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The non-linear dependence of flux on black hole mass and accretion rate in core-dominated jets

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ABSTRACT

We derive the non-linear relation between the core flux F_ν of accretion-powered jets at a given frequency and the mass M of the central compact object. For scale-invariant jet models, the mathematical structure of the equations describing the synchrotron emission from jets enables us to cancel out the model-dependent complications of jet dynamics, retaining only a simple, *model-independent* algebraic relation between F_ν and M. This approach allows us to derive the F_ν–M relation for any accretion disc scenario that provides a set of input boundary conditions for the magnetic field and the relativistic particle pressure in the jet, such as standard and advection-dominated accretion flow (ADAF) disc solutions. Surprisingly, the mass dependence of F_ν is very similar in different accretion scenarios. For typical flat-spectrum core-dominated radio jets and standard accretion scenarios, we find $F_\nu \sim M^{17/12}$. The 7–9 orders of magnitude difference in black hole mass between microquasars and active galactic nuclei (AGN) jets imply that AGN jets must be about 3–4 orders of magnitude more radio-loud than microquasars, i.e. the ratio of radio to bolometric luminosity is much smaller in microquasars than in AGN jets. Because of the generality of these results, measurements of this F_ν–M dependence are a powerful probe of jet and accretion physics. We show how our analysis can be extended to derive a similar scaling relation between the accretion rate $\dot{m}$ and F_ν for different accretion disc models. For radiatively inefficient accretion modes, we find that the flat-spectrum emission follows $F_\nu \propto (M\dot{m})^{17/12}$.

Key words: radiation mechanisms: non-thermal – galaxies: active – galaxies: jets – galaxies: nuclei – radio continuum: general – X-rays: binaries.

1 INTRODUCTION

Relativistic jets are collimated outflows from the innermost regions of accretion discs around black holes and neutron stars. Not all accreting compact objects form jets, but when they do, the jet synchrotron radiation dominates typically the radio spectrum of the compact object. Such objects are called radio-loud.

Compact objects span 9 orders of magnitude in central mass: in active galactic nuclei (AGN), it ranges from $M \sim 10^6\ \mathrm{M}_\odot$ to $M \sim$ few $\times\ 10^9\ \mathrm{M}_\odot$, whereas Galactic X-ray binaries extend this range down to a few $\mathrm{M}_\odot$. Yet, the jets formed by these objects appear morphologically remarkably similar, and their core emission follows typically the same flat power-law spectrum. This suggests that the process of jet formation is universal, and that jets from supermassive black holes of different masses are not qualitatively different from each other and from jets in X-ray binaries, called microquasars. If this is so, we can compare jets from objects of different mass M (which is measurable dynamically) and accretion rate $\dot{M}$ (which is proportional to the accretion luminosity L_{acc}) and determine how their observable characteristics change with M and $\dot{M}$ (Sams, Eckart & Sunyaev 1996).

The most readily available observable parameter is the jet flux at a given frequency, F_ν. In this Letter, therefore, we shall derive the relationship between F_ν and M from theoretical arguments. As we will show, the mathematical structure of the expression for the jet synchrotron flux F_ν enables us to contract all the model-dependent complications of jet physics into the formula for the observable spectral index α and thus remove them from the relation between F_ν and M. Thus for any observed value of α and for a set of boundary conditions delivered by accretion disc theory, we can formulate a *model-independent*, non-linear relation between F_ν and M.

A lot of effort has gone into searching for observational correlations between F_ν, M, and $\dot{M}$. Such measurements are difficult because of numerous selection effects. Nevertheless, some observational evidence of a non-linear dependence between AGN radio flux and black hole mass (Franceschini, Vercellone & Fabian 1998; Laor 2000; Lacy et al. 2001; McLure & Dunlop 2001) exists in the recent literature. However, other authors have found no such evidence (Ho 2002; Woo & Urry 2002). A systematic difference of radio-loudness

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between neutron star and black hole X-ray binaries has also been suggested (Fender & Kuulkers 2001).

Because of the large mass difference, any non-linearity between F_ν and M must be most apparent when comparing microquasars with AGN jets. Indeed, observations show that the radio-loudness parameter $R = L_{\rm R}/L_{\rm UV,X-ray}$, defined as the ratio of radio luminosity (emitted by the jet) to UV/X-ray luminosity (emitted by the accretion disc), is much smaller for microquasars during outburst than it is for radio-loud AGN (Falcke & Biermann 1996). In other words, the radio jet flux F_ν depends non-linearly on M.

In the following sections, we will argue that this observational non-linearity is not only consistent with but required by the *model-independent* F_ν–M relation that we will derive below.

2 SCALE-INVARIANT JETS

The two fundamental parameters that determine the conditions in the inner accretion disc are the accretion rate $\dot{M}$ and the mass M of the central object. All length-scales are proportional to the fundamental scale of the compact object, $r_{\rm g} \propto M$. The characteristic accretion rate of the disc is set by the Eddington rate $\dot{M}_{\rm Edd}$; for convenience, we will define the dimensionless accretion rate $\dot{m} \equiv \dot{M}/\dot{M}_{\rm Edd}$. All dynamically important variables are determined by these two parameters.

