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The NASA Monographs on Shell Stability Design Recommendations

A Review and Suggested Improvements

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Abstract

A summary of the existing NASA design criteria monographs for the design of buckling-resistant thin-shell structures is presented. Subsequent improvements in the analysis for nonlinear shell response are reviewed, and current issues in shell stability analysis are discussed. Examples of nonlinear shell responses that are not included in the existing shell design monographs are presented, and an approach for including reliability-based analysis procedures in the shell design process is discussed. Suggestions for conducting future shell experiments are presented, and proposed improvements to the NASA shell design criteria monographs are discussed.

Introduction

In the 1960's, the National Aeronautics and Space Administration (NASA) experience with spacecraft development indicated a need for uniform design criteria. This need led to the development of a series of monographs that provide design information and recommendations in the areas of environment; material properties and processes; stability, guidance, and control; chemical propulsion; and structures. One of the structures monographs, published in 1965 and revised in 1968, provides recommendations for the design of buckling-resistant circular cylindrical shell structures. This monograph is known throughout the aerospace industry as NASA SP-8007 (ref. 1). This monograph was followed in 1968 by NASA SP-8019 (ref. 2), which gives recommendations for the design of conical shells, and in 1969 by NASA SP-8032 (ref. 3), which gives recommendations for the design of doubly curved shells. These monographs primarily emphasize the behavior of thin-walled metallic shells subjected to axial compression, torsion, pressure, and bending loads, and to various combinations of these loads. Prior to the publication of these monographs, one of the most comprehensive collections of shell stability information available was the series of structural stability handbooks written by Gerard and Becker (refs. 4 through 6). The NASA monographs used and expanded the information provided in these handbooks.

The NASA structural stability monographs remain popular among designers primarily because they address one of the most important concerns associated with designing shells to satisfy stability requirements. Experience has shown that large discrepancies often occur between the classical shell stability analysis predictions for geometrically perfect shells and the corresponding results from experiments. The NASA monographs provide a reliable, but often overly conservative means of designing shells by using simple, linear analytical models and an empirical correction factor, referred to herein as a "knockdown factor." The format of the monographs was intended to satisfy the requirements of engineers and project managers concerned with the preliminary design of spacecraft. However, the amount of information presented in the NASA monographs is somewhat limited, and as a result, their range of applicability to the design of high-performance shell structures, such as those made of fiber-reinforced composite materials, is small.

Continued use of these NASA monographs by structural designers and technical specialists, and recent NASA experience with the development of launch vehicles and aircraft structures have indicated that the monographs on shell stability need to be updated and expanded. For example, the original NASA monographs contain practically no design information for lightweight, high-strength laminated composite shells subjected to mechanical or thermal loads. Such information could be used in the preliminary design of a high-speed civil transport aircraft or a single-stage-to-orbit reusable launch vehicle. The interest in updating the monographs is also influenced by the many advances in the state of the art of shell stability analysis that have taken place since the original monographs were published. Significant advances in computer technology and computational analysis tools since the late 1960's have made it possible to use much more sophisticated analytical models of nonlinear shell response. These tools have also enabled in-depth investigations of the effects of complicating structural details such as cutouts and other discontinuities on the buckling of shells and on their nonlinear behavior. In addition to advancements in analytical tools, many advancements have been made in experimental methods and techniques. For example, technology is now available to measure accurately the initial geometric imperfections of shell test specimens, and new combined-load test capabilities have been developed and used to provide more carefully controlled experiments and higher fidelity test results. Because of these technological advances and the large body of experimental data that has been amassed since the late 1960's, the development of modern versions of the shell stability monographs is being considered at Langley Research Center.

The present paper begins with a discussion of the approach commonly used to design buckling-resistant, thin-walled shells and describes how the approach evolved. Then, an overview of the NASA monographs on shell stability is given. Next, a discussion of some important issues that are presently confronting designers is presented, and two examples that illustrate some of these issues are described. The first example is the Space Shuttle superlightweight external liquid-oxygen (LO_2) tank. This contemporary thin-walled spacecraft structure was partially designed by using NASA SP-8007. The second example is a basic example that illustrates the effect of cutout size on the buckling behavior of a compression-loaded curved panel. Both examples illustrate shell behavior that is not addressed in the NASA monographs. The present paper includes a brief discussion of a state-of-the-art nonlinear shell analysis code and explains how it could be used to obtain a wide range of design information. In addition, a discussion of how to address design uncertainties and reliability in shell design is presented, and some suggestions for conducting future high-fidelity experiments are given. Finally, potential improvements to the NASA monographs on shell stability are discussed.

Common Approach to Stability Design

Prior to the late 1970's, the use of sophisticated analytical methods, such as the finite-element method, was not widespread, and shell stability calculations were done primarily with simple, specialized analytical models. These analytical models were typically formulated for regular geometries with uniform properties, uniform loading conditions, and uniform boundary conditions, and certain aspects of the response were neglected in order to obtain linear partial differential equations that could be solved readily. The simple analytical models typically neglected nonlinear prebuckling deformations, and simply supported boundary conditions were often used to reduce the computational effort needed to conduct parametric studies. This *linear* modeling approach, referred to more accurately as a linear bifurcation buckling analysis, came into use not only because of the computational considerations mentioned above, but also as the natural extension of the linear bifurcation buckling approach that had been used successfully for modeling columns and plates. Gradually, scientists and engineers learned that the buckling behavior of shells is fundamentally different from that of columns and plates.

The fundamental difference between the buckling behavior of columns and plates and the buckling behavior of shells was identified by von Kármán and Tsien (ref. 7) and was clarified by Donnell and Wan (ref. 8) and by Koiter (ref. 9). These references show that a major reason for the large discrepancy between the analytical predictions of shell buckling behavior and the corresponding experimental results is a sensitivity of shell buckling to initial geometric imperfections. This sensitivity was shown to be a consequence of the fact that shells are typically unstable at load levels equal to the bifurcation load. Because of the practical limitations of the analytical models and the sensitivity of shells to geometric imperfections, a stability design process evolved in which empirical "knockdown factors" were introduced to compensate for the differences observed between the results of theory and experiments. As part of this design process, a designer was faced with the need to conduct expensive experiments.

The NASA Monographs on Shell Stability

By 1960, many buckling tests of isotropic cylinders and curved panels had been conducted (e.g., see refs. 4, 5, and 6) as part of an effort by the technical community to establish a rational, practical approach for designing buckling-resistant shells. At that time, NASA conceived the shell stability monographs to make the results of these tests and many proposed tests for other shell geometries available to the aerospace structural design community and to establish practical design recommendations. The development of these monographs was a combined effort by members of industry, academia, and Langley Research Center. Much of the information given in these monographs is based on the research conducted by Seide, Weingarten, and Morgan (ref. 10). The initial emphasis on cylinders and cones and the format of the monographs were originally intended to satisfy the needs of engineers and project managers concerned with the preliminary design of launch vehicles and spacecraft. However, over time, it became evident that the monographs were also of great interest to structural stability specialists. The use of NASA SP-8007 was recently demonstrated in the shell analysis textbook by Vinson (ref. 11).

The NASA monographs provide design information in the form of empirical knockdown factors (referred to in the monographs as correction factors) and design recommendations for isotropic, orthotropic, ring- and stringer-stiffened, and sandwich shells. The important characteristics of various shell design problems, the sources of the design recommendations and their limitations, and discussions of how to proceed for cases with little known analytical and experimental data are also presented. In most cases, the knockdown factors are defined as empirical corrections to linear bifurcation buckling solutions for primarily elastic, simply supported shells. The knockdown factors are lower bounds to experimental data that were available at that time and are used to account for the large amount of scatter in the data. The knockdown factors consist of corrections that primarily account for initial geometric imperfections, nonlinear prebuckling effects associated with edge supports, and plasticity in some cases. The effects of edge boundary restraints (e.g., a simply supported versus a clamped boundary condition) are included in the knockdown factors so that edge restraints are treated as a random effect, in addition to the initial geometric imperfections. Plasticity correction factors are given only for cases in which there was a sufficient amount of data to characterize the behavior in a conservative manner. The basic recommendation given in the monographs is that any knockdown factor used for a design be substantiated by experiments. This recommendation applies for shell designs in which the restraint or boundary conditions are to be accounted for more accurately, or for designs with unusual surface geometries, modal interaction associated with optimization, cutouts, joints, or other irregularities, or where there are little or no test data and analytical results. A brief overview of the contents of each monograph follows.

