cathode drop are quite high, namely.

$$\varepsilon_{+\nu} = \sum_{\mu=2}^{\nu} e(U_{i\mu} - \varphi) + e U_c, \qquad (41)$$

$$\varepsilon_{+2} = 34 \ [eV]; \ \varepsilon_{+3} = 66 \ [eV]; \ \varepsilon_{+4} = 120 \ [eV].$$

Even though we have no detailed information about the accommodation coefficients of these processes these energies offer a basis for an explanation of the

surprising phenomenon that ions have been observed emerging from the cathode spot area with energies usually between $20 - 60$ [eV], i. e. with energies well above the total typical vacuum voltage [14].

Further study of this question, in particular the consideration of the mechanism by which the ions escape the spot area without loosing their energy, is in progress.

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The Non-Stationary Metal Vapour Arc

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This investigation studies the effect which the motion of the cathode spot has on its E-diagram. The motion is depicted as a sequence of steps of a finite residence time.

The spot motion affects essentially only the energy characteristics T_c which in comparison to the stationary characteristics T_{es} are shifted to smaller values. Hereby the critical currents I_0, I_1 are raised in comparison to the corresponding stationary limits I_{0s}, I_{1s}.

Particularly attractive are the phenomena found in connection with the dependence of the spot velocity v on the spot current I. If the spot velocity increases with the spot current stronger than $v \propto I^{1/2}$ then the E-diagram reveals the existence of an upper limit I_u for the spot current. This result can be used to explain qualitatively the experimentally observed phenomena of "spot multiplicity" and "spot extinction".

Quantitative conclusions are obstructed by the lack of knowledge about the velocity dependence on the spot current, $v(I)$. Experimental and theoretical studies to provide a better understanding of the physical background and the analytical laws describing the motion of the cathode spots are urgently needed.

Introduction

In an earlier investigation of the metal vapour arc [1] we considered the stationary cathode spot (SCS). Using the E-diagram technique [2, 3] we studied characteristics of charge conservation, particle conservation and energy conservation. Apart from a relatively close limitation of the current densities and the temperatures in the cathode spot we observed the occurrence of two different spot-modes (0, 1).

We further found a requirement for a minimum spot current (I_0, I_1) for the stationary operation of a cathodic onset in these modes.

In the consecutive treatment of the metal vapour or vacuum arc [4] we improved the understanding of the cathode spot mechanism by including simultaneously the analysis of the sheath and the space charge region and by taking into account multiply charged ions. In this way we found the variation of the electron temperature T_- and the cathode drop U_c with the spot surface temperature T and the total current density j.

In principal these new results ask for a recalculation of the E-areas derived in [1] utilizing the more accurate data of [4]. This analysis is in progress but

quite cumbersome. However, as was already argued in [4], it can be anticipated that this recalculation will approximately reproduce the E-areas of [1] and yield changes only in the rest of the $(j - T)$-diagram which is not directly relevant for the physical consequences studied here.

It seems noteworthy that neither the original investigation in [1] nor the improvement in [4] yield an upper limit for the total spot current which might be expected from the experimental evidence of the phenomenon of spot multiplicity. This leads us to suspect that our model is not complete yet.

In fact the stationary cathode spot presents an idealization of the true spot in that the true spot is practically always in motion. This non-stationarity may — as we already suspected in [1] — reduce the E-area, alter the critical minimum currents I_{cr} or even yield the expected upper limitation for I. Such influences might also have a bearing on the conclusions drawn for the chopping and stability qualities of the vacuum arc [5].

Principal Considerations of the Non-Stationary E-Diagram

As far as the basic features of this investigation are concerned, we refer the reader to the References [1-3].

As in [1, 6] we limit the existence area through "unidirected approximations", so called characteristics. Each of these characteristics yields a $T(j)$-dependence because it neglects one or a combination of the laws describing the spot mechanism. According to the large number of possible neglections, we have a large choice of possible characteristics.

We select here the same dependencies as in the treatment of the stationary spot where we restricted ourselves to three such characteristics and termed them for the reason of their physical background:

characteristic of charge conservation   (c),
characteristic of neutral particle conservation (n),
characteristic of energy conservation   (e).

Of course, under these circumstances it is advised to orientate our analysis of the nonstationary cathode spot (NCS) closely at the procedure used in [1]. In fact we will simply start with the equations of [1] and discuss to what extent alterations are required due to the non-stationarity:

We consider an arc-spot which moves with a certain constant velocity across the surface of the ca-

thode. The motion is interpreted on the basis of the idea that the seemigly continuous motion of the spot is actually a sequence of steps in the following sense. The arc operates in one place for a certain time τ_0, then discontinuously steps to a neighbouring site displaced about one spot diameter $(2\,a)$ and here operates again for a time τ_0, which we call the residence time. This "step motion" is supported by experimental evidence.

It is clear that within the frame of this model the relation

$$\tau_0 = O(a/v) = O(I/j\,v^2)^{1/2} \qquad (1)$$

holds, where (a) denotes the radius of the spot and (v) the spot velocity.

