The Node Degree Distribution in Power Grid and Its Topology Robustness under Random and Selective Node Removals

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***Abstract*—In this paper we numerically study the topology robustness of power grids under random and selective node breakdowns, and analytically estimate the critical node-removal thresholds to disintegrate a system, based on the available US power grid data. We also present an analysis on the node degree distribution in power grids because it closely relates with the topology robustness. It is found that the node degree in a power grid can be well fitted by a mixture distribution coming from the sum of a truncated Geometric random variable and an irregular Discrete random variable. With the findings we obtain better estimates of the threshold under selective node breakdowns which predict the numerical thresholds more correctly.**

***Index Terms*—Power grid, Topology Robustness, Node Degree Distribution**

I. Introduction

The electrical power grid belongs to the most critical infrastructures. Its reliable, robust, and efficient operation sustains our national economics, politics, and people's everyday life. A *Smart Grid* takes advantage of intelligent two-way digital communication to enable new control and management applications that will increase the efficiency and flexibility of the power distribution network. Investigating the intrinsic robustness of the power grid has the immediate benefit to understand the system characteristics. It also helps greatly the search for "smart" designs of the communication architecture that can enable control schemes able to enhance the robustness.

During the past decade more research efforts have been seen to study the robustness of power grids under random equipment breakdowns or intentional attacks. Dobson, Carresra *et al.* (2001) proposed an electrical power transmission model to study the dynamics of power grid blackouts in [4]. Carresras, Lynch *et al.* (2002) used the power transmission model from [4] to identify the critical points and transitions for cascading blackouts, based on a tree-topology network and the IEEE 118-bus network [5]. Ioannis and Konstantinos (2006) proposed a power transmission model which describes load demands and network improvements evolving on a slow timescale, and analyzed power grid blackouts in simple networks with ring or tree topology [7]. In [11] Wang and Rong (2009) applied a special load model to the western US power grid and studied its vulnerability to cascading failures. The model assumed that load is implemented on each bus and initially set to a power function of the product of its own node degree and the summation of the node degrees of all its immediate neighboring buses. If one node is attacked, its load would be proportionally redistributed to all its neighbors. In [10] and [9], Rosas-Casals, Valverde, Sole *et al.* (2007) studied the topology of European power grids and analyzed its robustness under random node breakdowns and intentional attacks. The adopted approaches include the numerical analysis of the relation between node removal and system global connectivity, and the analytical evaluation of the critical threshold of node removals to fragment a network.

[10] and [9] highlighted that the topology robustness of a network is closely related with its node degree distribution. In [12] we examined the node degree distribution of power grid based on available real-world US power grid data and found that it can be very well fitted by a mixture distribution from the sum of a truncated Geometric random variable and an irregular Discrete random variable. In this paper we applied these findings to analyze the topology robustness of the power grid under random and selective node removals, and to estimate the critical threshold of node removals to disintegrate a network. It was shown that the latter is able to predict the numerical results very well.

The rest of the paper is organized as follows. Section II discusses the effects of random and selective node breakdowns in a network. Section III presents our study of the node degree distribution in power grids. Section IV studies the topology robustness of power grids and compares the empirical results to the theoretical estimate. Section V concludes the paper.

II. Random and Selective Node Breakdowns in a Network

Assume the original network has a node degree distribution $P(k)$. After randomly breaking down a fraction f of the nodes, its node degree distribution is changed to:

$$\widetilde{P}(k) = \sum_{i=k}^{\infty} P(i) \binom{i}{k} f^{i-k}(1-f)^k \tag{1}$$

When f is below a certain threshold, $f \le f_c$, there still exists a large connected cluster spanning the entire network

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while its size is proportional to that of the entire network. However, if the node removal fraction exceeds that threshold, the network will disintegrate into small and disconnected parts. This phenomenon is called a *Percolation Transition* [1]-[3]. Cohen, Erez, ben Avraham, and Havlin, in their study of resilience of the Internet to random breakdowns [6], found that for networks whose nodes are connected randomly to each other so that the probability for any two nodes to be connected depends solely on their respective connectivity, its critical breakdown threshold can be found by the following criterion: if loops of connected nodes may be neglected, the percolation transition takes place when a node (i), connected to a node (j) in the spanning cluster, is also connected to at least one other node; otherwise the spanning cluster is fragmented.

$$\langle k_i | i \leftrightarrow j \rangle = \sum_{k_i} k_i P(k_i | i \leftrightarrow j) = 2 \tag{2}$$

with k_i being the node degree of node (i).

