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The Notional Category of Modality (2012)

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Modals and Conditionals

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INTRODUCING CHAPTER 2

The original version of *The Notional Category of Modality* is 30 years old. I gave it a thorough makeover for this collection, but left the original storyline intact. Among all the papers collected in this book, *The Notional Category of Modality* is the one that had the most impact on subsequent work in the semantics of modality and has triggered the most responses. This made it very difficult for me to update the old manuscript without dramatic changes. I decided to be responsive to at least some recent developments that go to the very core of the semantics of modals and thus present potential challenges for the analysis put forward in the original paper.

One of the conclusions of *What "Must" and "Can" Must and Can Mean* that was carried over to *The Notional Category of Modality* was that the interpretation of modals is relative to a conversational background that might be made explicit by adverbial phrases of various kinds. What I overlooked in the earlier work was that there are important differences between different adverbial phrases contributing conversational backgrounds for different types of modals. This is illustrated by the difference between the English sentence (1a) versus the German sentence (1b):

(1) a. Given the article in the Hampshire Gazette, Mary Clare Higgins
        must have been re-elected.

     b. Dem Artikel in der Hampshire Gazette nach, soll   Mary Clare
        The article in the Hampshire Gazette after modal Mary Clare

        Higgins wiedergewählt worden sein.
        Higgins re-elected   been   be.
        'According to the article in the Hampshire Gazette, Mary Clare
        Higgins was reportedly re-elected.'

An assertion of (1a) would commit me to the truth of what the article says, and continuing with (2) would be infelicitous:

(2)   ...but I wouldn't be surprised if she wasn't. The Gazette is usually too
      quick to draw conclusions from projected election results.

In contrast, an assertion of (1b) would not commit me to the truth of the report in the Gazette, and I could continue with (2) without contradicting

myself. The difference between (1a) and (1b) points to two different ways of interpreting modals in the "epistemic" or "evidential" family. In (1b), the accessible worlds are worlds that are compatible with the content of the report. The accessible worlds for (1a) are worlds with certain kinds of counterparts of the article in the Hampshire Gazette. The counterparts should have the same content as the original article and relate to reality in the same way. If the actual article was based on unreliable election projections, for example, so were all of its counterparts in the accessible worlds. The accessible worlds are also worlds that, by and large, function normally from the point of view of the actual world. For example, just as in the actual world, reports based on unreliable election projections might or might not be true. With accessibility relations of this kind, then, the truth of (1a) depends on how good the evidence for the Hampshire Gazette report actually was. If the evidence was shaky, Mary Clare Higgins became mayor in some, but not all of the accessible worlds. Only flawless evidence guarantees her being elected in all accessible worlds. As a consequence, I shouldn't assert (1a) unless I believed the evidence for the Gazette report to be highly reliable.

(1a) and (1b) show that modals in the epistemic/evidential family can have two types of interpretations: "strong" interpretations, which—at least with necessity modals—commit the speaker to the truth of the proposition the modal scopes over (von Fintel and Gillies 2010), and "weak" interpretations, which are relativized to the content of some source of information that may or may not be faithful to reality. Those two types of interpretations have figured prominently in the recent literature on the connection between epistemic modals and evidentials (Izvorski 1997; Faller 2002; Matthewson et al. 2007; Rullmann et al. 2008). For example, Rullmann et al. (2008) construe the modal alternatives for the St'át'imcets reportative modal *ku7* as the set of worlds where a relevant report was made, rather than the set of worlds where the content of such a report is true.[1] The result is a "given the report," rather than an "according to the report," interpretation, and *ku7* comes out as a "strong" epistemic modal that doesn't allow the speaker to distance herself from the content of the report. St'át'imcets *ku7* thus contrasts with the German reportative modal *sollen* illustrated in (1b), which relies on alternatives where the content of the relevant report is true, and hence is "weak."

Cross-linguistically, the invariant job of an evidential is to classify evidence for what is being said as direct, indirect, or hearsay (Willett 1988; de Haan

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[1] Page 350, definition 82.

1999; Garrett 2001; Faller 2002; Aikhenvald 2004; Speas 2008; Murray 2010). Direct evidence may come from direct perception or first-person experiences, like skin itching or headaches. Indirect evidence may come from reports, or inferences drawn from direct or indirect evidence. Rumors or legends may be classified as hearsay. The cross-linguistically invariant job of an epistemic modal is not to classify evidence, but to assess the truth of a proposition against a range of possibilities projected from a body of evidence. There are two distinct semantic jobs to be done, then: classify evidence versus assess the truth of a proposition against possibilities projected from a body of evidence. The two jobs often end up being carried by a single portmanteau item that might then be arbitrarily cataloged as modal or evidential. That evidential meaning components are in principle independent of modal meaning components, but can be bundled together with other meaning components in a single lexical item, was emphasized in Izvorski (1997). Izvorski points out that with finite verbs in the present tense, the Turkish perfect morpheme *mış* is interpreted as an indirect evidential. In non-finite environments and with future or past tense, *mış* only has a perfect, non-evidential, meaning. The evidential meaning component can't be contributed by *mış* itself, then, but seems to be a separate component spelled out in a portmanteau with the present tense. In Quechua and Korean, too, evidential meaning components can be attached to items that are commonly categorized as tenses (Faller 2004; Chung 2005, 2007; Lee 2009).

As a number of authors have pointed out, the English epistemic modal *must* also has evidential characteristics (Westmoreland 1998; Drubig 2001; von Fintel and Gillies 2010). Epistemic *must* excludes direct perceptual or irreducibly first-person evidence, for example, as illustrated by (3) and (4):

(3) a. # Your nose must be dripping. I can see it.
     b. You must have a cold. Your nose is dripping.

(4) a. # I must have a terrible headache. I feel lousy.
     b. The baby must have a terrible headache. He is screaming and pressing his hands against his temples.

English spells out evidential, modal, and temporal meaning components together as the single lexical item *must*, resulting in what we call a "present tense epistemic modal."

Natural languages show a grammatically significant split between so-called "root" and "epistemic" modals. Syntactically, root modals appear in low

positions in the line-up of verbal inflectional heads; epistemic modals appear in high positions. Semantically, root and epistemic modals differ with respect to the kinds of facts they depend on. The nature of that difference was a puzzle raised, but essentially left unsolved, in the original *The Notional Category of Modality*. I now believe that the impasse the older paper ran into was due to the erroneous assumption that the two types of modals semantically select modal bases with distinctive semantic properties: circumstantial backgrounds for root modals and epistemic backgrounds for epistemic modals. It now seems to me a hopeless enterprise to try to characterize formal objects like conversational backgrounds as "circumstantial" versus "epistemic." Both types of backgrounds are functions that map possible worlds to sets of factual premises. What is it that would allow us to single out some of those functions as epistemic, but not circumstantial, or the other way round? There don't seem to be any characteristic properties that could produce such a distinction (see Nauze (2008) for an insightful objection along those lines). We need to tell a different story about the source of the differences between root and epistemic modals. Hacquard (2006, 2010) has told such a story.

According to Hacquard, modal bases are projected from event arguments following very general recipes. Different types of possibilities become available in different places of the verbal projection spine because different types of event arguments appear in those places. The lower regions of the verbal projection spine provide access to the participants and spatio-temporal locations of the events described. According to Hacquard, the higher regions provide access to speakers' knowledge via a representation of the speech situation. Hacquard's work presents a major breakthrough in the theory of natural language modality. Her proposal does not only explain the existence of a surprising split between root and epistemic modals in the languages of the world. It also tells us how modal base dependencies might be represented in grammar: possibly only indirectly, via event arguments providing "anchors" from which modal bases can be projected. Hacquard's general vision can be fruitfully supplemented with insights from Hackl (1998), who shows that there is also syntactic variation within the class of root modals. Root modals, according to Hackl, may project control or raising structures, and may be anchored to entities of various types that are represented in the modals' specifier position, possibly as a result of overt or covert movement. Modal anchors do not necessarily have to be events, then, but can be entities of diverse types, including individuals and their stages, spatio-temporal locations, or situations—whatever entities might be represented in a modal's domain in the verbal projection spine.

The original version of *The Notional Category of Modality* accounted for graded and comparative notions of possibility by using ordering sources to induce orderings on the set of accessible worlds and the set of propositions, but didn't make any explicit connections with quantitative notions of probability or desirability. This shortcoming is repaired in the current version, which shows how quantitative notions of probability and desirability can emerge from comparative notions in a natural way: we need to look for suitable probability or desirability measures that preserve suitable relations of comparative possibility that an ordering semantics for modals provides. We may not necessarily find any such measures, but if we do, there are typically many that are potential candidates. This is as it should be, and no reason for concern. Our semantic knowledge alone does not give us the precise quantitative notions of probability and desirability that mathematicians and scientists work with. It seems to provide no more than conceptual launch pads for mathematical explorations to take off from. In fact, as Yalcin (2010) reminds us, Charles Hamblin (1959) thought that natural languages might not truly go beyond merely comparative notions of probability:

Metrical probability-theory is well-established, scientifically important and, in essentials, beyond logical reproof. But when, for example, we say "It's probably going to rain", or "I shall probably be in the library this afternoon", are we, even vaguely, using the metrical probability concept?[2]

In modal logic, modal operators come in duals. But even languages like English or German have modals without duals. The possibility of modals without duals was invoked by Robert Stalnaker (1981) for counterfactual *would*, and by Veronika Ehrich (2001) for the German weak necessity modal *sollen* ('be supposed to'). The issue rose to prominence when Hotze Rullmann, Lisa Matthewson, and Henry Davis (2008) reported that the Salish language St'át'imcets lacks dual modals altogether. The current version of *The Notional Category of Modality* suggests that at least some modals without duals might be neither possibility nor necessity modals, but degree expressions describing a high degree of desirability or probability.

At the time the first version of *The Notional Category of Modality* was written, the goal of compositionally interpreting hierarchical line-ups of inflectional heads was not yet commonly recognized. The theoretical landscape has changed dramatically in this respect. The place of modals in the verbal projection spine and their interactions with neighboring inflectional heads related to voice, aspect, tense, and mood is now much better understood

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[2] Hamblin (1959: 234).

through the work of Virginia Brennan (1993), Roumyana Izvorski (1997), Paul Portner (1998, 1999, 2003, 2009), Martin Hackl (1998), Rajesh Bhatt (1999 [2006]), Sabine Iatridou (2000), Veronika Ehrich (2001), Cleo Condoravdi (2002), Michela Ippolito (2002), Jonny Butler (2004), Tom Werner (2003), Timothy Stowell (2004), Ana Arregui (2005, 2007, 2009, 2010), Maria Bittner (2005, forthcoming), Cleo Condoravdi and Stefan Kaufmann (2005), Stefan Kaufmann (2005), Valentine Hacquard (2006, 2009, 2010), Kai von Fintel and Sabine Iatridou (2007, 2008), Lisa Matthewson et al. (2007), Hotze Rullmann et al. (2008), Katrin Schulz (2008), Elisabeth Villalta (2008), Henry Davis et al. (2009), Rebecca Cover (2010), Amy Rose Deal (2010a), Dorit Abusch (forthcoming), Aynat Rubinstein (forthcoming), among many others. I could not implement a truly compositional perspective in the new *Notional Category of Modality* without turning it into a book-sized manuscript. The hope is that whatever we may learn about modality all by itself may ultimately be of help when figuring out interactions with other inflectional heads.

Apart from a few stylistic changes, the sections on practical reasoning and conditionals of the original *The Notional Category of Modality* have been left intact, even though the discussion of conditionals is shorter and more condensed than it should be. Chronologically, it was preceded by Kratzer (1978), my dissertation, and by Kratzer (1979). To avoid too much overlap with later papers that share the same general approach to conditionals, but are more interesting from a modern point of view, I did not include Kratzer (1979) or passages from Kratzer (1978) in the present collection. Instead, I expanded and updated the (1986) paper *Conditionals,* which is based on a Chicago Linguistic Society paper and came out of the first seminar I taught on my older work on modals and conditionals after moving to the United States in 1985. The new version of *Conditionals* appears here as chapter 4.

There are three earlier published versions of *The Notional Category of Modality.* The first one appeared in H. J. Eikmeyer and H. Rieser (eds.), *Words, Worlds, and Contexts,* Berlin and New York: de Gruyter (1981), 38–74. The paper was reprinted unchanged in P. Portner & B. Partee (eds.), *Formal Semantics: The Essential Readings,* Oxford: Blackwell (2002), 289–323, and then again in Javier Gutierrez-Rexach (ed.), *Semantics: Critical Concepts,* London: Routledge (2003), vol. iv, 365–403.