This relation (1) allows an estimate of the time τ_0 which characterizes the non-stationarity and which will prove useful in discussing the extent of applicability of the SCS-theory to the NCS-analysis.

We know from experiment that for a Cu-Cathode the spot currents are in the range $I = O(10^2)$ [A] and their spot velocities in the range $v \leqq O(10^4)$ [cm/sec]. Further it follows from the E-diagram in [1] that the current densities are $j = O(10^6)$ [A/cm^2]. Therefore

$$\tau_0 \geqq O(10^{-6}) \; [\mathrm{s}] \qquad (2)$$

holds for the residence time of the spot.

Knowing τ_0 we now decide whether the laws describing the characteristics of charge conservation (c), neutral particle conservation (n) and energy conservation (e) relax to stationarity in a time $\tau_r \ll \tau_0$ or whether they require non-stationary adjustments.

Calculation of the Non-Stationary Characteristics

As stated, we orientate our calculations at the corresponding relations of the SCS-theory.

Characteristic of Charge Conservation

We use the equations presented in Ref. [1] in the form

$$j_e = e \int_{\varepsilon_m}^{\infty} D(E_c, \bar{\varepsilon})\, N(\bar{\varepsilon}, T, \varphi)\, \mathrm{d}\bar{\varepsilon}, \qquad (3)$$

$$D(E_c, \bar{\varepsilon}) = \left[1 + \exp\left(-\frac{2\,i}{\hbar} \int_{x_1}^{x_2} p(x)\, \mathrm{d}x \right) \right]^{-1}, \qquad (4)$$

$$N(\bar{\varepsilon}, T, \varphi) = \qquad (5)$$
$$[4\,\pi\,k\,T\,m_-/h^3]\ln[1 + \exp\{ -(\bar{\varepsilon} - \varphi)/k\,T\}],$$

$$E_c^2 = 4/\varepsilon_0\,(m_+\,U_c/2\,e)\,\{j_+\,(m_+/m_-)^{1/2} - j_-\}, \qquad (6)$$

$$j = j_+ + j_e(E_c\, T)\,,\qquad (7)$$

$$j_+/j \leqq U_c/(U_c + U_i)\,.\qquad (8)$$

Here j_e and j_+ designate respectively the absolute value (a. v.) of the electron emission density from and the ion current density to the cathode. D is the emission probability, N the supply function. $\bar{\varepsilon}$ designates the electron energy, p the momentum perpendicular to the electrode surface. x_1 and x_2 are defined through $p^2(x) = 0$. φ is the work function of the cathode material. E_c designates the a. v. of the electric field at the cathode surface. U_c is the cathode drop and U_i the ionization potential. The $(<)$-sign of Eq. (8) admits the existence of the spot in the range

$$T > T_{c1}(j)\qquad (9)$$

where the characteristic $T_{c1}(j)$ is the solution of Eqs. (3) − (8) using the $(=)$-sign in Equation (8).

The applicability of these relations (3) − (8) to the non-stationary moving spot can be judged by comparing the relevant relaxation times with the residence time τ_0.

The fastest process of the phenomena described in Eqs. (3) − (8) is the relaxation of the (T − F)-emission formulated in Eqs. (3) − (5). Its order of magnitude is characterized by

$$\tau_{c1} = l_m/\bar{v}_{em} = O(10^{-14})\ [\text{s}]\qquad (10)$$

where l_m is the characteristic length of the metal structure and $\bar{v}_{em}$ the average velocity of the electrons within the metal. Obviously τ_{c1} is many orders of magnitude smaller than τ_0 and consequently we treat the emission process quasi-stationary.

Next we consider Eq. (6) and characterize the corresponding relaxation time by

$$\tau_{c2} \leqq l_{sp}/v_{+s} \cong 10\,(k\,T_-/\varepsilon_s)^{1/2}\,\omega_{p+}{}^{-1} \approx \omega_{p+}{}^{-1}$$
$$\leqq O(10^{-8})\ [\text{s}]\qquad (11)$$

where l_{sp} designates the extension of the space charge region, v_{+s} the average velocity of the ions ini the space charge region, ε_s the corresponding energy and ω_{p+} the plasma ion frequency. Here we used the result from [4] that $l_{sp} = O(10\,\lambda_{-D})$ holds.

Obviously τ_{c2} is also much smaller than τ_0 so that we may use Eq. (6) unchanged.

The current continuity of Eq. (7) too is characterized in its relaxation by the quantity τ_{c2}. We can therefore use Eq. (7) for our moving spot.

Finally in judging Eq. (8) we remember that its background is the analysis of the ionization region.

Due to the assumption of quasi equilibrium the redistribution of this plasma caused by the moving spot is governed by ionization and recombination processes and we therefore have the relataxion time

$$\tau_{c3} = l_{pl}/\bar{v}_- \leqq O(10^{-9})\ [\text{s}]\qquad (12)$$

where l_{pl} characterizes an average mean-free-path for these processes and $\bar{v}_-$ the average electron velocity in the plasma. Here we have taken into account that the pressure of the neutral gas cloud is always near or above one atmosphere.

