The Numerical Evaluation Of The Levitation Force In A Hydrostatic Bearing With Alternating Poles

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Abstract. The magnetic field and the levitation force produced by a rotor with alternating magnetic poles in a magnetic fluid hydrostatic bearing is numerically evaluated. For the magnetic field computation has used a 3D-FEM program, MagNet 5.0 and for the numerical evaluation of the levitation force the MatLab program has been used.

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1 Introduction

The cylindrical bearing with magnetic liquid represents one of the applications of the second order magnetic levitation.

The bearing with magnetic fluid, alternating magnetic poles on the stator, and nonmagnetic rotor has been presented in [1] and analytically investigated in [2],[3]. A numerical computation of the force that acts on the rotor, using a 2D-FEM program under the hypothesis of a magnetic field with plane meridian symmetry, can be found in [4].

The bearing with magnetic fluid, alternating magnetic poles in rotor and nonmagnetic stator has been analyzed analytically in [5], [6], [7].

In the present paper, the magnetic field and the levitation force are evaluated numerically in a bearing with magnetic fluid, alternating magnetic poles in rotor and nonmagnetic stator. It has been used Magnet 5.0, a 3D program of Infolytica, based on the finite element method (FEM), and the MatLab medium.

The sketch of a bearing with magnetic fluid and alternating magnetic poles placed in rotor is presented in fig.1, where we have denoted:

1 – the rotor (the shaft) of r_1 radius and μ_0 permeability;

2 – the magnetic poles with h_p height, supposed to have a radial permanent magnetization $\mathbf{M_p}$ of zero divergence and $\mu_m=\mu_0\mu_{rm}$ permeability;

3 – magnetic liquid that fills the space between the rotor and stator, considered as a linear medium of permeability $\mu_l=\mu_0\mu_{rl}$;

4 – the nonmagnetic stator of μ_0 permeability.

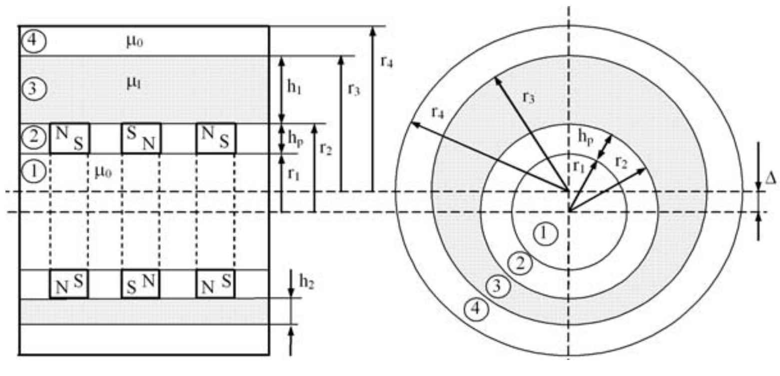

Fig.1 - The sketch of a bearing with alternating poles in rotor

The displacement between the rotor axes and stator axes will be denoted by Δ. Because of the displacement Δ, the magnetic field has not a radial symmetry, and on the rotor acts a magnetic force in the direction to bring the rotor in equilibrium (when it is centered in the bearing). The levitation force depends on the displacement Δ, the magnetic properties of the magnetic liquid, the permanent magnetization of the magnetic poles and on the geometrical design of the bearing. To evaluate numerically the levitation force, we need to know the magnetic field distribution produced by the magnetic poles.

2 Magnetic Field Equations And Finite Element Formulation

The bearing that will be analysed is considered to have a long extension along the z axis, fig.2.

