \bar{K}K$	$4/45\ (q/2\alpha)^4 A$	$1/9\ (q/2\alpha)^4 A$	8.8	11.0	
Case I $^3D_1$	$\psi'_{3.7} \to \psi_{3.1}\pi\pi$	*	*	$(0.11(g_s^2/4\pi)\beta^2)$ †)	$(0.28(g_s^2/4\pi)\beta^2)$ †)	0.5×30% (input) †)
	$\to \omega\pi\pi$	*	*	$19000\delta^2_{\psi'\omega}$	$19000\delta^2_{\psi'\omega}$	0.5×70%
	$\to \phi\pi\pi$	*	*	$42000\delta^2_{\psi'\phi}\tan^2\theta$	$42000\delta^2_{\psi'\phi}\tan^2\theta$	
	$\to \rho\pi$	$4/15(q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$4/3\ (q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$920\delta^2_{\psi'\omega}$	$4600\delta^2_{\psi'\omega}$	
Case II	$\to \omega\eta$	$4/135(q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$4/27(q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$100\delta^2_{\psi'\omega}$	$520\delta^2_{\psi'\omega}$	
$^3S_1$	$\to \omega\eta'$	$8/135(q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$8/27(q/2\alpha)^4 A\delta^2_{\psi'\omega}$	$200\delta^2_{\psi'\omega}$	$1000\delta^2_{\psi'\omega}$	
radial	$\to \phi\eta$	$16/135(q/2\alpha)^4 A\delta^2_{\psi'\phi}$	$16/27(q/2\alpha)^4 A\delta^2_{\psi'\phi}$	$440\delta^2_{\psi'\phi}$	$2200\delta^2_{\psi'\phi}$	
	$\to \phi\eta'$	$8/135(q/2\alpha)^4 A\delta^2_{\psi'\phi}$	$8/27(q/2\alpha)^4 A\delta^2_{\psi'\phi}$	$200\delta^2_{\psi'\phi}$	$1000\delta^2_{\psi'\phi}$	
	$\to \psi_{3.1}\eta$	$1/120(q/2\alpha)^4 A\beta'^2\sin^2\theta'$	$1/24(q/2\alpha)^4 A\beta'^2\sin^2\theta'$	$0.0007\beta'^2\sin^2\theta'$	$0.003\beta'^2\sin^2\theta'$	
	……	……	……	……	……	

$$\Gamma(\psi(3.7)\to\psi(3.1)+2\pi)$$
$$=\frac{1}{16(2\pi)^3}\int_{(M'^2-M^2+4\mu^2)/(2M')}^{M'-M} dW|T|^2\frac{\sqrt{(M'-W)^2-M^2}}{M'}\sqrt{\frac{M'^2-M^2-2WM'+4\mu^2}{M'^2-M^2-2WM'}}. \tag{3·7}$$

Using $m_\epsilon=700$ MeV and $\Gamma(\epsilon\to 2\pi)=600$ MeV, we get $g_\epsilon^2/4\pi=561\mu^2$ and

$$\Gamma(\psi(3.7)\to\psi(3.1)+2\pi)=\frac{(g_s\beta)^2}{4\pi}\times\begin{pmatrix}0.120\\0.301\end{pmatrix}\text{MeV}.$$

In order to determine g_s, we calculated the $\Gamma(\rho'(1.6)\to\rho+2\pi)$ in the same way and got

$$\Gamma(\rho'(1.6)\to\rho+2\pi)=\frac{g_s^2}{4\pi}\times\begin{pmatrix}1.98\\4.94\end{pmatrix}\text{MeV}.$$

From $\Gamma(\rho'(1.6)\to\rho+2\pi)\simeq\Gamma(\rho'(1.6)\to 4\pi)\simeq 400$ MeV and $\Gamma(\psi(3.7)\to\psi(3.1)+2\pi)\simeq 500\times 0.5$ keV, we get

$$\frac{g_s^2}{4\pi}=\begin{pmatrix}195\\79\end{pmatrix},$$
$$\beta=0.080. \tag{3·8}$$

The large value of $g_s^2/4\pi$ is compelled by the large $\Gamma(\rho'(1.6)\to\rho+2\pi)$ and is consistent with large $\pi\pi\epsilon$-coupling constant, since $g_\epsilon\simeq 4\cos\theta\mu g_s$ if the coupling of the composite pion and ϵ is calculated from (3·1). If we accept large $g_s^2/4\pi$, β has a reasonable magnitude! It seems to us that the large branching ratio of $\psi(3.7)\to\psi(3.1)+2\pi$ is due to large $g_s^2/4\pi$. This result corresponds to the fact that the dominant decay mode of $\rho'(1.6)$ is $\rho+2\pi$. It is to be noted that our model predicts small $\rho'(1.6)\to 2\pi$ decay width (see the Table) and dominant $\rho'(1.6)\to\rho+2\pi$.