Because jets are formed in the inner disc, it is reasonable to assume that the conditions in the inner jet are set by the conditions in the inner disc; thus, they will similarly depend on M and $\dot{m}$. However, it is possible that jets are powered by black hole spin extraction (Blandford & Znajek 1977), in which case all jet variables would also depend on the black hole spin parameter a. Thus, any dynamically important jet variable at the base of the jet will be determined by these three parameters.

2.1 Dependence on *M*

As mentioned in the introduction, observations suggest that the process of jet formation is universal and that no qualitative difference exists between jets from objects of different mass. This morphological and spectral similarity of jets from objects with fundamentally different black hole masses suggests that jet formation and propagation might be scale-invariant processes,[1] i.e. that there is one relevant length-scale in jet formation, which is $r_{\rm g}$, and that jet dynamics are invariant under changes in this length-scale.[2]

Scale invariance implies that the spatial variation of important jet quantities [such as the shape of the jet (i.e. its lateral cross-section), the orientation of magnetic field lines, the field strength, etc.] depends only on the dimensionless variable $\boldsymbol{r}/r_{\rm g}$. Thus, a given variable f should be proportional to a function $\psi(\boldsymbol{r}/r_{\rm g})$ which depends on $\boldsymbol{r}$ only through $\boldsymbol{r}/r_{\rm g}$. In other words, we can scale a jet model for mass M_1 by a length factor M_2/M_1 and some spatially independent normalization factor and arrive at a jet model for mass M_2.

In mathematical terms, this can be expressed as the condition that we can write any dynamically relevant quantity f, such as the magnetic field $B(r)$, as the product of two decoupled functions:

$$\begin{aligned} f(M, \dot{m}, a, r) &= \phi_f(M, \dot{m}, a)\psi_f\left(\frac{r}{r_{\rm g}}, \dot{m}, a\right) \\ &= \phi_f(M, \dot{m}, a)\psi_f(\chi, \dot{m}, a), \end{aligned} \quad (1)$$

where r is the distance to the central engine measured along the jet, ϕ_f describes the dependence of f on the central engine mass M, and ψ_f describes the spatial dependence of f on the similarity variable $\chi \equiv r/r_{\rm g}$ for a given set of $\dot{m}$ and a. Note that this is a *requirement* we place on the jet model, inspired by the observational similarity between jets from different kinds of objects. Not all possible jet model must necessarily satisfy this relation. However, those models that do satisfy it span an important subclass of jet models and all of them will obey the relations derived below. One important example of such a model is the Blandford & Königl (1979) model.

The normalization functions ϕ_f reflect the dependence of the conditions at the base of the jet on the central mass M. Because jets are launched above accretion discs, it is natural to assume that these functions ϕ_f can be adopted from accretion disc models.

For any geometric quantity, such as the jet diameter $R(r)$, the direct proportionality of R to $r_{\rm g}$ requires that $\phi_R \propto M$ (where we have contracted the constant of proportionality into ψ_R for convenience). As a dimensionless variable, it is reasonable to assume that the jet Lorentz factor Γ is entirely independent[3] of M. Although measuring the bulk velocity of jets directly is impossible in most cases, the existing observational limits suggest that jets from microquasars are not any more or less relativistic than their AGN counterparts, despite 9 orders of magnitude difference in M. We take this as sufficient evidence to assume in the following that Γ does not depend explicitly on mass, which allows us to write $\Gamma(r) = \psi_\Gamma(\chi, \dot{m}, a)$, i.e. $\phi_\Gamma = 1$.

The state of the inner accretion disc depends on the accretion rate $\dot{m}$. In all standard accretion disc models, the fundamental quantities take on a rather simple scaling with the black hole mass M. For high accretion rates, where electron scattering becomes the dominant opacity source and where radiation pressure dominates the energetics in the inner disc, the density and the pressure scale inversely with mass, $\varrho \propto P_{\rm rad} \propto M^{-1}$ (Shakura & Sunyaev 1976). The magnetic field might be of the same order as the total pressure, and thus $B \propto M^{-1/2}$ (Shakura & Sunyaev 1973). Recent numerical computations of the magneto-rotational instability in accretion discs seem to support this statement (Balbus & Hawley 1998). The same scaling arises in advection-dominated flows (Narayan & Yi 1995) and in any scenario in which dynamical terms dominate the cooling rate (e.g. convection- or outflow-dominated discs). Thus, we have

[1] The scale invariance assumed for the jet structure is only valid in the inner regions of the jet, where interactions with the environment are not important. On large scales, where this interaction dominates the jet dynamics, additional parameters independent of the inner accretion disc enter – most prominently, the external density and pressure. In this case, it is still possible to write down extended scaling relations (Heinz 2002), but not in the form of equation (1). However, because we restrict our analysis to the emission from the jet core, we can neglect these complications.

