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The network analysis of urban streets: a primal approach

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Abstract. The network metaphor in the analysis of urban and territorial cases has a long tradition, especially in transportation or land-use planning and economic geography. More recently, urban design has brought its contribution by means of the 'space syntax' methodology. All these approaches—though under different terms like 'accessibility', 'proximity', 'integration' 'connectivity', 'cost', or 'effort'—focus on the idea that some places (or streets) are more important than others because they are more *central*. The study of centrality in complex systems, however, originated in other scientific areas, namely in structural sociology, well before its use in urban studies; moreover, as a structural property of the system, centrality has never been extensively investigated *metrically* in *geographic* networks as it has been *topologically* in a wide range of other *relational* networks such as social, biological, or technological ones. After a previous work on some structural properties of the *primal* graph representation of urban street networks, in this paper we provide an in-depth investigation of *centrality* in the primal approach as compared with the dual one. We introduce *multiple centrality assessment* (MCA), a methodology for geographic network analysis, which is defined and implemented on four 1-square-mile urban street systems. MCA provides a different perspective from space syntax in that: (1) it is based on primal, rather than dual, street graphs; (2) it works within a metric, rather than topological, framework; (3) it investigates a plurality of peer centrality indices rather than a single index. We show that, in the MCA primal approach, much more than in the dual approach, some centrality indices nicely capture the 'skeleton' of the urban structure that impacts so much on spatial cognition and collective behaviours. Moreover, the distributions of centrality in *self-organized* cities are different from those in *planned* cities.

1 Introduction: which order for urban street patterns?

Since the dawn of modernity the power of Euclidean geometry has been immensely influential for "certain man using reason" (Descartes, 1994, page 27) such as an architect or urban designer, and it is almost an axiom when it comes to the design of streets, towns, and cities. Against uneven and windy street patterns ["le chemin des ânes" (the route of asses), Le Corbusier, 1994, pages 5 – 7] modernity has been diffusing grid-like and geometric structures ["le chemin des Hommes" (the route of men)] as the sign of a new era. Still today old neighborhoods are often underestimated in their most fundamental values: they might be considered picturesque, even attractive, but their *structure* is not so valuable: it is *disordered*. Against this modernist stigmatization, a whole stream of counterarguments have been raised since the early 1960s in the name of the 'magic' of old cities (Jacobs, 1993). The claim was not just about aesthetics: it was about livability. The modern city is hard to live in. The social success of an urban settlement *emerges* from the complex, uncoordinated interaction of countless different routes and experiences in a suitable environment. Is this a nostalgic claim to a prescientific era? Jane Jacobs argued, following Weaver (Jacobs, 1961; Weaver, 1948), that cities are complex-organized problems and, as such, in order to be understood, they require to be approached with a new science: only by means of the new science of complexity can the 'marvelous' complex order of the old city be revealed that, unlike the Euclidean geometry, is not visible at a first glance, is not imposed by any central agency, but, rather, sprouts out from the uncoordinated contribution of countless agents in time. That order, Jacobs concluded, is *the order of life*: that is why it fosters human life in cities; it is that order which builds the sustainable city of the future (Newman and Kenworthy, 1999).

So long evocated, clues to the complex order of life are now revealed. Following structural studies in biology and sociology, new insights have been gained which reveal that the most diverse of such systems do share astonishingly similar topological properties (Albert and Barabási, 2002; Barabási, 2002; Watts and Strogatz, 1998). Among others, our studies on *urban street networks* (Crucitti et al, 2006) have shown that the same properties actually rule those cases as well. These achievements allow us to acknowledge, *under* the seeming disorder of self-organized cities, a rule of preferential attachment and hierarchical topology that operates, in an *embedded* way, in the most diverse climatic, geographic, economic, social, and cultural conditions—an order shared with most nongeographic natural, biological, and social systems (Portugali, 2000; Ravasz and Barabási, 2003; Salingaros, 2003).

Here we make a step forward by defining the *multiple centrality assessment* (MCA), a methodology for the primal analysis of *centralities* on urban street systems. In section 2 a short review of centrality indices since the early 1950s is presented; a comparison is then addressed between 'space syntax', a well-known methodology for the dual analysis of street systems, and previously defined indices of centrality, which leads to the understanding of space syntax in the light of a broader framework and to the acknowledgement of its historical roots. In section 3 a brief discussion of the two different approaches—the primal and the dual—to the graph representation of urban street systems is presented. In section 4 selected indices of centrality are investigated over four cases of urban street networks *spatially*, through the presentation of thematic maps, and *statistically*, by plotting their cumulative distributions. The main message of this paper is then presented in section 5: the proposed MCA, grounded on a set of different centrality indices investigated over a primal, metric representation of street networks leads to an extended comprehension of the 'hidden orders' that underlie the structure of real, geographic spatial systems.

2 From structural sociology to space syntax: defining centrality indices

The basic idea in structural sociology is to represent a group of people as a network whose nodes are the individuals and whose edges are relationships between individuals (Wasserman and Faust, 1994). Bavelas was the first to realize that a *central* location in the network structure corresponds to *power* in terms of independence, influence, and control on the others (Bavelas, 1948). Freeman's masterworks on centrality (Freeman, 1977; 1979) reviewed and coordinated under the same roof previous researches addressed since the early 1950s (Bavelas, 1948; 1950; Leavitt, 1951; Shaw, 1954; 1964; Shimbel, 1953), and defined a first set of indices: degree (C^{D}), closeness (C^{C}), and betweenness (C^{B}) centralities.

More recently, new evidence has been obtained that complex networks in many different economic, social, natural, and man-made systems share some common structural properties. A first shared property is related to *distance and clustering*: in fact, it has been shown that most of those networks exhibit the *small-world* property, meaning that the average topological distance between a couple of nodes is small compared with the size of the network, despite the fact that the network exhibits a large local clustering typical of regular lattices (Watts and Strogatz, 1998). A second shared property is more related to centrality—that is, the distribution of a node's *degree*. The node's degree k is the number of its connections, nothing other than a centrality measure C^{D}. The study of a large number of complex systems, including networks as diverse as man-made systems such as the World Wide Web and the Internet (Pastor-Satorras and Vespignani, 2004), social networks such as the movie actors collaboration network or networks of sexual contacts (Liljeros et al, 2001), and many biological networks (Albert and Barabási, 2002), has shown that, in most of such cases, the degree distribution follows, for large degree k, a power law scaling $\mathrm{P}(k) = N_k/N \sim k^{-\gamma}$, with the exponent γ being between 2 and 3, and where N_k is the number of nodes having k links, and N is the total number of nodes. Networks with such a degree distribution have been named *scale free* (Albert and Barabási, 2002). The results found are particularly interesting in contrast with what is expected for random graphs (Erdös and Rényi, 1959). In fact, a random graph with N nodes and K edges (an average of $\bar{k}$ per node)—that is, a graph obtained by randomly selecting the K couples of nodes to be connected—exhibits a Poisson degree distribution centred at $\bar{k}$, with an exponential behavior and not a power law behavior for large values of k.