3-2. $\psi(3.1)$ *decay*

In our model, decay widths of $\psi(3.1)$ into various hadrons have values as is given in the Table, where $g_s^2/4\pi$ determined in § 3-1 was used for $\psi(3.1)\to\binom{\omega}{\phi}+2\pi$. In our opinion, the vector quantum emission effect must be taken into account in the single quark transition assumption[8] but not here. Therefore, numerical values in the Table should not be taken so severely but it will indicate the order of magnitudes. From $\delta_{\psi\omega}\simeq\delta_{\psi\phi}$ (see § 2) and $\Gamma(\psi(3.1)\to\text{hadrons})\simeq 80$ keV, we get

$$|\delta_{\psi\omega}|\lesssim\begin{pmatrix}0.0013\sim0.0017\\0.0017\sim0.0019\end{pmatrix}. \tag{3·9}$$

Tow cases (orbital and radial excitations) do not result in a sensible difference. The form factor obtained from the overlapping integral of harmonic oscillator wave function, though it is important for overall explanation of decay widths of ordinary meson resonances and possibly for the branching ratio of ψ decay, plays no role in explaining the narrowness of the width of $\psi(3.1)$. We are obliged to accept

$$\delta^2_{\phi\omega} \gg \delta^2_{\psi\omega} \sim \delta^2_{\psi\phi}\,.$$

Referring to Fig. 1, we can conclude that the value (3·9) is consistent with Okubo type $SU(4)$ mass formula. These very small $|\delta_{\psi\omega}|$'s≤ 0.006 are obtained for $753.9 \leq m_\rho \leq 759.9$.

For example, we have the following parameters which satisfy the mass formula

$$m_\rho = 758.7\,(\mathrm{MeV}), \quad m_{K^*} = 892.7\,,$$

$$m_\omega = 782.7\,, \quad m_\phi = 1019.8\,, \quad m_\psi = 3105.0\,,$$

$$m_{C_u} = 2256.8\,, \quad m_{C_s} = 2305.3\,,$$

$$\delta_{\psi\omega} = 0.00390\,, \quad -\delta_{\psi\phi} = 0.00292\,, \quad -\delta_{\phi\omega} = 0.0652\,.$$

We hope m_{C_u} and m_{C_s} conform with Niu's experiments.[10]

From $\Gamma(\phi \to \rho\pi) < 660$ keV, we get $\delta_{\phi\omega} < 0.07$ consistently with the mass formula.

§ 4. Summary

Okubo type $SU(4)$ mass formula is consistent with very small state mixing parameters $|\delta_{\psi\omega}|$ and $|\delta_{\psi\phi}|$. They can be simultaneously smaller than even 0.00005, if the large error of m_ρ is taken into account.

The decay widths of $\psi(3.1)$ and $\psi(3.7)$ are calculated by the naive quark model on the single quark transition assumption. $\Gamma(\psi(3.1) \to \mathrm{hadron}) \simeq 80$ keV requires

$$|\delta_{\psi\omega}| \simeq (|\delta_{\psi\phi}|) \lesssim 0.001 \sim 0.002\,,$$

which is consistent with the mass formula. Two assignments of $\psi(3.7)$ (orbital and radial excitations) do not result in a sensible difference. Relatively large $\Gamma(\psi(3.7) \to \psi(3.1) + 2\pi)$ is a result of large $\bar{Q}Q\epsilon$-coupling which also explains the fact that the dominant $\rho'(1.6)$ decay mode is $\rho + 2\pi$ and $\Gamma(\rho'(1.6) \to \rho + 2\pi) \simeq 400$ MeV. The breaking of the O-Z-I rule through ϵ is about 8% in amplitude (β).

The radiative decay of $\psi(3.1) \to$ (ordinary hadron) $+\gamma$ is expected to be smaller by the fine structure constant α than the ordinary hadron decay on the single quark transition assumption.

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Note added in proof: The large $\bar{Q}Q\epsilon$-coupling also explains the fact that g decays dominantly into 4π and is consistent with the decays of $\omega(1.67)$.