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## Abstract

*An optimizing model of a small open emerging market economy (SOEME) with dualistic labour markets and two types of consumers, is used to derive the natural interest rate, terms of trade and potential output. Shocks are classified into generic types that affect the natural interest rates. Since parameters depend on features of the labour market and on consumption inequality, the natural rates and the impact of shocks differ from those in a mature small open economy. Subsistence consumption is found to have the largest effect on the natural rates. It reduces the interest rate, raises natural output and the terms of trade. Technology and infrastructure backwardness reduce natural output. The implications for monetary policy are derived. The effect of managed exchange rates combined with different types of inflation targeting is examined through simulations. Endogenous terms of trade make the supply curve steeper in a SOEME, so partial stickiness of the real exchange rate can be beneficial. In general, domestic inflation targeting, with some weight on the output gap, delivers lower volatility. Output response is higher and volatility lower with fixed terms of trade, demonstrating the flatter supply curve. CPI inflation targeting also does well when terms of trade are credibly fixed.*

JEL codes: E52, F41

Key words: small open emerging market, optimal monetary policy, dualistic labour markets, natural interest rates, terms of trade, natural output

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<sup>1</sup> This paper uses the model developed in Goyal (2007) to address new questions. The earlier paper was presented at the IGIDR-RBI-Northwestern and the ISI conferences in 2007. I thank all those who contributed comments, including an anonymous referee, and T. S. Ananthi, Jayashree and Reshma Aguiar for secretarial assistance.

# The Natural Interest Rate in Emerging Markets

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## 1. Introduction

Forward-looking aggregate demand and supply curves are used to examine options for monetary policy. They are derived from an open economy dynamic stochastic general equilibrium (DSGE) model with imperfect competition<sup>2</sup> and nominal rigidities<sup>3</sup> as well as labour market features that reflect a populous emerging market. Since the behavioral equation coefficients are derived from basic technology, preferences and market structure, they are robust to policy changes, thus meeting the Lucas critique. The calibrated model allows estimation of indicative values for natural rates and the order of magnitude by which they differ from a small open economy (SOE).

In DSGE models optimizing labour supply decision drive unemployment—this cannot capture the dimensions of developing economy unemployment. The modeling of two types of labour makes it possible to capture a major aspect. The small open emerging market (SOEME) has a large share of less productive labour in the process of being absorbed into the modern sector.

The basic intertemporal optimization model tells us that the steady-state real interest rate must equal the representative consumer's time discount rate plus the rate of growth of population. The problem is that for a sustained period of transition and rapid catch-up the growth rate of such an economy can be above the growth rate of population. Does that mean that the natural interest rate should equal this growth rate and be considerably higher than benchmark international interest rates<sup>4</sup>? Since steady-state values are not of much guidance in transition periods, modern microfoundation-based models of monetary policy offer a good analytical framework in which to examine this question. The framework is also more relevant for the design of actual monetary policy.

The natural interest rate is defined as the equilibrium real rate, consistent with a zero or target rate of inflation, when prices are fully flexible. Shocks that change the natural rate open an output gap and affect inflation. Most Central Banks (CBs) have an operating target interest rate. This defines an operating rule, telling the CB how to change its interest rate in response to shocks. Inflation targeting is an example of such a rule.

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<sup>2</sup> Clarida et. al. (1999) surveys this literature, and Clarida et. al. (2001) extends it to an open economy. Woodford (2003) is a rigorous textbook treatment.

<sup>3</sup> Obstfeld and Rogoff (1995) is a textbook treatment of a large literature on the new open economy macroeconomics. Prices in their seminal contribution were determined one period in advance. Later treatments use variants of staggered prices, which allows smooth aggregate price adjustment.

<sup>4</sup> Such a view seems to guide Indian monetary policy making. For example a deputy governor of the Indian Reserve Bank, writes: "First, real GDP growth has recorded strong growth since 2003-04, averaging 8.6 per cent per annum over the four-year period ending 2006-07. This growth is significantly higher than world economic growth. This would suggest that equilibrium real interest rates for a country like India would be higher than world interest rates. Mohan (2007, pp.5)"

Shocks are of two generic kinds—those affecting demand and those affecting supply. They can be derived from the general equilibrium intertemporal optimization. The special labour market features of the SOEME introduce more shocks affecting the natural rate. Thus the SOEME model helps to identify these shocks, their difference from SOEs, and the implications for policy.

Two types of consumers and labour are distinguished in the SOEME, those above subsistence (R), and those at subsistence (P). While the first are able to smooth consumption using international markets, those at subsistence cannot. Their intertemporal elasticity of consumption, productivity and wages are lower and their labor supply elasticity is higher, compared to the first group<sup>5</sup>. These features follow from the key difference—high and low productivity. CES aggregation allows the micro diversity to be collapsed to macro aggregates, as is common in the literature.

A key difference between the two kinds of economies is that the real exchange rate for the SOEME is depreciated and appreciates over the long-run as development brings it closer to purchasing power parity (PPP). But there are short-run fluctuations. The stronger effects of income on the terms of trade in a SOE imply that fluctuations in the terms of trade make the aggregate supply curve steeper. This is particularly so for an almost closed economy with a large percentage of P. Therefore it may be better to manage the terms of trade, in response to temporary shocks, thus flattening aggregate supply. Variations in the exchange rate can still be used to counter shocks to aggregate supply.

A new exogenous variable in the model for the SOEME is consumption of the subsistence group. This has a large impact on the natural rate of interest, potential output, and the equilibrium terms of trade. In general, it reduces the effect of world output on these variables, compared to the SOE. A temporary shock to subsistence consumption imparts a strong negative shock to the natural rate, implying that policy should accommodate such a shock by decreasing the policy rate. The factors tending to decrease the natural rate are dominant. Subsistence consumption and the gap from world income levels, tend to raise natural output while technology and infrastructure backwardness reduce it.

Policy also has to accommodate permanent changes in natural output and equilibrium terms of trade as transition occurs. There are multiple steady-states on the way.

The welfare consequences of optimizing policy responses to shocks to the natural rate of interest are examined through simulations. Different types of inflation targeting are differentiated by weights in the policy objective function. Flexible targeting of domestic price inflation continues to deliver the best results, as in Goyal (2007). It involves active use of exchange rate policy to lower inflation, but volatility is lower than under consumer price targeting. The latter does almost as well as domestic inflation targeting when policy credibly fixes the terms of trade. The exchange rate is not itself a target but an instrument affecting both output and inflation. Constant terms of trade are consistent with variation in the nominal exchange rate.

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<sup>5</sup> This subsistence-based definition makes the model suitable for analyzing populous emerging markets such as India and China.

As more labour shifts above subsistence with development and the technology and infrastructure gap narrows, permanent changes occur in potential output, and in the equilibrium terms of trade. They eventually converge to world levels. By focusing on outcomes, inflation targeting allows policy to adjust to changes in potential output reflected in inflationary pressures. Changes in equilibrium terms of trade have also to be allowed.

Since too many things are going on in the real world, a model can serve as a valuable laboratory. But to have confidence that it captures the crucial aspects of the economy, its response to simple shocks must be similar to that of the real economy—then it can be trusted for more complex shocks (Christiano et al., 1999). The behavioral foundation, key structural aspects and the type of shocks modelled increase confidence that the paper does capture crucial aspects. In the Indian economy, however, interest and exchange rate flexibility is relatively recent, there is no formal inflation targeting, and policy optimization is only implicit. The simulation results can be understood as indicating what policy should do in such a structure. The results are intuitive both with respect to theory and structure. Policy has long been targeting a real effective exchange rate or constant terms of trade, it has recently moved to a more flexible nominal exchange rate, interest changes have been smooth and mild, as the simulations recommend. But policy has sometimes failed to accommodate natural rate shocks and kept the level of the policy rate too high. There has, however, been a tendency to accommodate agricultural shocks—which are shocks to the consumption of the poor.

The structure of the paper is as follows. Determinants of the natural interest rate are discussed in Section 2. The basic model (outlined in Appendix A) is adapted to an emerging market in Section 3, and its differences from the SOE model noted. Natural rates are derived in Section 4. Calibrated values of the natural rates, long and short-term optimal policy are obtained through simulations in Section 5. The results on optimal policy are assessed in the light of historical Indian macroeconomic policy in Section 6 before Section 7 concludes. Appendix B derives the employment subsidy that delivers the flexible price equilibrium in a SOEME.

## 2. The Natural Rate

The NKE school (Clarida et al 1999, 2001, Woodford, 2003) has developed forward-looking aggregate demand and supply curves from intertemporal optimization by representative consumers and firms<sup>6</sup>. The rational expectations equilibrium from which aggregate demand is derived comes from the basic consumption Euler, Eq. (1), as a result of household choice of the optimum timing of expenditure. Aggregate real expenditure,  $Y_t$ , and the price index,  $P_t$ , must satisfy the Euler condition (1) at all periods, where  $r_t$  is the riskless one period nominal interest rate controlled by the Central Bank (CB),  $G_t$  is government purchases, and  $U_C$  is the household's utility function. The exogenous disturbance,  $\xi_t$ , captures variations in the household's impatience to consume and  $\beta$  is its discount factor.