[2] Although the influence of the spin parameter a on jet formation is not clear, it is important to note that a second length-scale might be present in the process of jet formation, which is the light cylinder radius r_Ω of the black hole spin. However, r_Ω depends linearly on M and is thus a multiple of $r_{\rm g}$. The proportionality factor depends *only* on a, not on M, i.e. $r_\Omega = f(a)\, r_{\rm g}$. Thus, for a fixed spin parameter a, the only relevant length-scale that changes upon variations in M, which we are primarily concerned with here, is $r_{\rm g}$. This is the length-scale of relevance for changes in M, and we will assume henceforth that jet formation is invariant under changes in $r_{\rm g}$ for otherwise fixed parameters, as suggested by the observational similarity of jets from very different values of M. If a does indeed have significant relevance for the process of jet formation, then variations in a will simply introduce intrinsic scatter to the relations derived below, as all relations will be derived for fixed, but arbitrary, a.

[3] Comparison with young stellar object jets indicates that the specific velocity Γv of jets is related to the orbital/escape speed of the innermost regions of the disc, which is independent of M for black holes.

$\phi_P = M^{-1}$, $\phi_\varrho = M^{-1}$ and $\phi_B = M^{-1/2}$. Observations of X-ray binaries suggest that jet formation is linked to the so called *low–hard state* (Fender & Kuulkers 2001), which is believed to arise from an optically thin, geometrically thick accretion disc which follows these scalings. Therefore, we will adopt $\phi_P = \phi_\varrho = \phi_B^2 = M^{-1}$ as our fiducial values. Only if the innermost disc is of the standard gas-pressure-dominated Shakura & Sunyaev (1973) type (which might be the case in low-$\dot{m}$, low-α accretion discs in AGN – Frank, King & Raine 2002) does this scaling differ slightly: here, $P \propto B^2 \propto M^{-9/10}$ and $\varrho \propto M^{-7/10}$.

Jets emit synchrotron radiation from a power-law distribution of electrons:

$$\mathrm{d}n/\mathrm{d}\gamma = C\gamma^{-p} \tag{2}$$

within an energy range $\gamma_{\min} < \gamma < \gamma_{\max}$. Here, γ is the particle Lorentz factor, C is the normalization constant and p is the power-law index. The production of power-law distributions is a universal property of diffusive shock acceleration, and we will assume that the fundamental power-law parameters p and $\gamma_{\min}$ are universal in relativistic jets as well. Typically, the observations from the optically thin part of jet spectra give $p \gtrsim 2$. In the following, we will take $p = 2$ as our fiducial value for numerical examples. Because the high energy cut-off is dynamically unimportant for spectra with $p \geqslant 2$, we will not be concerned with its behaviour. C is then directly proportional to the pressure in relativistic particles and we can once again write $C = \phi_C(M)\psi_C(\chi, \dot{m}, a)$. It is reasonable and customary to assume that the relativistic power-law particle distribution is injected at some (unknown) fraction of equipartition with the magnetic field pressure, so $C \propto B^2$. Thus, for our fiducial values, we have $\phi_C = M^{-1}$.

The functions $\psi(\chi)$ can, in principle, take on rather complicated behaviour, depending on the specific jet model. We will not be concerned with the detailed nature of ψ, so long as they are mathematically well behaved (see Section 3.2 for a definition of what this means).

2.2 Dependence on $\dot{m}$

Although the main aim of this Letter is to derive the scaling relation between jet radio flux and black hole mass, it is interesting to consider the dependence on other accretion disc parameters, namely $\dot{m}$ (see also Section 3.3). The dependence of the fundamental disc parameters on $\dot{m}$ varies more significantly between different accretion models than the dependence on M.

Shakura & Sunyaev (1973) showed that in radiation-pressure-supported discs, the total and magnetic pressure are independent of $\dot{m}$. Thus, the magnetic pressure B^2 and the relativistic particle pressure C in the jet are also independent of $\dot{m}$. The mass density in the disc, on the other hand, should follow $\varrho \propto \dot{m}^{-2}M^{-1}$, which might or might not affect the mass loading and thus the Lorentz factor of the jet.

In mechanically cooled accretion discs (e.g. ADAFs), the pressure P and particle density ϱ are directly proportional to $\dot{m}$. In jets launched from such flows, we thus have $B^2 \propto C \propto \dot{m}M^{-1}$.

We can derive the same scaling if we assume simply that the mechanical jet power $W_{\rm jet}$ should be proportional to the disc luminosity $L_{\rm disc} \propto \dot{M} = M\dot{m}$. Because the jet power at injection is carried by internal energy, and because we can assume that the magnetic field and the relativistic particle pressure are related to each other by some form of dissipation (e.g. reconnection, shocks), both C and B are related to the jet power by $B^2 \propto C \propto W_{\rm jet}/R^2c \propto \dot{M}/M^2 \propto \dot{m}M^{-1}$. As mentioned above, we chose this parametrization as our fiducial case.