NASA SP-8007 (1968 Revision)

The 1968 revision of NASA SP-8007 consists primarily of discussions of research studies and design recommendations for elastic, isotropic, cylindrical shells. However, some information is provided for orthotropic and sandwich cylinders. Design recommendations are presented for isotropic cylinders subjected to axial compression, pure bending, uniform lateral pressure, uniform hydrostatic pressure, torsion, and combined loading conditions. The uniform lateral pressure loading condition does not include the compressive axial load caused by pressure acting at the ends of a cylinder. In contrast, the uniform hydrostatic pressure loading condition includes the lateral pressure load and the compressive axial load. Design recommendations for cylinders that are subjected to combined loading conditions are limited almost entirely to isotropic shells. The combined loading conditions consist of axial compression and pure bending; axial compression and lateral pressure or hydrostatic pressure; axial compression and torsion; internal pressure and axial compression; internal pressure and pure bending; and internal pressure, axial compression, and pure bending loads.

Design recommendations and buckling formulas that are lower bounds to experimental data for a wide range of radius-to-thickness ratios are given for isotropic cylinders subjected to axial compression or pure bending loads. For cylinders loaded by lateral or hydrostatic pressure, a single knockdown factor, which is a lower bound to the corresponding experimental data, is given for shells that buckle with more than two circumferential waves. An additional empirical knockdown factor is given for long shells that buckle into a one-half-wave oval shape. For torsion loads, a single knockdown factor that is a lower bound to the corresponding experimental data is given for moderately long cylinders. Because of limited experimental verification, design recommendations are given in the form of conservative, linear buckling interaction equations for shells subjected to combined axial compression and pure bending loads, combined axial compression and lateral pressure loads or hydrostatic pressure loads, and combined axial compression and torsion loads. For shells subjected to combined internal pressure and axial compression or combined internal pressure and pure bending loads, the buckling load is expressed as a combination of the load caused by the internal pressure, the buckling load for the unpressurized shell (including the appropriate knockdown factor), and an increase in the buckling load associated with the reduction in imperfection sensitivity caused by the internal pressure. Empirically determined increases in the buckling load, which are associated with the reduced imperfection sensitivity, are given for moderate ranges of internal pressures and radius-to-thickness ratios. Conservative, linear buckling interaction equations are also given for shells subjected to combined internal pressure, axial compression, and pure bending loads.

Results are also presented in NASA SP-8007 for elastic, orthotropic cylindrical shells subjected to axial compression, pure bending, uniform hydrostatic pressure, uniform lateral pressure, or torsion loads, and to combined axial compression and bending loads. The term "orthotropic" is used to indicate single-layer and multilayer composite monocoque shell wall constructions and stiffened shell wall constructions for which the rings and stringers are perpendicular. These results consist primarily of design recommendations because of the small amount of experimental data for orthotropic cylinders that was available at the time. Formulas for computing homogenized ("smeared") elastic, orthotropic stiffnesses for multilayered stiffened cylinders, isotropic stiffened cylinders, and ring-stiffened corrugated cylinders are presented.

An empirical formula for knockdown factors is presented for monocoque orthotropic cylinders loaded by axial compression. This formula is based on a small amount of experimental data and has a very limited range of validity. A similar formula is given for cylinders loaded by pure bending. A single knockdown factor, which is based on a small amount of experimental data, is given for cylinders that are subjected to axial compression or pure bending loads and that have closely spaced, moderately large stiffeners. A single knockdown factor that is also based on a small amount of experimental data is suggested for cylinders loaded by lateral or hydrostatic pressure or by torsion loads. In addition, because of a small amount of experimental data, a conservative, linear buckling interaction formula is suggested for use with cylinders loaded by combined axial compression and pure bending loads.

Design recommendations for sandwich cylinders with isotropic face sheets and with either an isotropic or an orthotropic core are also presented in NASA SP-8007. Design recommendations are given for shells loaded by axial compression, pure bending, uniform lateral pressure, or torsion loads. Knockdown factors are given only for shells with cores that have high transverse shear stiffness, and practically no experimental validation is described.

Analytical results and design recommendations are also presented in NASA SP-8007 for isotropic cylindrical shells that have an elastic core and that are subjected to axial compression, uniform lateral pressure, or torsion loads, or to combined axial compression and lateral pressure loads. Based on experimental data, the knockdown-factor formula given for compression-loaded cylinders without an elastic core is recommended for use with cylinders that have an elastic core. For cylinders loaded by lateral pressure, a single knockdown factor is given that is a lower bound to the corresponding experimental data. For the cylinders loaded by torsion, only design recommendations are given. Similarly, a conservative linear buckling interaction formula is recommended for cylinders loaded by combined axial compression and lateral pressure loads.

NASA SP-8019

NASA SP-8019 consists primarily of design recommendations for elastic, isotropic, conical shells subjected to axial compression, pure bending, uniform hydrostatic pressure, torsion, or combined loads. The design recommendations for cones subjected to combined loads are given for isotropic shells only. The combined loads consist of internal pressure and axial compression; internal pressure and pure bending; axial compression and pure bending; internal pressure, axial compression, and pure bending; uniform hydrostatic pressure and axial compression; torsion and uniform hydrostatic pressure; and torsion and axial compression.

Design recommendations and a single empirical knockdown factor that is a lower bound to experimental data are given for each of the single-component loading conditions. Only conservative design recommendations based on rational arguments are given for loading conditions that consist of combined internal pressure and axial compression and combined internal pressure and pure bending because of the very small amount of experimental data and the lack of analytical results that were available at the time. Conservative, linear buckling interaction equations based on experimental results are given for all other combined load conditions.

Results are also presented in NASA SP-8019 for elastic, orthotropic conical shells (constant-thickness orthotropic material and stiffened shells) subjected to uniform hydrostatic pressure or to torsion loads. These results consist primarily of design recommendations because of the very small amount of experimental data that was available at the time. Similarly, only design recommendations are given for sandwich cones with isotropic or orthotropic face sheets and with either an isotropic or orthotropic core.

NASA SP-8032

NASA SP-8032 consists primarily of discussions of research studies and results for elastic, isotropic, doubly curved shells. Design recommendations are given for spherical caps that are loaded by uniform external pressure, by a concentrated load at the apex, or by a combination of these loads. Buckling formulas that are lower bounds to experimental data are given for clamped spherical caps that are loaded by uniform external pressure or by a concentrated load at the apex. A lower-bound, empirical buckling formula is given for spherical caps that are loaded by a concentrated load at the apex and that have edges that are free to rotate and to expand in the direction perpendicular to the axis of revolution. No conclusive experimental results are given for spherical caps that are loaded by combined uniform external pressure and a concentrated load at the apex.

Design recommendations are also discussed for complete prolate and oblate spheroidal shells subjected to uniform external pressure and for complete oblate spheroidal shells subjected to uniform internal pressure. A single knockdown factor is given for the prolate spheroidal shells, and a lower-bound, empirical buckling formula is given for the oblate spheroidal shells. No experimental validation is given for the results for the oblate spheroidal shells subjected to uniform internal pressure. Design recommendations are also discussed for oblate spheroidal and torispherical bulkheads that have clamped edges and that are subjected to uniform internal pressure. An empirical knockdown factor is given for the torispherical bulkheads; however, no experimental validation is given for the oblate spheroidal bulkhead.