Conclusion: Since we have $(\tau_{c1},\ \tau_{c2},\ \tau_{c3}) \ll \tau_0$ the characteristic of charge conservation as calculated in the SCS-theory can be transferred to the E-diagram of the NCS-investigation.

Characteristic of Neutral Particle Conservation

In the SCS-treatment we considered two necessary conditions for neutral particle conservation corresponding to the limits that all or none of the neutral particles vapourized are ionized in the plasma cloud.

They read

$$j_+(j, T_{n1}) = \frac{e}{4}\,\frac{p_{ev}(T_{n1})}{(m_+ k\,T_{n1}/3)^{1/2}}\qquad (13)$$

the other

$$j = \left(\frac{I\,Q^2_{ei}}{a_n^2}\right)\frac{p^2_{ev}(T_{n2})}{(k\,T_{n2})^2\,\pi}\qquad (14)$$

where p_{ev} denotes the equilibrium vapour pressure, Q_{ei} the cross-section for ionizing electron-neutral collisions, I the total spot current and a_n an uncertainty factor $(a_n > 1)$. Equations (13), (14) together with Eqs. (3) − (7) provide two sets of characteristics T_{n1}, T_{n2} requiring

$$T \geqq T_{n1}(j),\qquad T_{n2}(j)\,.\qquad (15)$$

Having secured the quasi-stationary treatment of Eqs. (3) − (7) in the previous section the new relaxation time to be considered is the time τ_n of the adjustment of the neutral vapour density to a given surface temperature. This time τ_n is characterized by the average collision time of neutral cloud particles with the electrode surface

$$\tau_n = l_n/\bar{v}_n\,,\qquad (16)$$

where l_n describes the mean-free-path and $\bar{v}_n$ the average velocity of the neutrals in front of the cathode spot surface. Since all mean-free-paths − even

that of the electrons − are assumed to be small in comparison to the spot extension (a) and $\bar{v}_n \gg v$ holds, it is clear that we have $\tau_n \ll \tau_0$.

Consequence: We can use in the E-diagram of the movinig spot (NCS) the T_n-characteristics of the (SCS)-analysis.

Characteristic of Energy Conservation

The stationary energy characteristic was described by the relation

$$j \left| (U_c + U_i) \frac{j + (j, T_e)}{j} - \left(\varphi - \frac{0.32}{\sqrt{\pi}} I^{1/2} j^{1/2} \varrho(T_e) \right) \right| = \frac{\varepsilon}{4} \frac{p_{ev}(T_e)}{(m_+ k T_e/3)^{1/2}} + T_e \cdot w(T_e) \left(\frac{j \pi}{I} \right)^{1/2} \quad (17)$$

where the existence area is limited by

$$T < T_e(j, I). \quad (18)$$

In this relation ϱ and w denote respectively the specific resistivity and the heat conductivity of the cathode material. The relaxations of all processes entering into Eq. (16) where covered in the previous discussion of charge conservation and neutral particle conservation, − except one:

The relaxation time τ_e of the temperature distribution within the cathode.

To estimate this relaxation time τ_e consider a semi-infinite medium with $w(T)$ at an initial temperature T_0 (room temperature) with a plane surface to which a constant energy density flux J is applied over a circular area of radius (a). The time development of the temperature distribution in the medium can than be described [7] by

$$T = \frac{a J}{2 w} \int_0^\infty J_0(\lambda r) J_1(\lambda a) \left\{ e^{-\lambda z} \operatorname{erfc} \left[\frac{z}{2(\delta t)^{1/2}} - \lambda (\delta t)^{1/2} \right] - e^{\lambda z} \operatorname{erfc} \left[\frac{z}{2(\delta t)^{1/2}} - \lambda (\delta t)^{1/2} \right] \right\} \frac{d\lambda}{\lambda} \quad (19)$$

where δ designates the thermal diffusivity

$$\delta = w/\varkappa = w/c \, d, \quad (20)$$

d is the density and $\varkappa$, c describe the heat capacity per unit volume respectively unit mass. z is the co-ordinate perpendicular to the surface in the center of the circle, r the corresponding radial coordinate. J_s denotes the Bessel function of order s.

We can restrict our considerations to the z-axis $(r = 0)$ since for large distances the problem has spherical symmetry and so the z-axis is characteristic whereas for small distances the problem can be considered plane and our results hold across the whole (a)-area.

For $r = 0$ Eq. (19) reduces to

$$T = \frac{2 J}{w} (\delta t)^{1/2} \left\{ \operatorname{ierfc} \left(\frac{z^2}{4 \delta t} \right)^{1/2} - \operatorname{ierfc} \left(\frac{z^2 + a^2}{4 \delta t} \right)^{1/2} \right\} \quad (21)$$

where ierfc is the integral of the error function complementary.