The above criterion can be translated into a more obvious statement: when one randomly picks a link in the spanning cluster, the average node degree of its end nodes equals to 2, which can be written as

$$\langle k_i | i \leftrightarrow j \rangle = \overline{k} = \frac{\langle k^2 \rangle}{\langle k \rangle} = 2 \tag{3}$$

where $\langle k^2 \rangle$ and $\langle k \rangle$ are the second and the first moment of node degree respectively. This finally gives the critical breakdown threshold under random node removals as [6]:

$$f_c^{\text{rand}} = 1 - \frac{1}{\overline{k}_0 - 1} \tag{4}$$

where $\overline{k}_0$ is the average node degree of any picked link from the original intact network.

Sole, Rosas-Casals, Corominas-Murtra, and Valverde (2007) extended the study of Cohen and Erez *et al.* and studied the vulnerability of European power grids under intentional attacks [9]. That is, the node breakdowns are no longer random but selective with the node with largest connectivity being eliminated first. Therefore even a small fraction of node breakdowns may cause dramatic damages to the network structure. They also concluded that European power grids are sparsely connected with global average nodal degree of $\langle k \rangle = 2.8$ and the link distribution (i.e. nodal degree distribution) is exponential: the probability of having a node linked to k other nodes is $p(k) = \exp(-\frac{k}{\gamma})/\gamma$, with the constant $\gamma = \langle k \rangle$. Given this special node degree distribution, they translated the selective node breakdowns with fraction of f^{sel} into an equivalent random failure with a much larger fraction f_{eqv}^{rand}, which equals to the fraction of the lost links connected with the breakdown nodes.

$$f_{eqv}^{\text{rand}} = \int_{\tilde{K}}^{K} \frac{kP(k)}{\langle k \rangle} dk = \left(1 - \ln f^{\text{sel}}\right) f^{\text{sel}} \tag{5}$$

where K is the largest node degree in the original network and $\tilde{K}$ is the new largest node degree after node removals which satisfies $\int_{\tilde{K}}^{K} P(k)dk = f^{\text{sel}}$, therefore

$$\tilde{K} = -\gamma \ln f^{\text{sel}} \tag{6}$$

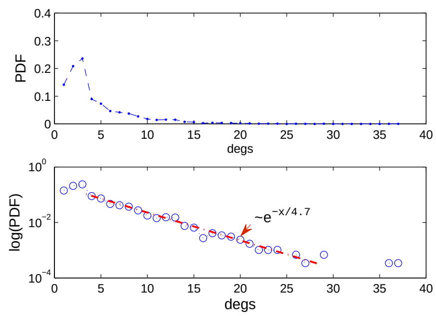


Fig. 1. Empirical PDF of Nodal Degrees in Real-world Power Grids (NYISO)

By substituting Equation (5) into (4), one gets the criterion for the critical threshold for selective node breakdowns as:

$$\left(1 - \ln f_c^{\text{sel}}\right) f_c^{\text{sel}} = 1 - \frac{1}{\overline{k}_0 - 1} \tag{7}$$

III. Node Degree Distribution in Power Grids

We examined the empirical distribution of nodal degrees $\underline{k} = \text{diag}(L)$ in the available real-world power grids. Fig. 1 shows the histogram probability mass function (PMF) of node degrees in the NYISO system[1]. Except for the beginning part for smaller node degrees, most part of the PMF is well fitted with a straight line in the semi-logarithmic plot (i.e., $\log(P(k))$ vs. k), which suggests a good fit with an exponential distribution, such as the Geometric distribution. However, for the range $k \leq 3$ in the NYISO case, the PMF function clearly deviates from that of Geometric distribution. As a matter of fact, this phenomenon is observable in many available data set that describe the topology of real-world power grids, both for the US [8][12] and for the European power grids data in [9] and [10]. As we will show in Section IV, this deviation from a pure Geometric distribution substantially affects the topology vulnerability of a network under intentional attacks.