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<sup>6</sup> Investment is not explicitly modeled in this framework but its effect comes in through exogenous variations in productivity—the analysis abstracts from the affect of investment on productive capacity and on marginal utility.

$$1 + r_t = \beta^{-1} \left\{ E_t \left[ \frac{U_c(Y_{t+1} - G_{t+1}; \xi_{t+1}) P_t}{U_c(Y_t - G_t; \xi_t) P_{t+1}} \right] \right\}^{-1} \quad (1)$$

Monetary policy responds through the interest rate. A natural way to think about monetary policy in this context is through the gap between the policy rate and the natural rate. The natural interest rate was a concept originally defined by Wicksell<sup>7</sup>, as the equilibrium real rate of return when prices are fully flexible. It is derived from the basic consumption Euler equation when output,  $Y_t$ , equals its natural level,  $\bar{Y}_t$ , and inflation is zero, so that prices are constant (or  $P_t = P_{t+1}$ ). In these conditions the policy rate in Eq. (1) equals the natural rate in Eq. (2):  $r_t = r_t^n$ .

$$1 + r_t^n \equiv \beta^{-1} \left\{ E_t \left[ \frac{U_c(\bar{Y}_{t+1}; \xi_{t+1})}{U_c(\bar{Y}_t; \xi_t)} \right] \right\}^{-1} \quad (2)$$

In the modern approach, the steady-state value of the natural interest rate is defined as the value consistent with a zero or target inflation rate. The natural rate is the real rate of interest that keeps aggregate demand equal to the natural rate of output. This equality follows from the firm's optimization. Output is at its natural rate when the real marginal cost of supplying each good equals the marginal revenue for any firm that is thinking of changing its price, when all firms charge identical prices. But when this condition holds no firm wants to charge a different price, so there is no inflation. Marginal revenue is the reciprocal of the desired gross mark-up. Since at both  $\bar{Y}_t$  and  $r_t^n$  inflation is at zero,  $\bar{Y}_t$  is used to define  $r_t^n$ . All these concepts are derived below in the context of our SOEME.

The shock or exogenous term  $\bar{r}r_t$  that enters the NKE aggregate demand is then the percentage deviation of the natural rate from its steady-state value. The deviation occurs due to real disturbances that change natural output. Log-linearizing Eq. 2, a real shock affecting utility gives  $\bar{r}r_t$ , which equals the expectation term on the RHS of Eq.2:

$$\bar{r}r_t \equiv \log(1 + r_t^n) + \log \beta \quad (3)$$

In case of non-zero target inflation  $\bar{\pi}$ , nominal interest rate must equal:

$$r_t = \bar{r}r_t + \bar{\pi} \quad (4)$$

A non-zero target inflation can be abstracted from since although it affects average values of output and nominal interest rates, it does not affect the latter's response to shocks upto a log-linear approximation. Since a log linearization of Eq. 1 implies that  $r_t^n = \bar{Y}_{t+1} - \bar{Y}_t$ , to understand the CB's response to shocks it is necessary to see how

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<sup>7</sup> The specific form of the interest rate rule Wicksell (1898) advocated for the implementation of price-level targeting was for the CB's interest rate to rise if prices rose, fall if they fell, and to remain unaltered at whatever level it was at, unless prices changed.

these shocks affect the natural output. A temporary shock to natural output changes the natural rate.

By substituting  $Y_t = \bar{Y}_t$  in the firm's marginal cost and log linearizing, an equation of the form (5) below can be derived. Log linear approximations of equilibrium conditions are adequate since the policy focus is on small fluctuations around a steady state. The generic disturbances that affect natural output then are<sup>8</sup>:

$$\bar{Y}_t = f(\hat{G}_t, \bar{c}_t, a_t, \varphi \bar{h}_t) \quad (5)$$

Each of these disturbances increases natural output. They can be grouped into those affecting demand, and therefore requiring variation in log output to maintain a constant marginal utility of real income, and those affecting supply and therefore requiring variation in log output to maintain a constant marginal disutility of labour supply. Given these shocks, the change in natural output depends on the intertemporal elasticity of substitution of private expenditure  $1/\sigma$ , and the elasticity of real marginal cost with respect to a firm's own output. In the first category are  $\hat{G}_t$  or the normalized deviation of government purchases from their steady-state level, and  $\bar{c}$  or shift in consumer preferences. Technology,  $a_t$ , and labour supply shocks  $\varphi \bar{h}_t$ , the latter due to shifts in the disutility of labour function, are in the second, or supply shock category.

The effect of each of these shocks on  $\bar{r}r_t$  can be obtained by substituting these solutions into Equation (3) for  $\bar{r}r_t$ :

$$\bar{r}r_t = g((1 - \rho_G)\hat{G}_t, (1 - \rho_c)\bar{c}_t, (1 - \rho_a)a_t, \varphi(1 - \rho_h)\bar{h}_t) \quad (6)$$

Each disturbance follows an independent first-order autoregressive process. For stability the respective autocorrelation coefficients (subscripted  $\rho$ s) must each be less than unity. The result is that  $\bar{r}r_t$  rises for any temporary demand shock and falls for any temporary supply shock. These equations and results are explicitly derived for our model in the section below. Variation in the generic shocks due to openness and underdevelopment is noted.

In rational expectations equilibria with stable inflation, interest rates must follow these exogenous variations in the natural rate of interest. Optimal policy requires insulating the output gap from these shocks, so that the CB's interest rate instrument should move in step with the natural rate. Thus the CB would accommodate supply shocks by lowering interest rates and offset demand shocks by raising interest rates. Since inflation may be stable even with a widening output gap if prices are preset, and output movements are the same under the different types of shocks but the optimal interest response varies, the CB may require information in addition to that contained in output and inflation to fully implement this policy. The required interest rate variation is higher the more temporary the shock.

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<sup>8</sup> See Woodford (2003) pp. 249 for a more specific functional form.

### 3. A Small Open Emerging Market Economy Model

Key features of microfoundation based SOE models used to derive optimal monetary policy are intertemporal optimization and labour-leisure tradeoff by consumers; monopolistic competition and product diversity so that producers have pricing power, and output is below the social optimum. The Calvo model of staggered prices generates the sticky prices required for monetary policy to have real effects on output. The optimization results in simple standard aggregate demand (AD) and supply curves (AS) with the difference that they include forward-looking variables. The curves are derived in Appendix A. They can be used to estimate the optimal policy response to shocks<sup>9</sup>.

The basic model has to be adapted to make it relevant to analyze monetary policy in emerging markets with large populations in low productivity employment. The steady-state full employment assumption of equilibrium models is far from adequate in these markets.<sup>10</sup> We consider a small open emerging market economy (SOEME) with two representative households consuming and supplying labour: above subsistence (R) and at subsistence (P). The product market structure, technology and preferences of R type consumers are the same across all economies. Productivity shocks differ since emerging markets are in transition stages of applying the new technologies becoming available. P type consumers are assumed to be at a fixed subsistence wage, financed in part by transfers from R types.

The government intermediates these transfers through taxes on R. It runs a balanced budget so that  $\eta T_{R,t} + M_t = - (1-\eta) T_{P,t}$  where a negative tax is a transfer.  $M_t$  is government revenue from its monetary operations. The subsidy is calculated to give P a subsistence wage if they work eight hours daily, but they are free to increase their wages by working longer hours. P types are willing to supply more labor hours to the modern sector at a wage epsilon above their opportunity cost or wages in the informal sector. Since each country is of measure zero, it takes world prices as given.

The intertemporal elasticity of consumption ( $1/\sigma_R$ ), productivity and wages ( $W_R$ ) of R are higher, their labour supply elasticity ( $1/\phi_R$ ) is lower compared to the P, and they are able to fully diversify risk in international capital markets.  $N_{i,t}$  denotes hours of labour supplied by each type.

Consumption of each type of good is a weighted average of consumption by the R and the P households, with  $\eta$  as the share of R. Since R and P consume home (H) and foreign (F) goods in the same proportion,  $C_t$  is distributed between R and P in the same proportion  $\eta$ , where  $\eta$  is the share of above subsistence households in consumption. The aggregate intertemporal elasticity of substitution,  $1/\sigma$ , and the inverse of the labour supply elasticity<sup>11</sup>,  $\phi$ , are also weighted sums with population shares of R and P as weights. Since P lack the ability to smooth consumption, their

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<sup>9</sup> Appendix A presents a simplified version of the Gali and Monacelli (2005, henceforth GM) small open economy model.

<sup>10</sup> This adaptation follows Goyal (2007). See the latter for detailed derivations, proofs, and systematic comparisons of the SOEME and the SOE.

<sup>11</sup> This is also the elasticity of price with respect to output in the aggregate supply curve derived. The labour supply elasticity of P can be expected to be high, and their intertemporal elasticity of consumption low. We normalize the latter at zero. Average  $\phi$  is taken as 0.25 in the simulations, implying a labour supply elasticity of 4.



intertemporal elasticity of consumption approaches zero, so the averaging is done with elasticities, rather than inverse elasticities.

The basic consumption Euler and household labor supply are derived for each type. Risk sharing can be derived only for R types. Payoffs D are taken as zero for P types, since they do not hold a portfolio of assets.