We are solely interested in the surface temperature at $z = 0$ and for that point Eq. (21) shows by means of a power expansion that the relaxation time to the stationary value is given by

$$\tau_e = a^2/\delta. \quad (22)$$

For a number of typical examples the relaxation times τ_e are calculated from the knowledge of the diffusivity and the spot size from the E-diagram.

	Fe	Cu	Mo	W	Cd
$O(\tau_e)$ [s]	10^{-4}	10^{-5}	10^{-6}	$5 \cdot 10^{-7}$	10^{-8}

$$(23)$$

These numbers show that in general the condition $\tau_e \ll \tau_0$ is *not fulfilled* so that we have to solve our energy conservation characteristic *non-stationary*.

Non-Stationary Energy Characteristic

As we know from the solution of the stationary problem the net-influx J to the spot area on the cathode surface is a function of the instantaneous cathode spot temperature. This temperature varies with time. Consequently J is an implicit function of time, $J[T(t)]$. Therefore it is our problem to solve the equation of heat conduction in the cathode

$$\partial \varkappa(T) \, T(\mathbf{x}, t)/\partial t = \nabla \{ w(T) \, \nabla [T(\mathbf{x}, t)] \} \quad (24)$$

subject to the boundary condition of a temperature dependent energy influx $J(T)$ in a circular area at the surface of a semi-infinite medium bounded by a planar surface. This provides us with an integro differential equation for the temperature if we use

the approach of Green's function. We neither know of any analytical solution of this problem [8] nor do we consider this as a task which we should tackle in this paper.

Instead we recall that for the E-diagram, it is sufficient to calculate a value T_e larger tan the true temperature T in the surface spot area

$$T_e > T . \qquad (25)$$

In this spirit let us then calculate the surface temperature of a model where the energy supply density $J[T(t)]$ is extended from the actual circular area (a) to the whole surface plane of the semi-space. The temperature T_e calculated in this way will certainly satisfy the relation (25). Moreover, the temperature T_e will even be a close approximation to T as long as the diffusion length $(\delta t)^{1/2}$ is smaller than the radius of the spot (a). This holds in the whole range of our interest since for a time $t = (a^2/\delta)^{1/2}$ we approach the stationary state. The assumption of a plane energy supply is therefore a good approximation for our calculation and completely save in the sense of Equation (25).

Still no general analytical solution can be found due to the temperature dependence of w and $\varkappa$. Again we may take advantage of the E-diagram technique by choosing w and $\varkappa$ constant, but such that the temperature T_e is increased in the direction of Equation (25).

With w and $\varkappa$ constant and a time dependent plane energy influx $J(t)$ the analytical solution is [11]

$$T(z,t) = \frac{1}{2(\pi\,w_0\,\varkappa_0)^{1/2}} \int_0^t \frac{e^{-z^2/4\delta(t-t')}\,J(t')}{(t-t')^{1/2}}\,dt' + T_0 \qquad (26)$$

where w_0 and $\varkappa_0$ denote the constant values as specified above. Since we are interested in the surface temperature only, Eq. (26) simplifies to equation

$$T_e(t) = \frac{1}{2(\pi\,w_0\,\varkappa_0)^{1/2}} \int_0^t \frac{J[T_e(t')]}{(t-t')^{1/2}}\,dt' + T_0 \qquad (27)$$

which is an integral equation for the function $T_e(t)$. A very awkward integral equation since the energy influx according to Eq. (17) is given by

$$J(j,I,T_e(t)) = j\left| (U_c + U_i)\frac{j_+(j,T_e)}{j} \right. \qquad (28)$$

$$\left. - \left(\varphi - \frac{0.32}{\sqrt{\pi}}\,I^{1/2}\,j^{1/2}\,\varrho(T_e)\right)\right| - \frac{\varepsilon}{4}\,\frac{p_{ev}(T_e)}{(m_+ \, k\,T_e/3)^{1/2}}$$

where $j_+(j,T_e)$ must be calculated from Equations (3) – (7).

Nevertheless it is possible to solve Eqs. (27), (28), (3) – (7) numerically by an iterative procedure and with that to find the characteristic $T_e(j,I,t)$ as a function of time having for $t = 0$ the value $T_e = T$ and approaching for $t \to \infty$ the stationary solutions T_{es}.

With that we know all three characteristics as a function of time and can therefore calculate the E-area as a function of time.

The Non-Stationary E-Area

From the relation (27) and the structure of the SCS-E-Diagram it is quite obvious that for $t = 0$ no self-sustained operation of the arc is possible. This means that immediately after a displacement — after a step — the spot cannot operate self-sustained. But during this initial phase it may still profit from the remains of the operation in the preceding step and consequently one can argue that self-sustainement is not required yet. On the other hand the moving arc certainly cannot operate continuously if it does not reach self-sustainement at any stage during its residence time τ_0. Consequently, for the continuous operation of a moving spot it is a *necessary requirement*:

A finite E-Area must be available in the non-stationary E-Diagram at least at the time $t = \tau_0$.