In formal terms a network can be represented as a *graph* $\mathrm{G} = (\mathrm{N}, \mathrm{K})$, a mathematical entity defined by two sets, N and K. The first set, N, is a nonempty set of N elements called *nodes*, *vertices*, or *points*, and K is a set of K elements containing unordered pairs of different nodes called *links* or *edges*. In the following discussion a node will be referred to by its order i in the set N, with $1 \leqslant i \leqslant N$. If there is an edge between nodes i and j, the edge being indicated as (i, j), the two nodes are said to be adjacent or connected. Sometimes it is useful to consider a *valued*, or *weighted* graph $\mathrm{G} = (\mathrm{N}, \mathrm{K}, \Omega)$, defined by three pairs of sets N, K, and Ω. The set Ω is a set of K elements, being the numerical values attached to the edges, and measuring the strengths of the tie. A graph $\mathrm{G} = (\mathrm{N}, \mathrm{K})$ can be described by a single matrix, the so-called adjacency matrix $\mathbf{A} = \{a_{ij}\}$, an $N \times N$ square matrix whose element a_{ij} is equal to 1 if (i, j) belongs to K, and 0 otherwise. A weighted graph $\mathrm{G} = (\mathrm{N}, \mathrm{K}, \Omega)$ can be described by giving two matrices, the adjacency matrix $\mathbf{A}$, defined as above, and a matrix $\mathbf{W}$ containing the edge weights. In the particular case of a *spatial* (or *geographic*) graph—that is, a graph whose nodes have a precise position in a two-dimensional or three-dimensional Euclidean space and whose links are real physical connections—we find it useful to work with lengths in place of weights, such that, instead of the weights matrix $\mathbf{W}$, we will consider the lengths matrix $\mathbf{L} = \{l_{ij}\}$, an $N \times N$ matrix whose entry l_{ij} is the metric length of the link connecting i and j (a quantity inversely proportional to the weight associated with the edge). In a valued graph the shortest path length d_{ij} between i and j is defined as the smallest sum of the edge lengths throughout all the possible paths in the graph from i to j, whereas in a nonvalued graph it is simply given by the smallest number of steps required to go from i to j.

The characteristic path length L (Watts and Strogatz, 1998) is defined as the average length of the shortest paths (with the average being calculated over all the couples of nodes in the network):

$$L = \frac{1}{N(N-1)} \sum_{i, j \in N;\, i \neq j} d_{ij} \,. \tag{1}$$

L is a good measure of the connectivity properties of the network. However, this index is not well defined for nonconnected graphs, unless we make the artificial assumption of a finite value for d_{ij} also when there is no path connecting nodes i and j. Thus a new index, the global efficiency E_{glob} (Latora and Marchiori, 2001), has been defined. As with the characteristic path length L, E_{glob} is a measure of how well the nodes communicate over the network, and it is based on the assumption that the efficiency ε_{ij} in the communication between two generic nodes i and j of the graph is inversely proportional to the shortest path length connecting the nodes—that is, $\varepsilon_{ij} = 1/d_{ij}$. In the case that G is unconnected and there is no path linking i and j, $d_{ij} = \infty$ and, consequently, $\varepsilon_{ij} = 0$. The global efficiency of graph G is defined as the average of ε_{ij} over all the couples of nodes:

$$E^{\mathrm{glob}}(\mathrm{G}) = \frac{1}{N(N-1)} \sum_{i,\, j \in N;\; i \neq j} \varepsilon_{ij} = \frac{1}{N(N-1)} \sum_{i,\, j \in N;\; i \neq j} \frac{1}{d_{ij}}. \tag{2}$$

The global efficiency is correlated to $1/L$, with a high characteristic path length corresponding to a low efficiency (Latora and Marchiori, 2003). By definition, in the topological (nonvalued graph) case, E^{glob} takes values in the interval [0, 1], and is equal to 1 for the complete graph [a graph with all the possible $N(N-1)/2$ edges]. In metric systems (translated into valued graphs), however, it is possible to normalize (Latora and Marchiori, 2001; 2002) such a quantity by dividing $E^{\mathrm{glob}}(\mathrm{G})$ by the efficiency $E^{\mathrm{glob}}(\mathrm{G}^{\mathrm{ideal}})$ of an ideal complete system in which the edge connecting the generic couple of nodes i, j is present and has a length equal to the Euclidean distance between i and j:

$$E^{\mathrm{glob}}(\mathrm{G}^{\mathrm{ideal}}) = \frac{1}{N(N-1)} \sum_{i \neq j \in \mathrm{N}} \frac{1}{d_{ij}^{\mathrm{Eucl}}}, \tag{3}$$

where d_{ij}^{Eucl} is the Euclidean distance between nodes i and j along a straight line—that is, the length of a virtual direct connection i–j. In this way we have $E_1^{\mathrm{glob}}(\mathrm{G}) = E^{\mathrm{glob}}(\mathrm{G})/E^{\mathrm{glob}}(\mathrm{G}^{\mathrm{ideal}})$. A different normalization has been proposed in Vragovìc et al (2004):

$$E_2^{\mathrm{glob}}(\mathrm{G}) = \left(\sum_{i,\, j \in N;\; i \neq j} \frac{d_{ij}^{\mathrm{Eucl}}}{d_{ij}} \right) \Big/ N(N-1). \tag{4}$$

We now have the setup to define and discuss the various measures of centrality. The three indices of centrality reported in Freeman (1977; 1979) can be roughly divided into two different *families* (Latora and Marchiori, 2004). Both C^{D} and C^{C} can be seen as belonging to the same concept of being central as being *near others* (Freeman, 1977; 1979; Nieminen, 1974; Sabidussi, 1966; Scott, 2003; Shimbel, 1953), and C^{B} measures can be viewed as being central in terms of being *between* (that is being the intermediary of *others* (Anthonisse, 1971; Freeman, 1977; 1979; Freeman et al, 1991; Newman and Girvan, 2003). After a number of revisions and applications through over four decades (Altman, 1993; Bonacich, 1972; 1987; 1991; Stephenson and Zelen, 1989), such indices have been changed and extended to different cases, but the basic families have not been changed so much. In transportation planning, for instance, the accessibility of a place is still intended to mean its 'ability' to be accessed within a short time from all other places, which is in essence—other than the fact that distance is measured by a much more complex notion of transportation cost—a kind of C^{C}.

The growth of interest in the network analysis of complex systems has led to new indices of centrality. For the purposes of this paper three of them, namely *efficiency*, *straightness*, and *information*, all based on global efficiency, are relevant. Efficiency centrality C^{E}, a kind of closeness, when applied to geographic graphs and normalized by comparing the length of shortest paths with that of virtual straight lines between the same nodes (Vragovìc et al, 2004), turns out to capture a new, inherently geographic concept that we term straightness centrality, C^{S}: being central as being *more directly reachable by all others* in the network. Information centrality C^{I} embeds both C^{C} and

C^{B} in a single quantity (Latora and Marchiori, 2004), and leads to another distinct concept of being central as being *critical for others*.

2.1 Being near others: degree and closeness centrality

Degree centrality is based on the idea that important nodes have the largest number of ties to other nodes in the graph. The *degree* of a node is the number of edges incident with the node, the number of first neighbours of the node. The degree k_i of node i is defined in terms of the adjacency matrix as $k_i = \sum_{j \in N} a_{ij}$. The degree centrality (C^{D}) of i is defined as (Freeman, 1979; Nieminen, 1974):

$$C_i^{\mathrm{D}} = \frac{k_i}{N-1} = \frac{\sum_{j \in N} a_{ij}}{N-1}. \tag{6}$$

The normalization adopted is such that C^{C} takes on values between 0 and 1, and is equal to 1 when a node is connected to all the other nodes of the graph. Degree centrality is not relevant in the primal representations, in which a node's degree (the number of streets incident in that intersection) is substantially limited by spatial constraints.

The simplest notion of closeness is based on the concept of minimum distance or geodesic d_{ij}—that is, the smallest sum of the edge lengths throughout all the possible paths in the graph from i to j in a weighted graph, or the minimum number of edges traversed in a topological graph. The closeness centrality of point i (Freeman, 1979; Sabidussi, 1966; Wasserman and Faust, 1994) is:

$$C_i^{\mathrm{C}} = L_i^{-1} = \frac{N-1}{\sum_{j \in N;\, j \neq i} d_{ij}}, \tag{7}$$

where L_i is the average distance from node i to other nodes. Such an index is meaningful for connected graphs only, unless one artificially assumes d_{ij} to be equal to a finite value when there is no path between two nodes i and j, and to take on values between 0 and 1 in the case of nonvalued graphs.