Design recommendations are discussed, and results are given for complete circular toroidal shells subjected to uniform external pressure, and for shallow, equatorial segments of complete toroidal shells. The toroidal shell segments, which consist of barrel-shaped shells that are bowed outward from the axis of revolution (positive Gaussian curvature) and waisted shells that are bowed inward (negative Gaussian curvature), are subjected to axial tension, to uniform lateral pressure, or to uniform hydrostatic pressure loads. Experimentally verified analytical results are given for complete circular toroidal shells for a small range of geometric parameters.

Similarly, an experimentally verified knockdown factor is given only for equatorial segments of toroidal shells that are loaded by axial tension and that are truncated hemispheres.

Essentially no experimentally validated design information is given for orthotropic shells or for sandwich shells that are doubly curved. Rational arguments are used to present design recommendations for specially orthotropic shells due to the absence of experimental data. No design recommendations are given for sandwich shells.

Shell Stability Issues

To adequately design a lightweight, buckling resistant, thin-walled shell structure, designers must understand several important shell stability issues, most of which are not addressed in the NASA monographs. Some of these issues are listed as follows, and a few are discussed subsequently.

Initial geometric imperfections
Nonlinear prebuckling deformations
Cutouts and joints
Boundary conditions
Load introduction effects
Thickness variations
Variation in material properties
Stiffener spacing
Local reinforcement
Combined loads
Variation of loads with time
Small vibrations
Laminate construction
Transverse shear deformation
Sandwich construction
Inelasticity and damage
Local eccentricities

Initial Geometric Imperfections

Sensitivity to initial geometric imperfections and the effects of nonlinear prebuckling deformations are two major issues in the design of isotropic shells. Experience has shown that initial geometric imperfections with a maximum amplitude on the order of one wall thickness can cause a reduction in the buckling load of a shell that is on the order of 60 percent of the buckling load calculated for the corresponding geometrically perfect shell. Thus, designing a minimum mass shell structure to be buckling resistant is a difficult task because a designer usually does not know the initial geometric imperfection shape and amplitude in advance. Because of this lack of knowledge, an assumed imperfection shape must be used to determine analytically a knockdown factor, or the design must be based on a knockdown factor that corresponds to the lower bound to the known relevant experimental data. Often, these data do not exist. In some cases, however, the shell manufacturing process may consistently produce a known imperfection shape with a known maximum amplitude. If so, this information can be used to determine a knockdown factor analytically.

Nonlinear Prebuckling Deformations

Nonlinear prebuckling deformations of shells are generally caused by the interaction between the compressive stresses in a shell and any localized bending deformations that arise, for example, from support conditions or from discontinuities in stiffness that are caused by abrupt changes in thickness or joints. The significance of the nonlinear prebuckling deformations was first identified by Stein for compression-loaded isotropic cylinders (refs. 12 and 13). As an isotropic cylindrical shell is compressed axially, it expands outward radially. At the supported edges, however, the radial expansion is restrained, which produces local bending deformations whose extent along a generator depends on the cylinder radius and thickness. A similar condition exists for compression-loaded isotropic truncated conical shells where the extent of local bending deformations along a generator also depends on the vertex angle. Generally, as a cone gets flatter, the extent of the boundary bending deformations grows. The local bending deformations that occur around a relatively large cutout in a compression-loaded cylinder or curved panel are another example of nonlinear prebuckling deformations. These bending deformations are manifested by the coupling between the in-plane and out-of-plane displacements in the strain-displacement relations for curved panels or shells.

A very important consequence of substantial nonlinear prebuckling deformations is that a linear bifurcation solution and a knockdown factor may be inadequate and uncharacteristic of the actual nonlinear response. One simple example of this deficiency is illustrated by the behavior of a ring-stiffened cylindrical shell loaded by axial compression or by external pressure (refs. 14 and 15). For these shells, a linear bifurcation analysis may not only overpredict the buckling load, but may also predict an incorrect buckling mode. Another, more complicated example is presented in reference 16 for the Space Shuttle superlightweight LO_2 tank shown in figure 1 and is discussed in the Examples section.

Cutouts

The effects of a cutout on the buckling behavior of a shell are another important shell stability issue for designers. The presence of a cutout may significantly alter the prebuckling stress distribution in a shell, depending on the type of loading and the cutout size, and may reduce its buckling load significantly. In addition, nonlinear prebuckling deformations that are local bending deformations near the cutout, may be present and can significantly affect the characteristics of the buckling behavior. A cutout may also have a significant effect on the imperfection sensitivity of a shell because as the cutout size increases, the amount of material removed by the cutout region, where imperfections may be very important, is reduced. Some effects of cutouts on the behavior of compression-loaded curved panels are also discussed in the Examples section.

Laminate Construction

Approximately 25 years ago, researchers realized that there is a great potential for reducing structural weight by using fiber-reinforced composite materials for structures. The increased use of composite materials for shell structures has led to additional shell stability issues for designers. For example, the effects of laminate construction (including sandwich construction) and transverse shear deformations on imperfection sensitivity are not well understood. Transverse shear flexibility tends to reduce the effective stiffness of a structure and can reduce its buckling load. Similarly, knowing that laminated shell wall construction can greatly affect the attenuation length of bending deformations implies that the effects of nonlinear prebuckling deformations may be severe for some laminate constructions.

Examples

The common approach to stability design described previously in the present paper is often used by industry in the preliminary design of shell structures. However, in some cases, the results of a linearized stability problem may not adequately represent the underyling physics of the actual response. Two examples that illustrate this potential pitfall are presented in this section. The first example is the Space Shuttle superlightweight LO_2 tank. This example of a contemporary thin shell structure that is subjected to combined loads illustrates complex nonlinear behavior that is dominated by local bending deformations. The second example is a much simpler "subcomponent-level" example, that is, a compression-loaded curved panel with a cutout. Because cutouts appear in nearly every kind of aerospace vehicle structure, designing properly for their effects on the buckling resistance of shells is very important. These two examples illustrate some physical behaviors that are not commonly understood and that are representative of problems that are dominated by effects that are currently not addressed in the NASA monographs.

Space Shuttle Superlightweight LO_2 Tank

The Space Shuttle consists of the orbiter, two solid rocket boosters (SRB's), and the external tank (ET), as shown in figure 1. The external tank consists of a LO_2 tank, a liquid hydrogen (LH_2) tank, and an intermediate structure called the intertank (fig. 1). Currently, NASA is engaged in the flight certification of a newly designed LO_2 tank that is referred to as the superlightweight LO_2 tank. This new LO_2 tank is significantly lighter than the one presently in service, and its buckling behavior is a significant concern in its design. The superlightweight LO_2 tank is a thin-walled monocoque shell that is made primarily of 2195 aluminum-lithium alloy. It consists of a nose cone, a forward ogive section, an aft ogive section, a cylindrical barrel section, and an aft elliptical dome section, as shown in figure 1. The intertank (fig. 1) is a right circular cylinder that is made from 2090 and 7075 aluminum alloys. Details and dimensions of the LO_2 tank and the intertank are given in reference 16.

An important loading condition that is illustrated by this example is the prelaunch loading condition for which the LH_2 and LO_2 tanks are full. Compressive stresses are present in the ogive sections of the (monocoque) LO_2 tank directly above the solid rocket booster attachment points for this loading condition. These compressive stresses are caused by the weight of the filled LH_2 and LO_2 tanks that is reacted at the two SRB attachment points. Both linear bifurcation and nonlinear analyses are presented in detail in reference 16. These results, which were obtained by using the Structural Analysis of General Shells (STAGS) nonlinear structural analysis code (ref. 17), are described briefly as follows.

The linear bifurcation solution yields a critical buckling load factor of $p_a = 3.78$, where a value of $p_a = 1.0$ corresponds to the magnitude of the operational loads. The corresponding buckling mode is shown in figure 2 and consists of a short-wavelength buckle in the forward part of the aft ogive that is essentially a wrinkle in the skin. The shortness of the wavelength is caused by the hoop tension that resists the LO_2 pressure.