To solve for  $S_t$  in terms of endogenous  $Y_t$  and exogenous variables, first substitute  $C_{R,t}$  and  $C_{P,t}$  for  $C_t$  in the aggregate demand equal to supply equation and then substitute out  $C_{R,t}$  using risk smoothing. This gives:

$$S_t = \left( \frac{Y_t}{Y_t^{*\eta} C_{P,t}^{1-\eta}} \right) \sigma_D \quad (7)$$

The terms of trade depreciate with a rise in  $Y_t$  and appreciate with a rise in  $Y_t^*$ ; but in a SOEME the former's effect is magnified.  $C_{P,t}$  also affects  $S_t$ , reducing the impact of

$Y_t^*$ . The multiplier factor  $\sigma_D = \frac{\sigma_R}{(\eta(1-\alpha) + \varpi\alpha)}$ , which affects only the SOEME, is

large because the intertemporal elasticity of substitution is small. If  $\sigma_R = 1$ , then  $\varpi = 1$ , and if  $1/\sigma_P = 0$ , then  $\sigma = \sigma_R/\eta$ . It also follows that  $\sigma_D < \sigma$ . Both rise as  $\eta$  falls or the proportion of P with low intertemporal elasticity of consumption ( $1/\sigma_P = 0$ ) rises. While  $\eta$  affects  $\sigma$ , both  $\eta$  and  $\alpha$  affect  $\sigma_D$ . As  $\alpha$  falls  $\sigma_D$  rises, and as  $\alpha$  approaches 0, or the economy becomes closed,  $\sigma_D$  equals  $\sigma$ , which is its upper bound. In a fully open economy  $\alpha$  approaches unity, and  $\sigma_D$  falls to its lower bound, the value unity in a SOE.

The dynamic aggregate supply, which gives domestic inflation as a function of the output gap  $x_t$ , now becomes:

$$\pi_H = \beta E_t \{ \pi_{H, t+1} \} + \kappa_D x_t \quad (8)$$

The slope for a SOEME is  $\kappa_D = \lambda(\sigma_D + \varphi)$ . The corresponding value for a closed economy is  $\lambda(\sigma + \varphi)$  and for a SOE is  $\lambda(\sigma_\alpha + \varphi)$ , where  $\sigma_\alpha = \frac{\sigma_R}{(1-\alpha) + \varpi\alpha}$ ,  $\sigma_R$  enters  $\sigma_\alpha$  since R in the SOEME are identical to the representative SOE consumer. The slope is reduced in an open compared to a closed economy since  $\sigma > \sigma_D > \sigma_\alpha$ , but the slope can be higher in the SOEME compared to a SOE, even though  $\varphi$  is lower for the SOEME, since  $\sigma_D > \sigma_\alpha$ . While  $\sigma_\alpha = 1$  if  $\sigma_R = 1$ ,  $\sigma_D$  always exceeds unity if  $\alpha < 1$ . Similar results hold for the more general case of  $\sigma_R \neq 1$ . Since the gap between  $\sigma$  and  $\sigma_D$  is large and varies with  $\eta$  and  $\alpha$ , the slope for the SOEME remains larger than in the SOE.

The dynamic aggregate demand (AD) equation for the SOEME is derived from the consumption Euler (see Appendix A). Writing it in terms of the output gap gives a term in change in natural output, which defines the shock to the natural rate  $\overline{rr}_t$ :

$$x_t = E_t \{ x_{t+1} \} - \frac{1}{\sigma_D} \left( r_t - E_t \{ \pi_{H, t+1} \} - \overline{rr}_t \right) \quad (9)$$

Where  $\overline{rr}_t = \rho - \sigma_D \Gamma(1 - \rho_\alpha) a_t - \sigma_D (1 - \eta + \Phi) E_t \{ \Delta c_{P, t+1} \} + \sigma_D (\Theta - \Psi) E_t \{ \Delta y_{t+1}^* \}$

and

$$\Theta = \alpha(\varpi - \eta), \quad d = \frac{1}{\sigma_D + \varphi}, \quad \Gamma = \frac{(1 + \varphi)}{\sigma_D + \varphi}, \quad \Psi = \eta(\sigma - \sigma_D)d,$$

$$\Phi = d((1 - \eta)(\sigma - \sigma_D))$$

Since  $\sigma_D > \sigma_a$ , the output gap is less responsive to the interest rate in the SOEME compared to the SOE.

Thus (8) and (9) are the two AS and AD equations for the SOEME.

#### 4. The natural rates of output and the terms of trade

The natural level of output  $\bar{y}_t$  is the level where marginal cost is at its desired steady-state level  $-\mu$  depending on the elasticity of demand. This has been derived for the SOE in the appendix and in our SOEME takes the value:

$$\bar{y}_t = \Omega + \Gamma a_t - \Psi y_t^* - \Phi c_{p,t} + d\sigma_D \kappa \quad (10)$$

Where  $\Omega = \frac{\nu - \mu}{\sigma_D + \varphi}$ ,  $d = \frac{1}{\sigma_D + \varphi}$ ,  $\Gamma = \frac{(1 + \varphi)}{\sigma_D + \varphi}$ ,  $\Psi = \eta(\sigma - \sigma_D)d$ ,  $\Phi = d((1 - \eta)(\sigma - \sigma_D))$

(11)

As in the case for marginal cost, the natural output for a SOE has  $\sigma_a$  instead of  $\sigma_\eta$  and no  $c_{p,t}$  and  $\kappa$  term. Since  $\sigma > \sigma_D > \sigma_a$ , but  $\sigma_R < \sigma_a$  when  $\sigma_R < 1$ ,  $y^*$  always has a negative effect on potential output in the SOEME, but has a positive effect in the SOE when  $\sigma_R < 1$ . The negative effect in the SOEME is reduced by the larger  $\sigma_D$ . The impact of technology on  $\bar{y}_t$  is also reduced. It can be negative if the SOEME technology parameter is below world levels normalized at unity. When  $C_{p,t}$  is below  $C^*$  normalized at unity,  $c_{p,t}$  raises the value of  $\bar{y}_t$ , since  $\log C_{p,t}$  is then negative. Potential is higher with higher  $\kappa$  to the extent that underdevelopment signifies an unrealized potential, although a different  $\nu$  neutralizes  $\kappa$ . As development occurs and the potential is realized  $\kappa$  goes to zero, and the coefficients of a SOEME approach that for a SOE.

Setting  $y_t = \bar{y}$  in Eq. (7), which gives the variables influencing  $s_t$ , we can derive the natural rate of the terms of trade,  $\bar{s}_t$ . Write Eq. (7) as:

$$\frac{1}{\sigma_D} s_t = y_t - \eta y_t^* - (1 - \eta) c_{p,t} - \kappa$$

Substitute for  $y_t = \bar{y}$  from Eq. (10), to get:

$$\bar{s}_t = \sigma_D (\Omega + \Gamma a_t - (\Psi + \eta) y_t^* - (\Sigma + 1 - \eta) c_{p,t} - (1 - d\sigma_\eta) \kappa)$$

The first three terms are similar to a SOE although the coefficients are different.  $y^*$  always exerts a negative effect, appreciating  $\bar{s}_t$ ; so does  $\kappa$  (since  $1 > d\sigma_D$ ). The terms are all magnified since  $\sigma_D > \sigma_a$ , the latter being the value in the SOE. The calibrations imply that the positive  $c_{p,t}$  term dominates. The result is intuitive since the  $c_{p,t}$  term captures the lack of maturity of the SOEME; it implies that  $\bar{s}_t$  is relatively depreciated compared to the value it will have when the  $c_{p,t}$  terms disappear and the SOEME has become a SOE. In the latter case natural terms of trade must be appreciated compared to their value when the consumption gap is positive since of underdevelopment. The

$c_{p,t}$  term captures the distance from world consumption levels that has to be overcome in the steady state.

## 5. Optimal policy

The model is calibrated for a SOEME and the calibration is used to examine the levels of the natural rates, the types of shocks affecting the natural rates, their relative sizes, how these differ in the SOEME compared to the SOE, and how the former collapses to the latter after development is completed. The calibrated natural rates are conditional on the consumption of the poor, the distance from world incomes, other structural features, and world incomes. There are longer-term implications for policy as the SOEME catches up and the rates change.

Short-term policy implications are also drawn for temporary shocks to the natural rate. The relative performance of different types of inflation targeting and of exchange rate intervention is examined through simulations. Because intervention has a rationale in the SOEME, leading to stickiness in the terms of trade, calibration and simulation was also done for this sticky S case. Uncovered interest parity (UIP) holds in some simulations but not when S or E is fixed. Limited capital account convertibility and some intervention that contribute to managing the exchange rates imply that UIP may not hold. The policy reaction function is estimated.