Chapter 2

The Notional Category of Modality

> It would be considered naïve today to attempt, as did Wegener (1885), to describe the semiotic stratification of human language with examples restricted to German, Greek and Latin. But it is remarkable how well Wegener's theory stands up now that the range of our evidence has been vastly broadened. It takes only a slightly more flexible calculus, I believe, to accommodate all the varieties of semiotic structure evident in ordinary discourse.
>
> Uriel Weinreich

2.1 Introduction

This chapter explores the notional category of modality as reflected in the modal vocabulary of German.[3] The main danger for anyone working on modals is to get utterly lost in the variety of interpretations one and the same expression can receive in different contexts. As a result, we may be tempted to develop sophisticated classifications and study the characteristics of major types of modals including ability, epistemic, or deontic uses. I am not really interested in such classifications. My main concern is to find answers to questions like the following:

- What is the logical nature of modal interpretations?
- What is their variability due to?
- How is the variability of modal interpretations restricted by the vocabulary of a language?
- How do graded and comparative notions of modality come about?
- How do graded and comparative notions of modality relate to quantitative notions of probability and desirability?
- What is the connection between modals and conditionals?

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[3] Many of the German examples in this article are directly inspired by, or adapted from, sentences and stories in Oskar Maria Graf's *Das Leben meiner Mutter* (*The Life of my Mother*, Graf 1946).

Traditionally, investigations of modality have focused on expressions like *necessarily, possibly, must, can, should,* or *may.* Little attention has been paid to the fact that natural languages have ways of grading and comparing possibilities and the path that leads from graded and comparative notions of possibility to the related quantitative notions of probability and desirability. Furthermore, conditionals are usually not considered in connection with modality. Yet, *if*-clauses often serve to restrict modals explicitly or implicitly (Kratzer 1978, 1979). In what follows, I will present a unified analysis of graded and non-graded varieties of modality that not only accounts for the variability and indeterminacy of modals, but also sheds light on the equally mystifying variability and indeterminacy of conditionals: since *if*-clauses often restrict modals, and since those modals are often unpronounced, complex modalized conditionals may be mistaken for simple conditionals consisting of just a binary connective joining two clauses. The variability and indeterminacy of modals and the variability and indeterminacy of conditionals have a common source. Once this possibility is recognized, insights gained in separate examinations of modals and conditionals fall out as special cases from a general theory of restricted modality.

2.2 Expressing modality in German

Modality has to do with necessity and possibility. In German, as in other languages, modal notions can be expressed in many ways.

Inherent modality

(1)   Niemand läuft in zehn Minuten von Andechs nach Aufhausen.
      Nobody   runs in ten   minutes from Andechs to   Aufhausen.

(2)   Dieses Auto fährt zwanzig Meilen pro Stunde.
      This   car goes twenty   miles   per hour.

(1) and (2) have modalized readings that can be paraphrased as in (1′) and (2′):

(1′)   Nobody is able to run from Andechs to Aufhausen in ten minutes.

(2′)   This car can go twenty miles an hour.

Suffixes on adjectives

German has two suffixes with modal meanings: *-lich* and *-bar.* Here are a few examples, some of which are borrowed from Paul (1920):

-lich

erblich	hereditary
umgänglich	sociable
zugänglich	accessible, approachable
käuflich	purchasable
zerbrechlich	fragile
sterblich	mortal
unsterblich	immortal
vergesslich	forgetful
untröstlich	inconsolable

-bar

zahlbar	payable
unfehlbar	infallible
brauchbar	useful, practicable
brennbar	combustible, inflammable
dehnbar	stretchable
denkbar	conceivable
essbar	eatable, edible
tragbar	portable, wearable
waschbar	washable

In general, the suffixes *-lich* and *-bar* express possibility. There are apparent exceptions like *zahlbar:*

(3)   Die Miete für das Haus auf dem Leoni-Acker beträgt
      The rent   for the house on the Leoni-Field amounts to

      zwanzig Gulden, zahlbar am ersten   Januar.
      twenty   guilders, payable on the first of January.

According to (3), it's not that the twenty guilders *can* be paid, they definitively *have to* be paid on the first of January.

Modal auxiliaries

must	muss	müsste
can	kann	könnte
may	darf	dürfte
shall	soll	sollte
will	wird	würde
may	mag	möchte

The exact meaning of some of these auxiliaries will be discussed in more detail as we go along. *Müsste, könnte, dürfte, sollte, würde, and möchte* are subjunctive forms of the corresponding verb on their left. They often have an independent, non-compositional, meaning, though.

Sentence adverbs and impersonal constructions

möglicherweise	possibly
notwendigerweise	necessarily
wahrscheinlich	probably
es ist möglich dass	it is possible that
es ist notwendig dass	it is necessary that
es ist wahrscheinlich dass	it is probable that

Adjectival phrases

imstande sein	to be able
in der Lage sein	to be in the position

The selection of modal expressions in this section makes clear that there is no syntactic category corresponding to the notional category of modality. What, then, is modality?

2.3 Basic notions

The following story highlights the core ingredients of the notional category of modality.

The murder

Much-Girgl has been murdered on his way home. There is an ongoing investigation. Conclusions about the circumstances of the crime are being drawn from the available evidence, and utterances of the following sentences might have occurred:

(4)   Der Kastenjakl kann der Mörder   sein.
       The Kastenjakl can   the murderer be.
       Kastenjakl may be the murderer.

(5)   Der Gauzner-Michl muss der Mörder   sein.
       The Gauzner-Michl must the murderer be.
       Gauzner-Michl must be the murderer.

In uttering (4), a police inspector may have claimed that given the available evidence, it is possible that Kastenjakl committed the murder. More evidence

might have become available at a later point, and the same inspector might then have been in a position to assert (5), expressing the opinion that the available evidence warranted the conclusion that Gauzner-Michl was indeed the murderer. The example shows that there are at least two ingredients involved in the interpretation of modals like *kann* or *muss*: a *conversational background* contributing the premises from which conclusions are drawn, and a modal relation determining the *force* of the conclusion. In his second utterance, the inspector drew a stronger conclusion than in his first. To make all of this more precise, I have to review a few notions from possible worlds semantics.

When Lenz says

(6) Bis jetzt   hab' ich dir genug   Bier weggesoffen.
     Up to now have I   you enough beer boozed away.

to the owner of Fink's pub, he expressed a proposition. Possible worlds semantics identifies propositions with subsets of a given universe of possible worlds W. Here are some standard definitions:

Definitions of the basic logical properties and relations

A proposition p is *true* in a world $w \in W$ iff $w \in p$. A proposition p *follows* from a set of propositions A iff $p \subseteq \cap A$. A set of propositions A is *consistent* iff $\cap A \neq \emptyset$. Finally, a proposition p is *compatible with* a set of propositions A iff $A \cup \{p\}$ is consistent.

In the imagined context for (6), the proposition expressed by Lenz's utterance is the set of possible worlds where Lenz has drunk enough of Fink's beer up to the day of his utterance. The meaning of a sentence is described by specifying which proposition(s) it expresses depending on relevant features of the utterance situation.

As in chapter 1, I take conversational backgrounds to be functions mapping possible worlds to premise sets—that is, sets of propositions. In a first approximation, modals express relations between conversational backgrounds and propositions. The most familiar modal relations are what we may call "simple" necessity and possibility. If f is a conversational background, a proposition is a *simple f-necessity* in a world w iff it follows from $f(w)$; it is a *simple f-possibility* iff it is compatible with $f(w)$.

The meanings of individual modals need to be linked to the right modal notions. For *necessarily*, for example, the link could be established as follows:[4]

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[4] Strictly speaking, rules like this would have to apply at a level of Logical Form, where all modal operators are propositional operators.

Necessarily

Suppose *u* is an utterance of a sentence of the form *necessarily* α such that the proposition *p* is expressed by α. Then

    (i) *u* expresses a proposition only if there is a unique conversational background for *u*

   (ii) if *u* expresses a proposition and *f* is the conversational background for *u*, then the proposition expressed is $\{w \in W: p$ is a simple f-necessity in $w\}$.

We may wonder why there should be a unique conversational background for a modalized sentence to express a proposition. This seems too strong. More often than not, conversational backgrounds for modals remain genuinely underdetermined and what speakers intend to convey is compatible with several choices of conversational backgrounds. In those cases, we might want to say that there are several propositions expressed—one relative to each background. It would then be part of the vagueness of modal expressions that, sometimes, it remains genuinely underdetermined which proposition was expressed (Lewis (1979a) and Pinkal (1977, 1979) have relevant proposals; now also von Fintel and Gillies (forthcoming)). There might also be problems if a sentence contains more than one modal, each requiring a conversational background of its own. To account for this, we would have to split up utterance situations further and consider separate utterances for each modal. The issue will be set aside here—in Kratzer (1978) I made an attempt to spell out the details of such an approach.

    As is, the proposed analysis of modals allows for one modal parameter to be fixed by the context of use. It implies that that parameter is responsible for the variety of interpretations modals can receive. In the murder example, a conversational background representing a piece of evidence created an epistemic interpretation of the modals in question. For further reference, I want to draw attention to a few kinds of conversational backgrounds that play a distinguished role in the semantics of modal constructions.

a. Realistic conversational backgrounds

    A realistic conversational background is a function *f* such that for any world *w*, $w \in \cap f(w)$. That is, *f* assigns to every possible world a set of propositions that are true in it.

b. Totally realistic conversational backgrounds

    A totally realistic conversational background is a function *f* such that for any $w \in W, \cap f(w) = \{w\}$. That is, *f* assigns to any world a set of propositions

that characterizes it uniquely. For each world, there are many ways of characterizing it uniquely. This is a major source of vagueness for counterfactuals, as argued in Kratzer (1981a; also section 2.9 below and chapter 3).

c. *The empty conversational background*

The empty conversational background is the function f such that for any w $\in W$, $f(w) = \varnothing$. Since $\cap f(w) = W$ if $f(w) = \varnothing$, empty conversational backgrounds are also realistic.

Realistic backgrounds for modals in natural languages all seem to track particular bodies of facts in the world of evaluation: that is, we invariably have functions f such that for each world w in the domain of f there is a particular body of facts in w that has a counterpart in each world in $\cap f(w)$. For so-called "root modality," the targeted facts relate to inherent properties or circumstances of individuals or spatio-temporal locations. It is those properties and circumstances that are "kept constant" in all accessible worlds. For so-called "epistemic modals" the targeted facts might correspond to what Hacking (1975) calls "evidence of things." Hacking illustrates this notion with an example by J. L. Austin, where pig-like marks in the ground, buckets of pig food, noises, and smell are taken to be evidence for the presence of pigs. Evidence of things consists of things in the world, including olfactory and auditory objects, which, according to Hacking, "are not private experiences, but rackets and stenches as public as pigsties."[5] However, private experiences should be able to function as evidence of things, too: experiences of seeing, hearing, or smelling—even experiences of illusions and hallucinations—can be actual events. Whatever exists in a world, including individuals, eventualities, and the world itself, should in principle qualify as potential evidence of things of that world.

Modals can also rely on backgrounds that are not realistic. They can depend on *informational backgrounds*, for example. Informational backgrounds represent the intentional content of sources of information.

d. *Informational conversational backgrounds*

An informational conversational background is a function f such that for any w in the domain of f, $f(w)$ represents the propositional content of some source of information in w.

Possible sources of information are things with intentional content: words, stories, books, reports, maps, testimony, perceptual experiences, and what have you. Sources of information have a double nature. They can function as

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[5] Hacking (1975: 32).

evidence of things for realistic backgrounds, or as sources of intentional content for informational backgrounds. To illustrate, if a testimony is the salient body of facts that a realistic background is about, the accessible worlds are those that have counterparts of that testimony. The actual existence of the testimony makes it a body of facts, and thus evidence of things, even if it is packed with lies. If that same testimony is the salient source of information feeding an informational background, the accessible worlds are those that are compatible with the intentional content of the testimony.