Results of nonlinear analyses presented in reference 16 are reproduced in figures 3 and 4. The solid lines shown in figure 3 represent the normal displacements along the length of the aft ogive shell wall for values of the applied load factor p_a approximately equal to 3.0, 4.0, and 5.0. Overall, negative values of the normal displacements are indicated by the left-hand-side ordinate for these three lines because of contraction of the aft ogive that is caused primarily by the LO_2 thermal load. The linear bifurcation mode is represented in the figure by the dashed line with the normalized amplitude given by the right-hand ordinate of the figure. The solid lines shown in figure 3 indicate a short-wavelength bending response in the aft ogive over the SRB attachment point (fig. 2) that is similar in shape to the corresponding linear bifurcation mode shape. The overall slope of the solid lines (obtained by fitting a straight line to each curve) is a result of outward displacements of the shell wall (indicated by less negative values) that are caused by the internal pressure and that are represented by a nonlinear analysis. This effect is not represented in the prebuckling stress state that is used in a linear bifurcation buckling analysis and, as a result, does not affect the overall slope of the dashed line.

The results presented in figure 3 predict a stable nonlinear response at load levels greater than the buckling load predicted by a linear bifurcation analysis. As the applied load increases, substantial bending deformations (indicated by the waviness of the curves) develop and grow in the shell wall. These bending deformations reduce the apparent meridional stiffness of the aft ogive. The nonuniformity of the bending deformations is caused by thickness variations in the ogive and the presence of circumferential weld lands. Similar results are presented in reference 16 which indicate that a geometric imperfection with a small negative amplitude and with the shape of the linear bifurcation mode greatly increases the severity of the stable bending deformations. This imperfection causes the growth of the bending deformations to begin at much lower load levels than the linear bifurcation buckling load.

The reduction in the apparent meridional stiffness of the aft ogive is shown more explicitly in figure 4. In this figure, the intensities of the largest bending deformations (indicated by the largest magnitude of the normal displacement amplitude) for the geometrically perfect shell and a geometically imperfect shell are given as a function of the load factor p_a. The amplitude Δw shown in figure 4 is the distance from the maximum value of the shell-wall displacement to the adjacent minimum value and represents the intensity of the local bending deformation in the response. The filled circles in the figure correspond to results for a geometrically perfect shell, and the unfilled squares correspond to results for geometrically imperfect shells with an imperfection-amplitude-to-wall-thickness ratio of A/t = 0.3 (t = 2.540 mm (0.100 in.)). The horizontal dashed line in the figure represents the linear bifurcation buckling load level.

The results presented in figure 4 indicate that the amplitude of the greatest local bending deformation grows with increasing load and that the amount of growth increases substantially with increasing geometric imperfection amplitude. The results predict that the shell can support loads greater than the critical buckling load predicted by the linear bifurcation analysis. Most importantly, the results show that the linear bifurcation analysis does not represent accurately the mechanics of the actual shell response. Moreover, a design based on the linear bifurcation analysis and a knockdown factor that was determined by using an intuitive approach likely would be overly conservative.

Compression-Loaded Curved Panel With a Cutout

Several tests of compression-loaded 6061-T6 aluminum singly curved panels with a central circular cutout were conducted at Langley Research Center. The panels had a nominal radius of curvature of R = 152.4 cm (60 in.) and a nominal thickness of t = 2.54 mm (0.10 in.). The length and arc-width of the panels were approximately 37.47 cm (14.75 in.) and 36.83 cm (14.5 in.), respectively. The panels were loaded slowly in axial compression by uniformly displacing the two opposite curved edges with a 1334-kN (300-kip)-capacity hydraulic testing machine. The loaded ends of a panel were clamped, and the unloaded edges were simply supported by a test fixture. The length and arc-width of the panels between the inside edges of the test fixture (unsupported area) were both 35.56 cm (14.0 in.). Electrical resistance strain gauges were used to measure strains, and direct current differential transformers were used to measure axial displacements and displacements normal to the panel surface. Shadow moiré interferometry was also used to monitor displacements normal to the panel surface.

Experimental results for load versus end shortening are presented in figure 5. The load is nondimensionalized by the linear bifurcation buckling load for a panel without a cutout P^{o}_{bif} = 62,988 N (14,161 lb) that was obtained from STAGS. This buckling load is based on a length L = 35.56 cm (14.0 in.), an arc-width W = 35.56 cm (14.0 in.), a nominal thickness of t = 2.54 mm (0.1 in.), a Young's modulus of E = 72.4 GPa (10.5×10^6 psi), and a Poisson's ratio of ν = 0.33. The end-shortening Δ is nondimensionalized by the nominal panel thickness t. The dashed line in the figure corresponds to a panel without a cutout, and the solid lines correspond to panels with cutout-diameter-to-panel-width ratios d/W = 0.3, 0.4, and 0.5.

The experimental results presented in figure 5 indicate that the character of the nonlinear response of a panel changes significantly as the cutout size increases. For example, the results indicate that the panels with d/W = 0 and 0.3 exhibit buckling behavior that involves a dynamic change from one stable equilibrium configuration to another. Similar results, not shown in the figure, were obtained for panels with d/W = 0.1 and 0.2. The results in figure 5 also indicate that the panels with d/W = 0.4 and 0.5 do not exhibit this type of behavior but exhibit stable, monotonically increasing nonlinear responses. The results show that the intensity of the dynamic buckling process decreases substantially as d/W increases from a value of 0 to 0.3. The intensity of the dynamic buckling response is indicated by the difference between the buckling load and the lowest stable postbuckling load.

The results presented in figures 6 through 9 provide additional insight into the effect of cutout size on the character of the nonlinear response. The results in these figures are shadow moiré patterns on the convex or outer surface of the panels. The shadow moiré patterns for the panel without a cutout are shown in figure 6 for values of P/P^{o}_{bif} = 0.86 (just before buckling) and 0.57 (just after buckling). The top pattern in figure 6 indicates that no significant nonlinear prebuckling deformations are present. This finding is consistent with the straightness of the initial portion of the dashed line shown in figure 5. The bottom pattern in figure 6 indicates that the stable postbuckling mode shape consists of a single half-wave along the panel length and across the panel width. The radial displacements of this postbuckling mode are inward.

Shadow moiré patterns for the panel with a cutout with d/W = 0.3 are shown in figure 7 for values of P/P^{o}_{bif} = 0.72 (just before buckling) and 0.67 (just after buckling). The top pattern in figure 7 indicates that significant nonlinear prebuckling deformations occur around the cutout, which is consistent with the deviation from straightness of the initial portion of the solid line shown in figure 5 for d/W = 0.3. The radial deformations around the cutout are outward. The bottom pattern in figure 7 indicates that the stable postbuckling mode shape consists of an outward deformation pattern on the left-hand side of the cutout, similar to the nonlinear prebuckling deformation pattern shown on the left side of the top pattern in the figure, and an inward buckle on the right-hand side of the cutout. This buckle consists of approximately a single half-wave along the panel length and across the panel half-width.

Shadow moiré patterns for the panel with a cutout with d/W = 0.4 are shown in figure 8 for values of P/P^{o}_{bif} = 0.46 and 0.71. The patterns in figure 8 and the corresponding curve in figure 5 indicate that significant outward nonlinear prebuckling deformations around the cutout dominate the response. There is no dynamic buckling response for this panel. Similarly, the shadow moiré patterns for the panel with a cutout with d/W = 0.5 that are shown in figure 9 for values of P/P^{o}_{bif} = 0.50 and 0.70, and the corresponding curve in figure 5 indicate the same type of response.

In summary, this simple example illustrates a response for compression-loaded curved panels that is typically not well understood, is not considered by designers, and is not addressed in the NASA monographs. The response trends change with loading, boundary conditions, and material systems, such as a laminated composite system. How these trends affect the cutout size at which the response changes its character is generally unknown. Information of this kind would be a valuable contribution to an updated shell design monograph.