Finally, in standard gas-pressure dominated discs (however, still dominated by electron scattering, appropriate for the inner gas-pressure dominated discs of AGN and X-ray binaries), the pressure follows $P \propto \dot{m}^{4/5}M^{-9/10}$; thus, for the jet plasma, we have $B^2 \propto C \propto \dot{m}^{2/5}M^{-9/20}$, whereas the mass density follows $\varrho \propto \dot{m}^{2/5}M^{-7/10}$.

After this excursion into the dependence of accretion disc and jet parameters on $\dot{m}$, now we proceed to investigate the radiative characteristics of self-similar jets.

3 THE NON-LINEAR SCALING OF JET FLUX WITH BLACK HOLE MASS AND ACCRETION RATE

3.1 Synchrotron emission from self-similar jets

The synchrotron self-absorption coefficient is

$$\alpha_\nu = A_p C B^{\frac{p+2}{2}} \nu^{-\frac{p+4}{2}}, \tag{3}$$

where A_p is a proportionality constant weakly dependent on p (Rybicki & Lightman 1979).

For ease of expression, we will present the following analysis in the case of a jet viewed from side on; however, extension to the case of arbitrary viewing angles is straightforward and the result we derive is fully general. In the perpendicular case, the expression for τ_ν takes on a particularly simple form:

$$\begin{aligned}\tau_\nu &= R_{\rm jet}\alpha_\nu = R_{\rm jet} A_p C B^{\frac{p+2}{2}} \nu^{-\frac{p+4}{2}}\\ &= A_p M \phi_C(M, \dot{m}, a)[\phi_B(M, \dot{m}, a)]^{\frac{p+2}{2}} \nu^{-\frac{p+4}{2}}\\ &\quad \times \psi_R(\chi, \dot{m}, a)\psi_C(\chi, \dot{m}, a)[\psi_B(\chi, \dot{m}, a)]^{\frac{p+2}{2}}\\ &= \Phi(M, \dot{m}, a, \nu)\Psi(\chi, \dot{m}, a),\end{aligned} \tag{4}$$

where we define

$$\Phi(M, \dot{m}, a, \nu) \equiv M\phi_C(M, \dot{m}, a)[\phi_B(M, \dot{m}, a)]^{\frac{p+2}{2}} \nu^{-\frac{p+4}{2}} \tag{5}$$

$$\Psi(\chi, \dot{m}, a) \equiv A_p \psi_R(\chi, \dot{m}, a)\psi_C(\chi, \dot{m}, a)[\psi_B(\chi, \dot{m}, a)]^{\frac{p+2}{2}}. \tag{6}$$

The optically thin synchrotron emissivity for a power-law distribution of electrons (well away from the lower and upper cut-off in the energy distribution) follows

$$\begin{aligned}j_\nu &= J_p C B^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}}\\ &= J_p \phi_C(M, \dot{m}, a)\,[\phi_B(M, \dot{m}, a)]^{\frac{p+1}{2}}\ \nu^{-\frac{p-1}{2}}\\ &\quad \times \psi_C(\chi, \dot{m}, a)[\psi_B(\chi, \dot{m}, a)]^{\frac{p+1}{2}},\end{aligned} \tag{7}$$

where J_p is a constant weakly dependent on p (Rybicki & Lightman 1979).

For simplicity, we will combine the dependence on the viewing angle ϑ due to Doppler beaming and optical depth effects into the function $\zeta(\vartheta)$. Because the viewing angle ϑ and the Lorentz factor Γ are independent of M, it follows that $\zeta(\vartheta)$ must also be independent of M, which justifies this approach in what follows. The jet surface brightness at a given frequency ν is then $S_\nu \sim \zeta(\vartheta)\, j_\nu\,(1 - \mathrm{e}^{-\tau_\nu})/\alpha_\nu$. The jet flux F_ν is then simply the surface integral over S_ν:

$$F_\nu = \int_{r_g}^{\infty} dr\, R(r) S_\nu(r) \approx \zeta(\vartheta) \int_{r_g}^{\infty} dr\, R(r) j_\nu(r) \frac{1 - e^{-\tau_\nu(r)}}{\alpha_\nu(r)}$$
$$\approx \zeta(\vartheta) \int_{r_g}^{\infty} dr [R(r)]^2 j_\nu(r) \frac{1 - e^{-\tau_\nu(r)}}{\tau_\nu(r)}$$
$$\propto \zeta(\vartheta) M^3 \phi_C \phi_B^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \int_1^{\infty} d\chi\, \psi_R^2 \psi_C \psi_B^{\frac{p+1}{2}} \frac{1 - e^{-\Phi\Psi}}{\Phi\Psi}$$
$$\propto M^3 \phi_C \phi_B^{\frac{p+1}{2}} \nu^{-\frac{p-1}{2}} \Theta[\Phi(M, \dot{m}, a, \nu), \dot{m}, a, \vartheta]. \quad (8)$$

The integral Θ depends on M and ν only through the combination Φ from equation (5).