The distinction between realistic backgrounds representing evidence of things and informational backgrounds representing information content plays an important role for so-called "evidentials" in natural languages. For example, the German reportative evidential *sollen* depends on informational conversational backgrounds: it reports the content of hearsay. In contrast, according to the characterization of Rullmann et al. (2008), the reportative evidential *ku7* of the Salish language St'át'imcets seems to depend on realistic backgrounds.[6] They give the example in (7):

(7)   *Context: There is a rumor going around that Roger was elected chief. Sometimes that kind of rumor is right, sometimes it's wrong. You really have no idea whether it's likely to be right or wrong. You tell me:*

    % aw-an-ém   ku7   kw s-Roger   ku cuz' kúkwpi7
        choose-DIR-PASS REPORT DET NOM-Roger DET going.to chief
    'Roger was reportedly elected to be chief.'
    Rullmann et al. (2008: example 79, 349)

Rullmann et al. report that judgments for (7) are variable and seem to depend on whether speakers think the rumor could be true. This would be unexpected if (7) just reported the content of the rumor, which is made clear in the example. What kinds of claims do sentences like (7) make, then? Contrasting the English example (8a) with the German example (8b) may point to a possible answer: they bring out the subtle difference between realistic

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[6] Once we make a distinction between modal bases and ordering sources, as proposed in 2.4, informational conversational backgrounds should be ordering sources, rather than modal bases, since they do not necessarily represent consistent information. If epistemic modals always have realistic modal bases and empty modal bases are special cases of realistic ones, German reportative *sollen* should have a realistic modal base and an informational ordering source. In contrast, St'át'imcets *ku7* would have a realistic modal base representing a salient piece of information functioning as evidence of things and an empty or stereotypical ordering source. As suggested in the original version of this chapter, we should also not exclude the possibility that the meanings of certain types of modals may have to be characterized by more than a single ordering source. Normalcy assumptions, for example, seem to play a role for informational modals, too.

backgrounds representing evidence of things and informational conversational backgrounds representing information content:

(8) a. Given the rumor, Roger must have been elected chief.

   b. Dem Gerücht nach, soll   Roger zum   Häuptling gewählt
      The rumor   after, MODAL Roger to-the chief   elected
      worden sein.
      been   be.
      'According to the rumor, Roger was reportedly elected chief.'

(8b) merely reports what the rumor says and allows the speaker to distance herself from it. (8a) suggests that the speaker considers the rumor a reliable source of information. This means that for (8a), the rumor is seen as feeding a realistic conversational background representing available evidence, not an informational one representing the content of the rumor. The claim is that in all relevant worlds that have a counterpart of that rumor, a counterpart of Roger was elected chief. In evaluating (8a) we seem to assume that the relevant worlds are worlds where the counterparts of the actual rumor not only say the same thing as the actual rumor does, but also were produced in the same way. Let me illustrate.[7] Suppose the rumor is a plain lie in the actual world. Its counterparts in the relevant accessible worlds should then be plain lies, too, and (8a) should wind up false. (8b) could still be true, as long as the rumor says that Roger was elected chief. Suppose now that the rumor happened to be true, but was based on shaky evidence, as rumors often are. Maybe your neighbor, who started the rumor, saw a banner with the words "Congratulations Roger," not knowing that it was for Roger's 70th birthday. The counterparts of such a rumor in the relevant accessible worlds would come into existence in the same way as the actual rumor in the actual world, with (a counterpart of) your neighbor spotting (a counterpart of) that sign and concluding that (a counterpart of) Roger was elected chief. The accessible worlds will differ in countless ways, but, most importantly for us here, they will differ as to whether or not Roger's counterparts did become chiefs. In some of the accessible worlds they did, in others not: shaky evidence might or might not produce a true rumor. (8a) winds up false, then. If the actual rumor is from a 100% reliable source, its counterparts in the relevant accessible worlds are, too, and (8a) is true.

Perceptual experiences, too, can feed both realistic and informational backgrounds. If the backgrounds are realistic, the accessible worlds all contain counterparts of the actual experience that come into existence in the

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[7] I am indebted to Seth Cable for his discussion of the analysis of evidentials in Rullmann et al. in his Fall 2008 UMass Amherst seminar.

same way and have the same content as the actual experience:[8] if the actual experience was an illusion or hallucination, so are all of its counterparts. On the other hand, if perceptual experiences feed informational backgrounds, the accessible worlds are worlds that conform to the information content of the experience. For instance, if I hallucinate unicorns that are approaching, the informationally accessible worlds determined by my hallucination are all worlds that have unicorns that are approaching.

If sources of information function as evidence of things feeding a realistic background, their counterparts in the relevant accessible worlds have to satisfy certain conditions, as we have seen: they have to carry the same information as the actual piece of information, and they have to come into existence in the same way. When sources of information function as evidence of things, their status as evidence is a highly relevant property and should thus play an important role in the choice of counterparts. Take Sewall's *Life of Emily Dickinson*, for example. If the corresponding accessible worlds were simply required to contain, say, duplicates of actual copies of Sewall's book, there would be some accessible worlds where the book is a piece of fiction, rather than a biography. If the book functions as evidence of things, the fact that it is a biography is essential, and hence should be preserved by all relevant counterparts. Moreover, if Sewall's book was based on authentic or forged letters, its counterparts should be based on counterparts of those letters, which would have to be authentic or forged just in case the corresponding actual letters were. All those properties of the actual book are essential for its status as evidence of things, hence need to be preserved by all relevant counterparts.

Informational backgrounds are not the only backgrounds that are not necessarily realistic. Other not necessarily realistic backgrounds may relate to norms of various kinds, and among those, backgrounds representing the normal course of events in the world of evaluation play a privileged role:

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[8] Lewis (1996) uses modal alternatives of this kind for his analysis of knowledge. "When perceptual experience E (or memory) eliminates a possibility W, that is not because the propositional content of the experience conflicts with W. (Not even if it is the narrow content.) The propositional content of our experience could, after all, be false. Rather, it is the existence of the experience that conflicts with W: W is a possibility in which the subject is not having experience E. Else we would need to tell some fishy story of how the experience has some sort of infallible, ineffable, purely phenomenal propositional content...Who needs that? Let E have propositional content P. Suppose even—something I take to be an open question—that E is, in some sense, fully characterized by P. Then I say that E eliminates W iff W is a possibility in which the subject's experience or memory has content different from P. I do not say that E eliminates W iff W is a possibility in which P is false" Lewis (1996: 553).

e. Stereotypical conversational backgrounds

A stereotypical conversational background is a function f such that for any world w, $f(w)$ represents what is normal in w according to some suitable normalcy standard for w.[9]

What is to count as normal? Definition (e) is deliberately vague and non-committal about what suitable standards of normalcy are and where they may come from. A simple illustration will have to do for now: in the world we live in, people normally die if they are exposed to certain amounts of arsenic. We might want stereotypical conversational backgrounds to represent this kind of normalcy. An example could be some background f such that $f(w_0)$ is consistent and all $w \in \cap f(w_0)$ are worlds where everyone dies who takes the critical amount of arsenic. Since there are a few actual people who have managed to build up tolerance for arsenic, the actual world w_0 itself is not a member of $\cap f(w_0)$, and f is not realistic. A person like Urquhart, who was able to consume large amounts of arsenic and survive in comfort, is not normal. That made him a very unlikely suspect in the murder case of Philip Boyes.[10]

Other instances of normative, and thus potentially non-realistic, conversational backgrounds are *deontic, teleological,* and *bouletic* conversational backgrounds:

f. Deontic conversational backgrounds

A deontic conversational background is a function f such that for any world w, $f(w)$ represents the content of a body of laws or regulations in w.

Teleological conversational backgrounds are related to goals and *bouletic conversational backgrounds* have to do with wishes.

It may now be tempting to try to characterize the semantic field of modal expressions along two axes: one specifying a modal relation (the modal force), and the other one specifying restrictions for admissible conversational

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[9] There is a legitimate question whether the best way to represent normalcy is via premise sets, rather than relying on basic, irreducible, relations that order worlds according to how normal they are from the point of view of a designated world. There is a related question about similarity: should similarity relations between worlds be induced via premise sets, or should they be basic and irreducible? The first question is raised in Yalcin (2010) and is still wide open. Counterfactuals have been a testing ground for the second question and answers have begun to emerge. Are empirical constraints on counterfactual reasoning best stated as constraints on premise sets or as constraints on orderings among worlds? It seems that, within a premise semantics, we can realistically aim for a theory that does not only cover the truth-conditions of counterfactuals, but also the process of drawing conclusions from inconsistent premises more generally, e.g. in completely unrelated areas like the computation of implicatures or the balancing of conflicting constraints in phonology.

[10] Dorothy Sayers: *Strong Poison.*

backgrounds. The following sections will show that this view is too simple. Realistic and normative conversational backgrounds need to be kept separate. They play distinct roles in generating the full range of possible modal meanings in natural languages. The most important argument in favor of such a separation is the fact that natural languages can express graded and comparative notions of possibility. Graded and comparative notions of possibility emerge when we rank worlds that are compatible with a body of facts according to how close they come to some norm or ideal. The gradability of modal notions is not only reflected in a range of different degree constructions that modal auxiliaries and adjectives participate in (2.4). It may also produce certain types of modals without duals (2.5).

2.4 Grades of possibility

Instead of sentences (4) or (5), the police inspector from the previous section might have uttered one of the following sentences:

(9)   Es **kann gut** sein, dass der Gauzner-Michl der Mörder   war.
       It can   well be   that the Gauzner-Michl the murderer was.
       There is a good possibility that Gauzner-Michl was the murderer.

(10) Es besteht aber immer noch eine **geringe Möglichkeit,** dass der
       There is however still   a   slight   possibility   that the
       Kastenjakl der Mörder war.
       Kastenjakl the murderer was.
       There is, however, still a slight possibility that Kastenjakl was the
       murderer.

(11) Der Gauzner-Michl **kann eher**   der Mörder sein als der Kastenjakl.
       The Gauzner-Michl can   rather the murderer be than the Kastenjakl.
       Gauzner-Michl is more likely to be the murderer than Kastenjakl.

(12) Es ist **wahrscheinlich,** dass der Gauzner-Michl der Mörder war.
       It is probable   that the Gauzner-Michl the murderer was.
       It is probable that Gauzner-Michl was the murderer.

The police inspector does not know what the real world is like. But he can draw conclusions from the growing evidence available to him. At any given time, this evidence partitions the set of worlds W into two subsets separating those worlds that are compatible with that evidence from those that are not. In the light of our earlier discussion, we know that compatibility with evidence can be understood in one of two ways. If the evidence has propositional content,

compatibility is logical compatibility with that content. With "evidence of things," compatibility amounts to co-existence with a counterpart of that evidence. Be this as it may, among the worlds that are compatible with the evidence in our case (in one sense or the other), some are more far-fetched than others. A world where Kastenjakl is the murderer is more far-fetched than one where Gauzner-Michl killed Girgl. Gauzner-Michl couldn't stand Girgl, but Kastenjakl got along very well with him. Even more far-fetched are worlds where someone from the other end of the world committed the crime. Far-fetched with respect to what? With respect to what is the case in the real world? No! Something that was almost impossible might very well turn out to be the case. This is precisely what happens in good detective stories. The most unlikely candidate turns out to be the murderer. What is far-fetched about someone from the other end of the world having killed Girgl is that such things do not correspond to the normal course of events. Normally, you don't meet people from the Antipodes in Girgl's village. And should someone show up who does not actually live in the neighborhood, he wouldn't just go and kill Girgl. Normally, people need a motive for killing someone. It couldn't have been for money, since Girgl wasn't robbed: all his money was found on him. Considering the normal course of events, it is far-fetched that someone from the other end of the world killed Girgl. And considering the normal course of events it is more far-fetched for Kastenjakl to be the murderer than for Gauzner-Michl.

In our example, let's assume that we have a realistic conversational background that determines the set of accessible worlds by tracking the actually available evidence in closely related worlds. It forms the *modal base*. There is a second, stereotypical, conversational background involved in the police inspector's uses of modals in (7) to (10). Stereotypical conversational backgrounds can be used to rank worlds according to how close they come to the normal course of events in the world of evaluation, given a suitable normalcy standard. In that case, they function as *ordering sources*.[11] Quite generally, a set of propositions A can induce an ordering $\leq_A$ on W in the following way:[12]

Inducing the ordering $\leq_A$

For all worlds w and $z \in W$: $w \leq_A z$ iff $\{p: p \in A$ and $z \in p\} \subseteq \{p: p \in A$ and $w \in p\}$.