We observe htat the restriction to a necessary requirement is again in the spirit of the E-diagram technique.

To evaluate the consequences of this necessary requirement, we plot the stationary characteristics $T_c(j) = T_{cs}(j)$ and $T_n(j) = T_{ns}(j)$ together with the non-stationary characteristics $T_e(j,I,\tau_0) < T_{cs}(j,I)$ into a diagram [12]. Due to the dependence of τ_0 on the spot velocity v [see Eq. (1)] the characteristics T_e must be expected to depend also on the velocity v, $T_e(j,I,v)$.

The characteristics $T_c = T_{cs}$ an $T_n = T_{ns}$ are shown in Figures 1 and 2.

The evaluation of the T_e characteristics requires the specification of the spot velocity v as a function of the spot current I.

The derivation of this relation $v(I)$ as a function of the experimental conditions cannot be subject of this paper. We should take this information either from experimental evidence or from earlier theoretical attempts. There is such experimental evidence
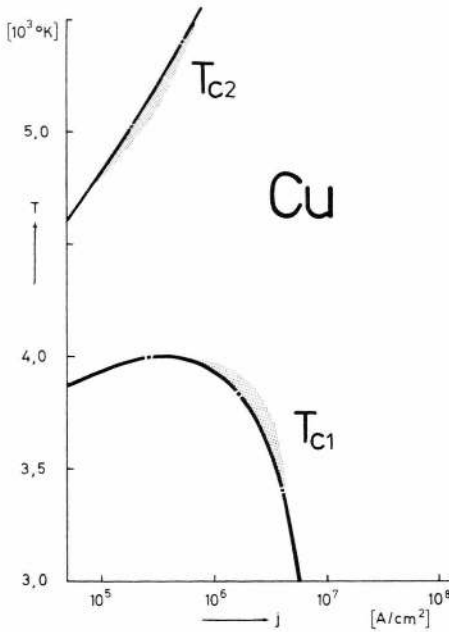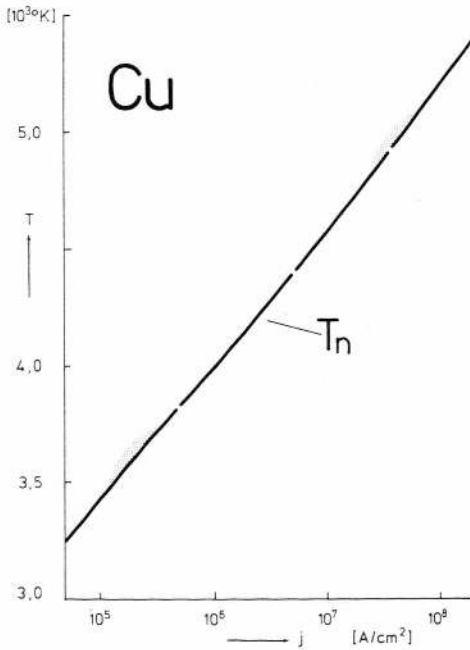
Fig. 1

Fig. 2

Figs. 1 and 2. Characteristics of charge and neutral particle conservation T_e, T_n identical with the corresponding stationary characteristics T_{es}, T_{ns} calculated in [1] of this investigation of the vacuum arc.

(Refs. [13-20]) but it is rather limited and not in the range of parameters where we would want it. There is an abundance of theoretical endeavours [21-29] but

most of them offer only a vague physical concept without analytical or even quantitative evaluation. Moreover, recent emotional dialectics contributed only to the general feeling of insecurity in this field [30].

In this situation we consider it reasonable not to use uncertain values of experimental or theoretical origin but rather to study the consequences of the spot motion for various cases of characteristic dependencies $v(I)$.

The velocity dependence on I is important since it influences the T_e-characteristics via the upper limit of integration in

$$T_e(j) = T_e(j, \tau_0) = \frac{1}{2(\pi w_0 \varkappa_0)^{1/2}} \int_0^{\tau_0} \frac{J[T_e(t')]}{(t-t')^{1/2}} \, dt' + T_0 \tag{29}$$

where $\tau_0 = (I/j \, v^2)^{1/2}$ holds.

It is obvious that we should distinguish two essentially different cases $(1, 2)$.

$$v = c \cdot I^v; \quad v \leqq 1/2; \tag{30}$$

$$v = c \cdot I^v; \quad v > 1/2. \tag{31}$$

C a s e   1 : As can be seen from Eq. (1) τ_0 increases or stays constant with increasing current. Under these circumstances it folows from the structure of Eq. (29) that the non-stationary $T_e(j, I)$-characteristics in their dependence on I form a set of curves shifting with increasing I monotonously to larger values of T. Of course the relation $T_e(j, I) < T_{es}(j, I)$ always holds.

