CHAPTER 11

THE OBSERVED JOINT DISTRIBUTION OF PERIODS AND HEIGHTS OF SEA WAVES

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ABSTRACT

Analysis is made of 89 records of surface waves for the joint distribution of the heights and periods of zero up-crossing waves. Records are classified into five groups according to the rank of the correlation coefficient between individual wave heights and periods, and the data of the joint distribution are presented for five groups separately. In comparison with the present data, the theory of Longuet-Higgins for a narrow band spectrum can describe the joint distribution in its upper portion with high waves when the spectral width parameter is fitted to the marginal distribution for wave periods, although the joint distribution in the lower portion with low waves shows deviation from the theory. Another theory by a group of C.N.E.X.O. based on the distribution of positive maxima can describe the general pattern of observed distribution better than the former theory, but the agreement remains qualitative. The present data also suggests that the joint distribution of wave periods and heights may be parameterized with the correlation coefficient between wave heights and periods.

INTRODUCTION

In the analysis of sea waves, the relationships between characteristic wave periods such as the highest, significant, and mean wave periods are often discussed, as there exists a growing demand for such information in the design of coastal and offshore structures. The problem is one aspect of the joint distribution of wave heights and periods. When the problems of irregular wave runups, overtopping, and wave forces are analyzed by the wave-by-wave method, the information of the joint distribution becomes vital for solving these problems.

The theory of the joint distribution of wave heights and periods was given by Longuet-Higgins [1] in 1975 in a closed form under the assumption of a narrow band spectrum. It was recapitulation of his previous work [2] on the statistical properties of random, moving surface. The theoretical distribution is characterized by having the axis of symmetry at $T = \bar{T}$ as demonstrated in Fig. 1, while sea waves generally demonstrate asymmetric distribution with respect to wave periods as exhibited by Chakrabarti and Cooley [3]. A measure of asymmetry is the correlation coefficient between wave heights and periods, which sometimes amounts to more than 0.7 among sea waves, while the theoretical distribution of Longuet-Higgins yields zero correlation.

The asymmetric pattern of the joint distribution of wave heights and periods was incorporated in the theory by Ahran, Cavanié, Ezraty,

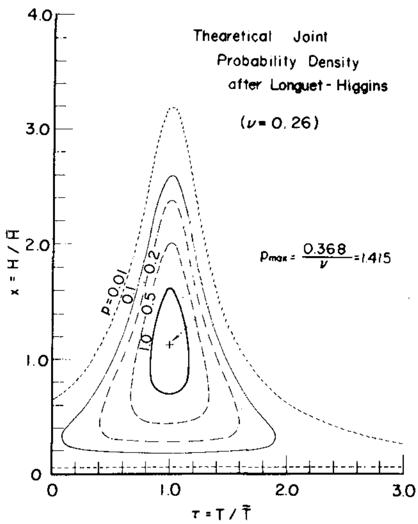


Fig. 1 Joint Probability Density by Longuet-Higgins' Theory (ν = 0.26)

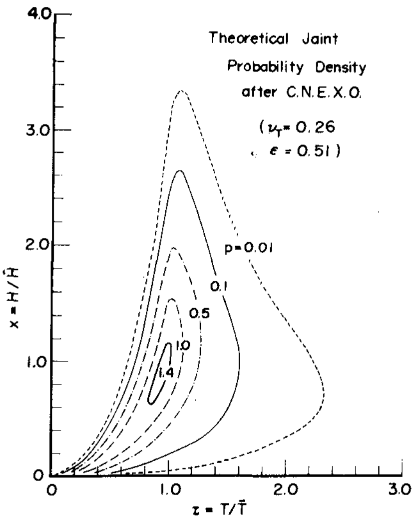


Fig. 2 Joint Probability Density by the Theory of C.N.E.X.O. (ε = 0.51 and ν = 0.26)