2.2 Being between others: betweenness centrality

Interactions between two nonadjacent nodes might depend on intermediate nodes that can have a strategic control or influence on them. This concept can be simply quantified by assuming that communication travels along only geodesics. Namely, if n_{jk} is the number of geodesics linking the two nodes j and k, and n_{jk} is the number of geodesics linking the two nodes j and k that contain node i, the betweenness centrality of node i is defined as (Freeman, 1979):

$$C_i^{\mathrm{B}} = \frac{1}{(N-1)(N-2)} \sum_{j,\, k \in N;\; j \neq k;\; j,\, k \neq i} \frac{n_{jk}(i)}{n_{jk}}. \tag{8}$$

C_i^{B} takes on values between 0 and 1 and reaches its maximum when node i falls on all geodesics. Here we just mention two other indices of betweenness that include contributions from nongeodesic paths: the *flow betweenness* and the *random paths betweenness*; however, in this study, we use the shortest paths betweenness defined in equation (8).

2.3 Being direct to the others: efficiency and straightness centrality

Efficiency and straightness centralities originate from the idea that the efficiency in the communication between two nodes i and j is equal to the inverse of the shortest path length d_{ij} (Latora and Marchiori, 2001). Thus, the efficiency centrality of node i is:

$$C_i^{\mathrm{E}} = \left(\sum_{j \in N;\, j \neq i} \frac{1}{d_{ij}}\right) \bigg/ \left(\sum_{j \in N;\, j \neq i} \frac{1}{d_{ij}^{\mathrm{Eucl}}}\right). \tag{9}$$

Straightness centrality is a variant of efficiency centrality, and originates from a different normalization (Vragovìc et al, 2004). The straightness of node i is:

$$C_i^{\mathrm{S}} = \left(\sum_{j \in N;\, j \neq i} \frac{d_{ij}^{\mathrm{Eucl}}}{d_{ij}}\right) \bigg/ (N-1)\,. \tag{10}$$

This measure captures how much the connecting routes from node i to all other nodes in the graph deviate from the virtual straight routes.

2.4 Being critical for all the others: information centrality

The information centrality of a node i is defined as the relative drop in the network efficiency caused by the removal from G of the edges incident in i:

$$C_i^{I} = \frac{\Delta E_2^{\mathrm{glob}}}{E_2^{\mathrm{glob}}} = \frac{E_2^{\mathrm{glob}}(\mathrm{G}) - E_2^{\mathrm{glob}}(\mathrm{G}')}{E_2^{\mathrm{glob}}(\mathrm{G})}, \tag{11}$$

where by G' we indicate the network with N points and $K - k_i$ edges obtained by removing from G the edges incident in the node i. Here we use the efficiency defined in equation (4). However, a generic performance parameter can be used in its place. The removal of some of the edges affects the communication between some of the nodes of the graph, thereby increasing the length of the shortest paths. Consequently, the efficiency of the new graph $E_2^{\mathrm{glob}}(\mathrm{G}')$ is smaller than $E_2^{\mathrm{glob}}(\mathrm{G})$. The index C_i^{I} is normalized by definition to take values in the interval [0, 1]. It can immediately be seen that C_i^{I} is correlated to all the other three standard centrality indices C_i^{D}, C_i^{C}, and C_i^{B}. However, C_i^{I} also depends on the lengths of the new geodesics, the alternative paths that are used once the node i is deactivated—no information about such new geodesics is contained in the other indices.

2.5 Space syntax in the field of centrality: integration and C^{C}

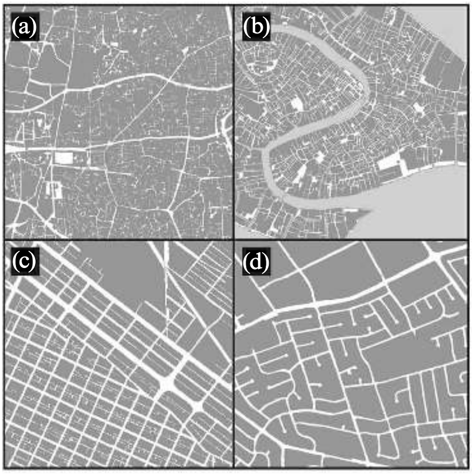
The network approach has been broadly used in urban studies. Since the early 1960s much research has been spent trying to model land uses, market behavior, or traffic flows on several topological and geometric characteristics of traffic channels (Larson, 1981; Wilson, 2000), or even the exchanges of goods and habitats between historical settlements in geographic space (Byrd, 1994; Peregrine, 1991; Pitts, 1965; 1979). The contribution of urban design has been mainly theoretical (Alexander, 1998; Batty, 2003; Batty and Longley, 1994; Salingaros, 1998) with one relevant exception: after the seminal work of Hillier and Hanson (1984), a consistent application of the network approach to cities, neighbourhoods, streets, and even single buildings has been developed under the notion of 'space syntax', thereby establishing a significant correlation between the topological accessibility of streets and phenomena as diverse as their popularity (pedestrian and vehicular flows), human wayfinding, safety against micro-criminality, retail commerce vitality, activity separation, and pollution (Penn and Turner, 2003). Though not limited to it alone, the core of the space syntax methodology, when applied to street networks, is the integration index, which is stated to be "so fundamental that it is probably in itself the key to most aspects of human spatial organization" (Hillier, 1996, page 33). The integration of one street has been defined as the "shortest journey routes between each link [or space] and all of the others in the network (defining 'shortest' in terms of fewest changes in direction" (Hillier, 1998, page 36). As such, integration turns out to be nothing other than a normalized closeness centrality (Jiang and Claramunt, 2004a), the above-mentioned closeness index defined in the early 1950s by structural sociologists and reviewed by Freeman in the late 1970s. A short comparison between formal definitions of integration—Hillier and Hanson (1984, page 108), Teklenburg et al (1993, page 35), Hillier (1996, page 36), Jiang and Claramunt (2002, pages 298–299), and many others—and that of C^{C} presented above in equation (7), fully confirms the assumption.

3 Primal versus dual: the 1-square-mile research

Networks of streets and intersections can be represented by spatial graphs in which zero-dimensional geographic entities (such as intersections) are turned into zero-dimensional graph entities (nodes) placed into a two-dimensional Euclidean space, and one-dimensional geographic entities (such as streets) are turned into one-dimensional graph entities (edges or links). Because of the coherence between the dimension of geographic and graph entities, this kind of representation is hereby termed 'direct', or *primal*; analogously, representations in which streets are turned into nodes and intersections are turned into edges, are hereby defined 'indirect', or *dual*—that is, the case of conventional space syntax analysis (Hillier, 1996; Hillier and Hanson, 1984). The network analysis, applied to territorial cases, has mostly followed a primal approach, which seems to be the most intuitive for systems in which distance has to be measured not just in topological terms (steps)—such as, for instance, in social systems—but rather in spatial terms (meters), such as in urban street systems. Traffic engineers and economic geographers or even geoarcheologists have mostly, if not always, followed the primal approach. The primal approach is also the world standard in geospatial dataset construction and diffusion: to date, an immense amount of information has been marketed already following the road-centerline-between-nodes rule, such as the huge TIGER (topologically integrated geographic encoding and referencing) database developed at the US Census Bureau. It might appear paradoxical, though, that space syntax, the flagship application of urban design, led in the opposite direction, being based on a dual representation of urban street patterns. In this representation, axial lines that represent generalized streets (more exactly: 'lines of sight' or 'lines of unobstructed movement' along mapped streets) are turned into nodes, and intersections between pairs of axial lines are turned into edges. Shortcomings as well as benefits of this approach have been often remarked (Batty, 2004a; 2004b; Crucitti et al, 2006; Desyllas and Duxbury, 2001; Hillier and Penn, 2004; Jiang and Claramunt, 2002, Ratti, 2004).