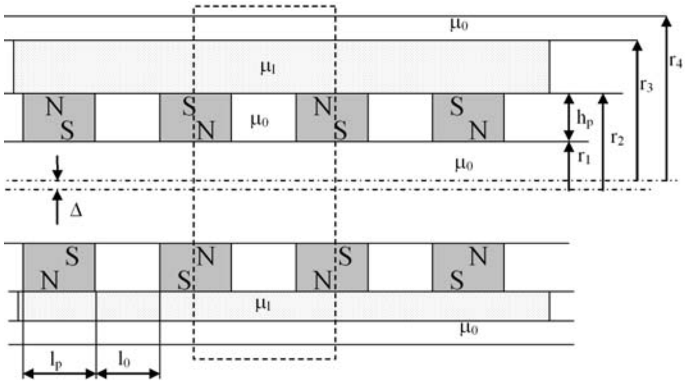

Fig.2 – The bearing heaving a long extension on z axes

The length of the 3D model has been denoted $\lambda/2=l_p+l_0$, a half of wavelength, [6]. It represents the length between two orthogonal planes on the z axes that pass through the middles of two consecutive poles, fig.3. The radial extension of the 3D model, a cylinder of r_{ext} radius, has been adopted so that the magnetic field is considered to vanish outside the model.
The Dirichlet condition A=0 has been considered for the boundary of the 3D model.
In all domains of the 3D model, fig.3, the magnetic potential vector **A** satisfies a Laplace equation:

$$\nabla^2 \mathbf{A} = 0 \tag{1}$$

as $div\mathbf{B} = 0$, $curl\mathbf{H} = 0$, $div\mathbf{M}_\mathbf{p} = 0$ and, $\mathbf{B} = \mu\mathbf{H} + \mu_0\mathbf{M}_\mathbf{p}$ in the magnetic poles, $\mathbf{B} = \mu_l\mathbf{H}$ in the magnetic liquid and $\mathbf{B} = \mu_0\mathbf{H}$ in the shaft, stator and outside the bearing.

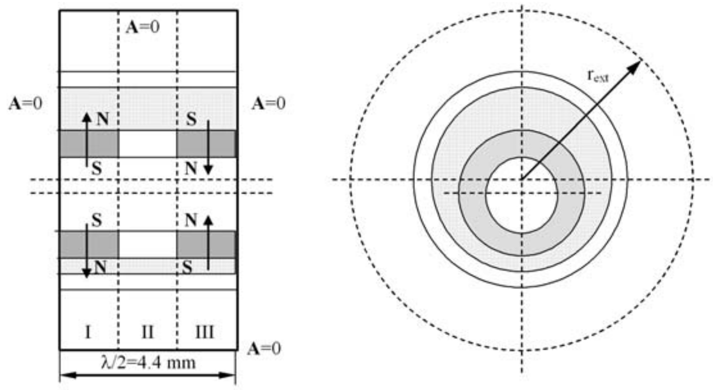

Fig.3 – The geometrical model used in 3D-FEM analyzes

The 3D-FEM program MagNet 5.0 of the Infolytica was used to solve numerically the field problem. The geometry and the finite elements in a orthogonal plane on the z axis of the model, generated by the MagNet is presented in fig.4.

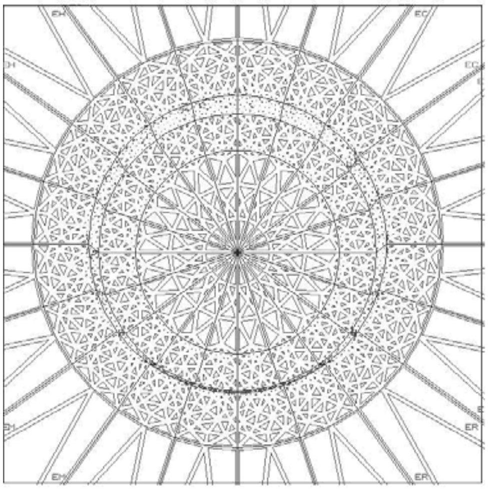

Fig.4 – The finite elements in a plane orthogonal on the z axis

To generate the 3D model, the extrusion principle has been used.