The calibration is loosely based on Indian stylized facts. Empirical estimations and the dominance of administered pricing in SOEME's suggest that past inflation affects current inflation (Fraga et. al., 2004), so a modification of the AS equation is made to accommodate such behaviour by imposing a share  $\gamma_b$  of lagged prices:

$$\pi_{H,t} = \gamma_f \beta E_t \{ \pi_{H,t+1} \} + \lambda \hat{m} c'_t + \gamma_b \pi_{H,t-1} \quad \gamma_f + \gamma_b = 1$$

In most simulations  $\gamma_b$  is set at 0.2 so  $\gamma_f$  is 0.8. Because of less than perfectly flexible interest rates, lagged interest rate also enters the AD with a weight of 0.2. The openness coefficient  $\alpha$  is set at 0.3; the proportion of R,<sup>12</sup>  $\eta$  at 0.4;  $\beta=0.99$  implies a riskless annual steady-state return of 4 percent; the price response to output,  $\phi$ , is set at 0.25, which implies an average labour supply elasticity of 4. Consumption of the mature economy and of the rich is normalized at unity, five times that of the poor so  $C_p = 0.2$ . Given  $\eta$ , this gives consistent C values of 0.75, K of 1.1 so that  $c_p = -1.6$  and  $\kappa=0.1$ . Initial conditions are normalized at unity so the log value is zero.

The natural output  $\bar{y}_t$  is derived from the flexible price equilibrium, with an employment subsidy  $\nu = -\log(1-\tau)$  set so as to correct for market power and for government temptation to change the terms of trade (GM, Section 4). In a SOEME it is also necessary to correct for the deviation from world income levels and poor infrastructure. The Appendix derives this value of the subsidy as  $\nu = \mu + \log(1-\alpha) - \kappa + \log \delta$ . The index of infrastructure  $\delta$  is taken as 0.5 less than the world level of unity<sup>13</sup>. An elasticity of substitution between differentiated goods,  $\varepsilon$

<sup>12</sup> GMM regressions of CPI inflation for India (Goyal, 2005) give a coefficient of expected inflation of 0.67. India's share of imports in GDP was about 20 percent in 2005, and the proportion of population in rural areas 60 percent. In GMM regressions of aggregate demand with monthly data, the one period forward index of industrial production was strongly significant with a coefficient of -0.42.

<sup>13</sup> The Hall and Jones (1999) index of social infrastructure gives 63 as the emerging market average compared to 14 as the developed country average, implying a  $\delta$  of about a quarter. Since there has been

equal to 6, implies a steady-state mark-up,  $\mu$ , of 1.2. The value of  $\nu - \mu$  derived from the value of  $\alpha$ ,  $\delta$  and  $\kappa$  is -0.9675. The price setting parameters are such that prices adjust in an average of one year ( $\theta = 0.75$ ), giving  $\lambda = 0.24$ .

Since  $\sigma_R = 1$  and  $1/\sigma_P=0$ , the implied average intertemporal elasticity of substitution is  $\eta(1-\alpha) + \alpha=0.58$ . A negative interest rate effect on consumption requires an intertemporal elasticity large enough so that the substitution effect is higher than the positive income effect of higher interest rates on net savers. Empirical studies have found real interest rates to have weak effects on consumption. Especially in low-income countries subsistence considerations are stronger than intertemporal factors. This is particularly so when the share of food in total expenditure is large. The elasticity Ogaki, Ostry and Reinhart (1996) estimate in a large cross-country study, varies from 0.05 for Uganda and Ethiopia to a high of 0.6 for Venezuela and Singapore. Our average elasticity compares well with these figures.

Burns (2008) estimates that the level of technology employed in developing countries is only one-fourth that in high income countries but technological progress increased 40-60 percent faster in the former than in the latter between the early 1990s and early 2000s.  $A_t$  is normalized at unity for the SOE. Since catch-up has been even faster in India in recent years we take its value to be 0.8 in the SOEME.

These calibrations allow the calculation of the three natural rates and the contribution of each of the exogenous components to the natural rates. Table 1 shows these and also the coefficient values of each exogenous term. For the SOEME natural output is lower than world output while the natural terms of trade are higher. It shows the component values are dominated by  $c_p$ . The distortions in a SOEME subsumed in the constant terms are also important. The coefficient value of the gap from world income levels is highest for natural output.

Table 1: Value of natural rates due to each component							
	Constant term	$a_t = -0.2231 = \log(.8)$	$y^* = 0$	$c_p = -1.6$	$\kappa = 0.1$	Log value of natural rates	Natural rates
Component values							
$\bar{y}$	-0.4901	-0.1413	0	0.3773	0.0873	-0.1667	$\bar{Y} = 0.85$
$\bar{s}$	-0.8450	-0.1413	0	1.3373	-0.0127	0.3384	$\bar{S} = 1.4$
$\bar{rr}$	0.01	0.0024	0	-0.0319		-0.0185	-0.0185
Coefficient values							
$\bar{y}$	-0.2313	0.6332	-0.1572	-0.2358	0.8734		
$\bar{s}$	-.3989	0.6332	-0.5572	-0.8358	-0.1266		
$\bar{rr}$	0.01	-0.0109	-0.00039	0.0193			

These results are intuitive since low wages and productivity require a depreciated terms of trade for output to be competitive in world markets. The gap signifies the

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extensive building of infrastructure and convergence in infrastructure over the past decade we take a value of 0.5.

potential catch-up raising natural output. Subsistence consumption and the gap from world income levels, tend to raise natural output while technology and infrastructure backwardness reduce it. Over the longer-run policy has to take account of the changes in natural rates. For example, as catch-up occurs, improvements in technology and infrastructure will raise natural output while a rise in subsistence consumption levels or a closing of the potential gap would lower it.

Policy makers have to be careful that they are using the correct natural output in their output gap variable so that they accommodate rather than choke changes in natural output. Permanent shocks affect natural output and terms of trade but not natural interest rates, since the change in natural output is permanent. Policy makers may raise interest rates with output if they do not realize that natural output has also risen so that the output gap is unchanged. Although lower rates help absorb labour in productive sectors, higher risk premiums in the SOEME may keep policy rates high.

Short-run policy has to respond to temporary shocks. Table 1 shows shocks to subsistence consumption have the largest size among shocks affecting the natural interest rate and tend to reduce it. We take the exogenous force driving the dynamic impulse response as a calibrated 0.1 shock to the period one natural rate. The latter equals  $\rho = \beta^{-1} - 1$ . The policy response is obtained under discretion with a central bank minimizing different weighted averages of inflation (domestic or consumer), output and interest rate deviations from equilibrium values normalized at unity for the simulations ( $L = q_Y y^2 + q_\pi \pi^2 + q_i i^2$ ). The weights attached to the different arguments of the loss function (qs) ensure stability since the weight on inflation exceeds unity. Under strict inflation targeting only inflation has a positive weight of 2. The exchange rate directly affects consumer inflation while it affects domestic inflation through its affect on marginal cost. Monetary policy affects domestic inflation directly by changing the output gap. Domestic inflation is a component of consumer inflation.

Table 2 reports some of the simulations. The benchmark set of parameters, for which sensitivity analysis is undertaken, is indicated. The unconditional standard deviations are reported. The square gives a measure of the welfare loss.

The effect of monetary policy depends on the lag structure imposed. Since there are many administered prices in the Indian consumption basket we assume:

$$\pi_t = P_t - P_{t-1}; \pi_{H,t} = P_{H,t} - P_{H,t-1}; \pi_t = \pi_{H,t-1} + \alpha(e_t - e_{t-1} - \pi_{H,t})$$

That is, consumer price inflation is a weighted average of lagged domestic prices and current depreciation. This structure seems to capture aspects of the relationship between Indian domestic and consumer price inflation, where the latter changes with a lag but is becoming more affected by current import prices as the economy opens out. Domestic inflation tends to lead consumer price inflation although the latter is normally more volatile.

A large number of simulations were done, with different variations of the model, to establish robustness. This enabled crosschecking to remove errors. The policy response is on expected lines. There is stability or convergence back to the initial state by the 12<sup>th</sup> period. The volatilities and initial simulated variable values (in brackets) in selected simulations are reported in Table 2. The first row reports results of Goyal

(2007) in which comparison of different types of discretionary targeting, under a cost shock impacting domestic inflation, showed that domestic inflation targeting (DIT) delivered the lowest volatility and therefore highest welfare. This result continues to hold under a generic shock to the natural interest rate, since DIT delivers lower inflation and exchange rate volatility with only slightly higher interest rate volatility. The initial rise in the policy rate is higher than with CPI inflation targeting (CIT) so that the rise in income is lower, but still volatility is higher with CIT. DIT also performs better than CIT under more kinds of exchange rate management. Therefore the DIT is used as the benchmark again. The reason for higher volatility with CIT is excessive use of the exchange rate channel, combined with lags in the CPI (Table 2 and Figures 1 and 3).

The generic response to a rise in the natural interest rate is a rise in the policy rate. But because initially the gap between the policy and the natural interest rate falls, output rises, this raises domestic inflation, but the accompanying currency appreciation reduces consumer price inflation. The rise in the policy rate covers the expected future depreciation and slowly brings output back to steady-state levels (Figure 1). The response to a fall in natural rates to  $-0.01$  (Figure 4 for DIT) is absolutely symmetric, with the signs reversed. Policy rates fall now. CIT has a lower increase in policy rates but greater volatility in every other variable.