3.2 The relation between F_ν and M

From equation (8), we can now work out the non-linear dependence of F_ν on the central engine mass M. The spectral index $\alpha \equiv -\partial \ln (F_\nu)/\partial \ln (\nu)$ of the jet emission is given by

$$\frac{\partial \ln (F_\nu)}{\partial \ln (\nu)} = -\frac{p-1}{2} + \frac{\partial \ln (\Theta)}{\partial \ln (\Phi)} \frac{\partial \ln (\Phi)}{\partial \ln (\nu)} \quad (9a)$$

$$= -\frac{p-1}{2} - \frac{\partial \ln (\Theta)}{\partial \ln (\Phi)} \left(\frac{p+4}{2}\right) \equiv -\alpha. \quad (9b)$$

Now taking the partial derivative of equation (8) with respect to M and substituting $\partial \ln (\Theta)/\partial \ln (\Phi)$ from equation (9b), we can write

$$\frac{\partial \ln (F_\nu)}{\partial \ln (M)} = 3 + \frac{\partial \ln \phi_C}{\partial \ln (M)} + \frac{\partial \ln \phi_B^{\frac{p+1}{2}}}{\partial \ln (M)} + \frac{\partial \ln (\Theta)}{\partial \ln (\Phi)} \frac{\partial \ln (\Phi)}{\partial \ln (M)}$$
$$= \frac{2p + 13 + 2\alpha}{p+4} + \frac{\partial \ln (\phi_B)}{\partial \ln (M)} \left(\frac{2p + 3 + \alpha p + 2\alpha}{p+4}\right)$$
$$+ \frac{\partial \ln (\phi_C)}{\partial \ln (M)} \left(\frac{5 + 2\alpha}{p+4}\right) \equiv \xi_M. \quad (10a)$$

Quite generally, the functions ϕ_C and ϕ_B will be simple powers of M – for our fiducial assumptions, $\phi_C = M^{-1}$ and $\phi_B = M^{-1/2}$, and thus the index ξ_M will be simply a constant:

$$\xi_M = \frac{2p + 13 + 2\alpha}{p+4} - \frac{1}{2}\left[\frac{2p + 3 + (p+2)\alpha}{p+4}\right] - \frac{5 + 2\alpha}{p+4}$$
$$\sim \frac{17}{12} - \frac{\alpha}{3} \approx 1.42 - 0.33\alpha, \quad (10b)$$

where the approximate expressions assume $p = 2$. Thus, for any given set of $\dot{m}$, a and ϑ, F_ν will follow a simple power-law relation in M with power-law index ξ_M:

$$F_\nu \propto M^{\xi_M} \sim M^{1.42 - 0.33\alpha}. \quad (11)$$

Variations in the other source parameters $\dot{m}$, a, the viscosity parameter $\alpha_{\rm visc}$ and ϑ will only cause a *mass-independent* scatter around this relation.

Remarkably, this result is entirely independent of the functions ψ_f. Given a set of functions ϕ_f which describe the dependence of the input conditions in the inner disc on M, and given an observed jet spectrum with spectral index α, equation (11) *predicts* the scaling of jet flux F_ν with M for any jet model that reproduces this spectral slope. The only assumptions that went into the derivation of this result are (a) that the relevant parameters can be decomposed following equation (1), (b) that the high-/low-energy cut-offs in the spectrum are far above/below the observed spectral band, and (c) that the function Θ is analytic. This is what was meant when we required the functions ψ_f to be mathematically well behaved in Section 2.

Typically, the radio emission from core-dominated jets follows a flat spectrum over many decades in frequency, i.e. $\alpha \sim 0$. In this case, it follows for our fiducial parameters that the radio flux F_ν depends non-linearly on the mass to the $\xi_M = 17/12 \sim 1.42$ power, once again *independent* of the jet model, which manifests itself only through ψ_f. Falcke & Biermann (1995) based their adaptation of the original Blandford & Königl (1979) model on the assumption that $B \propto M^{-1/2}$ and $C \propto B^2$. Indeed, for their specific choice of ψ_f, they found $\xi_M = 17/12$, which they already showed to be consistent with observations of flat-spectrum radio jets from AGN and microquasars (Falcke & Biermann 1996).

As we mentioned before, the fiducial $B^2 \propto C \propto M^{-1}$ scaling arises in a number of standard scenarios for the inner accretion disc – both in high-efficiency, radiation-pressure-dominated inner discs and in low-efficiency ADAFs. The value of $\xi = 17/12 - \alpha/3$ is therefore a very general result which depends only weakly on the spectral index α. In jets that are launched from standard gas-pressure-dominated discs that extend all the way to the innermost stable orbit, the change in the scaling to $\phi_B^2 = \phi_C = M^{-9/10}$ leads to a change in the mass index: $\xi_M = [143 + 22p - \alpha(14 + 9p)]/[20(p + 4)] \sim 1.56 - 0.23\alpha$, which is even more non-linear than the standard value of $\xi_M \sim 1.42 - 0.33\alpha$ from equation (10b). If the magnetic field responsible for spin extraction from black holes is supported by or anchored in the inner disc, the same considerations might hold for Blandford–Znajek (1977) jets.