According to this definition, a world w is at least as close to an ideal or norm determined by a set of propositions A as a world z iff all propositions of A

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[11] The term is inspired by what Raynaud (1974) calls "source" in French.

[12] The idea comes from David Lewis's work on ordering semantics; personal communication. Lewis's work on ordering semantics has since been published as Lewis (1981).

that are true in z are true in w as well. The relation $\leq_A$ is reflexive and transitive, but not necessarily connected. Technically, $\leq_A$ is a partial preorder, then. It is *partial* because worlds don't have to be comparable, and it is a *preorder* because it is not necessarily antisymmetric. The related relation $<_A$ is defined in the usual way: $w <_A z$ iff $w \leq_A z$, but not $z \leq_A w$. We can now define some additional modal relations that depend on a world w, a modal base f, and an ordering source g:

Necessity

A proposition p is a *necessity* in w with respect to f and g iff for all $u \in \cap f(w)$, there is a $v \in \cap f(w)$ such that

(i) $v \leq_{g(w)} u$

and

(ii)   for all $z \in \cap f(w)$: if $z \leq_{g(w)} v$, then $z \in p$.

Simplifying slightly, a proposition is a necessity just in case it is true in all accessible worlds that come closest to the ideal determined by the ordering source. Since the definition is neutral with respect to the so-called "Limit Assumption" (Lewis 1973) and thus does not presuppose that there are closest worlds, the definition of necessity is more complicated than might seem necessary. It is modeled after a definition David Lewis gives for counter-factuals.[13] Possibility is the dual of necessity:

Possibility

A proposition is a *possibility* in w with respect to f and g iff its negation (that is, its complement) is not a necessity in w with respect to f and g.

The new notion of necessity is weaker than the earlier notion of simple necessity. A necessary proposition is no longer required to be true in all accessible worlds. It is now sufficient for it to be true in the closest accessible worlds. On the other hand, the new notion of possibility is stronger than our earlier notion of simple possibility. For a proposition to be possible it is now no longer sufficient for it to be true in just some possible world.

Having ordered sets of accessible worlds makes it possible to define various notions of comparative possibility for propositions. There are many candidates and finding definitions that are right for different types of modals is not at all straightforward.[14] Notions of comparative possibility relating to probability

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[13] Lewis, personal communication; now Lewis (1981). See also Burgess (1981).

[14] Halpern (1997, 2003) and Yalcin (2010) have extensive discussion; see also Lassiter (2010). However, the critical assessments in Yalcin (2010) and Lassiter (2010) are not yet sufficiently respon-

are unlikely to be the same as notions of comparative possibility relating to desirability, for example. In the way of illustration, let us look at a notion of comparative possibility that establishes a connection to a plausible quantitative notion of probability, as we will see shortly: when comparing two propositions p and q, we disregard the worlds p and q have in common and compare p - q and q - p by checking whether there is any world in q - p that is higher ranked than every world in p - q. If not, p is at least as good a possibility as q. If q logically implies p, q - $p = \emptyset$ and p is automatically at least as good a possibility as q. More formally:

Comparative possibility (one option among many that should be considered)

A proposition p is *at least as good a possibility as* a proposition q in w with respect to f and g iff

$$\neg\exists u(u \in \cap f(w) \,\&\, u \in q\text{-}p \,\&\, \forall v(\,(v \in \cap f(w) \,\&\, v \in p\text{-}q) \to u <_{g(w)} v))$$

A proposition p is a *better possibility* than a proposition q in w with respect to f and g iff p is at least as good a possibility as q with respect to f and g, but the reverse does not hold.

The relation "is at least as good a possibility as" considered in the original 1981 version of this chapter was based on a different intuition: for p to be at least as good a possibility as q, it was required that for every world where q is true, there be a world where p is true that comes at least as close to the ideal provided by the ordering source. This definition has consequences that might be unwelcome for certain applications.[15] Suppose, for example, that there is a world w that is better than any other world. We would now predict that all propositions containing w are equally good possibilities. W and $\{w\}$ should be equipossible, then. The old definition may still do well in certain cases where the propositions to be compared can be assumed to be mutually disjoint, as is common in moral reasoning. We are assuming disjoint alternatives, for example, when we say that praying and doing good is better than *just* praying. The new definition is not without problems either. It might not deliver the desired result if the ordering of worlds allows ties or is not connected. Suppose three worlds w_1, w_2, and w_3 are all equally close to the ideal established by the ordering source. Then our (new) definition classifies $\{w_1\}$ and $\{w_2, w_3\}$ as equipossible, for example, which might or might not be a

sive to the important fact that we are very likely to need different notions of comparative possibility to account for different types of comparative modal operators in natural languages.

[15] See the critical and insightful discussion in Yalcin (2010).

welcome consequence. Or suppose that there is even a single world in $q - p$ that is not connected to any world in $p - q$. In that case, there cannot be a world in $p - q$ that is better than every world in $q - p$, and, consequently, p can never be a better possibility than q according to our definition. There may be good reasons, then, to carefully watch the kind of orderings among worlds that we may want to admit for modal expressions, a topic addressed for counterfactuals in Lewis (1981). The question of which notions of comparative possibility provide the best match with natural language expressions relating to comparative modal notions related to probability and preference is still open and in need of clarification.[16]

Portner (2009) observes that modal auxiliaries and adjectives like *possible* are not gradable in English. This is a language-specific fact, however. Modal auxiliaries and the counterpart of *possible* are gradable in German and other languages. German productively uses the adverb *eher* ('earlier') in comparative constructions with both the modal auxiliary *kann* ('can'; see examples (11) above and (58) below) and the modal adjective *möglich* ('possible'). There is also a corresponding superlative form *am ehesten*. Moreover, modal adjectives like *useful, stretchable, fragile, inflammable, soluble, prone to, able, capable*, etc. are all gradable even in English, and this means that any semantics for modals must in principle allow for graded notions of possibility.

A second issue raised by Portner (2009) is how notions of comparative possibility might relate to quantitative notions of probability. From the current perspective, we would want to understand under what conditions quantitative notions of probability can emerge from orderings induced by ordering sources. The project would be to try to find suitable probability measures that preserve suitable comparative possibility relations. Here is a toy example illustrating how the (new) notion of comparative possibility defined above might be linked to a plausible probability measure.

Suppose $\cap f(w_0) = W = \{w_0, w_1, w_2, w_3\}$ and $g(w_0) = A = \{ \{w_3\}, \{w_2, w_3\}, \{w_1, w_2, w_3\} \}$. The ordering $\leq_A$ induced on W is connected and has no ties, and we have: $w_3 <_A w_2 <_A w_1 <_A w_0$. We can now define a plausible probability measure P on the set of propositions $\wp(W)$ as follows:

$P(\emptyset) = 0$	$P(\{w_2\}) = 4/15$	$P(\{w_3\}) = 8/15$	$P(\{w_2, w_3\}) = 12/15$
$P(\{w_0\}) = 1/15$	$P(\{w_0, w_2\}) = 5/15$	$P(\{w_0, w_3\}) = 9/15$	$P(\{w_0, w_2, w_3\}) = 13/15$
$P(\{w_1\}) = 2/15$	$P(\{w_1, w_2\}) = 6/15$	$P(\{w_1, w_3\}) = 10/15$	$P(\{w_1, w_2, w_3\}) = 14/15$
$P(\{w_0, w_1\}) = 3/15$	$P(\{w_0, w_1, w_2\})$ $= 7/15$	$P(\{w_0, w_1, w_3\})$ $= 11/15$	$P(\{w_0, w_1, w_2, w_3\})$ $= 15/15$

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[16] I am grateful to Aynat Rubinstein for discussion of those issues.

P is one of many probability measures that preserve the relation "is a better possibility than" defined above and satisfy the standard conditions on probability measures: *P* assigns a number between 0 and 1 to every proposition in $\wp(W)$, it assigns 1 to *W*, and for any disjoint propositions $p, q \in \wp(W)$, $P(p \cup q) = P(p) + P(q)$. Using the table above, the reader can verify that for all $p, q \in \wp(W)$, *p* is a better possibility than *q* iff $P(p) > P(q)$.

To turn our toy example into a more realistic example, we could think of the four worlds w_0, w_1, w_2, w_3 as representatives of suitably chosen equivalence classes. Suppose the possible suspects in Girgl's murder case are Michl, Jakl, Hansl, and Seppl. The set of possible worlds that are compatible with our evidence can then be partitioned according to which one of the four men killed Girgl. If the question "who did it?" is the only issue we are interested in, all other differences between accessible worlds can be neglected, and we end up with a four-cell partition of the set of accessible worlds. Suppose furthermore that, given certain normalcy standards, Michl is the most likely murderer, Jakl is next, Hansl is third, and Seppl is last. To find a plausible probability measure in this case, we can pick one representative from each of the four cells in the partition of accessible worlds and proceed as illustrated above: w_3 could represent the worlds where Michl murdered Girgl, w_2 could stand for the Jakl-worlds, w_1 for the Hansl-worlds, and w_0 for the Seppl-worlds. The probabilities that *P* above assigns to the singletons $\{w_0\}$, $\{w_1\}$, $\{w_2\}$, and $\{w_3\}$ could now be taken to correspond to the probabilities of the four respective cells in the partition of accessible worlds.

This section showed how a separation of realistic and normative backgrounds can in principle lead to plausible comparative and quantitative notions of possibility, probability, and preference. We saw that comparative notions of possibility might provide conceptual jump-off points for the development of corresponding quantitative notions by experts able to push beyond the limits of what the faculty of language provides for everyone.

2.5 Modals without duals

According to the definition in the previous section, the orderings premise sets induce on sets of possible worlds are allowed to be partial and to have ties. Worlds can come equally close to the ideal or norm represented by the ordering source, and they are not even required to be comparable at all. Do modals ever truly care about such properties of orderings? Are there any modals that do not tolerate incomparabilities or ties for the orderings they rely on, for example? These are momentous questions, because disallowing both incomparabilities and ties might mean loss of the distinction between

necessity and possibility as we have defined it. If we add the Limit Assumption, which many authors accept, the distinction between necessity and possibility collapses. The toy example from the previous section is a good illustration: all propositions with a probability of 8/15 or higher wind up as both possible and necessary; all propositions with a probability of 7/15 or lower come out as neither possible nor necessary. There is no longer a distinction between what is possible and what is necessary, then. Stalnaker (1981) argues that English counterfactual *would* is a collapsed possibility/ necessity modal in this sense: contrary to appearance, *might* is not the dual of counterfactual *would* for Stalnaker—*would* has no dual. For Stalnaker, a conditional is true in a world *w* just in case its consequent is true in the closest world to *w* where its antecedent is true. The assumption is that there is just one such closest world, and this leaves no room for distinguishing counterfactual necessity and possibility.

Both Stalnaker (1981) and Lewis (1981) emphasize that, at least for counterfactuals, the difference between systems that allow orderings with incomparabilities and ties versus those that do not is not as dramatic as it may seem. An order that has incomparabilities or ties can be matched with a multiplicity of orders that disagree precisely in how they resolve those incomparabilities or break those ties. It would then be part of the notorious context dependency of counterfactuals that there might be unresolved indeterminacy about which ordering was intended.

To see what indeterminacy of orderings might mean for the typology of modals more generally, let's construct another toy example. As before, assume that $\cap f(w_0) = W = \{w_0, w_1, w_2, w_3\}$, but we are now comparing a single ordering O_3, which has a tie, with a pair of orderings O_1 and O_2, which resolve the tie of O_3 in opposite ways:

Option 1: indeterminacy between two orders without ties:
O_1: $w_3 < w_2 < w_1 < w_0$
O_2: $w_2 < w_3 < w_1 < w_0$

Option 2: order with a tie:
O_3: $w_2, w_3 < w_1 < w_0$

Assuming O_3, the following propositions wind up as necessary according to our definition: $\{w_0, w_1, w_2, w_3\}$, $\{w_1, w_2, w_3\}$, $\{w_0, w_2, w_3\}$, and $\{w_2, w_3\}$. Those are also the propositions that are necessary with respect to *both* O_1 and O_2— that is, those are precisely the propositions that wind up as necessary, regardless of how we resolve the indeterminacy between O_1 and O_2.