Considering the cases of exchange rate management, only the exchange and inflation rates change under DIT. With S fixed the only change from the benchmark is a lower appreciation and therefore lower fall in consumer price inflation (Figure 3). With E fixed there would be a slight rise in these prices for a positive shock (Figure 4). They fall with a negative shock. The large response in the policy rate under CIT limits the rise in output, without much gain in further inflation reduction (Figure 1). CIT targeting delivers a much lower rise in policy rates under perfectly flexible exchange rates but a much higher rise under fixed exchange rates, compared to DIT, so that DIT is more robust. CIT does almost as well only in the case of fixed terms of trade, since the exchange rate channel contributes to stabilizing inflation but excessive volatility is avoided. Under fixed exchange rates the exchange rate channel is not available at all leading to large policy rate response and more volatility.

Table 2: Simulations and volatilities						
Simulations	Parameters	Standard deviations of (in percentages):				
Benchmark:	$\eta=0.4, \alpha=0.3, \phi=0.2$ 5	Consumer inflation	Output	Domestic inflation	Exchange Rate	Interest rate (initial response)
DIT (cost shock)	$qy=0.7, q\pi_H=2, qi=1$	0.58	0.36	1.08	0.82	0.70(0.0256)
DIT, $0.01r_n$	$qy=0.7, q\pi_H=2, qi=1$	0.46(-0.02)	0.16(0.006)	0.31	1.59 (-0.054)	0.39 (0.0133)
DIT, $0.01r_n$	S Fix	0.31(-.0036)	0.16	0.31	1.02	0.39(0.0133)
DIT, $0.01r_n$	E Fix	0.21(0.0068)	0.16	0.31	0.00	0.39 (0.0133)
CIT, $0.01r_n$	$qy=0.7, q\pi = 2, qi=1$	0.45(-0.004)	0.43(0.0161)	0.59	1.85 (-.0585)	0.38 (0.0063)
CIT, $.01r_n$	S Fix	0.33	0.05(0.0006)	0.22	1.05	0.54 (0.02)
CIT, $.01r_n$	E Fix	0.54	0.30	0.77	0.00	0.84(0.0296)
DIT, $.01r_n$	S sticky model, S fix	0.12	0.32	0.10	0.38	0.11(0.0039)
CIT, $.01r_n$	S sticky model,	0.10	0.32	0.09	0.40	0.08(0.0001)
CIT, $.01r_n$	S sticky model, S fix	0.12	0.33	0.11	0.39	0.12(0.0041)

Note: The bracketed terms give the value of the variable in the first period of the simulation

In the simulations above, market expectations are that the terms of trade will adjust to their natural level. If the policy commitment to fixed or managed rates is credible, factoring in fixed terms of trade changes the coefficients of the equations. Simulations were also conducted with this S sticky model. Notable is the much lower rise in the policy rate, much higher output response, and lower inflation and exchange rate volatility. The supply curve is much flatter, supporting the theoretical result of a flatter supply curve with S fixed (Figure 5, 6, and Table 2). In the S sticky model with S fixed both DIT and CIT have very similar effects. Table 2 gives results for CIT with S fixed and flexible respectively although the latter is not consistent with a credible belief in fixed terms of trade. CIT does best if the terms of trade are credibly fixed. Although in general the performance of CIT improves with S fixed, its highly variable performance, suggests that DIT is more robust.

Simulations	Parameters	Coefficients of:			
		Shock dummy/ Output gap	Domestic inflation	Exchange Rate	Interest rate
Benchmark:	$\eta = 0.4, \alpha = 0.3, \phi = 0.25$				
DIT, $.01r_n$	$qy=0.7, q\pi_H=2, q_i=1$	0.0133	0.1282		-0.0659
DIT, $-0.01r_n$	E Fix	-0.0133	0.1282		-0.0659
CIT, $0.01r_n$	$qy=0.7, q\pi = 2, q_i=1$	0.0063	0.2310	- 0.0710	-0.0345
CIT, $-0.01 r_n$	$qy=0.7, q\pi = 2, q_i=1$	-0.0063	0.2310	- 0.0710	-0.0345
CIT, $.01r_n$	S Fix	0.0200	0.0397	0.0728	-0.1036
CIT, $.01r_n$	E Fix	0.0296	0.0192	0.0470	-0.0795
DIT, $.01r_n$	S sticky model, S fix	0.0039	0.0254		-0.0206
CIT, $.01r_n$	S sticky model,	0.0001	0.2837	-0.1266	-0.0137
CIT, $.01r_n$	S sticky model, S fix	0.0041	0.0005	0.0086	-0.0207

Table 3 gives the reaction functions in the different cases. Noteworthy is the fall in the weight on the output gap in the S sticky model, again demonstrating the flatter supply curve. There is a weight on the exchange rate under CIT since CPI is a weighted average of the exchange rate and domestic inflation. Under S fix, when the change in exchange rate is moderated, the sign becomes negative while it is positive with flexible exchange rates.

The simulation results should be taken as only indicative since the model is not estimated, and is idealized in many respects. To give inputs for actual policy the lag structures specific to an economy have be built in. Even so, the structural SOEME features together with the microfoundations give useful insights for policy.

## **6. Indian Episodes**

India has largely followed a monetary targeting approach. In the late nineties there was a switch to a multiple indicator approach. There is no formal inflation targeting but the policy statements give both inflation control and facilitating growth as key objectives. A specific value of 5 percent is given as the desirable rate of inflation, with the aim to bring it even lower in the long-term. Although the exchange rate was said to be market determined after the reforms and two-stage devaluation of the early nineties, massive RBI intervention continued in order to absorb foreign inflows. Trend depreciation was allowed all through the nineties in order to cover the inflation differential and maintain the real effective exchange rate set in the early nineties. There was some appreciation due to the weakening of the dollar from 2002, and two-way movement of the nominal exchange rate was allowed from 2004. Foreign exchange reserves had been accumulating steadily since the opening out, but accelerated in this period. Inflation fell in the late nineties and continued low despite

high growth and firming international oil prices, but it peaked in March 2007 and 2008. Throughout this period, gradual financial reforms deepened markets; most interest rates stopped being administered, and became an effective policy instrument<sup>14</sup>. With the implementation of the liquidity adjustment facility (LAF) in 2001 policy was successful in keeping call money rates between the LAF bands determined by the policy repo and reverse repo rates, which began to be changed frequently and smoothly.

The economy has suffered from frequent cost shocks, either from a failure of the rains or from international oil price shocks. Since there are administered interventions in the price of food and of oil, their point of impact on prices is known. Administered prices are normally raised after monetary interventions to bring down inflation rates (Bhattacharya and Bhattacharyya, 2001). Political sensitivity to the consumption of the poor, (in a democracy where they are still about 30 percent of the population, and 50 percent of average consumption basket is spent on food) has normally implied monetary accommodation of government expenditure during a drought, with a tightening immediately afterwards (Dash and Goyal, 2000).

Our optimal monetary policy model does imply that policy should accommodate a temporary shock to the consumption of the poor, but instead of relying solely subsequent monetary tightening to bring down inflation, more nuanced policies that shift down the supply curve could be followed. Because of greater interest and exchange rate flexibility more policy options have now become feasible. We illustrate past policy responses and the variants suggested by our model below.

The drought and terms of trade shocks over 1965-67, led to a fiscal tightening, with a cut in budget deficits and public investment. Monetary policy was non-accommodating but not severe. Fiscal and monetary policies were closely linked, as the budget deficit was automatically financed. The oil price plus agricultural supply shock over 1973-75 led to severe monetary and fiscal measures. In both cases there was an unnecessary loss of output. A focus on expanding food supply would have been more effective (Joshi and Little, 1994). The lesson had been learned by the 1979-80 crisis. There was no cut in public investment, no sudden monetary tightening, no long-term adverse effects on output, and a rapid recovery.

But the populist fiscal response to supply shocks was having a cumulative effect in widening the revenue deficit. The response to the early nineties balance of payments crisis included a cut in public investment, an artificial agricultural supply shock as procurement prices for food grains were raised, and a monetary tightening to sterilize capital inflows in 1992-93. Growth revived in 1993-94, and monetary policy was accommodating, but exchange rate volatility in 1995 led to a monetary squeeze that precipitated a slowdown. The monetary stance was relaxed, but reversed again at the first sign of exchange rate volatility. Inflation fell, with the improvements in productivity, and the influence of low global inflation in a more open economy, but industrial growth did not revive until 2003, when Indian interest rates followed falling global interest rates. And even with higher growth, inflation remained low despite an extended period of high global oil prices.

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<sup>14</sup> Agarwal (2008) establishes this empirically by examining monetary transmission in different post reform periods.



Macroeconomic policy was vitiated by the fiscal authority's populism and the monetary authority's tendency to squeeze demand, until it was rescued by falling global interest rates. The fiscal responsibility and budget management act (FRBM) enacted in 2003 was not designed to protect investment while controlling populism and inefficiency, so the fiscal authority continued excessive populism while being conservative with productive expenditure. Off balance sheet items like oil bonds destroyed the spirit of the act while satisfying the letter. The Reserve Bank had more autonomy, since it no longer had to automatically finance deficits; fiscal populism pushed it towards conservatism in order to reduce inflationary expectations. But since populism raised inefficiencies and therefore costs the supply curve shifted up, while monetary tightening reduced demand, resulting in a large negative effect on output for little gain in lower inflation.