In our previous work [12], we analyzed the probability generation function (PGF) to search for an accurate model for the node degree distribution in power grids. The PGF of a random variable X is defined as $G_X(z) = \sum_k \text{Pr}_{(x=k)} z^k$. Our working hypothesis is that the degree distribution was most likely well fitted by a mixture model, one providing most of the mass for the tail of the distribution and one responsible for the lower degrees probability mass.

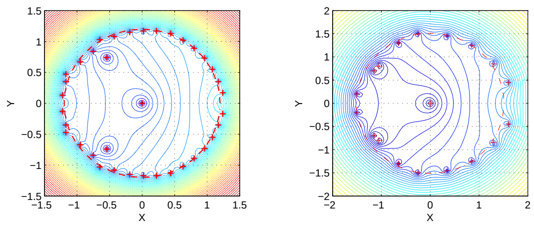
The examination of PGFs indicated that the node degree distribution in power grids can be very well approximated by a sum of two independent random variables, that is,

$$\mathcal{K} = \mathcal{G} + \mathcal{D}, \tag{8}$$

where $\mathcal{G}$ is a truncated Geometric with the threshold of $k_{\max}$,

$$\text{Pr}_{(\mathcal{G}=k)} = \frac{p(1-p)^k}{1-(1-p)^{k_{\max}+1}}, \quad k = 0, 1, 2, \cdots, k_{\max} \tag{9}$$

[1]NYISO stands for New York Independent System Operator, which is the operator of the New York electric power grid. The power grid topology we used to represent the NYISO transmission network contains 2935 nodes and 6568 links, with the average node degree $\langle k \rangle = 4.47$.



(a) (b)

Fig. 2. The Contour Plot of $E(z^X)$ of Node Degrees: (a) the NYISO system; (b) the WSCC system; the zeros are marked by red '+'s.

and $\mathcal{D}$ is an irregular Discrete random variable with probability masses $\{p_1, p_2, \cdots, p_{k_t}\}$,

$$\Pr_{(\mathcal{D}=k)} = p_k, \quad k = 1, 2, \cdots, k_t \tag{10}$$

Therefore the PMF of $\mathcal{K}$ is

$$\Pr_{(\mathcal{K}=k)} = \Pr_{(\mathcal{G}=k)} \otimes \Pr_{(\mathcal{D}=k)} \tag{11}$$

And the PGF of $\mathcal{K}$ can be written as

$$G_{\mathcal{K}}(z) = \frac{p\left(1-((1-p)z)^{k_{\max}+1}\right)\sum_{i=1}^{k_t} p_i z^i}{(1-(1-p)^{k_{\max}+1})\,(1-(1-p)z)} \tag{12}$$

The equation (12) indicates that the PGF $G_{\mathcal{K}}(z)$ has $k_{\max}$ zeros evenly distributed around a circle of radius of $\frac{1}{1-p}$ which are introduced by the truncation of the Geometric $\mathcal{G}$ (because the zero at $\frac{1}{1-p}$ has been neutralized by the denominator $(1-(1-p)z)$ and has k_t zeros introduced by the irregular Discrete $\mathcal{D}$ with $\{p_1, p_2, \cdots, p_{k_t}\}$.