The situation is different for possibility, however. For possibility, option 1 and option 2 truly come apart. On option 1, the necessary propositions are the same as the possible ones. Consequently, the propositions that are necessary regardless of how the indeterminacy between O_1 and O_2 is resolved are precisely the propositions that are possible regardless of how the indeterminacy is resolved. Option 2 presents a rather different picture. According to our definitions, the following propositions come out as *merely* possible: $\{w_0, w_1, w_3\}, \{w_1, w_3\}, \{w_0, w_3\}, \{w_3\}, \{w_0, w_1, w_2\}, \{w_1, w_2\}, \{w_0, w_2\}, \{w_2\}$. Only on option 2 can we draw a distinction between necessary and possible propositions, then. The two notions collapse into each other on option 1. Crucially, this is so even on a super valuation approach, where modal claims are true just in case they wind up true no matter how ordering indeterminacies are resolved.[17] Whether we can or cannot have the familiar dual pairs of modals in a language crucially depends on the orderings the modals tolerate, then.

Rullmann, Matthewson, and Davis (2008) document that not every language draws a lexical distinction between possibility and necessity modals of the kind found in the familiar Indo-European languages. From the current perspective, this could mean that some languages might generally disallow incomparabilities or ties for their ordering source induced orderings. It would then be literally impossible for those languages to have the familiar dual necessity and possibility modals. But there are other possibilities that need to be considered for modals without duals.

In an ordering semantics for modals, ordering sources are used as domain restrictions for the set of accessible worlds: not all, but only the "closest" accessible worlds matter for what is possible or necessary. As the domain of accessible worlds shrinks, necessity modals become weaker and possibility modals become stronger. In the most extreme case, the distinction between necessity and possibility collapses. In less extreme cases, necessity and possibility may still be formally distinguishable, but a language may nevertheless choose not to lexicalize dual pairs of modals in some or all modal domains. The retained modals might all be possibility modals, for example. Being weaker than the corresponding necessity modals, possibility modals could be used to describe situations where English might use *must* or *may*. Peterson (2008) proposes that the modals in the Tsimshianic language Gitksan (spoken in North-Western British Columbia) are possibility modals of precisely this kind. Deal (2010b) makes a similar point for the modal suffix *o'qa* in Nez Perce.

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[17] Stalnaker (1981) made this point for counterfactuals.

Rather than being just a possibility modal or a collapsed possibility/ necessity modal, a modal without dual could also be a degree expression covering the upper end of a scale of degrees of probabilities or preferences. Such upper-end degree modals could correspond to notions like, "it is (somewhat) probable," or, "it is (somewhat) desirable." We would then expect there to be a certain amount of vagueness with respect to the lower bound of the range of probabilities allowed. For epistemic degree modals admissible probabilities might range from, say, around 50% to 100%, for example.[18]

Here is a toy example illustrating what an upper-end degree modal may do. As before, suppose $\cap f(w_0) = W = \{w_0, w_1, w_2, w_3\}$, but this time round, $g(w_0) = A = \{ \{w_2, w_3\}, \{w_1, w_2, w_3\} \}$. The ordering $\leq_A$ induced on W is O_3 from option 2 above: $w_2, w_3 < w_1 < w_0$. There is a tie between w_2 and w_3, then, which has the consequence that the distinction between necessity and possibility does no longer collapse. Below is a table displaying a probability measure P on $\wp(W)$ that assigns probabilities to the singleton sets in a way that respects O_3. However, since there are ties, P no longer preserves the $O3$-induced notion of comparative possibility between propositions defined above.

$P(\emptyset) = 0$	$P(\{w_2\}) = 4/11$	$P(\{w_3\}) = 4/11$	$P(\{w_2, w_3\}) = 8/11$
$P(\{w_0\}) = 1/11$	$P(\{w_0, w_2\}) = 5/11$	$P(\{w_0, w_3\}) = 5/11$	$P(\{w_0, w_2, w_3\}) = 9/11$
$P(\{w_1\}) = 2/11$	$P(\{w_1, w_2\}) = 6/11$	$P(\{w_1, w_3\}) = 6/11$	$P(\{w_1, w_2, w_3\}) = 10/11$
$P(\{w_0, w_1\}) = 3/11$	$P(\{w_0, w_1, w_2\})$ $= 7/11$	$P(\{w_0, w_1, w_3\})$ $= 7/11$	$P(\{w_0, w_1, w_2, w_3\})$ $= 11/11$

As before, P is just one of many probability measures satisfying our current, rather weak, requirements on suitable probability measures. In this example, the necessary propositions are all those that contain both w_2 and w_3. Those are the propositions whose probability is at least 8/11. The possible propositions are all those that contain w_2 or w_3. Those are the propositions whose probability is at least 4/11. An upper-end degree modal might cover a probability range from, say, 5/11 or 6/11 to 11/11. Such a modal could thus be

[18] The official weather forecast for Bergen (Norway) extends *somewhat probable* even further: "Within these fields, it is considered most probable (50 percent) that the development hits the dark part. Still it is somewhat probable (30 percent) that it hits the lighter part outside the dark." <http:// www.yr.no/place/Norway/Hordaland/Bergen/Bergen/long.html>. A systematic investigation of the probability ranges people attach to expressions of uncertainty in English is Mosteller and Youtz (1990). They report, for example, that the unmodified adjective *probable* tends to be associated with a probability range of about 60 to 80 percent.

used in situations where English would sometimes use *must*, and at other times *may* or *might*. Marginally, such a modal might even be used with two propositions that are negations of each other. In the way of illustration, consider the propositions $p = \{w_0, w_3\}$ and $\neg p = \{w_1, w_2\}$ from our toy example. The probability of both propositions is around 50%: $P(p) = 5/11$ and $P(\neg p) = 6/11$. Marginally, they could both be said to be somewhat probable, then.

The data and observations presented in Rullmann et al. (2008) invite the conjecture that in the modal system of the Salish language St'át'imcets, upper-end degree modals with the force of (*somewhat*) *probable* or (*somewhat*) *desirable* might take the place of necessity and possibility modals. The St'át'imcets modal system lacks a lexical distinction between necessity and possibility modals. The distinction is not only missing for epistemic modals, but for all modals, including deontic, irrealis, and future modals.[19] Here are some of Rullmann et al.'s examples with the inferential epistemic modal *k'a*:

(13) *apparently / it seems:* **k'a**
      Wa7 k'a qwenúxw.
      '*He must be sick.*' or '*I guess that he is sick.*'
      Rullmann et al., example (4): 320.[20]

(14) a. t'ak   k'a   tu7 kents7á ku míxalh
         go.along INFER then DEIC   DET bear
         'A bear must have gone by around here.'

      b. nilh k'a   kw s-Henry   wa7 pegwpegwtsám'
         FOC INFER DET NOM-Henry IMPF knock.repeatedly
         'That'll be Henry knocking.'

      c. *Context: You have a headache that won't go away, so you go to the doctor. All the tests show negative. There is nothing wrong, so it must just be tension.*
         nilh k'a   lh(el)-(t)-en-s-wá (7)-(a)   ptinus-em-sút
         FOC INFER from-DET-1SG.POSS-NOM-IMPF-DET think-MID-OOC
         'It must be from my worrying.'

      d. wa7 k'a   séna7   qwenúxw
         IMPF INFER COUNTER sick
         'He may be sick.' (*Context: Maybe that's why he's not here.*)

[19] See 2.7 for an illustration of how ordering sources produce graded notions of possibility for root modals, too. We would then expect to find upper-end degree modals among the root modals as well.

[20] Rullmann et al. credit the example to Alexander, C., B. Frank, G. Ned, D. Peters Sr., C. Shields, and R.A. Whitley. 2006. In Henry Davis (ed.), *Nqwal'luttenlhka'lha: English to St'át'imcets Dictionary*, vol. ii: *Intermediate*. Lillooet, BC: Upper St'át'imc Language Culture and Education Society.

e. *Context: His car isn't there.*
plan   k'a   qwatsáts
already INFER leave
'Maybe he's already gone.'
Rullmann et al., example (5): 321.

(15) is particular telling, since it involves two incompatible propositions:

(15) k'a   lh-zúqw-as   tu7 ni7   na núkw-a   qelhmín
INFER COMP-die-3CONJ then DEMON DET other-DET old.person
smúlhats k'a   lh-mím'c-as   tu7 nka7
woman   INFER comp-move-3CONJ then where
'Maybe the other old woman died or maybe she moved somewhere.'

Rullmann et al., example (13): 324.

Rullmann et al. take St'át'imcets modals to be necessity modals that can be contextually weakened by domain restrictions. They do not discuss the hypothesis that St'át'imcets modals might be possibility modals that can be contextually strengthened by domain restrictions. And they do not consider the possibility for languages to have upper-end degree modals, which are neither possibility nor necessity modals. Rullmann et al. posit a special mechanism of domain restriction via choice functions, but since ordering sources already function as domain restrictors and are independently needed, the default assumption would be that they are the main source for additional domain restrictions, and thus the main source for variable modal force in St'át'imcets. No further mechanism for domain restriction seems to be needed. An analysis of St'át'imcets modals as upper-end degree modals seems to predict the data and observations presented in Rullmann et al. correctly. First, bilingual speakers translate St'át'imcets modals as English possibility or necessity modals, depending on context. Second, conjunctions of impossibles, as in (15) are acceptable, but only marginally so. Rullmann et al. report that speakers' judgments are not consistent with examples of this kind. This would not be expected if St'át'imcets modals were simply possibility modals. And, finally, there is a clear preference for St'át'imcets modals to describe necessary, rather than merely possible, states of affairs. This preference would again be surprising for possibility modals, but is expected for upper-end degree modals. As illustrated in our last toy example, all necessary, but not all possible, propositions are clear cases of somewhat probable propositions. Quite generally, necessary propositions are always covered by an upper-end degree modal. Since the lower bounds of what are acceptable degrees of probabilities, preferences, tendencies, propensities, etc.

are genuinely underdetermined, there might be questions about which possible propositions are covered, too.

The interpretations of the modals discussed in this section depend on two conversational backgrounds, rather than just one. Does this mean that for different types of modals, a different number of parameters has to be fixed by the utterance context? Would we still want to say that there is a class of modals that express relativized "simple necessity" or "simple possibility," as hypothesized in 2.3? In other words, are there any modals whose interpretations depend on just a modal base, rather than on both a modal base and an ordering source? We will see shortly that the interpretations of apparently "simple" modals like *muss, kann, it is necessary that*, etc. can depend on ordering sources, too. So a better view would be to assume that, quite generally, the interpretations of modals depend on both a modal base and an ordering source, but either parameter can be filled by the empty conversational background. The full range of possible modal meanings expressed in natural languages can now be characterized by conditions on three parameters: modal base, ordering source, and modal force. The available modal forces depend on the properties of the orderings induced by the interaction of modal base and ordering source, as we have seen: apart from the familiar dual pairs of possibility and necessity modals, languages may also have "collapsed" possibility/necessity modals, just existential modals, or simple or complex degree modals with modal forces derived from some notion of comparative possibility in one way or other: *is a better possibility, is a good possibility, is a slight possibility, is somewhat probable, is somewhat preferable*, and so on. Given the rich inventory of possible modal meanings, figuring out what kind of meaning is suitable for a particular modal in a given language is now no longer a simple task. The lack of dual pairs is an important clue for both the researcher and a language-learning child, and so is apparently variable modal force, as documented in the St'át'imcets examples. If, depending on context, a modal shows chameleon-like behavior in allowing both possibility and necessity interpretations, but with a preference for necessity interpretations, a degree modal might be your best bet.

2.6 Root versus epistemic modals

The previous section suggested that in modal reasoning, a conversational background may play the role of a modal base or an ordering source. The modal base determines the set of accessible worlds, and the ordering source induces an ordering on it. This section is about a major dichotomy seen in the modal bases for modals in natural languages. Modal vocabularies often draw a

distinction between so-called "epistemic" versus "root" modals. The termin-
ology is well established in the linguistic literature, so I will continue to use it
here, even though it's not clear what "root" is meant to refer to, and "epi-
stemic" modals do not have any necessary connection to knowledge. In my
older work on modality, I coined the term "circumstantial" modals for root
modals, and this is still the term I prefer and use when circumstances in a wider
sense are at stake. The term makes clear that with *can*, for example, the
intended range of uses does not just include ability interpretations, but also
so-called "metaphysical" modalities (Condoravdi 2002), which Abusch (forth-
coming) showed to be cut from the same cloth as other cases of circumstantial
modality. In the end, those terms are all likely to be problematic in one way or
other, though, and it's ultimately the analysis that will tell us what the
grammatically significant types of modality are. Established terms for different
types of modalities pick out pretheoretical distinctions that are useful at the
beginning of an investigation, but may not survive careful theorizing.