Concept for New Design Recommendations

Development of new, expanded versions of the NASA monographs is now possible because of significant technological advances and advances in the understanding of shell stability. In particular, advances in computers and analysis tools have increased greatly the ability to solve complex shell stability problems. Thus, a brief description of the capabilities of an advanced, state-of-the-art analysis tool that could be used to obtain a wide range of analytical results that could be included in expanded versions of the NASA monographs is presented in this section.

Before embarking on an endeavor to revise the NASA monographs, a two-part question remains to be addressed; that is, "What kind of an approach to stability design should be used, and how should problem uncertainties be addressed?" A basic, first-approximation answer to this question is suggested later in this section. The approach is based on the premise that many of the shell response parameters are not necessarily probabilistic in nature and that a completely probabilistic approach may tend to obscure the physical understanding of behavior. Thus, a hybrid approach to shell stability design is under consideration and will be discussed briefly in this section.

Another major consideration in the formulation of new design recommendations for a revised set of NASA monographs is experimental testing. With shell buckling behavioral trends established analytically, selective experiments can be identified and conducted to establish credible design recommendations. This selective testing approach, made possible by advanced analysis tools, is particularly important when considering the costs of conducting experiments and the costs of test specimens such as those made of fiber-reinforced composite materials. Moreover, to establish the best possible design recommendations, it is imperative to use high-fidelity experimental results. This step is necessary to prevent the introduction of excessive conservatism through the use of poor-quality experimental results. Some suggestions on how to obtain high-fidelity experimental results are also given in this section. Finally, some specific suggestions for improving the NASA monographs are presented.

Capabilities of an Advanced Analysis Tool

Advances in the finite-element method during the last 15 years have improved the capability for analyzing complex nonlinear shell problems and for obtaining accurate buckling and nonlinear response predictions. For example, an advanced, state-of-the-art structural analysis code has been used to conduct in-depth nonlinear analyses of the Space Shuttle superlightweight LO_2 tank (refs. 16 and 17). This code was chosen for analyzing this problem because of its robust state-of-the-art nonlinear-equation solution algorithms and its general user-input capability that is convenient for modeling branched shells typically used for launch vehicles. The code uses both the full and modified Newton methods to obtain an accurate nonlinear solution, and large rotations in a shell are represented by a co-rotational algorithm at the element level. The Riks arc-length projection method is used to continue a solution past limit points, and the Thurston (ref. 18) equivalence transformation processor is used for solution-branch switching in the vicinity of a bifurcation point. The code also permits complex geometries, loading conditions, boundary conditions, and initial geometric imperfections to be modeled in a direct manner by using user-written subroutines. These subroutines are essentially independent of the mesh discretization and provide analysts with a great deal of flexibility for modeling complex structural configurations (e.g., see ref. 16) and conducting mesh refinement studies.

Advanced analysis tools with the capabilities mentioned above make it possible to determine accurate analytical estimates of the sensitivity of a shell buckling load to initial geometric imperfections or other destabilizing irregularities. Thus, state-of-the-art nonlinear shell analysis codes can be used to establish shell buckling behavioral trends deterministically for a wide range of system parameters and to identify any unusual, possibly unexpected nonlinear behavior that designers should consider.

Basic Approach to Stability Design

Modern, high-fidelity nonlinear shell analysis codes, such as STAGS, have enabled accurate predictions of the nonlinear response and buckling loads of thin-shell structures. The response of a shell can be determined accurately when its dimensions and properties are known to sufficient precision. For example, the effects of initial geometric imperfections can be dealt with by measuring the true shape of the shell and by modifying the shell analysis model to represent the true measured geometry. Such deterministic analyses are valuable for identifying and isolating important contributions to the nonlinear response and for systematically quantifying the effects of changes in structural and material design parameters.

The reliability of current shell design procedures can be improved by using these more accurate deterministic tools, provided that accurate information on the dimensions and material properties is available. If some dimensions and properties are not well known, however, it should be possible to modify the design process to include such uncertainties. By coupling a probabilistic representation of uncertain dimensions, tolerances, and material properties with a deterministic analysis that incorporates the better-known parts of the design problem, a hybrid design process could be developed. A typical result of the process might be a stiffened shell with a prescribed buckling load, complete with a rationally obtained confidence interval. The hybrid approach could also serve as the basis for a reliability-based design procedure.

Suggestions for Future Experiments

The determination of meaningful knockdown factors for shell buckling depends greatly on high-fidelity experimental results. Some of the scatter in the post-1930's test data for buckling loads of isotropic cylindrical shells can be attributed to nonuniform load introduction or to a poor simulation of the boundary conditions by the test fixture. When questionable test results are used to determine knockdown factors from lower bound curve fit approximations to the test data, the knockdown factor is likely to be overly conservative. Thus, it is very important to know the pedigree of a given set of test data.

To obtain high-fidelity experimental results, several issues must be addressed and several tasks must be performed. Prior to conducting an experiment, initial geometric imperfections of the shell surface, the wall thickness distribution, unevenness of the loaded edges, and the material properties should be measured. Knowledge of these quantities is extremely important for obtaining good correlation between theory and experiment. The instrumentation for a test should be planned adequately to facilitate the correlation between theory and experiment and to provide enough data to help one understand the expected behavior. The data sampling rate should be high enough to capture adequately the shell response. The instrumentation should include back-to-back strain gauges for monitoring bending strains and local nonlinear deformations; direct-current differential transformers (DCDT's), or other similar devices, for monitoring displacements normal to the shell surface; and shadow moiré interferometry for qualitatively monitoring buckle patterns. In many cases, the amount and type of instrumentation needed can be determined from preliminary analyses. It is important to reiterate that for some shell stability problems, a linear bifurcation analysis may not adequately represent the shell behavior, and as a result, may be inadequate for planning instrumentation. For experiments that involve load introduction by displacing a platen of a loading machine, proper alignment of the platens should be verified, and DCDT's, or other similar devices, should be used to define the plane of the loading platen and to detect any load introduction anomaly. The loaded edges of compression-loaded shells should be measured to ensure that the edges are as close to flat and parallel as possible. A loading rate that is consistent with the goals of the test should be selected. Details of the test fixture and its relationship to the desired boundary conditions should be clearly defined when reporting test data; all instrumentation locations that correspond to the reported results should be indicated clearly.

For experiments that involve thermal loading or combined mechanical and thermal loading, additional issues must be considered. An in-depth discussion of several of these issues has been presented by Blosser (ref. 19), and some of the information needed to characterize experimental results adequately is summarized as follows. First, the temperature distribution of the structure and its test fixture, as well as the heat flux at all the surfaces, needs to be recorded adequately to facilitate the correlation between theory and experiment. In addition, any difference in coefficient of thermal expansion of the specimen and the test fixture, any heating of the loading platens, and all locations of insulated surfaces and heat conduction paths should be recorded. Complete descriptions of the thermal test fixture components, including coolant passages and cavities, should be given, and any interaction of the thermal components with the components used to introduce mechanical load should be identified. Other important details that should be recorded are the air temperature in the area surrounding the test specimen, the method of heating or cooling used for the specimen and test fixture, and changes in material properties of the specimen and test fixture with temperature.

Potential Improvements to the NASA Monographs

Certainly one of the most significant improvements to the NASA monographs would be the inclusion of design recommendations for laminated composite shells that are based on the analytical and experimental studies that have been conducted over the past 25 years. Another improvement would be to base knockdown factors on accurate analytical models of "nominally perfect" shells (such as shells free of initial geometric imperfections and material variances) that include the proper boundary conditions (as opposed to only simply supported boundary conditions, which are used to a large extent in the current monographs) and possibly the effects of nonlinear prebuckling deformations. These tasks can be done for a wide range of parameters by using specialized codes such as BOSOR4 and DISDECO, which compute bifurcation buckling loads that include the effects of nonlinear prebuckling deformations and various boundary conditions by solving a nonlinear eigenvalue problem (refs. 20 and 21). Isolating the effects of nonlinear prebuckling and boundary conditions are essential steps to understanding the shell behavior and to obtaining reliable knockdown factors that are not overly conservative.