Monetary policy can use its knowledge of structure to fight inflation. Policy has to tighten only if there is excess demand. But supply shocks have been the dominant source of inflation. During a catch-up period of rapid productivity growth, potential output becomes more uncertain. Excess demand can be removed without output cost if agents are forward looking, but a cost shock creates a short-run tradeoff between inflation and output variability.

Since food inflation has high welfare costs, where food is still a large part of the consumption basket, this can be countered in the short-term by exchange rate policy, changes in tax rates, or other fiscal measures. Rise in wages in response to food prices has been important in second round propagation of Indian inflation. Inflation targeting would prevent the second-round inflationary wage-price expectations from setting in that can imply a permanent upward shift in the supply curve from a temporary supply shock. Optimal policy can aim to achieve a stricter inflation target only over the medium-term in order to allow time for temporary supply shocks to peter out. Short-term inflation targeting should be flexible.

If two-way movement of the nominal exchange rate is synchronized with temporary supply shocks, and the exchange rate appreciates when there is a negative supply shock, it would lower intermediate goods and food prices. This differs from fixing the exchange rate to bring down high levels of inflation, which often leads to real appreciation and ends in a crisis, as in Latin American exchange-based stabilization episodes. Two-way movement only pre-empts the effect of temporary supply shocks on the domestic price-wage process.

The nominal exchange rate reacts to temporary shocks, and the terms of trade to permanent. Productivity improvements would be required to tackle shocks like a permanent rise in global oil prices. In our model, the natural terms of trade must appreciate with a rise in the consumption of the poor. Unless it is productivity driven, such an appreciation would come through inflation. Equilibrium appreciation of natural rates gives policy more freedom to change nominal rates.

## **7. Conclusion**

The optimizing model of a SOEME, with dualistic labour markets and two types of consumers, delivers a tractable model for monetary policy. The basic structure of the forward-looking aggregate demand and supply equations is the same as for the SOE

and the closed economy, but the parameters depend on features of the labour market and on consumption inequality. These parameters also affect the natural rates. The SOEME collapses to the SOE model as inequality disappears.

Although the slope of the aggregate supply curve is lower in an open compared to a closed economy and more elastic labor supply in the SOEME curve also lowers the slope, the SOEME supply curve is normally steeper than the SOE curve when the terms of trade are endogenous. The narrow section of the population sharing international risk magnifies the changes in the terms of trade due to its determinants, which now also include subsistence consumption.

The lower intertemporal elasticity of consumption also magnifies the coefficients of the SOEME natural terms of trade. The positive term due to subsistence consumption levels dominates, implying that the current terms of trade is depreciated and reduction of the consumption gap between the SOEME and the SOE will appreciate the natural terms of trade.

The subsistence consumption term is the largest determinant of the natural output as well as the terms of trade. Policy has to be careful to accommodate permanent changes in natural rates taking place as development occurs and the SOEME approaches the SOE. It must make sure it is targeting the output gap and not output as natural output changes. Multiple steady-states occur on the development path.

Temporary shocks affect the natural interest rate. Frequent shocks to the consumption of the poor tend to decrease the natural interest rate. Optimal policy requires insulating the output gap from such shocks, so that the CB's interest rate instrument should move in step with the natural rate. Thus the CB would accommodate supply shocks by lowering interest rates and offset demand shocks by raising interest rates.

Comparing the performance of different targeting regimes in response to a temporary shock to the natural rate shows that DIT delivers lower inflation and exchange rate volatility with only slightly higher interest rate volatility. CIT delivers a much lower rise in policy rates under perfectly flexible exchange rates but a much higher rise under fixed exchange rates so that DIT is more robust. Moreover, the volatility of all other variables is higher under CIT. Excessive use of the exchange rate channel under CIT causes higher volatility. CIT does as well only in the case of fixed terms of trade, since the exchange rate channel contributes to stabilizing inflation but excessive volatility is avoided. Under fixed exchange rates the exchange rate channel is not available at all leading to large policy rate response and more volatility.

If the policy commitment to fixed or managed rates is credible, factoring in fixed terms of trade changes the coefficients of the equations. Notable in this S sticky model is the much lower rise in the policy rate, much higher output response, lower change in inflation, and in the exchange rate. That is, the supply curve is much flatter, supporting the theoretical result of a flatter supply curve when S is not endogenous.

Goyal (2007) had established that monetary policy does best, in the presence of cost shocks, by flexible targeting of domestic inflation, giving some weight to the output gap and interest rate smoothing. A similar result holds here for generic shocks to the natural rate. Middling exchange rate regimes that imply terms of trade stay close to

natural rates do best, and both DIT and CIT work well in such regimes. Limited flexibility of the nominal exchange rate contributes to reducing inflation, but aggressively using the direct exchange channel at short-horizons is not optimal. Neither is fixing the exchange rate.

Limiting capital account convertibility can help implement the required exchange rate policy. In particular impediments are required on short-term arbitrage so that the UIP does not hold. The cost of reserve accumulation will imply a higher tax on the rich but there are gains from the reduction in volatility. Lower inflation volatility will especially benefit the poor. The terms of trade can be fixed only in the short-run, but over time have to accommodate changes in the natural terms of trade.

The simulations give results in line with theory, but much work remains to be done. First, estimations and analysis for India (Goyal, 2005) suggest that CPI inflation may be forward-looking but domestic inflation is not. This could be expected due to the increasing impact of exchange rates. It will be useful, therefore to simulate the model imposing this restriction. Second, the implications of pricing to market or in importer's currency can also be explored although this may not be so relevant for commodity imports. Including capital markets may imply a greater role for interest smoothing. Third, SOEMEs have different kinds of nominal and real wage rigidities. It would be particularly useful to model the consequences of real wages rigid in terms of food prices (Goyal, 2005), since this is a feature of populous low-per capita income SOEMEs, and of forward-looking wage setting. Fourth, to explore the consequences of relaxing simplifying assumptions, including on elasticities and on uncorrected steady-state distortions, such as deviations from PPP, and on asset accumulation through the current account. A non-zero current account implies that multiple steady states can exist. Fifth, derive optimal weights for the loss function specifically for a SOEME, given its characteristics. This will allow a more robust welfare analysis.

## References

Agarwal A., 2008, 'Inflation Targeting in India: An Explorative Analysis', PhD thesis submitted to IGIDR.

Bhattacharya K and I Bhattacharyya, 2001: "Impact of Increase in Oil Prices on Inflation and Output in India", *Economic and Political Weekly*, Vol. XXXVI, No.51 (December 22-28, 2001), 4735-4741.

Burns A., H. Timmer, E. Riordan, and W. Shaw, 2008, Global Economic Prospects 2008: Trends In Technology Diffusion in Developing Countries, World Bank Report, accessed from <http://go.worldbank.org/TC26UFESJ0> on 16th January, 2008.

Christiano, L.J., M. Eichenbaum and C.L.Evans, 1999, 'Monetary Policy Shocks: What have we learned and to what end?' J.B.Taylor and M. Woodford eds. *Handbook of Macroeconomics*, Vol.1, 65-148, Elsevier: North Holland.

Clarida, R., Gali, J., and Gertler, M., 1999, 'The Science of Monetary Policy: A New Keynesian perspective', *Journal of Economic Literature*, 37 (4), 1661-707.

- Clarida, R., Gali, J., and Gertler, M., 2001, 'Optimal Monetary Policy in Closed Versus Open Economies: An integrated approach', *American Economic Review*, 91 (2), May, 248-252.
- Dash, S. and A. Goyal, 2000, 'The Money Supply Process in India: Identification, Analysis and Estimation', *Indian Economic Journal*, July-September, 48(1).
- Fraga, A., I. Goldfajn, and A. Minella, 2004, 'Inflation Targeting in Emerging Market Economies', in Gertler M. and K. Rogoff (eds.) *NBER Macroeconomics Annual*, Cambridge: MIT Press.
- Gali, J. and T. Monacelli, 2005, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies*, 72(3): 707-734.
- Goyal, A., 2007, 'A General Equilibrium Open Economy Model for Emerging Markets,' paper presented at ISI International Conference on Comparative Development, available at <http://www.isid.ac.in/~planning/ComparativeDevelopmentConference.html> on 26th May 2008. An earlier version is available as IGIDR working paper WP-2007-016.
- Goyal, A., 2005, 'Incentives from Exchange rates in an Institutional Context,' IGIDR working paper available at [www.igidr.ac.in/pdf/publication/WP-2005-002-R1.pdf](http://www.igidr.ac.in/pdf/publication/WP-2005-002-R1.pdf) on 26th May 2008, forthcoming *Journal of Quantitative Economics*.
- Hall, R.E, and C.I. Jones, 1999, 'Why Do Some Countries Produce So Much More Output per Worker Than Others?' *Quarterly Journal of Economics*, 114(1): 83-116.
- Joshi, V. and I.M.D. Little (1994) *India: Macroeconomics and Political Economy 1964-1991*, Oxford University Press, Delhi.
- Mohan, R., 2007, 'Monetary Management in Emerging Market Economies: Concerns and Dilemmas', Comments by Deputy Governor, Reserve Bank of India at Policy Panel at the NBER Conference on International Dimensions of Monetary Policy at S'Agaro, Catalonia, Spain on June 12. Accessed on January 11, 2008 from [http://www.rbi.org.in/scripts/BS\\_SpeechesView.aspx?Id=362](http://www.rbi.org.in/scripts/BS_SpeechesView.aspx?Id=362)
- Obstfeld, M. and K. Rogoff, 1996, *Foundations of International Macroeconomics*, Cambridge, Massachusetts: MIT Press.
- Ogaki, Masao, Jonathan Ostry, and Carmen M. Reinhart, 1996, 'Savings Behaviour in Low-and Middle-Income Countries: A comparison', *IMF Staff Papers* 43 (March): 38-71
- Woodford, M, 2003, *Interest and prices: Foundations of a Theory of Monetary Policy*, NJ: Princeton University Press.