Epistemic and root modals differ syntactically. Epistemic modals occupy
high positions in the hierarchy of verbal inflectional heads, root modals
appear in lower positions. Both types of modals can have non-empty realistic
modal bases. However, if they do, the facts relied on seem to be different in a
way that has proven difficult to characterize in formal terms. Here are a few
examples.

Root modals

(16) Sie wollte   schreien und konnte nicht, gewann   aber
     She wanted to scream   and could   not,   regained however
     endlich die   Herrschaft über ihre erlahmten Glieder.
     finally the   control   over her paralyzed limbs.

Genovev was so terrified that she was unable to move.[21]

(17) Der Jani-Hans schimpfte nie,   fluchen konnte er gar nicht.
      The Jani-Hans scolded   never, curse   could he at all not.

Jani-Hans had such a mild character that he just wasn't capable of getting
angry.[22]

(18) Hier können die Tomaten gedeihen.
      Here can   the tomatoes prosper.

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[21] Adapted from Graf (1978[1946]: 25).
[22] Adapted from Graf (1978[1946]: 32).

(19) Wer nichts hat dem   kann man auch nichts   nehmen.[23]
      Who nothing has, from whom can one also nothing take away.

Epistemic modals

(20) Es kann nur einer   gewesen sein, der sich im   Haus
      It can   only someone been have,   who REFL in the house
      auskennt hat.
      at home was.

The Heimraths have been burgled and Girgl is trying to find out who might
have been the thief. It must have been someone who was familiar with the
house.[24]

(21) Sie   hatten den Befehl, den jungen König zu suchen, der sich
      They had   order   the young king to look for, who REFL
      in einer seiner Jagdhütten   aufhalten musste.
      in one of his   hunting huts stay   must.PAST

The young king has disappeared, and given the evidence available, he must be
hiding in one of his hunting huts.[25]

(22) Soweit   wir wissen, muss es   für sie   nie   etwas   anderes
      As far as we know, must there for them never anything else
      gegeben haben als Geborenwerden, Aufwachsen, unermüdliche Arbeit
      been   have but being born,   growing up, tireless   work
      und Sterben.
      and dying.

Oskar Maria Graf draws this conclusion from the historical sources about the
life of the Heimrath family some centuries ago.[26]

  There is a subtle semantic difference between the two kinds of modals I
grouped under the two headings. It is a difference in the kind of facts relied
on. Root modals are typically future oriented and are used to talk about
propensities and potentials of people, things, and spatio-temporal locations,
given their current circumstances. Usually, circumstances permit or prevent
events from happening. Only sometimes do they necessitate events: we have
to die, cough, vomit, laugh, cry, or realize that we are lost.

  To see the difference between root and epistemic modals with non-empty
realistic modal bases more clearly, compare the (a)- and (b)-sentences in (23)
and (24):

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[23] Adapted from Graf (1978[1946]: 57).   [24] Adapted from Graf (1978[1946]: 66).
[25] Adapted from Graf (1978[1946]: 37).   [26] Adapted from Graf (1978[1946]: 12).

(23) a. Aus dieser Kanne Milch kann die Kathl ein Pfund   Quark
         From this can of milk can   the Kathl one pound of cottage cheese
         machen.
         make.

     b. Es kann sein, dass die Kathl aus dieser Kanne Milch ein Pfund
         It may be   that the Kathl from this can of milk   one pound
         Quark   macht.
         cottage cheese makes.

(24) a. In dieser Gegend können Zwetschgenbäume wachsen.
         In this   area   can   plum trees   grow.

     b. Es kann sein, dass in dieser Gegend Zwetschgenbäume wachsen.
         It may be   that in this   area   plum trees   grow.

The modal *kann* ('can') in (23a) and (24a) can be a root or an epistemic modal. For sentences (23b) and (24b), the epistemic interpretation is prominent. Given a circumstantial interpretation for the (a)-sentences and an epistemic interpretation for the (b)-sentences, we can imagine situations where I speak truly when uttering an (a)-sentence, but falsely when uttering the corresponding (b)-sentence. Take (23): given the cottage cheese production methods and tools available to Kathl, it would be possible for her to produce a pound of cottage cheese from the milk in the can. She has other uses for the milk in the can, however, and never uses the whole can for the production of cheese: a bit of the milk goes into her coffee, a bit into her porridge, a bit goes to the cat, and whatever remains is used for her cheese. The likelihood that Kathl will in fact produce a pound of cottage cheese from the milk in the can might thus be close to zero.

When using a root modal, we neglect certain kinds of facts, even though we might be aware of them. Suppose I am traveling in an exotic country and discover that soil and climate are very much like that in my own country, where plum trees prosper everywhere. In such a situation, an utterance of (24a) in its circumstantial sense would probably be true. But (24b) might very well be false, given that that country has had no contacts whatsoever with Western civilization and the vegetation is altogether different from ours. The available evidence rules out the possibility that plum trees grow in this area.

The kind of facts we take into account for root modality are a rather slippery matter. This may give rise to misunderstandings and jokes.[27] I once heard a philosopher say that one of the defining properties of a cup is that

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[27] Horgan (1979) and Lewis (1979a) have more illustrations of this point.

you can pour things like coffee in it. A student objected to this in pointing out that—if this were true—a cup which has coffee in it already would not be a cup anymore.

When we talk to each other, we hardly ever make explicit in view of what circumstances something should be necessary or possible. We may give hints. Usually people understand. And they usually understand in pretty much the same way. Take (25):

(25)   Ich kann nicht Posaune   spielen
     I   can   not   trombone play.

Depending on the situation in which I utter (25), I may say rather different things. I may mean that I don't know how to play the trombone. I am sure that there is something in people's minds that becomes different when they start learning to play the trombone. A program is filled in. It is in view of that program that it is possible for me to play the trombone. Or suppose that I suffer from asthma. I can hardly breathe. In view of my physical condition I am not able to play the trombone, although I know how to do it. I may express this with (25). Or else imagine that I am traveling by sea. The ship sinks and so does my trombone. I manage to get to a lonely island and sadly mumble (25). I could play the trombone in view of my head and my lungs, but the trombone is out of reach. There are more conceivable types of situations covered by (25), but most of them bring in norms in addition to the facts. That is, most of them involve a non-empty ordering source. I'll discuss such cases in the following section.

A distinction between circumstances concerning the outside world, the body or the mind of a person plays a role in the semantic development of *können*. According to Deggau (1907), the Old High German equivalent of that modal was first used for intellectual capacities. Then, it could also express possibilities in view of outside circumstances. Only later was it used for talking about physical abilities. Kiefer (1983) shows that similar distinctions are made in Hungarian. In Hungarian, the verbal suffix *-hat/-het* expresses possibility. In its root reading, it can only be used for possibilities in view of outside circumstances. Taking up some of Kiefer's further observations, consider a phrase like *imstande sein* ('to be able'). I could say

(26)   Ich bin nicht imstande, Posaune   zu spielen.
     I   am not   able   trombone to play.

if I have asthma or weak nerves, or if I have no talent. I doubt whether I could say it in a situation where I haven't learnt how to play the trombone. And I could never say it on the island with my trombone lost at sea. The relevant

circumstances for *imstande sein* are concerned with the strength of the body, character, or intellect. For *kann,* there is another restriction:

(27)   # Dieses Messer kann nicht schneiden.
       This   knife   can   not   cut.

(28)   # Dieser Hut kann den Kopf warmhalten.
       This   hat can   the   head keep warm.

(29)   # Dieser Ofen kann nicht richtig   heizen.
       This   stove can   not   properly heat.

(27) to (29) sound funny. They suggest that the knife, the hat, or the stove are agents taking an active part in the cutting, the warming of the head, or the heating. To avoid this effect, we would have to say:

(27')   Dieses Messer schneidet nicht.
       This   knife   cuts   not.

(28')   Dieser Hut hält   den Kopf warm.
       This   hat   keeps the   head warm.

(29')   Dieser Ofen heizt nicht richtig.
       This   stove heats not   properly.

One of the factors responsible for the deviance of (27) to (29) relates to agency: the knife is not an agent, but an instrument for cutting something. The hat is not an agent, but an instrument for warming the head. And the stove is not an agent, but an instrument for heating a room. Some machines, like music boxes, can do things all by themselves, thus functioning as agents. This seems to rescue (30):

(30)   Diese Spieluhr   kann "La Paloma" spielen.
       This   music box can   "La Paloma" play.

In this section, I have examined realistic modal bases for the two major types of modals in natural languages: root (or circumstantial) versus epistemic modals. In both cases, realistic modal bases target relevant bodies of facts in the evaluation world and track them via counterpart relations in all accessible worlds. The kind of facts that are targeted by the two types of modals are different in kind, though: external or internal circumstances of people, things, or places that determine their possible futures contrast with evidence of things implying or suggesting the presence of other facts in the past, present, or future.

Formally, empty modal bases wind up as limiting cases of realistic ones. This is a welcome consequence. Natural languages have not come up with special vocabulary for logical and mathematical necessity and possibility. Root or epistemic modals are used in those cases, too, even if no non-trivial facts are involved. We can now hypothesize that both root and epistemic modals have realistic modal bases. If all modals are either root or epistemic, it follows that *all* modals have realistic modal bases. Potentially non-realistic conversational backgrounds must then function as ordering sources.

The distinction between root and epistemic modality is evident in the vocabulary of German. Verbs with inherent modality, modal adjectives ending in -*lich* and -*bar*, and phrases like *imstande sein* or *in der Lage sein* never express epistemic modality. Sentence adverbs like *wahrscheinlich* or *möglicherweise* and auxiliaries like *wird* or *dürfte* always express epistemic modality. The neutral auxiliaries *müssen* and *können* can express root or epistemic modality, depending on their syntactic position.

If root and epistemic modals occupy different positions in the hierarchy of verbal inflectional heads, we may wonder whether the subtle semantic differences between the two types of modals can be derived from their syntactic differences. Valentine Hacquard's work (Hacquard 2006, 2010) points to a positive answer. Hacquard observes that during semantic composition, different regions of a verb's extended projection manipulate different kinds of semantic objects from which modal bases can be systematically projected. Argument structure is built in the lower regions of extended verbal projections, for example, and, consequently, modals in those regions can target the potentials and propensities of events, event participants and event locations. In higher regions, inflectional heads like tense shift the perspective to the speech event and the speaker. This is why, according to Hacquard, the possibilities of modals appearing in higher regions are keyed to the epistemic possibilities of speakers. While the last conclusion does not seem to be correct, Hacquard's general idea to derive the core differences between root and epistemic modals from their different syntactic positions is a major step towards explaining a distinction that has long resisted analysis.

2.7 Approaching norms and ideals with root modals

Root modals have realistic modal bases that interact with normative ordering sources to produce deontic, bouletic, teleological, or propensity interpretations. As with epistemic modals, ordering sources for root modals induce orderings on the set of accessible worlds that allow us to define suitable notions of necessity, possibility, and comparative possibility. Some root

modals tolerate a wide range of ordering sources. Others are submitted to tighter restrictions.

Können and dürfen

(31) Du kannst doch nicht nur Häuser bauen oder Semmeln backen und wenn
     You can   not only houses build or   rolls   bake   and when
     du dann gestorben bist, ist alles   aus,   alles.   weggewischt.
     you then dead   are is everything finished, everything wiped out.

Shortly before his death, old Graf realizes that according to some conception of an ideal life, you should do more than just care for your property or do your daily work.[28]

(32) Sagen kannst gewiss   nicht, dass ich dir einmal schlecht geraten hab'.
     Say   can you certainly not   that I   you once   bad advice given have.

Jani Hans always advised the Heimrath widow well. Given this fact, it goes against ideals of truthfulness and trust that she say anything to the contrary.[29]

(33) Dieses Brot kann man ja   direkt   seiner Majestät empfehlen.
     This   bread can   one indeed straight away to his Majesty   recommend.

This bread is good. If you recommend something good to him, the King will be pleased. If you recommend something bad to him, however, the King will hate you. Given these facts, it is compatible with a desirable future where the King loves you that you recommend this bread to him.[30]

(34)   Kann ich jetzt gehen?
       Can   I   now   leave?