It is worth noting that this analysis holds even for the case of jets composed of discrete ejections or internal shocks, if we define $F_{\rm r}$ as the time-averaged flux or the peak flux. In fact, because the derivation of equations (8)–(10a) did not assume any specific jet-like geometry, they hold for any synchrotron-emitting plasma with power-law spectra if the source parameters can be described by equation (1).

3.3 The relation between F_ν and $\dot{m}$

An interesting feature of the derivation of equation (10a) is that it is modular: any fundamental accretion disc parameter that enters into the dynamical description of the jet in an invariant fashion following equation (1) such that it only appears in the functions ϕ_f and not in ψ_f leads to such a relation. For example, if we can separate the dependence of any dynamical quantity f on the accretion rate $\dot{m}$ into a function ϕ_f so that $f(M, \dot{m}, a, r) = \phi(M, \dot{m}, a)\psi(\chi, a)$, we can derive a non-linear relation between $F_{\rm r}$ and $\dot{m}$ of the form[4]

$$\frac{\partial \ln (F_\nu)}{\partial \ln (\dot{m})} = \frac{\partial \ln (\phi_B)}{\partial \ln (\dot{m})} \left[\frac{2p + 3 + \alpha(p+2)}{p+4}\right]$$
$$+ \frac{\partial \ln (\phi_C)}{\partial \ln (\dot{m})} \left(\frac{5 + 2\alpha}{p+4}\right) \equiv \xi_{\dot{m}}, \quad (12a)$$

following the same derivation as in equation (10a).

For our fiducial assumption that $\phi_C \propto \phi_B^2 \propto \dot{m}$ (from ADAF-type accretion, or the Ansatz $W_{\rm jet} \propto L_{\rm disc}$), we get

[4] Note that the assumption that the jet Lorentz factor $\Gamma_{\rm jet}$ is independent of $\dot{m}$, which is implicit in combining line-of-sight effects into an $\dot{m}$-independent function $\zeta(\vartheta)$, is not necessarily given. In the case where a strong dependence of $\Gamma_{\rm jet}$ on $\dot{m}$ arises, the complications introduced by Doppler beaming will introduce an $\dot{m}$-dependent scatter in the $F_\nu - \dot{m}$ relation, which could skew the distribution away from the mean scaling index $\xi_{\dot{m}}$ expected from equation (12a).

Table 1. The dependence of B and C on M and $\dot{m}$, and the scaling indices ξ_M and $\xi_{\dot{m}}$ for different accretion modes (rows 1–3), and for the Ansatz that the mechanical jet luminosity $W_{\rm jet}$ should be proportional to the disc power $L_{\rm disc}$ (row 4), assuming $p = 2$.

	Injection mode	$B^2 \propto C$	ξ_M	$\xi_{\dot{m}}$
1	ADAF	$\dot{m}/M$	$17/12 - \alpha/3$	$17/12 + 2\alpha/3$
2	rad. press. disc	M^{-1}	$17/12 - \alpha/3$	0
3	gas press. disc	$\dot{m}^{4/5}M^{-9/10}$	$(187 - 32\alpha)/120$	$(17/12 + 2\alpha/3)4/5$
4	$W_{\rm jet} \propto L_{\rm disc}$	$\dot{m}/M$	$17/12 - \alpha/3$	$17/12 + 2\alpha/3$

$$\xi_{\dot{m}} = \frac{2p + (p+6)\alpha + 13}{2(p+4)} \sim \frac{17}{12} + \frac{2\alpha}{3} \approx 1.42 + 0.67\alpha, \qquad (12b)$$

where the approximate expressions assume that $p = 2$. Note that for flat-spectrum sources ($\alpha = 0$), the dependence on $\dot{m}$ is the same as that on M, as found by Sams et al. (1996).

Using the accretion disc scaling relations discussed in Section 2.2, we have presented the power-law indices for the jet scaling relations with mass M and accretion rate $\dot{m}$ expected for these different accretion modes in Table 1. The last two columns in this table show the scaling indices ξ_M and $\xi_{\dot{m}}$ such that

$$F_\nu \propto M^{\xi_M} \dot{m}^{\xi_{\dot{m}}}. \qquad (13)$$

If jet production by black hole spin extraction is invariant under changes in a in the sense of equation (1), i.e. if it depends only trivially on a, we can write a relation between F_ν and a by simply replacing $\dot{m}$ by a in equation (12a).

3.4 Optically thin versus optically thick emission

Synchrotron self-absorption is stronger at lower frequencies. Thus, at high frequencies, the jet must be optically thin even at the location where it is injected. Because equations (10a) and (12a) were derived without any restrictions on τ_ν, we can use them to infer the scaling of radiation at high, optically thin frequencies as well, as long as the other assumptions made above still hold. The only assumption which might be violated is that radiative cooling of the electron spectrum is negligible. Because modifications by radiative cooling should be visible as spectral breaks or cut-offs, we will continue to neglect them here, assuming that a spectral band can be chosen where the spectrum is optically thin yet unaffected by cooling.