Another significant improvement to the NASA monographs would be to establish practical nondimensional parameters that contain the appropriate geometric and material variables and that enable concise representations of behavioral trends and sensitivity of the response to variations of the parameters (e.g., see ref. 22). Guidelines for including damage tolerance and the sensitivity of a design to load introduction effects would be valuable additions to the monographs. One of the most significant improvements that can be made immediately is to provide insight into, and quantitative results for, the true nonlinear interaction of combined loads that has been treated very conservatively in the NASA monographs as a linear interaction. Furthermore, providing design recommendations for thermal loads and for combined mechanical and thermal loads would be a significant improvement.

Another issue that must be addressed to obtain a new set of useful and practical design monographs is design uncertainties. A significant contribution to this area can be made by providing guidelines for determining which shell stability issues are more adequately handled in a deterministic rather than in a probabilistic manner. From a practical viewpoint, this information indicates approximately the number of experiments and analyses needed to establish meaningful design recommendations and reliable, but not overly conservative, knockdown factors. Ultimately, the improvements to the NASA monographs should be focused on the practical needs of industry structural designers and chief engineers and should reflect the scientific advances that have been made over the last 25 years. The end result of such an effort would be a collection of scientifically based knockdown factors and design recommendations.

Concluding Remarks

A summary of the existing National Aeronautics and Space Administration (NASA) monographs for the design of buckling resistant thin-shell structures has been presented. Improvements in the analysis of nonlinear shell response have been reviewed, and current issues in shell stability analysis have been discussed. Examples of nonlinear shell responses that are not included in the existing NASA shell design monographs have been presented, and an approach for including reliability-based analysis procedures in the shell design process has been discussed. Suggestions for conducting future shell experiments to obtain high-fidelity results have been presented, and proposed improvements to the NASA shell design criteria monographs have been discussed.

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Hampton, VA 23681-2199
November 3, 1997

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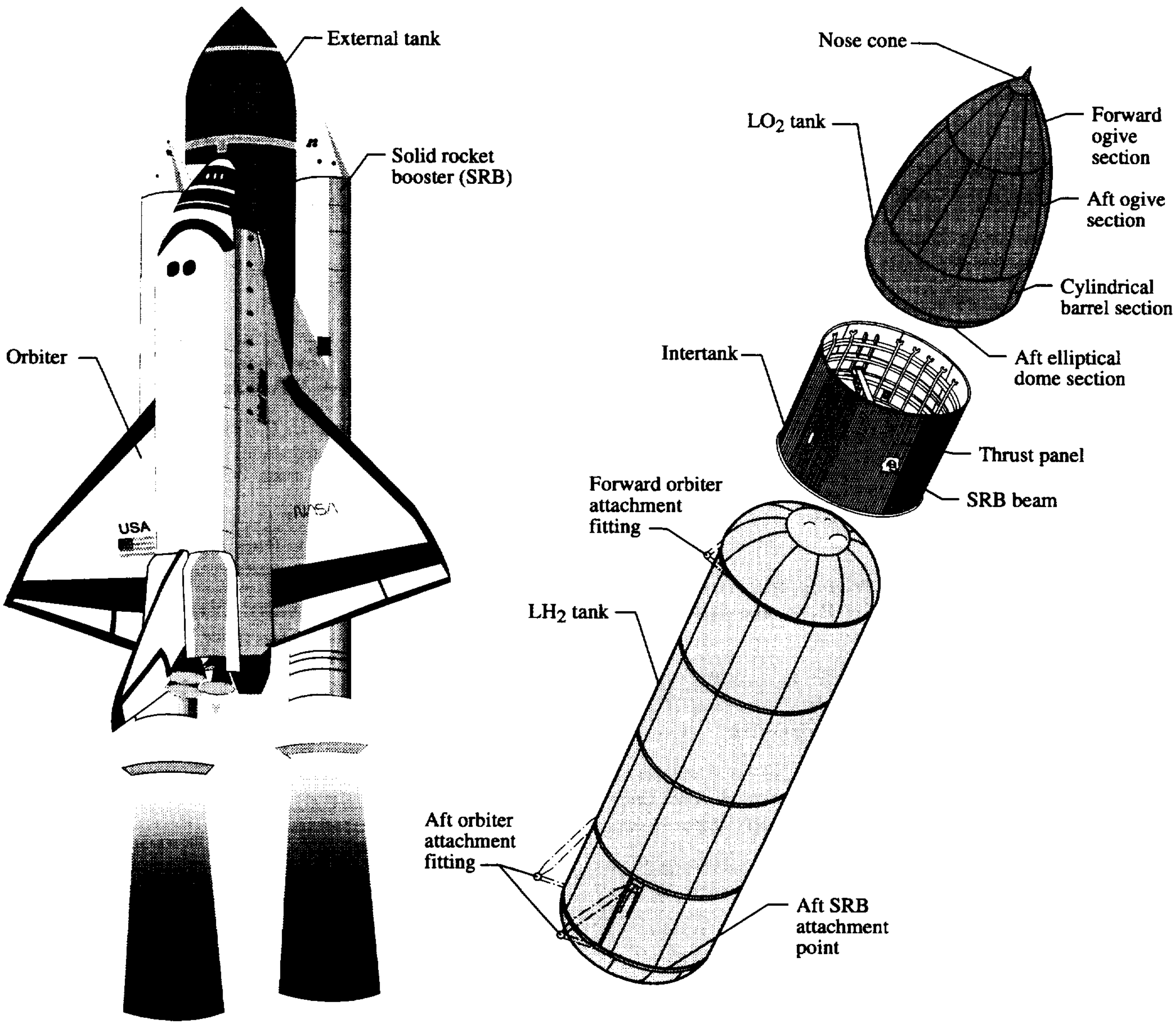


Figure 1. Space Shuttle external tank components.