The first order tetrahedrons were used in all the regions of the model except the region of the liquid where second order tetrahedrons have been used. In the analysed cases there has been used a number of 25000-30000 nodes and 130000-150000 tetrahedrons.

3 The Numerical Force Evaluation

The force that acts on the rotor is, [8], [6]:

$$\mathbf{F} = -\oint_{\Sigma} \left(\int_0^H \mathbf{B} \cdot \mathbf{dH} \right) \mathbf{dS} + \oint_{\Sigma} \mathbf{H}(\mathbf{B} \cdot \mathbf{n})\, dS \tag{2}$$

where Σ is the cylindrical interface surface between the magnetic liquid and rotor and $\mathbf{dS} = dS\,\mathbf{n}$ is the surface unit vector oriented outwards the rotor, fig.5.

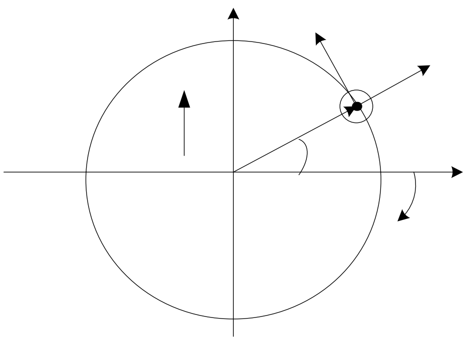

Fig.5 – The cylindrical system of coordinates

Due to the symmetry of the magnetic field, the integrals on the two bases of the cylinder has been neglected. Considering the magnetic liquid as a linear medium of $\mu_l = \mu_0\, \mu_{rl}$ permeability, we have: $\int_0^H \mathbf{B} \cdot \mathbf{dH} = \mu_l \int_0^H \mathbf{H} \cdot \mathbf{dH} = \frac{1}{2} \mu_l H^2$, and the expression (2) becomes:

$$\mathbf{F} = -\frac{1}{2} \mu_l \int_{S_l} H^2 \mathbf{dS} + \mu_l \int_{S_l} \mathbf{H}(\mathbf{H} \cdot \mathbf{n})\, dS \tag{3}$$

where S_l is the surface of the cylinder of r_2 radius and $\lambda/2$ height taken in the points of the magnetic liquid.
Using the cylindrical coordinates (r, θ, z), fig.5, we have successively:

$\mathbf{H} = H_r \mathbf{u_r} + H_\theta \mathbf{u}_\theta + H_z \mathbf{u_z}$, $\mathbf{dS} = dS\,\mathbf{n} = dS\,\mathbf{u_r}$, $H = \sqrt{H_r^2 + H_\theta^2 + H_z^2}$, and

$\mathbf{H}(\mathbf{H}\cdot\mathbf{n}) = (H_r \mathbf{u_r} + H_\theta \mathbf{u}_\theta + H_z \mathbf{u_z})H_r = H_r^2 \mathbf{u_r} + H_\theta H_r \mathbf{u}_\theta + H_z H_r \mathbf{u_z}$.

The force expression (3) becomes:

$$\mathbf{F} = \mu_l \int_{S_l} \left(H_r^2 - \frac{H^2}{2} \right) dS\,\mathbf{u_r} + \mu_l \int_{S_l} H_r H_\theta \, dS\,\mathbf{u}_\theta + \mu_l \int_{S_l} H_r H_z \, dS\,\mathbf{u}_z \tag{4}$$

As the global levitation force that acts on the rotor is different of zero only in the direction of the maximum displacement, the levitation force is:

$$F_m = F_r \sin\theta + F_\theta \cos\theta = \mu_l \int_{S_l} \left[\left(H_r^2 - \frac{H^2}{2} \right) \sin\theta + H_r H_\theta \cos\theta \right] dS \tag{5}$$

The intersection between the 3D mesh of tetrahedron and the cylindrical slice of r_2 radius and $\lambda/2$ height generates a 2D mash of triangles as finite elements.