## **Appendix A**

*Deriving aggregate demand and supply in a SOE*

The generic form of the objective function the representative consumer maximizes is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \quad (\text{A1})$$

Consumption,  $C$ , increases and labour,  $N$ , decreases the discounted present value of utility with  $\beta$  is the discount factor. Underlying the macro variables is CES aggregation, over  $i \in [0, 1]$  countries, and  $j \in [0,1]$  product varieties. Aggregate consumption,  $C$ , is derived from CES aggregation of consumption of home and foreign goods ( $C_H, C_F$ ). If the elasticity of substitution between H and F goods is equal to unity, the CES aggregation collapses to Cobb-Douglas:

$$C_t = k C_{H,t}^{1-\alpha} C_{F,t}^{\alpha} \quad (\text{A2})$$

Where  $k = \frac{1}{(1-\alpha)^{1-\alpha} \alpha^{\alpha}}$  is a constant and  $\alpha$  is an index of openness. The associated consumer price index (CPI) is:

$$P_t = (P_{H,t})^{1-\alpha} (P_{F,t})^{\alpha} \quad (\text{A3})$$

$C_{H,t}$  is itself an index of consumption of domestic goods derived by CES aggregation with elasticity of substitution  $\varepsilon > 1$  over  $j$  domestic varieties.  $C_{F,t}$  is an index of imported goods, derived by CES aggregation with elasticity of substitution  $\gamma = 1$  over imported goods  $j$ , from  $i$  countries of origin,  $C_{i,t}$ . Thus  $C_{i,t}$  is an index over  $j$  goods imported from country  $i$  and consumed domestically. There is also CES aggregation with elasticity of substitution  $\varepsilon > 1$  between  $j$  varieties produced within any country  $i$ .

The other great simplification in a SOE is that foreign variables are independent of home country action, and can be taken as given. Variables with a superscript \* indicate foreign countries.

The specific form of the utility function is:

$$u(C_i, N_i) = \frac{C_{i,t}^{1-\sigma_i}}{1-\sigma_i} - \frac{N_{i,t}^{1+\phi_i}}{1+\phi_i} \quad (\text{A4})$$

Since each country  $i$  is assumed to have identical preferences the subscript  $i$  can be dropped. The objective function is maximized subject to the period budget constraint:

$$P_t C_t + E_t \left\{ \frac{D_{t+1}}{R_t} \right\} \leq D_t + W_t N_t + T_t \quad (\text{A5})$$

Where  $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$  and  $R_t$  is the gross nominal yield on a riskless one-period discount bond paying one unit of domestic currency in  $t+1$  so  $\frac{1}{R_t}$  is its price.

Security markets are complete.  $D_{t+1}$  is the random payoff of the portfolio purchased at  $t$ .

Differentiating with respect to the two arguments  $C$  and  $N$  and over time gives the intratemporal optimality condition:

$$C_t^\sigma N_t^\phi = \frac{W_t}{P_t} \quad (\text{A6})$$

And intertemporal optimality or the consumption Euler:

$$\beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^\sigma \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1 \quad (\text{A7})$$

Log- linearized forms of these FOC's are:

$$w_t - p_t = \sigma c_t + \phi n_t \quad (\text{A8})$$

$$c_t = E_t \{c_{t+1}\} - \frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\} - \rho) \quad (\text{A9})$$

Where  $\rho$ , the discount rate, equals  $\beta^{-1}-1$  and  $\pi_t$ , CPI inflation, is given by  $\pi_t = p_t - p_{t-1}$ . Small letters normally denote log variables.

Optimal allocation of expenditure between domestic, H, and imported goods, F, gives:

$$C_{H,t} = (1 - \alpha) \frac{P_t}{P_{H,t}} C_t \quad (\text{A10})$$

$$C_{F,t} = \alpha \frac{P_t}{P_{F,t}} C_t \quad (\text{A11})$$

Identities and relationships between different types of inflation and real exchange rates are also required. Log- linearization of CPI gives:

$$p_t = (1 - \alpha) p_{H,t} + \alpha p_{F,t} \quad (\text{A12})$$

The effective terms of trade is:

$$s_t = \frac{P_{F,t}}{P_{H,t}} \quad (\text{A13})$$

Or in log terms:

$$p_{F,t} = s_t + p_{H,t}$$

Substituting in CPI (Eq. A12) gives:

$$p_t = p_{H,t} + \alpha s_t \quad (\text{A14})$$

Or

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t$$

That is, CPI inflation is a weighted average of domestic inflation and the terms of trade. The real exchange rate, Q, is related to the terms of trade as follows:

$$Q = \frac{EP^*}{P}$$

$$q_t = e_t + p_t^* - p_t \quad (\text{A15})$$

$$\begin{aligned}
&= s_t + p_{H,t} - p_t \\
&= (1 - \alpha)s_t \\
Q &= S_t^{(1-\alpha)}
\end{aligned}$$

The identity  $p_{F,t} = e_t + p_t^*$  is used in the derivation.

*International risk sharing:*

The consumption Euler for any other country  $i$ , with its prices translated into home country prices using the nominal exchange rate, is:

$$\beta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\varepsilon_t^i}{\varepsilon_{t+1}^i} \right) = \frac{1}{R_t} \quad (\text{A16})$$

Using the equivalent Euler equation for the home country, the definition of  $Q$ , and integrating over  $i \in [0, 1]$  countries to get  $C_t^*$ , gives:

$$\begin{aligned}
C_t &= \nu C_t^* Q^{\frac{1}{\sigma}} \\
c_t &= c_t^* + \frac{1}{\sigma} q_t \\
&= c_t^* + \frac{(1-\alpha)}{\sigma} s_t
\end{aligned} \quad (\text{A17})$$

Symmetric initial conditions and zero net foreign holdings are assumed so that  $\nu = 1$ . In the symmetric steady state with PPP,  $C = C^*$  and  $Q = S = 1$  would also hold.

*Aggregate demand and output equality:*

For goods market clearing in the SOE, domestic output must equal domestic and foreign consumption ( $C_H^*$ ) of home goods:

$$Y_t = C_{H,t} + C_{H,t}^* \quad (\text{A18})$$

We show below that substituting the allocation FOCs (A10) and (A11) in (A18) and simplifying, with  $\sigma = 1$ , this demand supply equality reduces to:

$$Y_t = S_t^\alpha C_t \quad (\text{A19})$$

The allocation of foreign consumption to goods produced in the SOEME is the same as FOC (A11) with  $P_t^* C_t^*$  instead of  $P_t C_t$ . Multiplying and dividing by  $P_{F,t}^*$  and converting the numerator  $P_{F,t}^*$  into SOEME prices using the nominal exchange rate gives:

$$C_{H,t}^* = \alpha \frac{\varepsilon P_{F,t}^*}{P_{H,t}} \frac{P_t^*}{P_{F,t}^*} C_t^*$$

Of the two relative prices, the first one compares the price of SOEME goods to all other foreign goods translated into SOEME prices. The second relative price compares the foreign country price index to the price index of all other foreign goods. Thus more SOEME goods are imported as a function of these two relative prices, the

weight of foreign goods in the consumption basket, and aggregate foreign consumption.

Multiplying and dividing by  $P_t$  and substituting  $Q_t$ :

$$C_{H,t}^* = \alpha Q_t \frac{P_t}{P_{H,t}} C_t^*$$

Substituting the FOC for the SOEME consumer (A10) and that just derived for the foreign consumer, in the aggregate demand = supply Eq. (A18) for the SOEME ( $Y_t = C_{H,t} + C_{H,t}^*$ ), gives:

$$Y_t = \frac{(1-\alpha)P_t C_t}{P_{H,t}} + \alpha Q_t \frac{P_t}{P_{H,t}} C_t^*$$

Substituting out  $C_t^*$  using risk sharing Eq. (A17):

$$Y_t = \frac{(1-\alpha)P_t C_t}{P_{H,t}} + \alpha Q_t \frac{P_t}{P_{H,t}} C_t Q^{-\frac{1}{\sigma}}$$

Simplifying and assuming  $\sigma = 1$  gives:

$$Y_t = \frac{P_t}{P_{H,t}} C_t (Q^{1-\frac{1}{\sigma}})$$

$$C_t = S^\alpha C_t^* Q^{1-\frac{1}{\sigma}}$$

Which simplifies to (A19)

*Determinants of the terms of trade:*

Substituting risk sharing again (with  $\sigma = 1$ ) in aggregate demand equals supply Eq. (A19), we get:

$$Y_t = S^\alpha C_t^* Q$$

(A20)

Substituting  $Q_t = S_t^{1-\alpha}$

$$Y_t = S_t^\alpha Y_t^* S_t^{1-\alpha}$$

$$S_t = \frac{Y_t}{Y_t^*}$$

(A21)

That is, the terms of trade depreciate with a rise in home output relative to world output.