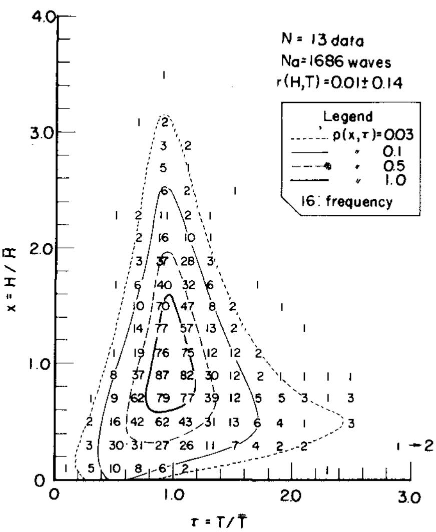


Fig. 3 Example of Observed Joint Probability Density for a Group of Wave Data with r(H,T) = -0.25 ∿ 0.19

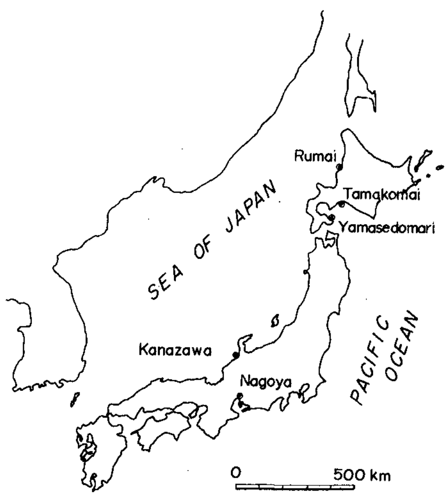


Fig. 4 Location Map of Wave Observation Stations

and Laurent, i.e., a group of C.N.E.X.O. [4,5,6]. They formulated the joint distribution of the amplitudes and quasi-periods of positive maxima on the basis of Cartwright and Longuet-Higgins [7]. The quasi-periods of positive maxima were estimated from the second derivatives of surface elevations at the maxima by fitting a sinusoidal wave profile. They further presumed that it could be applicable to the joint distribution of the heights and periods of zero up-crossing waves by replacing the amplitude of positive maximum with one-half wave height and the quasi-period of positive maximum with the zero crossing wave period. The theory was found to agree with the ocean wave data except for the region of large nondimensional wave periods where the theory tends to overestimate the probability density.

Sea waves are known to exhibit large variability in their statistical parameters such as the correlation coefficient and spectral width parameters. It is readily conceived that the characterisitics of the joint distribution of wave heights and periods could be examined in detail by making a rankwise analysis according to the magnitude of statistical parameters. Following this concept, reanalysis of the available surface wave records was made for the joint distribution of wave heights and periods after classifying the data according to the rank of the correlation coefficient of individual wave heights and periods. The analysis has yielded several findings on the characteristics of the joint distribution as it will be seen in the subsequent chapters.

Note: Throughout the present paper a wave is defined by the zero up-crossing method, and no reference is made of a wave defined by the crest-to-trough method except for the number of maxima in a wave record.

FORMULAE OF JOINT PROBABILITY DENSITY

The joint probability density of wave heights and periods by the theory of Longuet-Higgins can be written as in the following form after normalizing wave heights and periods with their mean values, or $\bar{H}$ and $\bar{T}$, respectively:

$$p(x,\tau) = \frac{\pi x^2}{4\nu} \exp\{-\frac{\pi}{4}x^2[1+\frac{(\tau-1)^2}{\nu^2}]\}, \quad (1)$$

where,

$$x = H/\bar{H} \quad \text{and} \quad \tau = T/\bar{T}.$$

The parameter ν is a measure of spectral bandwidth defined by

$$\nu = [m_0 m_2/m_1^2 - 1]^{1/2} \equiv \nu_S , \quad (2)$$

where,

$$m_n = \int_0^{\infty} f^n S(f)\, df . \quad (3)$$

An example of the joint probability density is shown in Fig. 1 for the case of $\nu = 0.26$.

The marginal distribution of wave heights is shown to be the Rayleighan, by integrating Eq. 1 with respect to τ from $-\infty$ to ∞.

The marginal distribution of wave periods is similarly derived as

$$p(\tau) = \int_0^\infty p(x,\tau)\,dx = \frac{\nu^2}{2[\nu^2 + (\tau-1)^2]^{3/2}} \;. \qquad (4)$$

The distribution has a tail in the region of $\tau < 0$ which is unrealistic, but the assumption of narrow band spectrum with the condition $\nu << 1$ yields the probability of $\tau < 0$ practically nil.