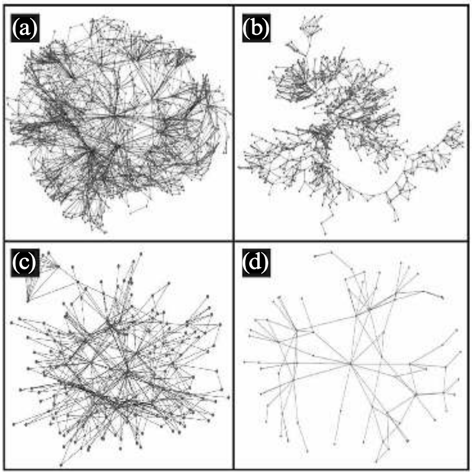
3.1 The 1-square-mile research

Recently we addressed a systematic evaluation of different centrality indices distributions over eighteen 1-square-mile samples of urban fabrics drawn from a previous work of Allan Jacobs (1993) in a primal geographic framework (Crucitti et al, 2006). Four of those cases [figure 1(a)–(d), panel 1, over], namely Ahmedabad, Venice, Richmond, CA, and Walnut Creek, CA, are here given closer focus in order to frame the comparison between the primal and the dual approach. Moreover, whereas Ahmedabad and Venice are typical *self-organized* patterns, in that they 'spontaneously' emerged from a historical process outside of any central coordination, Richmond and Walnut Creek are *planned* patterns, developed following one coordinating layout in a relatively short period of time. In the primal approach centrality scores are calculated on nodes over the primal graphs. Primal graphs [figure 1(a)–(d), panel 2] are constructed by following a road-centerline-between-nodes rule: real intersections are turned into graph nodes and real streets are turned into graph edges; all graph edges are defined by two nodes (the endpoints of the arc) and, possibly, several vertices (intermediate points of linear discontinuity); intersections among edges are always located at nodes; edges follow



Panel 1

Panel 2



Panel 3

Figure 1. Four 1-square-mile cases of urban patterns as they appear in original maps [(a) – (d), panel 1], reduced to primal road-centerline-between-nodes graphs [(a) – (d), panel 2], and dual generalized graphs [(a) – (d), panel 3]. Two cases [(a) Ahmedabad; (b) Venice] are mostly self-organized patterns, while the other two cases [(c) Richmond, CA; (d) Walnut Creek, CA] are predominantly planned patterns. However, all cases are strikingly different after all other economic, historical, cultural, functional, and geoclimatic conditions are considered. In particular, Ahmedabad is a densely interwoven, uninterrupted urban fabric, whereas Venice is dominated by the Grand Canal separation which is crossed in just two points (the Rialto and Accademia bridges); moreover, Richmond shows a traditional gridiron structure whereas Walnut Creek has a conventional 'lollipop' layout typical of postwar suburbs. These geographic peculiarities, which are well featured in the primal valued (metric) representation, get lost in the dual representation, in which just the topological properties of the systems are retained.

the footprint of real streets as they appear on the source map; all distances are calculated metrically. After the computation of centrality scores on primal nodes, analogous primal layouts (red-and-blue maps) are produced with reference either to *node* or *edge centrality*; in the latter case, because in the primal graph one edge is defined by just one pair of ending nodes by which the edge 'participates' in the topology of the network as a whole, centrality on one edge is simply equated to the average of the centralities of its defining pair of nodes. An example of node-referenced layout is given in figure 3, and in figure 4 an edge-referenced layout is offered. Both are the result of the same MCA primal approach.

In the dual approach centrality scores are calculated on nodes over the dual graph [figure 1(a) – (d), panel 3]; here, streets are turned into nodes and intersections are turned into edges of the dual graph; the distance between two nodes (streets) is equated to the number of intervening edges (intersections) along the shortest connecting path: it is a topological, nonmetric concept of distance which accounts for how many 'steps' one node is positioned from another, no matter the length of those steps. Subsequently, color-coded primal layouts (red-and-blue maps) are drawn from the dual graph, in which, because nodes in the dual graph represent streets, the centrality scores of a dual node are associated with the corresponding street in the primal layout. Beside some dissimilarities, including a different generalization model, this is a conventional space-syntax approach.

3.2 Generalized versus direct graph representation

A key question in the dual representation of street patterns is whether a 'principle of continuity' can be found to extend the identity of a street over a plurality of edges; this can be referred to as a problem of 'generalization'. A generalization model is a process of complexity reduction used by cartographers when reducing the scale of a map; in a first step, this is based on merging single street segments into longer 'strokes' (Thomson, 2004).

In space syntax research, axial mapping acts as a generalization model in which the principle of continuity is the *linearity* of the street spaces [figure 2, row (a), over]. However, Batty and Rana (2002) found nine different methodologies for axial mapping, each generating different results, which pose a problem of subjectivity. Jiang and Claramunt, after a first primal attempt based on characteristic nodes and visibility (Jiang and Claramunt, 2002), have recently proposed a dual model under a 'named-street approach' (Jiang and Claramunt, 2004a; 2004b) in which the principle of continuity is the street name [figure 2, row (b)]: two different arcs of the original network are assigned the same street identity if they share the same street name. The main problem with this approach is that it introduces a nominalistic component in a pure spatial context, which results in a loss of coherence of the process as a whole: street names are not always meaningful nor reliable, and street-name databases are not always available in all cases or at all scales; moreover, the process of embedding and updating street names into geographical information systems seems rather costly for large datasets.

In building our dual graphs, we introduce a generalization model based on a different principle of continuity, one of 'good continuation' (Thomson, 2004), which is based on the preference to go straight at intersections, a well-known cognitive property of human wayfinding (Conroy Dalton, 2003; Dalton, 2001; Dalton et al, 2003). The model [figure 2, row (c)], which we term *intersection continuity negotiation* (ICN), is purely spatial in the sense that it excludes anything that cannot be derived by the sole geometric analysis of the primal graph itself. ICN runs in three steps:

(1) All nodes are examined in turn, beginning at random. At each node the continuity of street identity is negotiated among all pairs of incident edges: the two edges forming the largest convex angle are assigned the highest continuity and are coupled together; the two edges with the second largest convex angle are assigned the second largest continuity and are coupled together, and so forth; in nodes with an odd number of edges, the remaining edge is given the lowest continuity value.

(2) A street identification code is assigned to the edge and, at relevant intersections, to the adjacent edges coupled in the first step.