*Deriving aggregate supply:*

A simple log-linear production function where output increases with labour input and its productivity, gives marginal cost Eq. (A23) as a function of unit labour costs, from the firms' optimization,

$$y_t = a_t + n_t$$

(A22)



$$(A23) \quad mc_t = -\nu + w_t - p_{H,t} - a_t$$

The employment subsidy  $\tau$  or  $\nu = -\log(1 - \tau)$ , guarantees the optimality of the flexible price outcome, since it induces firms to increase employment to the social optimum. Adding and subtracting  $p_t$ :

$$mc_t = -\nu + (w - p_t) + (p_t - p_{H,t}) - a_t$$

Substituting the intratemporal FOC (A8) and CPI (A14):

$$mc_t = -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \quad (A24)$$

Substituting risk sharing (A17), production function (A22), and from

$$(A21) y_t^* = y_t - s_t:$$

$$\begin{aligned} mc_t &= \nu + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t \\ &= -\nu + (\sigma + \varphi) y_t - (1 + \varphi) a_t \end{aligned}$$

$$(A25)$$

The log of gross mark up in steady state,  $mc$ , falls as elasticity of demand rises:

$$mc = -\log \frac{\varepsilon}{\varepsilon - 1} \equiv -\mu$$

The difference of actual from this optimal marginal cost is:

$$\hat{mc}_t = mc_t - mc$$

Under Calvo-style staggered pricing, where  $(1 - \theta)$  percent of firms change prices in a period, the firm's optimal price-setting can be shown to give the dynamics of domestic inflation as a function of real marginal cost and discounted expected future inflation (GM Appendix B):

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \lambda \hat{mc}_t \quad \lambda \equiv \frac{(1 - \beta\theta)(1 - \theta)}{\theta} \quad (A26)$$

The deviation of marginal cost from its optimum is related to the output gap,  $x_t \equiv y_t - \bar{y}_t$ , or the deviation of  $y$  from steady state  $\bar{y}_t$ . The latter is derived from  $mc_t$  (A25) by imposing  $mc_t = -\mu$  and solving for  $y_t$ . If  $\sigma = 1$  then:

$$\bar{y}_t = \frac{\nu - \mu}{1 + \varphi} + a_t$$

Subtracting  $y_t$  from  $\bar{y}_t$ , substituting for  $y_t$  from the  $mc_t$  equation (A26) and for  $\bar{y}_t$  from above shows how the deviation of  $mc$  from its optimal rises with the output gap:

$$\hat{mc}_t = (\sigma + \varphi) x_t$$

Combine with the price setting equation (A26) to get aggregate supply:

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa x_t \quad \kappa = \lambda(1 + \varphi) \quad (A27)$$

This is the New Keynesian Phillips Curve. It differs from the standard Phillips Curve in including forward-looking variables, which enter since it is derived from microfoundations with optimization over time. Similarly for aggregate demand derived below.

Substituting c in the Euler Eq. (A9) with y from the aggregate demand equal to supply Eq. (A19) and log-linearizing gives:

$$y_t = E_t \{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t \{\pi_{t+1}\} - \rho) - \frac{\alpha}{\sigma} E_t \{\Delta s_{t+1}\}$$

Converting to domestic prices using  $\pi_t = \pi_{H,t} + \alpha \Delta s_t$ ,

$$y_t = E_t \{y_{t+1}\} - \frac{1}{\sigma} (r_t - E_t \{\pi_{H,t+1}\} - \rho)$$

Writing in terms of output gaps,

$$x_t = E_t \{x_{t+1}\} - \frac{1}{\sigma_\alpha} (r_t - E_t \{\pi_{H,t+1}\} - r\bar{r}_t)$$

(A28)

Aggregate demand is less interest elastic in an open compared to a closed economy, since  $\sigma_\alpha$  equals unity if  $\sigma = 1$  and  $\sigma_\alpha < \sigma$  otherwise.

$$r\bar{r}_t = \rho - \sigma_\alpha (1 - \rho_a) a_t + \chi E_t \Delta y_{t+1}^* \quad (\text{A29})$$

If  $\sigma = 1, \chi = 0$  so  $y^*$  drops out of the equation. World income then does not affect aggregate demand. All the exogenous shocks affecting AD now come through the  $\bar{r}_t$  term.

## Appendix B

*To derive the unemployment subsidy*

The flexible price equilibrium in the SOEME, with variables denoted by an upper bar, must satisfy:

$$1 - \frac{1}{\varepsilon} = \overline{MC}_t \quad (\text{B1})$$

Where  $\varepsilon$  is the elasticity of demand. Real optimal or steady state marginal cost defined above must also equal real unit wage costs:  $-\frac{(1-\tau)}{A_t} \frac{\delta \bar{W}}{\bar{P}_{H,t}}$ . The subsidy  $\tau$

decreases unit labour cost and costs imposed by poor infrastructure increases it. If world infrastructure quality is unity then  $\delta < 1$  measures poor infrastructure in the SOME which adds to the marginal cost facing a firm.

The first order condition from the consumer's choice, the definition of the terms of trade and the aggregate demand equal to supply identity in the SOEME give:

$$\frac{W}{P_t} = - \frac{U_N(\bar{C}_t, \bar{N}_t)}{U_C(\bar{C}_t, \bar{N}_t)}$$

$$\bar{S}^\alpha = \frac{\bar{P}_t}{\bar{P}_{H,t}} = \frac{\bar{Y}_t}{\bar{C}_t K}$$

from

$$\bar{Y}_t = \bar{C}_t \bar{S}^\alpha K$$

Substituting the relations above in (B1) we get:

$$1 - \frac{1}{\varepsilon} = -\frac{\delta(1-\tau)}{A_t} \frac{\bar{W}}{S_t} \frac{\bar{P}_t}{\bar{P}_t} = \frac{\delta(1-\tau)}{A_t} \frac{\bar{Y}}{K\bar{C}_t} \frac{U_N}{U_C}$$

Substituting the value of the derivatives of the utility function

$$1 - \frac{1}{\varepsilon} = \frac{\delta(1-\tau)}{A_t} \frac{\bar{Y}_t}{K\bar{C}_t} \bar{N}_t^\varphi \bar{C}_t \quad (\text{B2})$$

In a SOEME, which takes world output and consumption as given the optimal allocation must satisfy:

$$-\frac{U_N}{U_C} = (1-\alpha) \frac{C_t}{N_t}$$

And from the derivatives of the utility function  $-\frac{U_N}{U_C} = C_t N^\varphi$

Equating the two gives:

$$N = (1-\alpha)^{\frac{1}{1+\varphi}} \quad (\text{B3})$$

Substituting (B3) in (B2) using the definition of the production function gives:

$$1 - \frac{1}{\varepsilon} = \frac{\delta(1-\tau)(1-\alpha)}{K}$$

Taking logarithms:

$$\nu - \mu = \log(1-\alpha) - \kappa + \log \delta \quad (\text{B4})$$

Where  $\mu = \log(\varepsilon/(\varepsilon-1))$ . The optimal marginal cost or log of the gross mark-up in a flexible price economy is  $-\mu$ , so setting  $\nu = -\log(1-\tau)$  such that (B4) holds gives the equivalent of the optimal flexible price equilibrium.

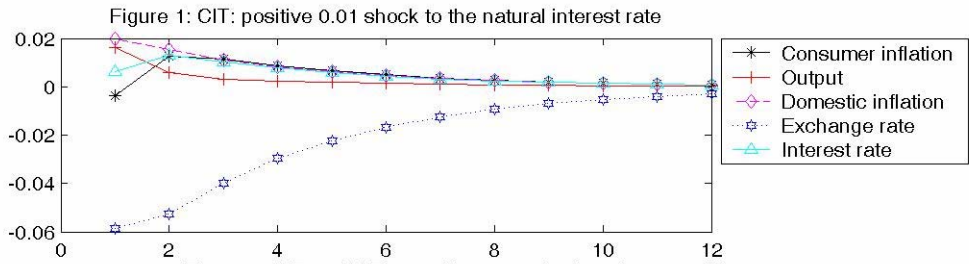


Figure 2: CIT and E Fix: positive 0.01 shock to the natural interest rate

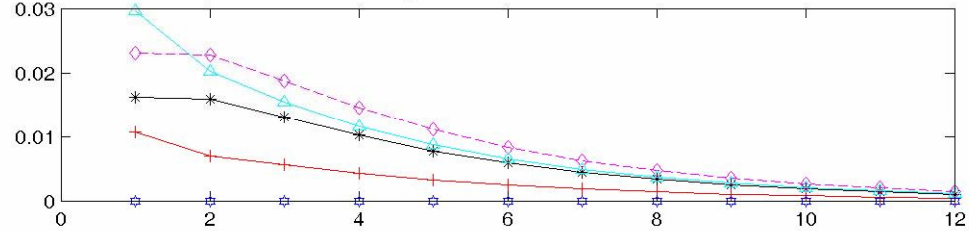


Figure 3: DIT: positive 0.01 shock to the natural interest rate