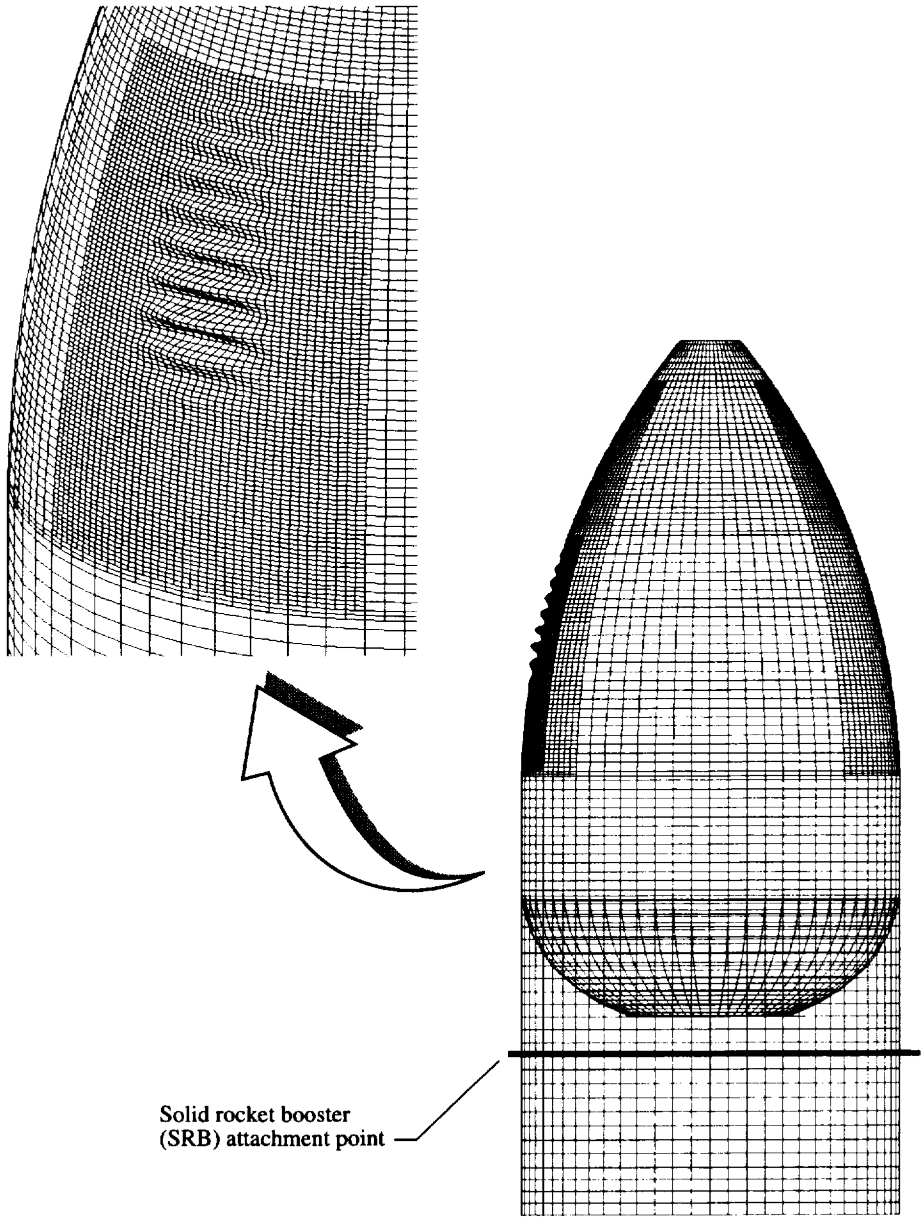


Figure 2. Linear bifurcation buckling mode for a 99000 degree-of-freedom model (load factor $p_a = 3.78$).

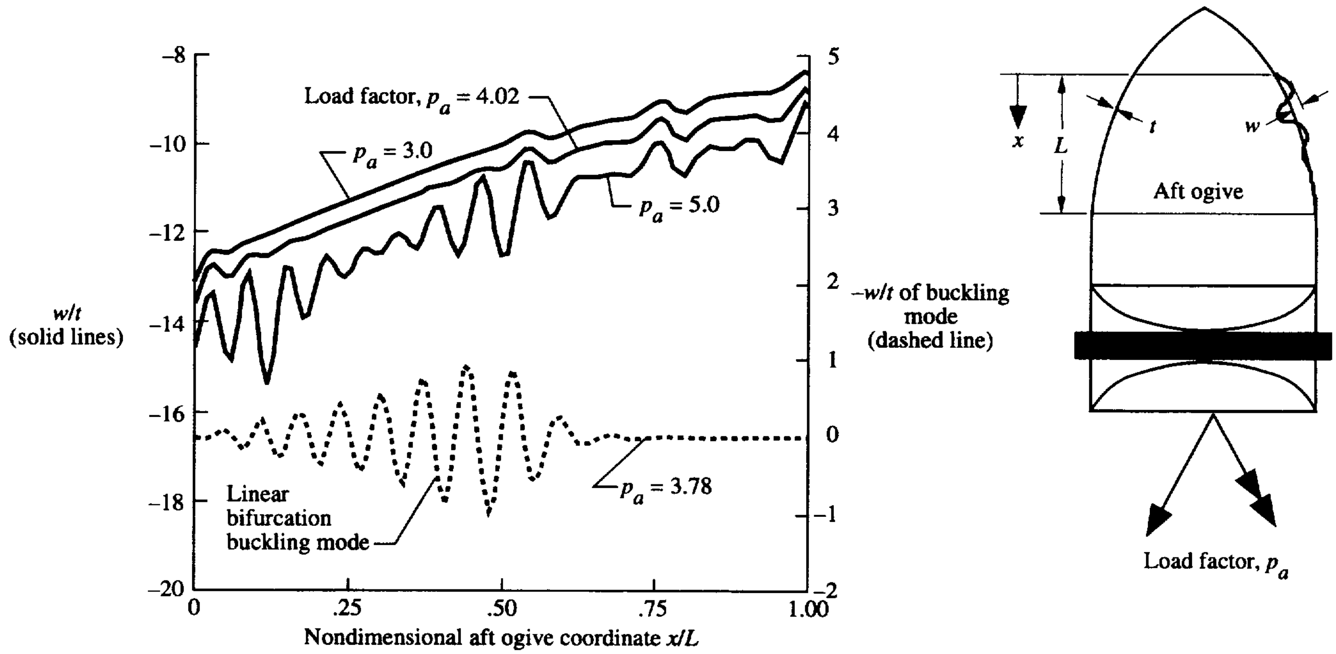


Figure 3. Predicted nondimensional normal displacement w/t of aft ogive of a geometrically perfect shell for increasing LH_2 interface loads.

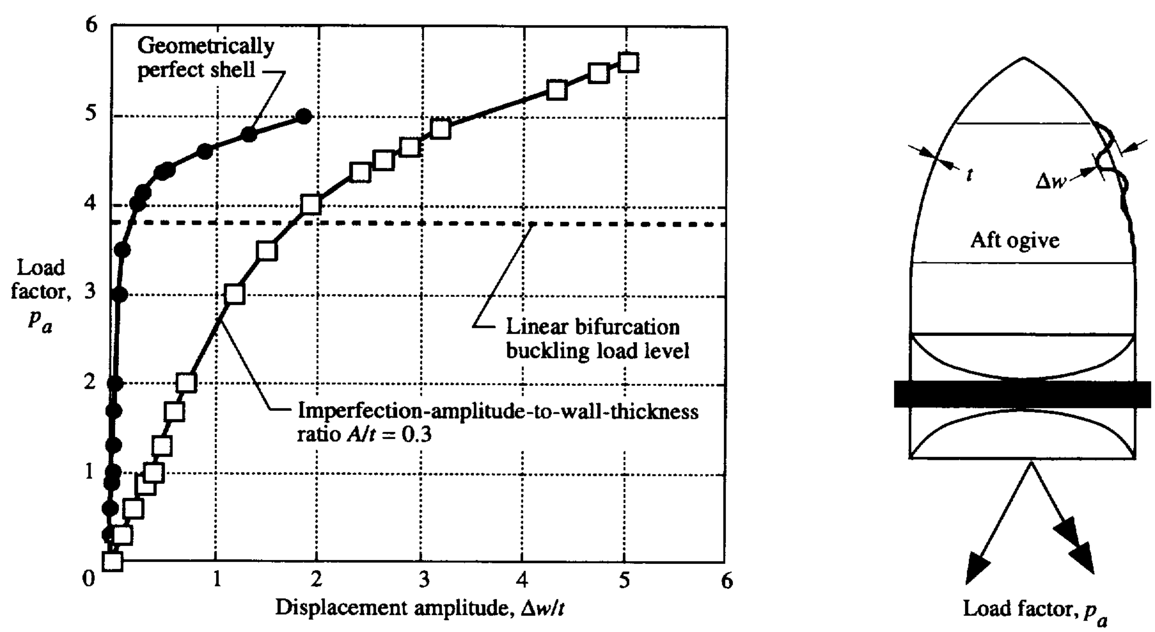


Figure 4. Predicted local nondimensional displacement amplitude $\Delta w/t$ of aft ogive surface for increasing LH_2 interface loads; geometrically perfect and geometrically imperfect shells.