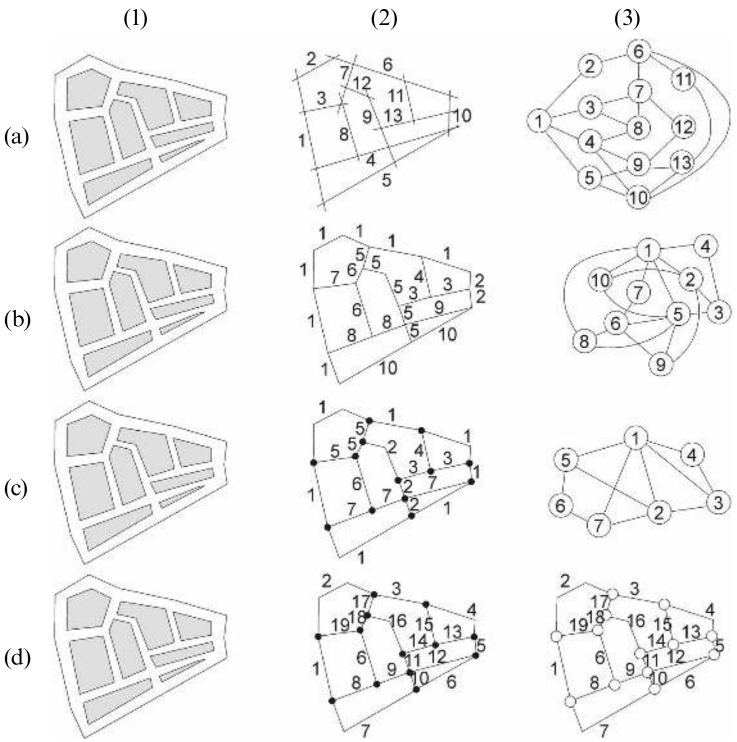


Figure 2. A comparison of various mapping approaches for a fictive urban system [column(1)], showing the resultant primal network models [column (2)] and dual connectivity graphs [column (3), except for row (d)]. The numbers are the identity codes of network edges in column (2), and in column (3) they correspond to nodes in the dual graphs and to edges in the primal graphs. (a) The dual, space-syntax approach, after Hillier and Hanson (1984). (b) The dual, named-street approach, after Jiang and Claramunt (2004a; 2004b). (c) The dual, intersection continuity negotiation (ICN) approach in which the direct representation of the urban network is properly a graph, such that intersections are turned into nodes and street arcs into edges; edges follow the footprint of real mapped streets (a linear discontinuity does not generate a vertex); the ICN process assigns the concatenation of street identities throughout nodes following a principle of 'good continuation' (Thomson, 2004). (d) The primal nongeneralized approach and its direct representation [columns (2) and (3)] in a primal graph: columns (2) and (3) are identical; the nongeneralized graph gets much more fragmented. This is the traditional geomapping way, the world standard in transportation planning. Immense information resources are currently available and continuously updated in this format. The approaches shown in rows (c) and (d) were the formats used for the dual and primal cases, respectively, in the present research.

(3) The dual graph is constructed and overlaying double edges in the dual graphs are eliminated. The main scope of ICN, in this context, is to make it possible to derive the dual case from the same source graph of the primal, which allows comparison.

3.3 Metric versus step distance

In the process of building the dual graph, which means reducing streets into nodes, what gets lost is something very relevant—aside from its somehow questionable importance for the human *cognitive* experience of spaces (Penn, 2003)—for any human *sensorial* experience of space (Hall, 1966): distance. No matter its real length, one street will be represented in the dual graph as one point. Moreover, as long as a generalization model is run and the 'identity' of one real street is extended over a conceptually unlimited number of real intersections, in the dual graph one node (street) can exhibit a conceptually unlimited number of edges (intersections), a number which heavily depends on the length of the street. Thus, the longer the street in reality, the more central (by degree) it is likely to be in the dual graph, which counters the experiential concept of accessibility that is conversely related to how close a destination is to all origins, such as in transportation models. Moreover, the dual-generalized reduction makes it impossible to account for the variations that so often characterize one generalized street, variations that may become very significant for lengthy streets that cross large urban areas; this is the case, for instance, for the via Etnea in Catania, Sicily, a roughly 3 km long, perfectly straight 17th-century street that runs from the baroque city core to the countryside beneath the Etna volcano, a street that exhibits radically different social, economic, demographic, and environmental conditions across seemingly all possible urban landscapes on Earth.

In more structural terms metric distance has been recognized as the key feature of road networks, which, exactly because of this fact, need to be dealt with as a new, specific family of networks (Gastner and Newman, 2004); the crucial nature of geographic Euclidean distance at the core of such systems leads to other key features, namely the planar nature and the extremely reduced variance of a node's degree, whose distribution can never recall any particular scale-free behavior. However, as mentioned above, when processed through the dual representation and a generalization model, the same road network is freed of such limitations: the loss of any limitation to the degree of a node in the dual graph makes the dual-generalized street systems structurally analogous to all other topological systems recently investigated in other fields, systems which in fact do not exhibit any geographic constraint; this leads, for instance, to the recognition of scaling rules in the degree distribution (Jiang and Claramunt, 2004a; Rosvall et al, 2005). Hence, the dual representation and the generalization model—the two pillars of the space-syntax castle—actually push urban street systems, in strict structural terms, out of the geographic domain. Though other means can be investigated in order to introduce geographic distance into such a dual representation (Batty, 2004b; Salheen, 2003; Salheen and Forsyth, 2001), if a role to geographic distance has to be recognized in a straight and plain manner the primal road-centerline-between-nodes representation of street patterns appears the most valuable option.

3.4 Many centrality indices, or how to overcome the C^{C} border effect

Implemented on primal graphs, the spatial flow of C^{C} is dominated by the so-called 'border effect', in the sense that higher C^{C} scores consistently group around the geometric center of the image. To some extent less evident in less dense cases such as Walnut Creek, the border effect is overwhelming in denser urban fabrics such as those of Ahmedabad and Venice [figures 3 and 4(a) (over)]. However, in all cases the border effect affects C^{C} spatial flow enough to disable the emergence both of central routes and of focal spots in the city fabric—a crucial feature for urban analysis—such that it leads to results which are, to a large extent, meaningless.

Though the border effect dominates the primal representation, it is somehow minimized in the dual approach [figure 4(b) (over)], owing to the combined impact both of the loss of metrics and, on the other hand, of the generalization model, which makes the network *less fragmented*. In so doing, the generalization model actually plays a vital role in that it allows us to limit, to some extent, the border effect. On the other hand, the identification of continuous routes across the urban fabric is performed *before* centrality analysis rather than being one of its outcomes: as such, the results of centrality analysis get deeply affected by principles that do not belong to any concept of centrality, but belong rather to the algorithm embedded in the generalization model (straightness at intersections in ICN, uninterrupted linearity in axial mapping, or others). Thus our dual analysis, like that of space syntax, can be referred

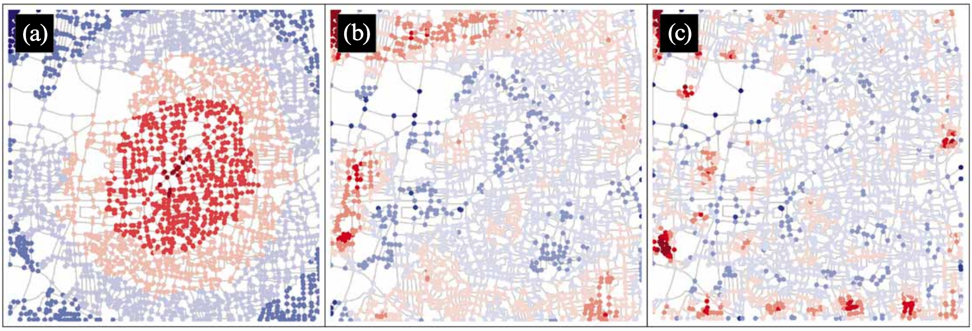


Figure 3. The primal approach. Closeness centrality (C^{C}) spatial flow in Ahmedabad: scores are calculated on the nodes of the primal weighted graph, where weights are the metric lengths of edges. (a) Global closeness: C^{C} is calculated on the whole network; (b) local closeness: C^{C} is calculated on the subnetwork of nodes at distance $d < 400$ meters from each node; (c) local closeness: $C^{\mathrm{C}}$is calculated on the subnetwork of nodes at distance $d < 200$ meters from each node. Here color nodes are attributed to the centrality of *nodes*, though in other cases it may be preferable to code the centrality of *edges*, as in figure 4(a).