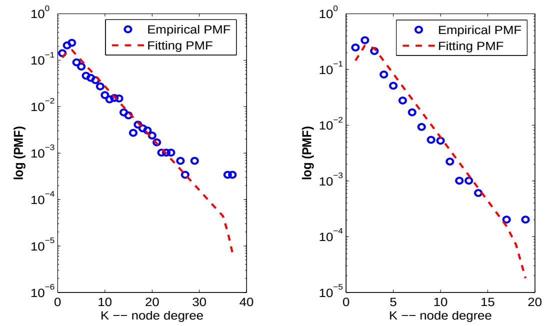
Fig. 2 shows the contour plots of PGF of node degrees in the NYISO system and the WSCC[2] system. Three interesting observations are supported by Fig. 2: (a) Clearly each plot contains evenly distributed zeros around a circle, which match what predicted by the term $\frac{(1-((1-p)z)^{k_{\max}+1})}{1-(1-p)z}$ in Equation (12), indicating a truncated Geometric component; (b) Besides the zeros around the circle, the contour plots also have a small number of other zeros (off the circle), which come from the factor $\sum_{i=1}^{k_t} p_i z^i$ associated with the irregular Discrete component; (c) The contour plots for the two systems have zeros with similar pattern but different positions. This implies that their node degrees have similar distribution functions but with different parameters.

From the contour plots one can easily locate the zeros in PGF, and further determine the parameters of corresponding distribution functions. The estimated parameters for the nodes in the NYISO and the WSCC systems are listed in the Table I. Fig. 3 compares the probability mass function (PMF) with estimate parameters and the empirical PMF for both systems and shows that the former matches the latter with quite a good approximation. The results validated our assumption of node degree distribution in power grids, i.e., that it can be expressed as a sum of a truncated Geometric random variable and an irregular Discrete random variable. And the results also demonstrated the effectiveness of the proposed method of analyzing node degree distribution by using the probability generation function which, to the best of our knowledge, has not been employed before.

[2]The WSCC system represents the electrical power grid of the western United States. The topology contains 4941 nodes and 6954 links, with the average node degree $\langle k \rangle = 2.67$.

TABLE I
ESTIMATE PARAMETERS OF THE TRUNCATED GEOMETRIC AND THE IRREGULAR DISCRETE FOR THE NODE DEGREES IN THE NYISO AND WSCC SYSTEM

node groups	max($\underline{k}$)	p	$k_{\max}$	k_t	$\{p_1, p_2, \cdots, p_{k_t}\}$
NYISO	37	0.2269	34	3	0.4875, 0.2700, 0.2425
WSCC	19	0.4084	16	3	0.3545, 0.4499, 0.1956



(a) (b)

Fig. 3. Comparing the Empirical and Fitting PMF of Node Degrees: (a) the NYISO System; (b) the WSCC System

IV. ROBUSTNESS EXPERIMENTS ON REAL-WORLD POWER GRIDS

In this section we perform numerical tests to determine the robustness on the IEEE model systems, the NYISO and the WSCC systems under random and selective breakdowns. The former eliminates a fraction of nodes from the system randomly, i.e., the selection of breakdown nodes is independent of the network structure. While the latter intends to disintegrate the network in one of the most effective ways, as in an intentional attack, with the nodes with largest node degrees eliminated first. And, for the nodes with the same node degrees, their chance of breakdown is uniform. After a fraction f of node removal (f is a fraction to the original intact network size), the relative size of the largest connected spanning cluster in the remaining network, S_{inf}, is evaluated to indicate the effects of the node breakdowns. The critical breakdown threshold is computed by using the criterion as Equation (3). Fig. 4 shows the experiment results: comparing (a) and (b), one can clearly see that S_{inf} in the remaining network is brought down much more effectively under the selective node breakdowns than that at random.

Table II presents the critical thresholds under random and selective breakdowns in IEEE model systems and real-world power grids such as the NYISO and the WSCC systems. As expected f_c^{sel} is much less than f_c^{rand} which means that the removal of a small fraction of nodes with larger degrees in the network will disintegrate the system very quickly.