Imagine a student who says (34) to his teacher. The teacher is the source of law and order for him. What she wants is ordered and nothing is ordered unless she wants it. The boy wants to know whether it is compatible with his teacher's orders that he leaves. In this case, *kann* is deontic. For *darf*, a deontic ordering source is common, but not obligatory. Suppose two burglars are trying to enter a farmhouse and whisper to each other:

(35)   Jetzt dürfen wir keinen Lärm machen.
       Now   may   we no   noise make.

It is not that they are not allowed to make a noise. They can't make a noise in view of their goal to burgle the farmers without getting caught. *Kann* and

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[28] Adapted from Graf (1978[1946]: 114).   [29] Adapted from Graf (1978[1946]: 60).
[30] Adapted from Graf (1978[1946]: 94).

darf have similar meanings. Both express possibility. But there are differences. *Darf* requires an ideal according to which possibilities are assessed. *Kann* is more neutral. With *kann*, possibilities may depend on brute facts alone—that is, the ordering source may be empty. *Darf* doesn't admit ordering sources related to normalcy standards. Suppose I have a horrible headache and say with a deep sigh:

(36) Ich kann das nicht aushalten.
      I   can this not   bear.

This use of *kann* involves standards concerning normal tolerance thresholds for pain. I couldn't convey the same meaning by uttering

(37) Ich darf das nicht aushalten.
      I   may this not   bear.

Kann may have difficulties with bouletic ordering sources. Imagine that tomorrow is the coronation of the King and I say:

(38) Morgen   darf es nicht regnen.
      Tomorrow may it not   rain.

What I say here is roughly that according to what we all want, it shouldn't rain tomorrow. I couldn't get this interpretation by uttering:

(39) Morgen   kann es nicht regnen.
      Tomorrow can it not   rain.

I conclude that there are restrictions for *kann* and *darf* that concern the admissible ordering sources.

Müssen and sollen

(40) Wegen der Lola Montez hat er dem   Thron entsagen müssen.
      Because of Lola Montez has he the.DAT throne abdicate must.INF

Ludwig I of Bavaria loved Lola Montez. People became angry. Revolution broke out. Respecting the public interest he had to resign.[31]

(41) Es muss mir   gehören, es muss.
      It must to me belong,   it must.

Kastenjakl is desperate to buy a piece of land from the Heimraths. His own wishes dictate that it must belong to him.[32]

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[31] Adapted from Graf (1978[1946]: 39).
[32] Adapted from Graf (1978[1946]: 78).

(42) Lump muss man sein, nur als Lump zwingt   man die lumpige Welt.
     Crook must one be,   only as crook conquers one the crooky world.

Lenz presents his goal in life in the second part of the sentence. Given the
facts of the actual world, you must be a crook if you want to conquer the
world.[33]

(43) Arbeiten haben wir bis jetzt   müssen, arbeiten werden
      Work   have we up to now must.INF, work   will
      wir auch weiter   müssen.
      we also in future must.INF.

The Heimraths are peasants. Given their social status, they have to work if
they are aspiring to a decent and honest life where they aren't beggars or
burglars.[34]
    Like *kann*, *muss* accepts a wide range of ordering sources. The ordering
source may be empty, too. This is suggested by sentences like:

(44) Er musste   husten.
      He must.PAST cough.

Like *darf*, *soll* requires a non-empty ordering source.

(45) Ein Richard Wagner Festspielhaus sollte   nach   den
      A   Richard Wagner festival hall shall.PAST after   the
      Entwürfen des   Architekten Semper gebaut werden.
      designs   of the architect   Semper built be.

According to the plans of King Ludwig II of Bavaria, a Richard Wagner
festival hall was to be built after the designs of the architect Semper.[35]

(46) Ich bitt' euch gar schön, der hochwürdige Herr Pfarrer soll kommen.
      I   ask you very nicely, the reverend   Sir priest shall come.

Gauzner Michl is dying. He wants a priest to come and see him.[36] In Luther's
translation, God uses *sollen* a lot when he talks to Moses.

(47) Sechs Tage soltu   erbeiten und alle deine Werck thun.
      Six   days shalt thou labor   and all thy   work do.

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[33] Adapted from Graf (1978[1946]: 82).
[34] Adapted from Graf (1978[1946]: 57).
[35] Adapted from Graf (1978[1946]: 41).
[36] Adapted from Graf (1978[1946]: 103).

According to what God wants, it is necessary that you work six days a week. In some societies, what God wants is commanded. In other societies, what God wants is good and recommended, but not commanded. If I lived in a society of the first kind, I would most naturally say:

(48)   Ich muss sechs Tage arbeiten und alle meine Werke tun.
      I   must six   days work   and all   my   work do.

If I lived in a society of the second kind, however, I would prefer to say:

(49)   Ich soll   sechs Tage arbeiten und alle meine Werke tun.
      I   shall six   days work   and all   my   work do.
      I am supposed to work for six days and to do all my work.

Sollen might express the weakened kind of necessity that comes with a non-empty ordering source, as proposed in the original paper. But it could also be a degree modal covering the upper end of a scale induced by an ordering source corresponding to what is good, planned, or recommended, or by what someone wants, plans, or recommends.[37] Actually, it is not just what *anyone* wants, plans, or recommends. The one who does so cannot be identical with the individual referred to by the subject of the sentence in which *sollen* occurs, for example. I can't say

(50)   Ich soll   ein Bäcker   werden.
      I   shall a   baker   become.
      I am supposed to become a baker.

if it is mine but no one else's wish that I become a baker. Compare this with Gunnar Bech's characterization in Bech (1949): "*sollen*...bezeichnet einen nicht dem Subjekt innewohnenden Willen," "*sollen* refers to a will which is not inherent in the subject." Since in a passive sentence like (51), *er* is not the "logical" subject, (51) is not a counterexample to Bech's principle:

(51)   Er soll   in Ruhe gelassen werden.
      He shall in peace left   be.

I could use (51) for expressing that it is according to his own wishes that he shouldn't be bothered. *Muss* is neutral with respect to who wants me to become a baker.

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[37] The assumption that *sollen* might be an upper-end degree modal has interesting consequences that deserve to be explored in a separate investigation—in particular because of the intimate connection between *sollen*-type modals and imperatives. We would expect absence of a dual and apparently variable modal strength, for example. The extant literature on *sollen*-type modals and imperatives looks promising for a degree modal analysis.

(52)   Ich muss ein Bäcker werden.
        I   must a   baker become.

(52) can be used if I, myself, or someone else wants me to become a baker.

The suffixes *-bar* and *-lich* allow all kinds of ordering sources, depending on the adjective they are attached to.

-bar and -lich

(53)   Dieses Eintrittsbillet   ist nicht übertragbar.
        This   admission ticket is not   transferable.

According to the regulations, it is not possible to give this ticket to anyone else.

(54)   Diese Tasse ist zerbrechlich.
        This cup   is fragile.

(54) has a realistic modal base and an empty ordering source. It is in view of certain properties inherent in the cup that it is possible for it to break.

(55)   Dieser Vorschlag ist annehmbar.
        This   proposal is acceptable.

Given our common goals, it is possible to accept this proposal.

(56)   Diese Lage   ist unerträglich.
        This situation is intolerable.

Every night, Marie-Louise's living room becomes the meeting place for all the cats in the neighborhood. This is intolerable in view of normal standards concerning property, noise, and smell. We may add a phrase like *for Marie-Louise* to indicate that the standards involved are more subjective.

(57)   Für Marie-Louise ist diese Lage unerträglich.
        For Marie-Louise is this situation intolerable.

Ordering sources permit the grading of possibilities:

(58)   Ich kann eher   Bäcker als   Stellmacher werden.
        I   can rather baker than cartwright   become.
        'It is more possible for me to become a baker than a cartwright.'

Maxl was wounded during the war against the Prussians. Given this, becoming a baker is a better possibility for him than becoming a cartwright: becoming a baker would require less effort for him, and he would therefore also do a better job as a baker.

Kann eher.... als expresses comparative possibility. Comparative possibility was the main motivation for introducing a distinction between modal bases and ordering sources in 2.4. We saw that for epistemic modals, the interaction between stereotypical ordering sources and modal bases projected from pieces of evidence produces comparative and quantitative notions of epistemic probability. It seems that for root modals, the interaction between stereotypical ordering sources and modal bases projected from the current circumstances of individuals and spatio-temporal locations produces comparative and quantitative notions of propensity—sometimes referred to as "aleatory probability." To illustrate, a fair coin has the propensity to land heads 50% of times when it is tossed in a fair setting. The coin has that propensity even if it is never tossed—it's one of its inherent properties. Hacking (1975) quotes a passage by Richard von Mises, which uses various comparative forms of the German adjective *möglich* ('possible') to illustrate the difference between epistemic and aleatory probability:[38]

Ordinary language recognizes different degrees of possibility or realizability. An event may be called possible or impossible, but it can also be called quite possible or barely possible (*schwer oder leicht möglich*) according to the amount of effort that must be expended to bring it about. It is only 'barely possible' to write longhand at 40 words per minute; impossible at 120. Nevertheless it is 'quite possible' to do this using a typewriter [...] In this sense we call two events equally possible if the same effort is required to produce each of them. This is what Jacques Bernoulli, a forerunner of Laplace, calls *quod pari facilitate mihi obtingeri possit* [...] But this is not what Laplace's definition means. We may call an event 'more possible' [*eher möglich*] than another when we wish to express our conjecture about whatever can be expected to happen. There can be no doubt that equipossibility as used in the classical definition of probability is to be understood in this sense, as denoting equally warranted conjectures [1951, p. 78].

Hacking explains that "according to Mises the epistemic concept of probability corresponds to an epistemic concept of possibility, while the aleatory concept of probability corresponds to a concept of physical possibility."[39] The two major interpretations of probability thus seem to be intimately linked to the fundamental difference between root and epistemic modality.

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[38] Quoted from Hacking (1975: 123–4).
[39] Hacking (1975: 124).

In the following section, I will discuss practical inferences, which provide an additional argument in favor of separating realistic and normative conversational backgrounds.

2.8   Practical reasoning

Capturing the semantics of modals via two interacting conversational backgrounds has interesting consequences for what has been called "practical inference."[40] A practical inference may have the following form:

I want to become mayor.
I will become mayor only if I go to the pub.

Therefore I should go to the pub.

Spelling out hidden assumptions:

All I want is to become mayor.
The relevant circumstances are such that I will become mayor only if I go to the pub.

Therefore, considering the relevant circumstances and what I want, I should go to the pub.

In this section, I will tentatively assume that *should* is a necessity modal, rather than an upper-end degree modal—an assumption that will ultimately have to be submitted to further scrutiny. The phrase *the relevant circumstances* in the second premise of the inference above contributes a modal base f that maps a world w to the set of propositions that correspond to the relevant circumstances in w. *What I want* contributes an ordering source g that maps a possible world w to the set of propositions that correspond to what I want in w. Let's assume that $f(w)$ contains just one proposition in our example: the proposition that I will become mayor only if I go to the pub. Assume furthermore that $g(w)$ only contains the proposition that I will become mayor. The union of $f(w)$ and $g(w)$ is consistent, then. It follows that the proposition that I go to the pub is a necessity in w with respect to f and g iff $f(w) \cup g(w)$ implies that I do so. Since the implication holds, the inference comes out valid.

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[40] Anscombe (1957), von Wright (1963, 1972).

Let us now look at a slightly more complicated example:

All I want is two things, namely avoid going to the pub and become mayor. The relevant circumstances are such that I will become mayor only if I don't avoid going to the pub.

Therefore, considering the relevant circumstances and what I want,

Conclusion one:	I should go to the pub.
Conclusion two:	I should avoid going to the pub.
Conclusion three:	I could avoid going to the pub and still become mayor.
Conclusion four:	I could go to the pub.
Conclusion five:	I could avoid going to the pub.

This is the horrible story of someone who wants something but rejects the necessary means leading to the fulfillment of her desires. Which conclusion can we draw in such a case? The first three conclusions are out, but the last two are fine. Our current analysis predicts this. Suppose *should, the relevant circumstances*, and *what I want* are interpreted as in the previous example and *could* expresses possibility. This time round, $g(w)$ contains two propositions: that I will become mayor and that I avoid going to the pub. If $\cap f(w)$ is the set of worlds accessible from w, then:

(a) For all worlds $v \in \cap f(w)$:
   If I avoid going to the pub in v, I won't become mayor in v.