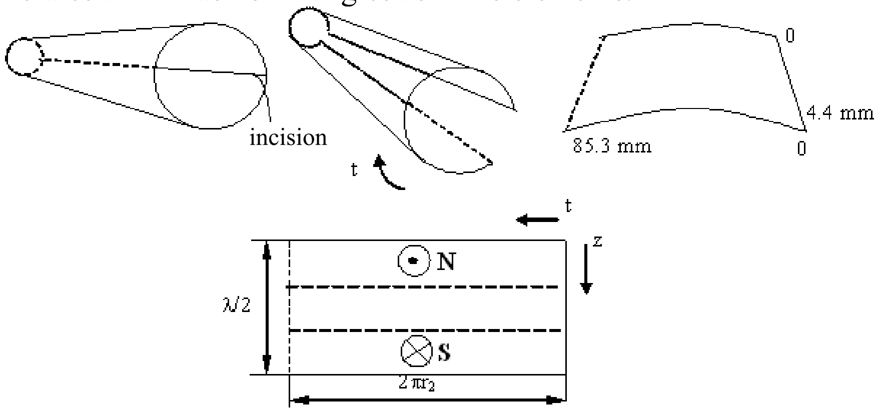

Fig.6 – How the cylinder "unwraps" for a 2D computation

For an incision made at θ=0, MagNet unwrapped the cylinder of r_2 radius, peeling it open from the incision line in the manner illustrated in fig.6.
The resulting rectangular surface has about 20000 elements (triangles). As the elements are very small, the magnetic field and its components has been considered constant on each element, and (5) becomes:

$$F_m = \mu_l \sum_i \left[\left(H_{r_i}^2 - \frac{H_i^2}{2} \right) \sin\theta_i + H_{r_i} H_{\theta_i} \cos\theta_i \right] \Delta S_i \tag{6}$$

The levitation force that acts on the rotor for the analysed model has been computed numerically, using (6) and MatLab medium.
The levitation force that acts on an unit length of rotor, is:

$$F_m^* = \frac{F_m}{\frac{\lambda}{2}} = \frac{2F_m}{\lambda} \tag{7}$$

The nodes coordinates and the magnetic field components generated by MagNet (as text files) has been used in numerical computation of the levitation force.

4 The Numerical Results

The values of the geometrical quantities used in the numerical computation are: r1=10mm, r2=13.57mm, δ=1mm, r3=14.57mm, λ=8.8mm, lp=2.93mm, l0=1.466mm and rext=100mm, where δ represents the maximum displacement.

The results of the numerical simulation of the magnetic field inside the bearing are presented in a graphical form. They have been generated by the MagNet 3D-FEM program.