Fig. 6 shows the empirical results compared to the theoretical thresholds from Equation (4) and (7), given $\overline{k}_0$ of the original network. An interesting discovery is that the empirical critical thresholds under random breakdowns matches the theoretical f_c^{rand} with good approximation; while the empirical critical thresholds under selective breakdowns deviates from the theoretical value substantially. The reason is that Equation (4) holds in general for any node degree distribution as long as it is random; Equation (7), instead, is derived from the assumption of a pure Geometric (or equivalently Exponential) distribution for the node degrees. As we pointed out in Section III, the node degree in power grids fits well a mixture model which includes the sum of a truncated Geometric random variable $\mathcal{G}(p, k_{\max})$ plus an irregular Discrete random variable $\mathcal{D}(p_1, p_2, \ldots, p_{k_t})$. In the following paragraphs we will provide a intuitive quantitative argument showing that it is the discrepancy in the assumed node degree distribution model that causes the substantial deviations observed in the empirical and the theoretical f_c^{sel}.

The probability distribution of the node degrees $\mathcal{K}$ in a power grid, equal to the convolution of that of the two components, can be expressed as following by using Equation (9) and (10).

$$\text{Pr}_{(\mathcal{K}=k)} = \begin{cases} \alpha \sum_{i=1}^{k} pq^{k-i}p_i \ , & k < k_t \\ \alpha \sum_{i=1}^{k_t} pq^{k-i}p_i \ , & k_t \le k \le k_{\max}+1 \\ \alpha \sum_{i=k-k_{\max}}^{k_t} pq^{k-i}p_i \ , & k_{\max}+1 < k \le K \end{cases} \tag{13}$$

with $q = 1-p$, $K = k_t + k_{\max}$, and $\alpha = \frac{1}{1-q^{k_{\max}+1}}$ as constants.

On the other hand, a pure Geometric with truncation at K has $\text{Pr}_{(\widetilde{\mathcal{K}}=k)} = \alpha p q^{k-1}$, with $k = 1,\ 2,\ \ldots,\ K$. Dividing $\text{Pr}_{(\mathcal{K}=k)}$ by $\text{Pr}_{(\widetilde{\mathcal{K}}=k)}$, we have the ratio as

$$r_{\mathcal{K}/\widetilde{\mathcal{K}}}(k) = \frac{\text{Pr}_{(\mathcal{K}=k)}}{\text{Pr}_{(\widetilde{\mathcal{K}}=k)}} = \begin{cases} \sum_{i=1}^{k} \frac{p_i}{q^{i-1}} \ , & k < k_t \\ \sum_{i=1}^{k_t} \frac{p_i}{q^{i-1}} \ , & k_t \le k \le k_{\max}+1 \\ \sum_{i=k-k_{\max}}^{k_t} \frac{p_i}{q^{i-1}} \ , & k_{\max}+1 < k \le K \end{cases} \tag{14}$$

Fig. 5 plots the ratio $r_{\mathcal{K}/\widetilde{\mathcal{K}}}(k)$ growing as a function of the node degree k, with the parameters obtained from Table I for the NYISO system and the WSCC system respectively. Our study on the node degree distribution of the power grid in Section III has shown that a real-world power grid usually has $k_{\max} \gg k_t$ (refer to Table I). As a result, the ratio curve has a very short monotonically-increasing section at the beginning and a very short monotonically-decreasing section at the tail, each with a length of $k_t - 1$; while for the much longer middle interval, i.e., where $k \in [k_t, k_{\max}+1]$, the ratio stabilizes at its upper bound:

$$r^{\max} = \sum_{i=1}^{k_t} \frac{p_i}{q^{i-1}} \tag{15}$$

Furthermore, due to the short length of the tail section and due to the negligible probability mass of a Geometric random variable in that portion (that is, with the PMF decreasing exponentially, its probability mass dies out very quickly as k increases), it should be reasonable to roughly assume that the ratio $r_{\mathcal{K}/\widetilde{\mathcal{K}}}(k) \equiv r^{\max}$ as long as $k \ge k_t$. And this approximation will not cause substantial numerical errors in the evaluation of integral probability to estimate the critical node breakdown threshold as in Equation (5) and (16).