Given the definition of possibility, it follows right away that conclusion three is false. Wishes cannot override facts. Consider now the ordering source $g(w)$. It induces a three-cell partition of $\cap f(w)$ as follows:

- A is the set of all worlds in $\cap f(w)$ where I go to the pub and become mayor
- B is the set of all worlds in $\cap f(w)$ where I avoid going to the pub and won't become mayor
- C is the set of all worlds in $\cap f(w)$ where I go to the pub, but still won't become mayor.

The reader can verify that all of the following statements are true:

(b) A, B, and C are not empty, they are pairwise disjoint and $A \cup B \cup C = \cap f(w)$.
(c) If $v \in A$ and $z \in B$, then neither $v \leq_{g(w)} z$ nor $z \leq_{g(w)} v$.
(d) For all v and $z \in A$: $v \leq_{g(w)} z$.

(e) For all v and $z \in B$: $v \leq_{g(w)} z$.

(f) If $z \in C$ and $v \in A \cup B$, then $v \leq_{g(w)} z$, but not $z \leq_{g(w)} v$.

All worlds in A are worlds where I go to the pub, and for none of those worlds is there any world where I avoid going to the pub that is at least as close to what I want. This makes the proposition that I go to the pub a possibility. Consequently, conclusion two is predicted to be false and conclusion four is predicted to be true. All worlds in B are worlds where I avoid going to the pub, and for none of those worlds is there any world where I do go to the pub that is at least as close to what I want. This makes the proposition that I avoid going to the pub a possibility. Consequently, conclusion one is predicted to be false and conclusion five is predicted to be true. Separating modal bases and ordering sources allows us to make the correct predictions in practical inferences, then.

The separation of modal bases and ordering sources also leads to an insightful analysis of conditional modality. In Kratzer (1978, 1979), I was not able to offer a general recipe for how *if*-clauses modify modals. I had to give meaning rules for each modal separately. In so doing, I missed a generalization about how *if*-clauses restrict modals of various strengths and flavors. In the following section, I will present a sketch of a theory of conditional modality as a first step towards a general account of conditionals.

2.9 Conditionals

In Kratzer (1978, 1979), I argued that many conditionals involve modals in an explicit or implicit way. The logical forms of such conditionals conform to the following rough schema, where an adjoined *if*-clause modifies a sentence that has a modal sitting in its left periphery:

(If . . . .), (necessarily . . . . . .)
(If . . . .), (possibly . . . . . .)
(If . . . .), (probably . . . . . .)
etc.

The matrix clauses of such conditional constructions are overtly or covertly modalized sentences of the kind we have been discussing. The job of *if*-clauses in modalized conditionals is simple: they restrict the modal base of the associated modal in the matrix clause. In a first approximation, this proposal can be spelled out as follows:

Conditional modality

Suppose *u* is an utterance of a sentence of the form (*if* α) β, where u_1 is the part of *u* where the *if*-clause is uttered, u_2 is the part of *u* where β is uttered, and the proposition expressed by α is *p*. We have then:

> (i) u_1 requires one, and only one, modal base and one, and only one, ordering source to be felicitous[41]
> (ii) if *f* is the modal base and g the ordering source for u_1, then f^+ is the modal base and *g* the ordering source for u_2, where for any world *w*, $f^+(w) = f(w) \cup \{p\}$.

There are various possibilities for fleshing out the informal characterization of conditional modality above, and more research is needed to see which possibility is right. Much depends on how exactly conversational backgrounds enter into the interpretation of sentences with modals. Are they arguments of modals that might be syntactically represented? Or are they contextual or evaluation parameters without syntactic expression? As we have seen, there are also reasons to believe that modal bases might only be represented indirectly in grammar via suitable modal anchors. To have at least a preliminary definition to hold on to, here is a slightly more formalized characterization of conditional modality. Modal base dependencies might ultimately have to be derived from anchor dependencies, but this potential inaccuracy does no harm for our current purposes. *If*-clauses do target modal bases in one way or other, if not directly, then at least indirectly:

Conditional modality (alternative definition, still not the last word)

For any conversational backgrounds *f* and *g*:

Correct $$[[if\ \alpha\ \beta]]^{f,g} = [[\beta]]^{f^+,g}, \text{where for all } w \in W, f^+(w)=f(w) \cup \{[[\alpha]]^{f,g}\}.$$

Different kinds of conditionals emerge from different settings for the parameters *f* and *g*. In what follows, consider utterances of sentences realizing the schema *(if* α *)*, *(necessarily* γ*)* and suppose that *p* and *q* are the propositions expressed by α and γ respectively and that *necessarily* expresses necessity as defined in 2.4. Material implication emerges as the special case where the modal base is totally realistic and the ordering source is empty:

Material implication

A material implication is characterized by a totally realistic modal base *f* and an empty ordering source *g*. Sketch of proof:

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[41] Instead of the uniqueness condition, a solution along the lines of Lewis (1979a) or Pinkal (1977, 1979) would be preferable here as well. There is quite a bit of vagueness surrounding conditionals.

Case one: Suppose p is true in w. Then $\cap f^+(w) = \{w\}$. But then q is a necessity in w with respect to f^+ and g iff q is true in w.

Case two: Suppose p is false in w. Then $\cap f^+(w) = \emptyset$. But then q is trivially a necessity in w with respect to f^+ and g.

Strict implication

A strict implication is characterized by an empty modal base f and an empty ordering source g.

Sketch of proof: since $g(w)$ is empty, we have $u \leq_{g(w)} v$ for all worlds u and v in $\cap f^+(w)$. Since $f^+(w) = f(w) \cup \{p\} = \{p\}$, q is a necessity in w with respect to f^+ and g iff q is true in all worlds of $\cap \{p\}$. But that means that p logically implies q.

Counterfactuals

A counterfactual is characterized by an empty modal base f and a totally realistic ordering source g.

David Lewis (personal communication[42]) showed that the above analysis of counterfactuals is equivalent to the one in Kratzer (1981a; reprinted here as chapter 3). Here is the idea behind the proposed account of counterfactuals: all possible worlds where the antecedent p is true are ordered with respect to their being more or less near to what is actually the case in the world under consideration. What is actually the case is a vague concept. There are many ways of uniquely characterizing a world. Put formally, there are many functions g such that $\cap g(w) = \{w\}$ for a given world w. The differences between them cannot possibly make a difference when they are used as modal bases, but they become important when they function as ordering sources. To illustrate, consider conversational backgrounds g_1 and g_2 such that $g_1(w) = \{r, s\}$ and $g_2(w) = \{r \cap s\}$. Even though $\cap g_1(w) = \cap g_2(w)$, $g_1(w)$, and $g_2(w)$ can induce different orderings. Take two worlds u and v such that r is true and s is false in u, and r and s are both false in v. Then $v \leq_{g_2(w)} u$, but not $v \leq_{g_1(w)} u$. Since realistic contexts are unlikely to pick out unique ordering sources for counterfactuals, counterfactuals are predicted to be inherently vague. Since counterfactuals *are* inherently vague, this is the right result (see chapter 3 or Kratzer (1981a) for more discussion). No such vagueness is expected for material implications, where totally realistic conversational backgrounds function as modal bases.

As a last example, I will briefly discuss *deontic conditionals*, which have non-empty ordering sources like counterfactuals. Imagine my uttering the following sentences:

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[42] Now Lewis (1981).

(59) Jedem   Menschen muss Gerechtigkeit widerfahren.
      To every person   must justice   be given.

(60) Wenn jemand   ungerecht behandelt wurde, muss das Unrecht
      If   someone unjustly   treated   was,   must the injustice
      gesühnt   werden.
      amended for get.

(61) Wenn jemand   ungerecht behandelt wurde, muss das Unrecht
      If   someone unjustly   treated   was,   must the injustice
      belohnt   werden.
      rewarded be.

Traditional approaches to conditionals would have to analyze (60) and (61) as modalized material implications and assign them the logical form *necessarily* $(\alpha \rightarrow \beta)$. This leads to trouble.[43] Suppose for convenience that we also follow tradition in relativizing modals to a mere accessibility relation. The proposition expressed by (59) would now be true (in the actual world) iff there is no injustice in any morally accessible world. The proposition expressed by (60) would be true iff any injustice there may be is amended for in all morally accessible worlds. And the proposition expressed by (61) would be true iff any injustice there may be is rewarded in all morally accessible worlds. The problem is that, supposing that the proposition expressed by (59) is true, the propositions expressed by (60) and (61) come out vacuously true. If there is no injustice in any morally accessible world, anything you like is true in morally accessible worlds where there is injustice. The proposed analysis of conditionals avoids this problem. Simplifying somehow, assume that the modal base f is empty and the ordering source g corresponds to what is morally good. The proposition expressed by (59) is now true (in the actual world) iff there is no injustice in any world that comes closest to what is morally good. The proposition expressed by (60) is true iff whatever injustice there is is amended for in all worlds with injustice that come closest to what is morally good. And the proposition expressed by (61) is true iff whatever injustice there is is rewarded in all worlds with injustice that come closest to what is morally good. On this analysis, it is possible for the first two propositions to be true, and the third one to be false. For us, a world where injustice is amended for is not good, since there is no injustice in a good world. But it is still closer to what is good than any world where injustice is

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[43] Hansson (1969), van Fraassen (1972), Lewis (1973a) have detailed discussions of the problem.

rewarded. These truth conditions are in essence those that David Lewis derives in Lewis (1973a).

Whether an analysis of conditionals is correct is often assessed by examining its predictions for inference patterns like "transitivity," "strengthening the antecedent," or "contraposition."[44] The analysis I am proposing predicts that the three inference patterns can't be expected to be valid for all those types of conditionals that involve a non-empty ordering source. In the literature, the failure of these inference patterns is usually discussed in connection with deontic conditionals, probability conditionals and counterfactuals. If we analyze these conditionals in the way suggested here, their specific behavior in inferences is an automatic consequence of the analysis.

2.10 Conclusion

This chapter has tried to elucidate what a speaker has to know to master the semantics of modal constructions in her language. Most importantly, she needs to make the right distinctions between modals by keeping track of three parameters: modal base, ordering source, and modal force. To monitor settings for the three parameters, she needs to pay attention to the lexical properties of individual modals, the syntactic structures they appear in, and the discourse context. She might also have to rely on general cognitive mechanisms for projecting modal bases from suitable modal anchors made available in various places along the verbal projection spine. Modal bases and ordering sources are complicated formal objects and she needs to know how to manipulate them in computing the denotations of complex modal constructions, including degree constructions and conditionals. Finally, she has to come to terms with the fact that realistic utterance contexts rarely provide unique modal bases or ordering sources for modal constructions. She needs to be able to manage a high degree of context-dependency and vagueness, then.

Acknowledgments

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[44] See for example Lewis (1973a), Kratzer (1978, 1979).

INTRODUCING CHAPTER 3

At the end of *What 'Must' and 'Can' Must and Can Mean*, a challenge is raised for theories of conflict resolution within a premise semantics. The way conflicts are resolved in a premise semantics depends on the way the premises are divided up and lumped together. Are there deep and non-trivial principles guiding this process that might be worth exploring? This question has set the agenda for much of my work in premise semantics. In his 1981 paper on ordering semantics, David Lewis described the task then ahead of me as follows:

> We must be selective in the choice of premises...By judicious selection, we can accomplish the same sort of discrimination as would result from unequal treatment of premises. As Kratzer explains..., the outcome depends on the way we lump items of information together in single premises or divide them between several premises. Lumped items stand or fall together, divided items can be given up one at a time. Hence if an item is lumped into several premises, that makes it comparatively hard to give up; whereas if it is confined to a premise of its own, it can be given up without effect on anything else. This lumping and dividing turns out to be surprisingly powerful as a method for discriminating among worlds - so much so that...premise semantics can do anything that ordering semantics can. Formally, there is nothing to choose. Intuitively, the question is whether the same premises that it would seem natural to select are the ones that lump and divide properly; on that question I shall venture no opinion.
>
> (1981: 220–1)

What I embarked on, then, was no longer a purely logical enterprise. As Lewis made clear, there is now the empirical question "whether the same premises that it would seem natural to select are the ones that lump and divide properly." In *Partition and Revision*, the truth of a counterfactual depends on a set of premises that represent the facts of the world of evaluation. Counterfactual assumptions conflict with the facts, and the conflict is resolved by the very same mechanism of making the best of an inconsistent set that was invoked in chapter 1. *Partition and Revision* explores the hypothesis that variability in the way the facts of a world "hang together" might yield a correct and explanatory characterization of the vagueness and indeterminacy