C a s e   2 : Again $T_e(j, I) < T_{es}(j, I)$ holds. However, the decrease of τ_0 with increasing I now causes an important change. For small values of I the T_e-curves shift with increasing I to larger values of T as in case 1 and as in the stationary case. However, above a certain current I the characteristics start to move down again due to the decrease in τ_0. As we will see, this effect has decisive consequences with respect to the E-area.

As a typical example for the structure of case 1, Fig. 3 demonstrates the T_e-characteristics for a constant velocity $(c = 10 \; [\text{m/s}], \; v = 0)$. For the purpose of comparison Fig. 4 presents the corresponding T_{es} curves.

The principal features of case 2 are shown in Fig. 5 where we have chosen $(c = 1/32 \; [\text{m/s A}], \; v = 1)$. The figures clearly demonstrate the predicted rise and fall of the characteristics with increasing total current I.
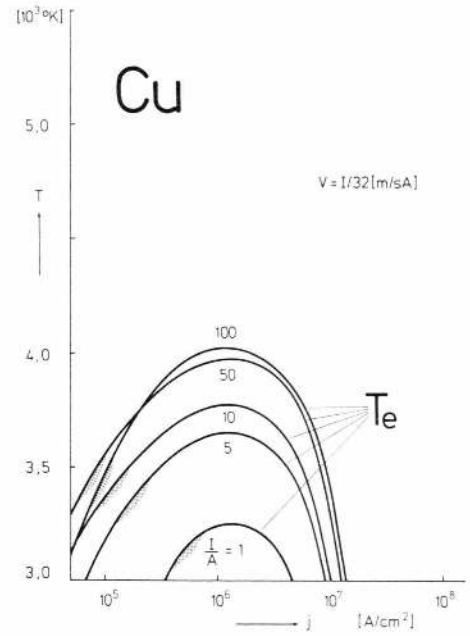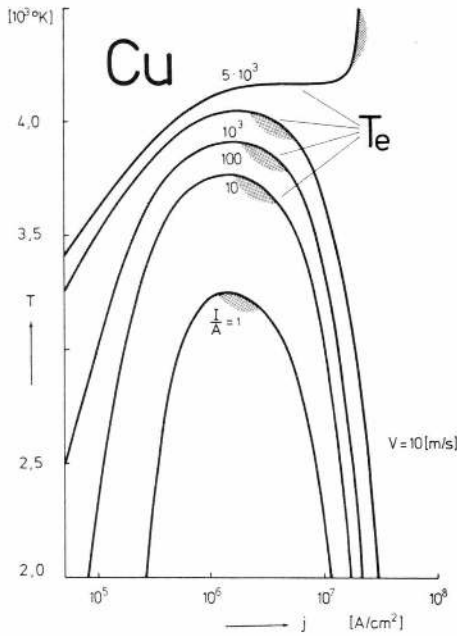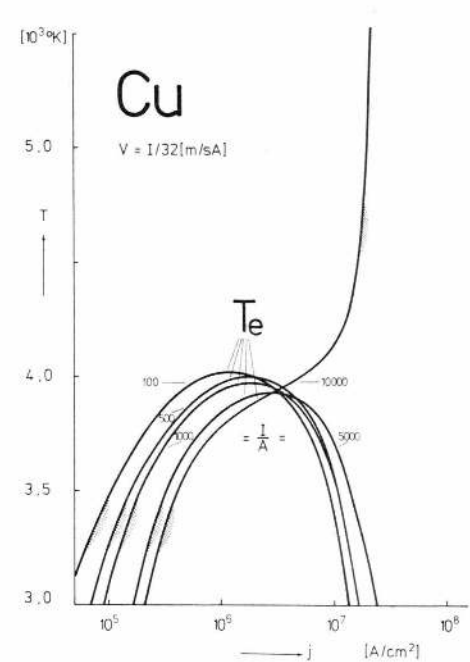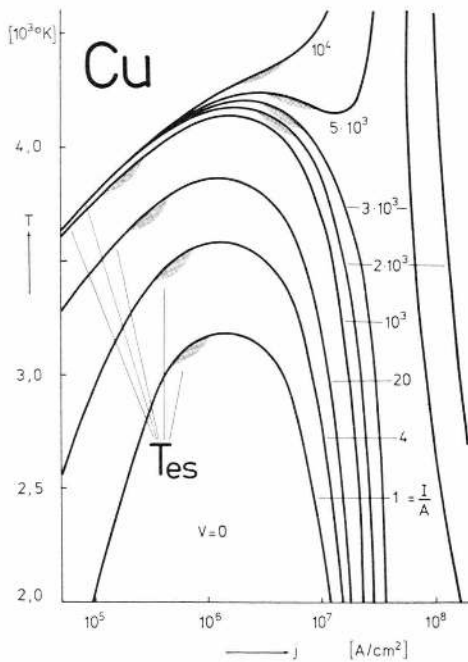
Fig. 3

Fig. 4

Figs. 3 and 4. Non-stationary energy characteristics T_e for a spot velocity $v=10$ [m/s] and stationary spot characteristics T_{es} as calculated in [1].