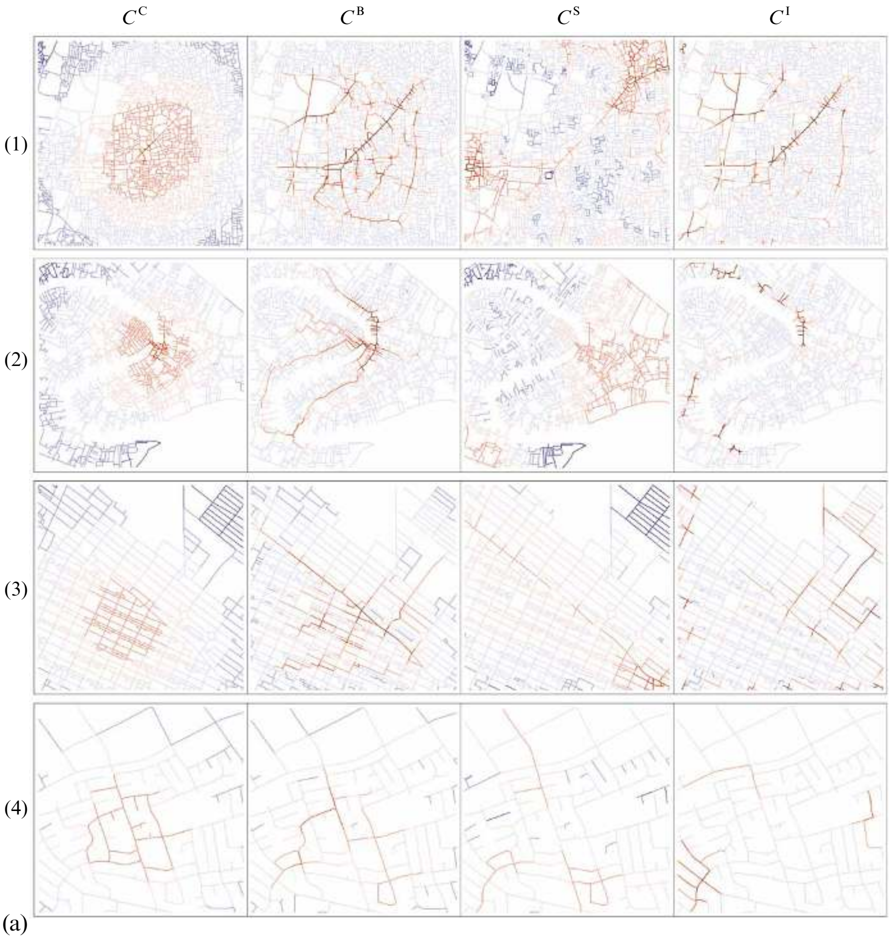


Figure 4. Comparison of the primal (a) and dual (b) approaches through the use of four different indices of centrality (C^{C} denotes closeness; C^{B} denotes betweenness; C^{S} denotes straightness; and C^{I} denotes information) for four 1-square-mile sample cases [(1) Ahmedabad, (2) Venice, (3) Richmond, CA, and (4) Walnut Creek, CA].

to as the combined result of two diverse and autonomous rationales, the first drives the generalization process, and the second drives the spatial flow of centrality. This finding confirms that of a previous work in which a conventional space-syntax dual analysis, applied without any generalization model on a segmentally represented street network, was found to be misleading for the overwhelming impact of the border effect (Dalton et al, 2003). Again, this effect is not due to some hidden structure of the urban phenomenon, but to the inherent character of the chosen index: integration, or C^{C}, is quite affected by the border effect as a result of its deep-seated nature; it does not lead by itself to any legible description of urban routes or focal areas unless the system is artificially defragmented throughout a generalization process and the study area is widened in order to leave the most border-affected parts out of the picture, which seems a scarcely efficient—though truly effective—solution.

In the light of this evidence one has two options. The first option: we can persist with the dual-generalized approach [figure 2, rows (a)–(c)] and stress to its limits the empowerment of the generalization model—that is, by automating axial mapping in the new field of 'visibility analysis' (Batty, 2001; 2004a; 2004b; Batty and Rana, 2002; Carvalho and Batty, 2004; Dalton et al, 2003; Fisher-Gewirtzman et al, 2005; Turner et al, 2001)—or by new principles of street identity (Jiang and Claramunt, 2000; 2004b; Penn et al, 1997); this is going to be fertile to the extent that it finds a solution

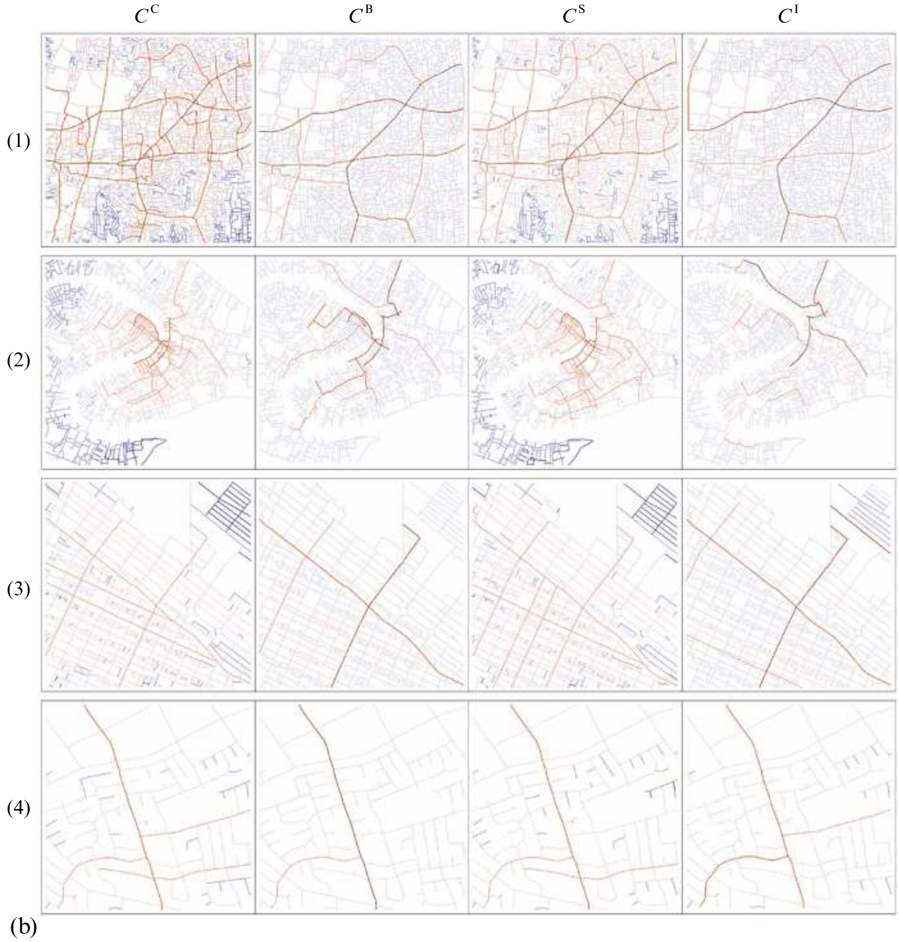


Figure 4 (continued).

to the persisting problem of subjectivism. The second option: we can embrace a primal approach [figure 2, row (d)], thereby riveting everything to metric distance as measured on a road-centerline-between-nodes graph. This latter option would allow us to reach a much finer characterization of even the longest streets; to abandon generalization models, with a relevant advantage in process feasibility, objectivity, and legibility; to access endless readily available and constantly updated information resources. This approach would also lead to a great enhancement in the realism of calculations and representations, in the sense that it pairs the topological properties with the metric properties of the system, thus comprehending both the cognitive and the proxemic dimension of collective behaviors in space. These are evidently striking benefits. But there is one problem and one question.