On the other hand, we can see that $r_{\mathcal{K}/\widetilde{\mathcal{K}}}(1) = p_1 < 1$, and $r^{\max} > 1$, which means the irregular Discrete random variable component embedded in the node degree distribution function reduces the relative mass of probability for lowest degree values, while magnifying the probability of large node degrees. This leads to the conclusion that, compared to a network with pure Geometric node degree distribution, the power grid is more vulnerable to intentional attacks when nodes with large degrees become first targets of the attack; this happens because the number of links lost due to the node breakdowns statistically increases, even if the fraction of node removal is kept the same as that dictated by the pure Geometric distribution model.

Following the estimation method of f_c^{sel} in [9] and taking into account the correct node degree distribution for power grid in Section III, we derived a new cut-off node degree $\tilde{K}^{\text{new}}$ after node removal which satisfies $r^{\max} \int_{\tilde{K}^{\text{new}}}^{K} P(k)dk = f^{\text{sel}}$, therefore

$$\tilde{K}^{\text{new}} = -\gamma \ln \frac{f^{\text{sel}}}{r^{\max}} \tag{16}$$

and a new equivalent f_{eqv}^{rand} given the same f^{sel}. With

$$f_{eqv}^{\text{rand}} = \int_{\tilde{K}^{\text{new}}}^{K} \frac{k\text{Pr}_{(\mathcal{K}=k)}}{\langle k \rangle} dk \tag{17}$$

$$= r^{\max} \int_{\tilde{K}^{\text{new}}}^{K} \frac{k\text{Pr}_{(\widetilde{\mathcal{K}}=k)}}{\langle k \rangle} dk \tag{18}$$

$$= \left(1 - \ln \frac{f^{\text{sel}}}{r^{\max}}\right) f^{\text{sel}} \tag{19}$$

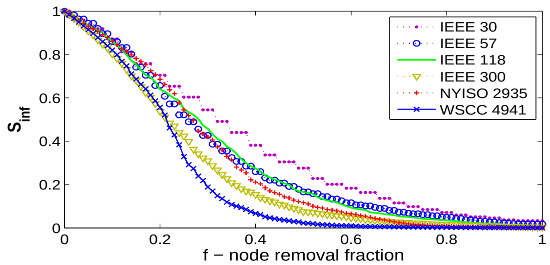
with $\text{Pr}_{(\widetilde{\mathcal{K}}=k)}$ representing the pure Geometric (or Exponential) distribution assumed in [9]. Under selective node breakdowns, only a small fraction of nodes need to be eliminated in order to disintegrate the network (refer to Table II with $0.15 < f_c^{\text{sel}} < 0.40$), which means $\tilde{K}$ is big enough therefore $\tilde{K} > k_t$ (as a matter of fact $\tilde{K} = 5$ *or* 6 for the NYISO and the WSCC). Therefore we set the ratio to its maximum value. Consequently the critical breakdown threshold for power grid under selective node removal can obtained by solving below equation:

$$\left(1 - \ln \frac{f_c^{\text{sel}}}{r^{\max}}\right) f_c^{\text{sel}} = 1 - \frac{1}{\bar{k}_0 - 1} \tag{20}$$

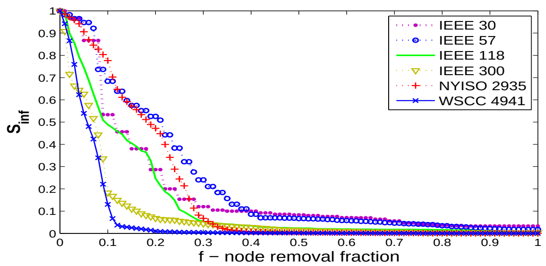
Obviously, Equation (20), compared to (7), implies a much smaller critical node removal threshold due to the presence of $r^{\max}$. With the maximum ratios $r_{\text{NYISO}}^{\max} = 1.4074$ and $r_{\text{WSCC}}^{\max} = 1.9690$, the new theoretical f_c^{sel} curves have been drawn in Fig. 6. It shows that these curves match the empirical results much more closely than that obtained from Solé's model as in Equation (7), which verifies our analysis above.