If the jet is injected at a distance $\chi_{\rm i}$ from the black hole (expressed in dimensionless units), then the frequency $\nu_{\tau=1}$ at which the jet spectrum becomes optically thin is given by equation (4) by demanding that $\tau_\nu(\chi_i, M, \dot{m}, a) = 1$:

$$\nu_\tau = \left[M\phi_C \phi_B^{\frac{p+2}{2}} \Psi(\chi_i, ...) \right]^{\frac{2}{p+4}} \propto \left(M\phi_C \phi_B^{\frac{p+2}{2}} \right)^{\frac{2}{p+4}}. \qquad (14)$$

Above ν_τ, the spectrum is optically thin and has a spectral index of $\tilde{\alpha} = p - 1/2$ [as can be seen from equation (8) by setting $\tau_\nu \ll 1$]. Here and below, we will denote optically thin values by a tilde. For example, $F_{\tilde{\nu}}$ is the flux at an optically thin frequency $\tilde{\nu}$.

To derive the scaling indices $\tilde{\xi}_M$ and $\tilde{\xi}_{\dot{m}}$ in the optically thin case, we could go through the same arguments as in equations (8) through (12b), now imposing that $\tau_{\tilde{\nu}} = \Phi\Psi \ll 1$. More easily, however, we can derive $\tilde{\xi}_{\rm M}$ and $\tilde{\xi}_{\dot{m}}$ by simply replacing α in equations (10a) and (12a) by the optically thin value $\tilde{\alpha} = (p-1)/2$. This gives

$$\tilde{\xi}_M = 3 + \frac{\partial \ln(\phi_C)}{\partial \ln(M)} + \frac{p+1}{2}\frac{\partial \ln(\phi_B)}{\partial \ln(M)} \left[\sim \frac{5}{4} \right], \qquad (15)$$

where the expression in square brackets is valid for $\phi_C \propto \phi_B^2 \propto M^{-1}$ and $p = 2$.

For optically thin jets with $p \sim 2$, Sams et al. (1996) suggest that the observed brightness temperature $\tilde{T}_{\rm b,obs}$ in microquasar and AGN jets decreases with bolometric luminosity as $(M\dot{m})^{-0.76}$. The optically thin radio flux then goes as $F_{\tilde{\nu},\rm obs} \propto R_{\rm jet}^2 \tilde{T}_{\rm b,obs} \propto M^2 \tilde{T}_{\rm b,obs} \propto M^{1.24}$. Thus, the observations give $\tilde{\xi}_{M,\rm obs} = 1.24$, which coincides well with the theoretical value of $\tilde{\xi}_M = 1.25$.

For the scaling of optically thin flux $F_{\tilde{\nu}}$ with $\dot{m}$, we find that

$$\tilde{\xi}_{\dot{m}} = \frac{\partial \ln(\phi_C)}{\partial \ln(\dot{m})} + \frac{p+1}{2}\frac{\partial \ln(\phi_B)}{\partial \ln(\dot{m})} \left[\sim \frac{7}{4} \right], \qquad (16)$$

where the expression in square brackets is valid for $\phi_C \propto \phi_B^2 \propto \dot{m}$ and $p = 2$.

For fixed M, a change in $\dot{m}$ results in a change to both the optically thick flux F_ν and the optically thin flux $F_{\tilde{\nu}}$. We can thus relate these changes to see how the optically thick flux varies as a function of the optically thin flux:

$$\xi_{\tilde{F}} \equiv \left. \frac{\partial \ln(F_\nu)}{\partial \ln(F_{\tilde{\nu}})} \right|_{M=\rm const.} = \frac{\xi_{\dot{m}}}{\tilde{\xi}_{\dot{m}}}, \qquad (17a)$$

where $\xi_{\dot{m}}$ is given in equation (12a) and $\tilde{\xi}_{\dot{m}}$ in equation (16).

If we impose a unique relation between ϕ_C and ϕ_B (the most reasonable assumption here is a fixed fraction of equipartition between relativistic particles and B-field at the base of the jet, which reproduces our fiducial assumption of $\phi_C \propto \phi_B^2$), we can actually rewrite $\xi_{\tilde{F}}$ as

$$\xi_{\tilde{F}} = \frac{1}{p+4} \left(5 + 2\alpha + \frac{1 - p + 2\alpha}{1 + p + 2\frac{\mathrm{d}\ln\phi_C}{\mathrm{d}\ln\phi_B}} \right), \qquad (17b)$$

where $\dot{m}$ is now only an implicit parameter. This implies that the relation holds for any variation in the jet parameters (whether it is caused by a change in $\dot{m}$ or any other parameter), as long as it does not affect the geometry or dimensions of the jet [because both the functions ψ_f and $R_{\rm jet}$ were kept fixed when deriving equation (17b)].