The problem: as we have just shown, the C^{C} integration index simply does not work on such primal graphs because C^{C} is vulnerable to the border effect; moreover, primal graphs are much more fragmented than dual generalized. But C^{C} is not the only option—centrality is a multifold concept and we have many indices at hand. Thus, to overcome this problem we can limit the analysis of C^{C} to a local scale, at which it maintains a good potential (figure 3), and simply begin to test other centrality indices, such as the previously mentioned C^{B}, C^{S}, and C^{I}. A review of our findings is offered in the next section.

The question: we know that the more general stream of the network analysis of complex real-world systems has found a particular scale-free order recursively emerging in the distribution of node degree centrality C^{D}, and we also know that in our primal approach to street systems we face a strong limitation of the same C^{D} range of variance to scores between 3 and 6 (roughly the number of streets per intersection in real urban patterns); is the primal approach therefore pushing the network analysis of road systems out of the larger domain of the 'new sciences of networks'? Again, C^{D} is also not the only option—we will show that, once the analysis of the statistical distribution of centrality over the network is extended from C^{D} to the other centrality indices, consistent scaling behaviors come to light that provide a much deeper insight into the complex nature of real street networks (and into geographic systems in general).

The evaluation of multiple centrality concepts and measures, it turns out, is the key to being able both to perform a pure, primal road-centerline-between-nodes spatial analysis and to reenter the network analysis of geographic systems into the mainstream of the 'new sciences of networks'.

4 Spatial flow and statistical distribution of centrality indices

Differences and correlations among the many indices of centrality in social networks have been investigated in a significant flurry of literature over the last decades (Bell et al, 1999; Bolland, 1988; Cook et al, 1983; Donninger, 1986; Markovsky et al, 1988; Mullen et al, 1991; Nakao, 1990; Poulin et al, 2000; Rothenberg et al, 1995; see, for a review, Wasserman and Faust, 1994). The goal was to understand the real nature of those indices when applied to human groups or organizations. The implementation of centrality indices in territory-related cases—though not always geographic—has been, in this respect, much less tested, with some exceptions (Byrd, 1994; Faust et al, 1999; Irwin, 1978; Irwin-Williams, 1977; Peregrine, 1991; Pitts, 1965; 1979; Rothman, 1987; Smith and Timberlake, 1995). In space syntax, for instance, the link between the C^{C}integration core index (as well as the ancillary C^{D}connectivity index) and centralities in social networks has been only very recently acknowledged (Hillier and Iida, 2005; Jiang and Claramunt, 2004b), thus there is apparently a poor comparison with other families of centrality indices.

Our primal studies on 1-square-mile samples of urban street networks (Crucitti et al, 2006) reveal that the four families of centrality— 'being near' (C^{C}), 'being between' (C^{B}), 'being direct (C^{S}), and 'being critical' (C^{I})—exhibit highly diverse spatial flow patterns [figure 4(a)]. In this sense two conclusions occur. On one hand, no single index gives the whole picture, as they tell strikingly different stories; on the other hand, C^{I} emerges as the most comprehensive single index of the whole set, and gathers properties of all the other indices that we have taken into consideration (section 2.4). Viewed in greater detail, C^{C} (performed globally) fails to individuate a hierarchy of central routes or areas and is therefore of no help in urban analysis: the border effect overwhelms it unless the system itself is isolated in the urban context, such that borders carry a real territorial meaning (an island, an external campus, a hamlet in the desert, etc); C^{B} is mostly effective in letting centrality emerge along even lengthier urban routes, but is still being affected, to some extent, by the border effect; C^{S} gives the most unexpected results, clearly mapping areas of higher centrality as well as central routes, and has no apparent problems with the border effect; C^{I} nicely captures the criticality of edges that play a 'bridging' role in keeping the network connected, and at the same time partially retains the behavior of C^{B}. In general, the particular effectiveness of the analysis to account for the variations in centrality levels within the same route, as in the case of C^{B} in Venice, C^{S} in Ahmedabad, or C^{I} in Richmond, should be highlighted, especially considering that those routes emerge 'naturally' as a pure convergence of centrality across street segments, without any exogenous intervention of rationales of a different kind, such as that of a generalization model. As such, MCA suggests that centrality can play a distinct role in the 'organic' formation of a 'skeleton' of most practiced routes as the cognitive framework for wayfinding in a complex urban environment (Kuipers et al, 2003).

In figure 4(b) an analogous assessment of a dual-generalized representation of the same cases is presented. The analysis of C^{C} gives one good result, which is for Ahmedabad; however, it clearly appears to be affected by the border effect as the street pattern, as in the case of Venice, becomes more fragmented. The ICN-driven construction of generalized streets, which is preliminary to the process of centrality calculation, deeply impacts on the final results in all cases, and leads to a more artificial picture of real systems and to less differentiated information among indices. Although in the primal approach continuous routes or subareas emerge in the urban fabric as a result of the natural 'convergence' of centrality over a chain of single streets across a number of intersections, in the dual approach we have routes that are identified before centrality enters the scene and are *then* attributed a value of centrality. This leads to less univocal results, in the sense that it is impossible to distinguish the actual centrality of a single street from that of the whole generalized unit (named street, axial line, etc) with which it has been associated in the course of the generalization process.

The primal representations display consistent behaviors for the same index across both cases, though different behaviors emerge for different indices (figure 5, top panel, see over). $\mathrm{P}(C^{\mathrm{C}})$ and $\mathrm{P}(C^{\mathrm{S}})$ are mainly linear; $\mathrm{P}(C^{\mathrm{B}})$ has a single scale and the dashed lines in the linear – logarithmic plot show an exponential distribution $\mathrm{P(C)} \sim \exp(-\mathrm{C}/s)$ for self-organized cities (such as Ahmedabad, $s_{\mathrm{Ahm}} = 0.016$), and a Gaussian distribution $\mathrm{P(C)} \sim \exp(-1/2\,\mathrm{C}^2\sigma^2)$ for mainly planned ones (such as Richmond, $\sigma_{\mathrm{Rich}} = 0.049$). Such distinction is more emphasized in C^{I}, which follows a power-law distribution $\mathrm{P}(C) \sim C^{-\gamma}$ in self-organized cities ($\gamma_{\mathrm{Ahm}} = 2.74$), and an exponential distribution $\mathrm{P(C)} \sim \exp(-\mathrm{C}/s)$ in planned cases ($s_{\mathrm{Rich}} = 0.002$). The dual representations $\mathrm{P}(C^{\mathrm{C}})$ and $\mathrm{P}(C^{\mathrm{S}})$ (figure 5, bottom panel), are S-shaped in a linear – linear scale. Both C^{B} and C^{I} exhibit many-scale distributions in Ahmedabad and single-scale distributions in Richmond. Although it is possible to intuit the kind of