Fig. 5. Rising and falling non-stationary energy characteristics T_e for a spot velocity v depending on the spot current I according to the relation $v=I/32$ [m/s A].

Noteworthy is the fact that the (1)-mode is practically precluded in both cases. In case 1 this mode disappears for all currents below the unphysically high limit of about 5000 A. In case 2 it disappears altogether [31].

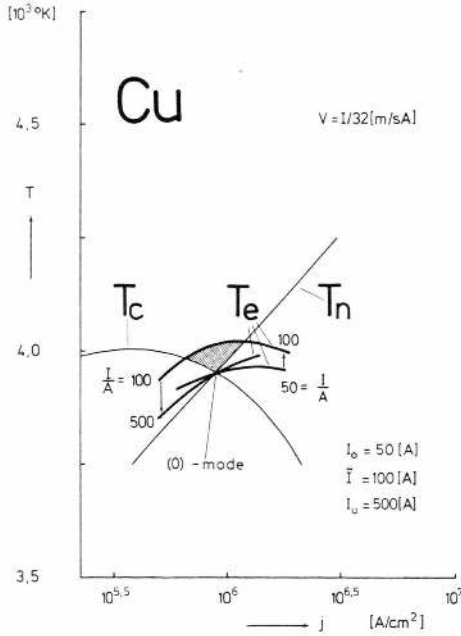The construction of the existence area for an example of case 2 results in Figure 6. For reasons of

Fig. 6. Structure of the E-area of a cathode spot moving with the velocity $v = I/32$ [m/s A]. The arrows indicate the shift of the T_e-characteristics with increasing current I. I_0 designates the minimum current required for existence, $\tilde{I}$ the current at which the shift of the T_e-characteristics changes its sign, I_u the maximum current for existence.

comparison Fig. 7 shows the corresponding stationary existence area.

We note that the critical minimum current I_0 below which we expect fluctuations and extinction is in the non-stationary case larger than in the stationary case, — as expected. Most important however, the continuous increase of the existence area observed in the stationary diagram is not recovered in the non-stationary diagram of case 2. In fact the existence area in the non-stationary case first increases with increasing current and then passing a certain current $\tilde{I}$ decreases reaching again zero at a critical upper current I_u. Spot velocities in the range of case 2 produce an upper limitation I_u for the spot current.

Since these limiting currents I_0, I_u are of central interest, we calculated their values as a function of the parameter c for $\nu = 1$.

The result is shown in Figure 8. It shows that the upper and lower current limit approach each other with increasing velocity. When they intersect we have reached a veocity where spot existence is excluded altogether.
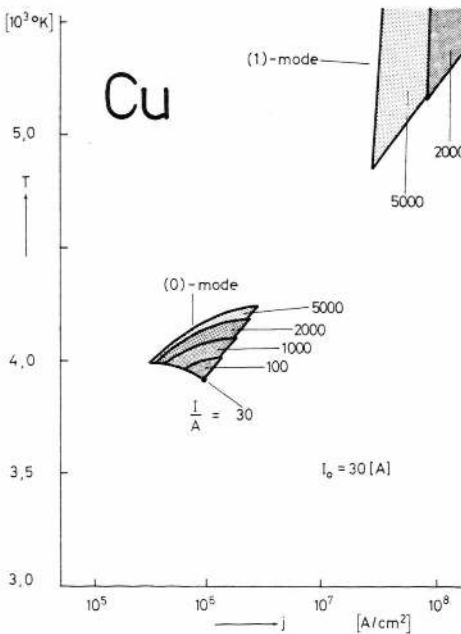

Fig. 7. Existence diagram of the stationary spot as calculated in [1]. I_0 is the minimum current required for the existence of a spot at rest.
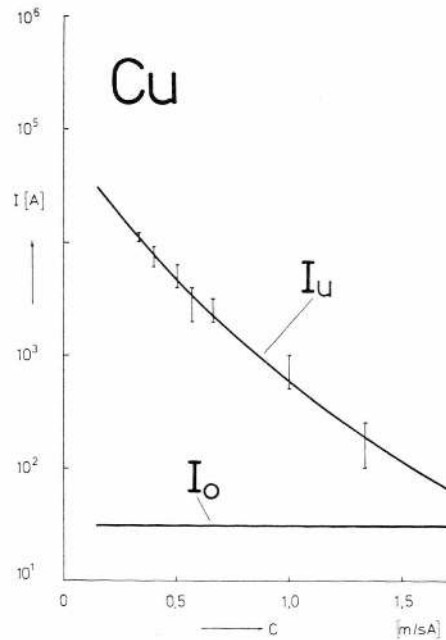

Fig. 8. Maximum and minimum critical current I_u, I_0 required for the existence of a spot moving according to the relation $v = c\,I$ as a function of the constant c.

Summary and Discussion

In this investigation we considered the non-stationarity of the cathode spot as caused by its motion across the cathode surface. The motion is depicted as a sequence of steps of finite residence time τ_0.