V. Conclusion

In this paper we numerically study the topology robustness of power grids under random and selective node breakdowns, and analytically estimate the critical node-removal thresholds to disintegrate a system, based on the available US power grid



(a)



(b)

Fig. 4. Effects of Random and Selective Node Breakdowns on Real-world Power Grids: (a) Random Breakdowns; (b) Selective Breakdowns

TABLE II
CRITICAL THRESHOLDS UNDER RANDOM AND SELECTIVE BREAKDOWNS IN REAL-WORLD POWER NETWORKS

	(N, m)	$\langle k \rangle$	$\overline{k}$	f_c^{rand}	f_c^{sel}
IEEE-30	(30,41)	2.73	3.44	0.5298	0.1618
IEEE-57	(57,78)	2.74	3.18	0.4680	0.1892
IEEE-118	(118,179)	3.03	3.84	0.6278	0.2062
IEEE-300	(300, 409)	2.73	3.60	0.6114	0.2088
NYISO	(2935,6567)	4.47	7.92	0.8470	0.3595
WSCC	(4941, 6594)	2.67	3.87	0.6545	0.1685

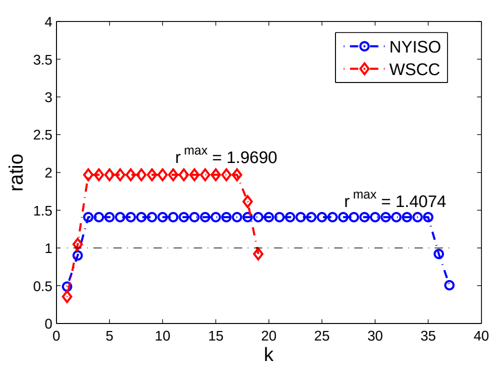


Fig. 5. The Ratio $r_{\mathcal{K}/\widetilde{\mathcal{K}}}(k)$ vs. Node Degree k

data. It is evident that selective node breakdowns is much more effective to fragment a network because the nodes with largest degrees become first targets of attack and even a small fraction of node removal can cause dramatic damage. Although the empirical thresholds under random node breakdowns match the theoretical values very well, the thresholds under selective breakdowns obviously deviates from the predicted results from [9] which assumed that node degree in a power grid follows a

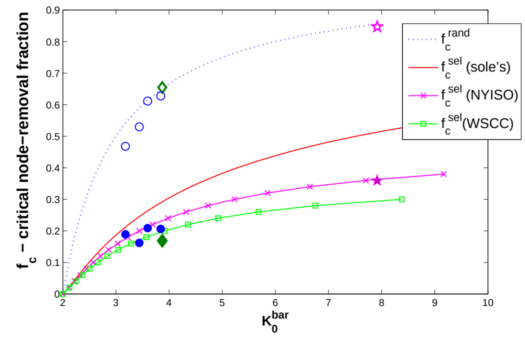


Fig. 6. The Theoretical and Empirical Critical Breakdown Thresholds vs. $\overline{k}_0$. f_c^{rand}: theoretical(blue dotted line), IEEE model system (circles), WSCC (hollow diamond), NYISO (hollow star); f_c^{sel}: theoretical-Solé's model [9] (red solid line), theoretical-our model with $r^{\max}$-NYISO (x-marked pink line), our model with $r^{\max}$-WSCC (square-marked green line), IEEE model system (solid circles), WSCC (solid diamond), NYISO (solid star)

pure Geometric distribution. Our analysis shows that the node degree distribution of power grids is not a pure Geometric probability mass function but that it can be well fitted by a mixture distribution coming from the sum of a truncated Geometric random variable and an irregular Discrete random variable. Our findings give better estimates of the threshold for a disintegrated topology under selective node breakdowns which we compared to the numerical thresholds obtained with real grid data. The study results provide a deeper understanding of the intrinsic robustness of power grid and will help us search for the "smart" designs of communication architecture and control schemes in order to compensate for the network intrinsic vulnerability and to enhance its robustness.

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