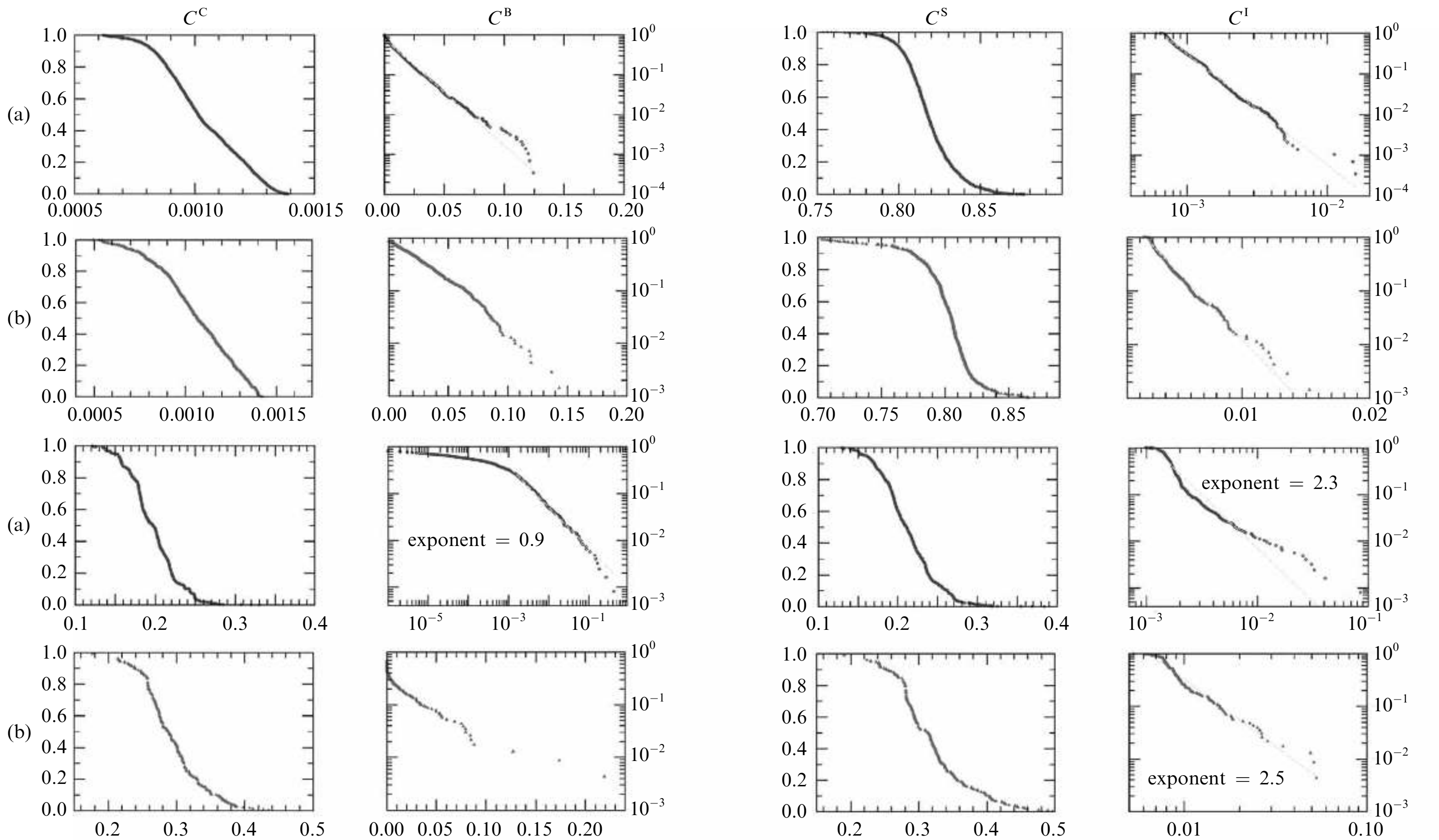


Figure 5. The cumulative distribution of the four indices of centrality (C^{C}—closeness centrality, C^{B}—betweenness centrality, C^{S}—straightness centrality, C^{I}—information centrality) in the primal (top panel) and in the dual (bottom panel) graph representations of Ahmedabad (a) and Richmond (b). Cumulative centrality distribution $P(C)$ are defined by $P(C) = \int_{C}^{\infty} N_{C}/N \, dC'$ where N_C is the number of nodes having centrality equal to C.

law that would better represent each distribution, the deviation from the analytic curves is often very large. In short, the statistical analysis of centrality distributions on primal graphs confirms that the cumulative distributions of C^{C}, C^{B}, C^{S}, and C^{I} consistently follow characteristic behaviours; an interesting result comes from C^{I}, which is distributed according to an exponential curve for planned cities, whereas for self-organized cities it follows a power law.

Homogeneity and heterogeneity in the allocation of the centrality 'resource' among nodes, investigated by the calculation of the Gini coefficient (Dagum, 1980) of each centrality distribution, have been demonstrated to be sufficient for a broad classification of different cities through a cluster analysis that groups together cities with similar urban patterns (Crucitti et al, 2006). This confirms that, by means of the primal representation and a set of different centrality indices, it is possible to capture basic crucial properties of real urban street systems for an appropriate classification of cities.

Contrary to the case of the primal approach, in dual-generalized graphs the statistical distribution of C^{D} is a relevant feature because the number of intersections per street is conceptually unlimited. However, centrality analysis is hereby extended to the other centrality indices (figure 5, bottom panel), and reveals that, as in the primal representation, C^{C} and C^{S} have an S-shaped distribution in the dual approach, C^{B} and C^{I} seem to follow many-scale distributions for Ahmedabad and single-scale distributions for Richmond. Nevertheless, in the dual-generalized analysis, distributions are much less clear than in the primal analysis (notice for instance the deviation from the analytic fit in the C^{I} of Ahmedabad), thereby confirming once more that the primal representation has a greater capability to extract such hidden order from urban patterns.

5 Conclusions: benefits of the primal approach and multiple centrality assessment

A network analysis of four 1-square-mile samples of urban street systems has been performed over primal and dual graphs. The results show that it is possible to distinguish between homogeneous and heterogeneous patterns—that is, planned versus self-organized cities—although it is worth noting that the particular dimensional limitations of the chosen samples suggest we may have to wait for conclusive considerations. However, within the limits of this study, findings strongly support the primal approach as a more comprehensive, objective, realistic, and feasible methodology for the network analysis of geographic systems such as those of streets and intersections. Being based on a world standard data format, the primal approach is suitable for making the best use of huge information resources developed and available in a broad variety of different fields. This, in turn, significantly reduces subjectivism—and enhances feasibility—in data processing by excluding the implementation of any generalization model. Although, in the dual-generalized approach, centrality statistical distributions, aside from the case of C^{D} in systems of relevant size, exhibit curves that significantly deviate from analytical fits, such rules clearly and consistently emerge in the primal approach. In particular, scaling behaviors emerge for C^{I} in self-organized urban patterns, whereas in planned patterns they do not—a feature which parallels some of the major achievements in the study of nongeographic self-organized complex systems to date. This seems to be inherently linked to what is by far the most relevant difference between the primal and the dual approaches: whereas the primal approach allows a metric computation of distance without abandoning the topology of the system, the dual-generalized approach leads to only a topological computation of distance, which makes indices and processes fundamentally more abstract, in the sense that they appear to miss a relevant part of the causal factors of collective behaviors in space.

Finally, our work also shows that centrality is not just one single thing in spatial systems. Centrality is a multifaceted concept that, in order to measure the 'importance' of single actors, organizations, or places in complex networks, has led to a number of different indices. We show that such indices, at least those mentioned in this paper, belong to four different concepts of being central as being near, being between, being straight to, and being critical for the others: the diversity of these 'families' is witnessed by the consistently different distributions of centrality scores in considered cases, both in terms of *spatial flows* as mapped in red-and-blue layouts and in terms of *statistical distributions* as shown in cumulative plots. We also show that such indices, when applied to geographic networks, capture different ways for a place to be central, ways that seem to be always working together, often in reciprocal contradiction, in shaping our perception, cognition, and usage of urban spaces.

A new approach to the network analysis of centralities in geographic systems is therefore appearing. Its three pillars are (1) primal graphs; (2) metric distance; (3) many different indices of centrality. As such, we may well name it multiple centrality assessment. Offering a set of multifaceted pictures of reality, rather than just one, MCA leads to more argumentative, thus less assertive, indications for action.

On this basis, further research may well proceed in three directions. First, significant achievements are likely to be gained after establishing correlations between centralities *of* the networks and dynamics *on* the networks (such as land uses, real-estate values, the location of social groups, crime rates, community retail vitality, and pedestrian and vehicular flows—that is, by recoding centrality indices in the context of spatial networks (Crucitti et al, 2006). Finally, an effort should be made to apply MCA to systems at different scales, from the macroregional to the microarchitectural.

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