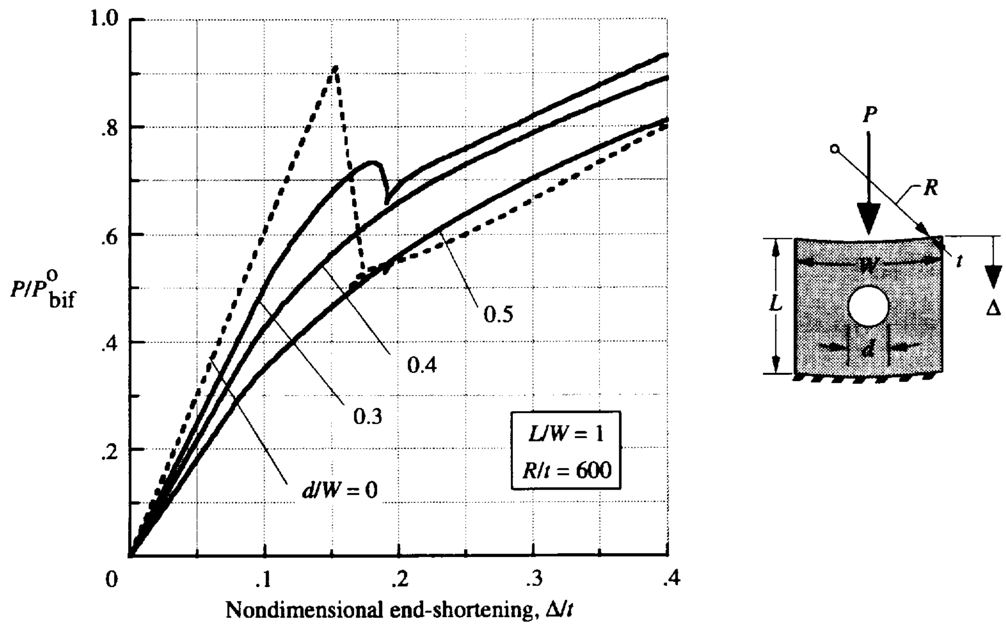
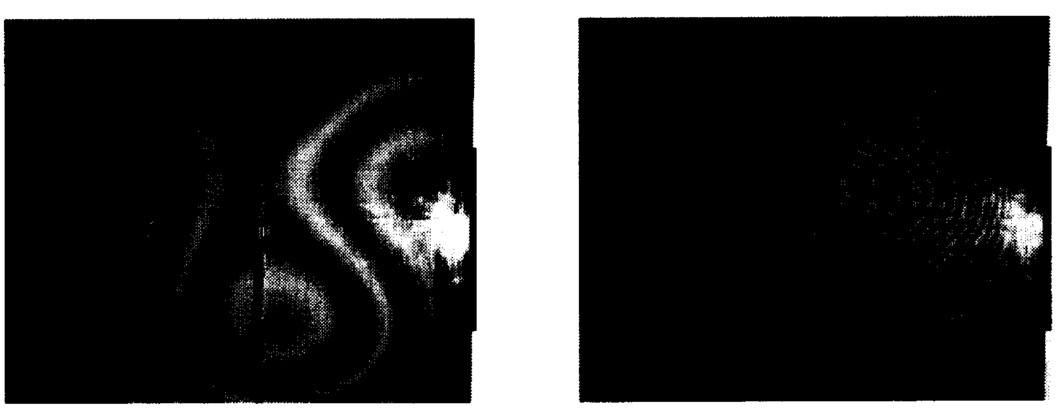
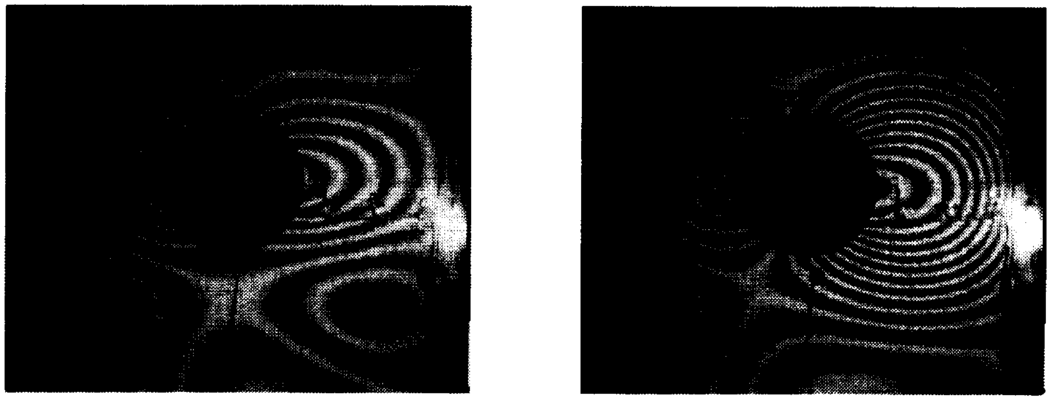


Figure 5. Nondimensional load versus end-shortening curves for aluminum curved panels with a central circular cutout; P^{o}_{bif} is the analytical prediction of the linear bifurcation buckling load for the panel without a cutout.



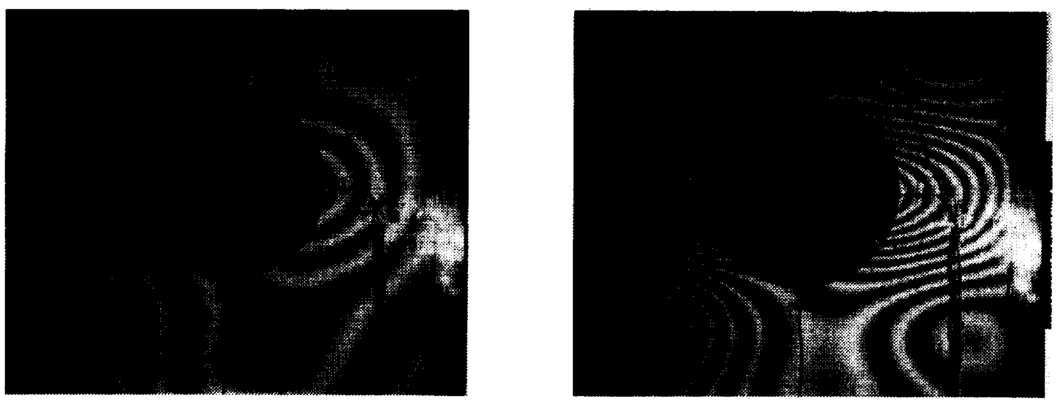
$P/P^{o}_{bif} = 0.86$ (before buckling) $P/P^{o}_{bif} = 0.57$ (after buckling)

Figure 6. Shadow moiré patterns for aluminum curved panels without a cutout.



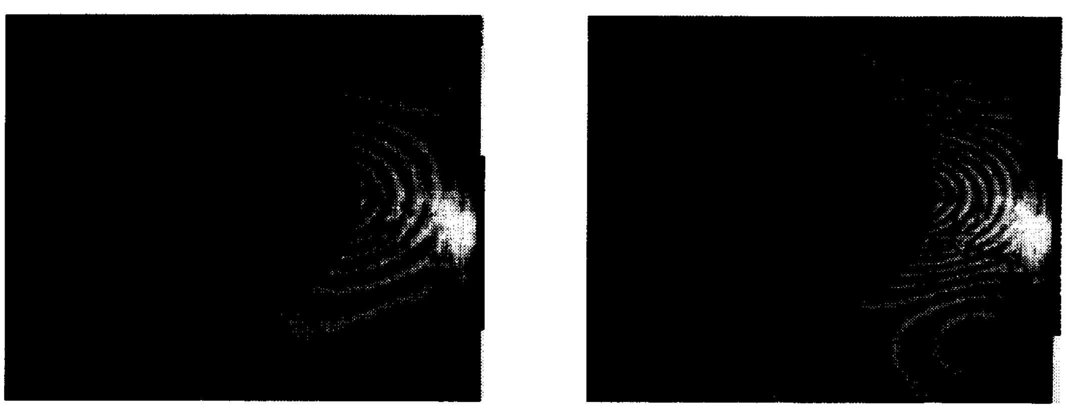
$P/P^{o}_{bif} = 0.72$ (before buckling) $P/P^{o}_{bif} = 0.67$ (after buckling)

Figure 7. Shadow moiré patterns for aluminum curved panels with a central circular cutout ($d/W = 0.3$).



$P/P^{o}_{bif} = 0.46$ $P/P^{o}_{bif} = 0.71$

Figure 8. Shadow moiré patterns for aluminum curved panels with a central circular cutout ($d/W = 0.4$).



$P/P^{o}_{bif} = 0.50$ $P/P^{o}_{bif} = 0.70$

Figure 9. Shadow moiré patterns for aluminum curved panels with a central circular cutout ($d/W = 0.5$).

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13. ABSTRACT *(Maximum 200 words)*

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