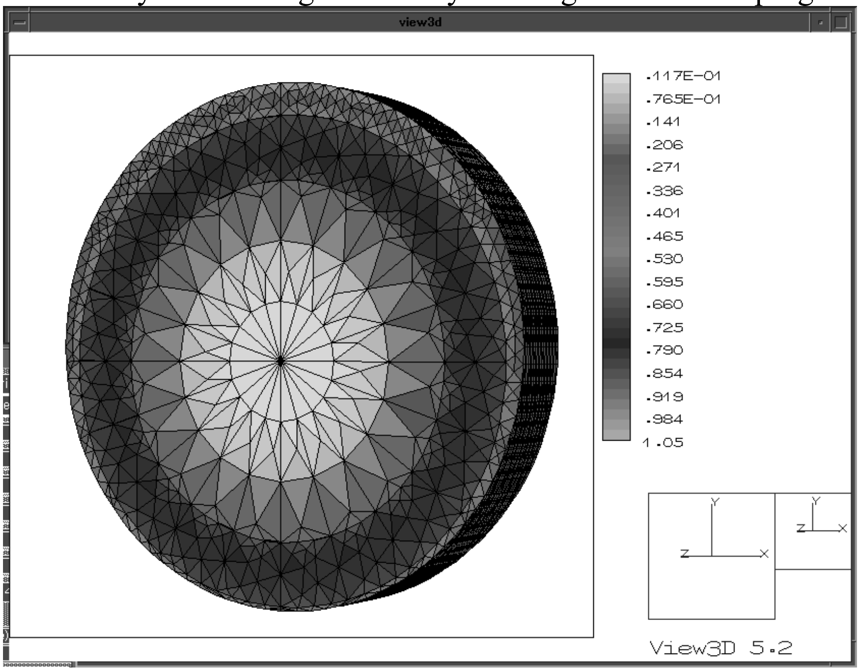

Fig.7 – The magnetic flux density distribution in the rotor and magnetic liquid

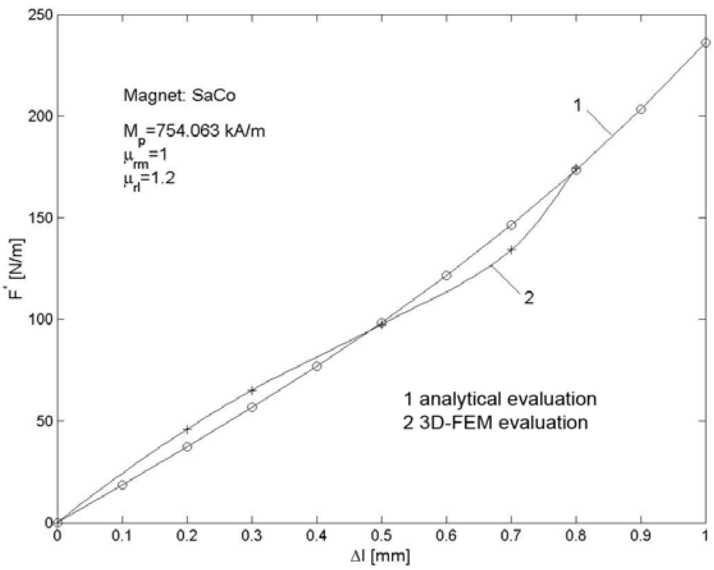

Fig.8 – The levitation force versus the displacement Δ

Fig.7 presents the magnetic flux density distribution (in Tesla) in the rotor with magnetic poles and the magnetic liquid. There has been used a Sa-Co permanent magnet with M_p=754 KA/m, μ_{rm}=1 and a liquid with μ_{rl}=1.2 for a displacement Δ=0.8 mm. The higher values of the flux

density in the liquid takes place where the liquid layer is the thinnest (the bottom part of the liquid).
In fig.8 the levitation force that acts on the unit length of rotor has been rpesented versus the displacement Δ for a Sa-Co permanent magnet with M_p=754 KA/m, μ_{rm}=1 and a liquid with μ_{rl}=1.2.
The curve denoted by (1) has been obtained using a analytical approximate expression for the levitation force, [5], [6], and the curve (2) used the numerical 3D-FEM computation. The two curves match quite well.

5 Conclusion

In [6], [7] there has been proposed an analytical expression for the levitation force that acts on the rotor with magnetic poles of a hydrostatic bearing with magnetic liquid. The magnetic liquid has been considered as a linear medium and for the magnetic field has been adopted a plan-parallel model. The relation for the second order levitation has been used for the force evaluation.
The numerical computation developed in present paper has adopted a 3D model for the magnetic field and the levitation force has been calculated using an expression derived from the Maxwell tensor.
The results for the levitation force calculated in present paper match quite well with the corresponding results obtained by using the analytical expression. As a consequence, the analytical expression of the levitation force could be used to analyse and design cylindrical bearings with alternating poles placed in rotor and magnetic liquid.
The numerical computation developed in present paper is applied also in nonlinear cases (considering the magnetic liquid as a nonlinear medium), where the analytical solution is no more possible.

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