For the fiducial assumption of $\phi_C \propto \phi_B^2$, equation (17b) reduces to

$$\xi_{\tilde{F}} = \frac{1}{p+4} \left(4 + 2\alpha + \frac{6 + 2\alpha}{p+5} \right) \left[\sim \frac{17 + 8\alpha}{21} \right], \qquad (17c)$$

where the expression in square brackets holds for $p = 2$.

Thus, we have a relationship between optically thick and optically thin flux under the condition of fixed M of the form

$$F_\nu \propto F_{\tilde{\nu}}^{\xi_{\tilde{F}}} \sim F_{\tilde{\nu}}^{(17+8\alpha)/21}, \qquad (18)$$

which is remarkably close to the observed correlation between the flat spectrum radio flux and the X-ray flux observed in the Galactic source GX 339-4 (Corbel et al. 2000; 2003) of $\xi_{\tilde{F},\rm obs} \sim 0.7$.

Once again, all of these relations arise independently of the specific jet model, so long as it produces a spectral index of α in the optically thick part of the spectrum. In fact, Markoff et al. (2003) find exactly the same result for $\xi_{\tilde{F}}$ when applying their jet model (Markoff, Falcke & Fender 2001) to GX 339-4 as equation (17c) for the case of $\alpha = 0$ and $p = 2$.

4 DISCUSSION

4.1 Observational consequences

Because the relations of equations (10a) and (12a) are model-independent (they only depend on the boundary conditions at the base of the jet, not on ψ_f), measurements of ξ_M, $\xi_{\dot{m}}$ and $\xi_{\tilde{F}}$ *cannot* be used to distinguish between different jet models. However, the generality of this result makes such measurements an even stronger probe of the underlying nature of jet physics, given below.

(i) Observational confirmation of equations (10a) and (12a) would prove that jet formation is scale-invariant. On the other hand, if observations can rule out a any correlation which can be described by these equations, this would argue strongly against scale invariance.

(ii) Measuring the values of ξ_M and $\xi_{\dot{m}}$ would provide diagnostics of the conditions in the inner disc and at the base of the jet, i.e. measuring ξ_M and $\xi_{\dot{m}}$ could be used to put limits on ϕ_f.

(iii) Because the accretion rate cannot be measured directly in low-efficiency accretion, it might not be possible to establish a direct observational correlation between F_ν and $\dot{m}$. However, if the above relations hold, any correlation of the optically thick, flat-spectrum jet radio emission with emission at higher frequencies could be used to constrain the high-energy emission processes (e.g. optically thin jet emission or bremsstrahlung from an ADAF).

(iv) Measuring the residual spread of F_ν around the predicted relation could provide a handle on the relative importance of orientation effects, and thus on the measurement of the mean jet Lorentz factor Γ.

5 CONCLUSIONS

We have derived the non-linear relation between the observed jet flux at a given frequency F_ν and the black hole mass M. For scale-invariant jets, the nature of the expression for the jet synchrotron emission makes it possible to contract all the model dependence into the observable spectral index α. Thus, for any observed value of α, the derived F_ν–M relation is now *model-independent* – any jet model that produces the observed jet spectrum automatically satisfies this relation. Given a prescription of the input conditions at the base of the jet, provided by accretion disc theory, we can thus *predict* the scaling of jet flux with M. Most accretion scenarios produce a scaling relation of the form $F_\nu \propto M^{17/12-\alpha/3}$. Thus, for the optically thick flat spectrum radio emission from core-dominated jets, we find that $F_\nu \sim M^{17/12}$, while for optically thin emission with $\alpha \sim 0.5$, we find $F_\nu \sim M^{5/4}$. *Due to the large range in black hole mass, this non-linearity makes AGN jets much more radio-loud than microquasar jets.*

This analysis can be extended to any fundamental accretion parameter (e.g. accretion rate or black hole spin) if jet dynamics are invariant with respect to changes in this parameter. For example, for ADAF-like boundary conditions at the base of the jet, the scaling with accretion rate $\dot{m}$ follows $F_\nu \sim (M\dot{m})^{17/12+2\alpha/3}$, and in the flat spectrum case of $\alpha = 0$, the dependence on $\dot{m}$ and M is the same: $F_\nu \propto \dot{M}^{17/12}$. Because this result is model-independent, observational measurements of the non-linear scaling of F_ν with M and $\dot{m}$ are powerful probes of the behaviour of the underlying accretion flows and of the nature of the energy and matter supply to jets from compact objects.

Clearly, the physics of jet formation is extremely complicated. For example, we still do not understand the nature of the radio-loudness dichotomy in AGN, and even though they contain black holes of similar mass, GRS 1915+105 is much more active in the radio band than Cyg X–1. Nevertheless, independent of the complicated physics of jet formation, the arguments presented in this Letter show that the radio-loudness of jets increases with increasing black hole mass, and thus that the radio emission from microquasars should be a much smaller fraction of their bolometric luminosity than that of radio-loud AGN.

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