A discussion of the relaxation times shows that the $T_{es}(j)$- and $T_{ns}(j)$-characteristics of the stationary spot can be transferred to the moving spot. The same is not true for the $T_{es}(j)$-characteristics.

The T_e-characteristics are determined through a complicated integral Equation (28), (29). All solutions of this equation satisfy the relation $T_e(j, I) < T_{es}(j, I)$.

It is noteworthy that the velocity of the spot enters into the calculation (28), (29) through the upper integral limit in the form $I/j\,v^2$. Due to this fact the dependence $v(I)$ has a decisive influence on the T_e-characteristics. Considering the relation $v = c\,I^\nu$ the residence time τ_0 increases with increasing current I and the T_e-characteristics rise monotonously with increasing current if we choose $\nu \leq 1/2$ (case 1). In case 2 with $\nu \geq 1/2$, however, the residence time τ_0 decreases with the current and as a consequence the T_e-characteristics rise only up to a certain current $\tilde{I}$ and then descend again.

In case 1 the principal structure of the E-area is not changed in comparison to the stationary state. There is still the (0)- and the (1)-mode. For all currents $I > I_0$ we have finite E-areas increasing with the current I. The critical current of the (0)-mode I_0 is slightly shifted to higher values in comparison to I_{0s}. The critical current of the (1)-mode may be raised drastically to unphysically high values.

In case 2 we do still have a critical current $I_0 > I_{0s}$ for the (0)-mode. However, we find in comparison to the stationary E-area spectacular changes.

The (1)-mode does not exist any more [32].

The most striking phenomenon, however, is the fact that now the E-area opens up at I_0, increases with increasing current $I > I_0$ up to a certain value $\tilde{I}$ and then starts to decrease when the current is monotonously raised beyond $\tilde{I}$. Finally at an upper limiting value of the current I_u the E-area vanishes again.

In this second case therefore the spot current is limited to a finite range

$$I_0 < I < I_u \,. \tag{32}$$

We studied the dependence of the two limiting currents (I_0, I_1) on the proportionality constant (c) of the velocity law. The result shows that with increasing (c) — that is with increasing velocity for a constant current — the interval $\Delta I = I_u - I_0$ shrinks and eventually reaches the value zero. From that point on existence of the spot is precluded altogether.

A *physical interpretation* of these analytical results may be useful to deepen the understanding:

In the case of the stationary spot we understood that the minimum current I_{cr} has its physical origin in the fact that below this current the heating of the cathode is insufficient to produce the surface temperature which is necessary to provide the metal vapour and the plasma conditions which in turn yield the ion current necessary for the surface heating. For $I > I_{cr}$ there is an E-area available which increases with the current. Within this "uncertainty area" lies the point of solution where our physical conditions can be met.

If the spot moves the energy balance is disfavoured and consequently T_e is shifted to lower values. This cannot surprise since — contrary to the stationary spot — the moving spot after each step must built-up the surface temperature from the value T_0. In this way it will reach the stationary temperature T_{es} only after an infinitely long residence time. The smaller the residence time in a step, the more will the T_e-characteristic shift to smaller values, $T_e < T_{es}$.

This has an interesting consequence in connection with the dependence of the velocity on the current, $v = c\,I^\nu$. In the case where we have $\nu > 1/2$ the residence time decreases with increasing current. This tends to shift the T_e-characteristics with increasing current to smaller values. Depending on the parameters, this disfavouring effect may overcome the in principal favourable influence of increasing spot current on the energy balance. In this way, we can understand the rise and the descend of the T_e-characteristics in case 2. At the same time we understand the disappearance of the E-area at un upper current limit I_u.

The results described here offer themselves only to *qualitative conclusions* since the lack of knowledge of the velocity dependence $v(I)$ on the spot current precludes quantitative comparison:

The existence of an upper limit of the spot current I_u in case 2 of the velocity dependence not only could explain but would demand the phenomenon of *spot multiplicity*. If the total arc current surpasses the critical value I_u then there is no other choice for the discharge but to form additional spots

at the cathode. The results also offer a possible explanation for the observed phenomenon of *spot extinction* when the spot approaches the rim of the cathodic area. Due to the influence of magnetic interaction we can expect — and experimental observation shows — that the spot velocity increases towards the rim of the cathodic discharge area. Consequently we must expect that the spot moves in the direction of increasing values of (c) when it approaches the rim. The results of Fig. 8 would then predict the suicide of the spot when it reaches the critical value (c_{cr}) where the curves I_u and I_0 intersect.

In *summarizing* we may say that our results in this paper show that the motion of the spot does not change the principal conclusions drawn in [1] and [4] of this treatment. It introduces however some additional features which offer a possibility to explain the phenomenon of "spot multiplicity" and "spot extinction". The results with regard to the latter two phenomena cannot be conclusive due to the insufficient knowledge of the laws of spot motion. Additional experimental and theoretical information to this latter point would be useful in many respects.

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