As the distribution is symmetric with respect to $\tau = 1$, the mean of heightwise ranked period remains at $\bar{\tau}_H = 1$. The standard deviation of heightwise ranked period is calculated from the conditional joint probability density as

$$\sigma_H(\tau) = \frac{\nu}{\sqrt{\pi/2}\,x} \;. \qquad (5)$$

Because this diverges as x goes to zero, the overall standard deviation of wave periods cannot be defined. As an alternative measure of the dispersion of wave periods, Longuet-Higgins [1] introduced the interquartile range of the marginal distribution of nondimensional wave periods and correlated it with the spectral width parameter ν. Althernatively, ν can be estimated from the interquartile range of nondimensional wave period as

$$\nu = \frac{\sqrt{3}}{2}\,IQR(\tau) \equiv \nu_T \;. \qquad (6)$$

In order to avoid confusion, ν_T estimated by Eq. 6 is henceforth called the period bandwidth parameter.

The applicability of Longuet-Higgins' theory to waves with a narrow band spectrum can be proved, for example, with the data of numerically simulated random wave profiles [8]. The upper limit of ν may be taken at about 0.1 if it is to be estimated from the spectrum [9]. It will be later shown however that the theory can be partially applied to sea waves with broad band spectra as well if ν is estimated from the bandwidth of period distribution by Eq. 6.

The joint probability density of wave heights and periods by the group of C.N.E.X.O. has the following form:

$$p(\xi,\zeta) = \frac{\alpha^3\xi^2}{4\sqrt{2\pi}\,\varepsilon(1-\varepsilon^2)\zeta^5}\exp\{-\frac{\xi^2}{8\varepsilon^2\zeta^4}[(\zeta^2-\alpha^2)^2 + \alpha^4 a^2]\} \;, \qquad (7)$$

where,

$$\xi = H/\sqrt{m_0}, \quad \zeta = \bar{\zeta}\tau = \bar{\zeta}T/\bar{T}, \quad \alpha = \frac{1}{2}(1+\sqrt{1-\varepsilon^2}), \quad \text{and} \quad a = \varepsilon/\sqrt{1-\varepsilon^2}. \qquad (8)$$

Though Battjes [10] recommends to introduce the relationship between the mean interval of positive maxima and that of zero up-crossings into the term of ζ so as to have theoretical consistency, the original form is employed in the subsequent calculation as it produces fairer agreement with observation data.

The parameter ε is a measure of spectral bandwidth introduced by Cartwright and Longuet-Higgins [7] as

$$\varepsilon = [1 - m_2{}^2/m_0m_4]^{1/2} \equiv \varepsilon_S \;. \qquad (9)$$

This parameter is very sensitive to the Nyquist frequency of spectral analysis relative to the frequency of spectral peak when applied to sea waves [11,12]. The group of C.N.E.X.O. recommends the use of the following parameter for ε :

$$\varepsilon = [1 - N_0{}^2/N_c{}^2]^{1/2} \equiv \varepsilon_T , \qquad (10)$$

in which N_0 and N_c denote the numbers of zero up-crossings and maxima within a wave record, respectively. Though ε_S and ε_T should give the same value from theoretical point of view, sea waves usually produce ε_T less than ε_S without a definite interrelation. In this sense, ε_T estimated by Eq. 10 should be treated separately from ε_S estimated by Eq. 9, and it is henceforth called the apparent spectral width parameter.

The marginal distribution of wave heights derived from Eq. 7 is nearly the Rayleighan when ε is not large. The marginal distribution of wave periods is obtained as

$$p(\zeta) = \frac{\alpha^3 a^2 \zeta}{[(\zeta^2-\alpha^2)^2 + \alpha^4 a^2]^{3/2}} \cdot \qquad (11)$$

The mean value of ζ which is estimated by numerically integrating Eq. 11 remains close to 1.0 for the range of $0 < \varepsilon < 0.95$. The period bandwidth parameter ν_T can also be estimated numerically from Eq. 11. It is interesting to note that there exists an approximate relation of

$$\nu_T \fallingdotseq 0.5\varepsilon_T + 0.023\varepsilon_T{}^2 \qquad : 0 < \varepsilon_T < 0.85, \qquad (12)$$

which nealy coincides the relation of $\nu = \frac{1}{2}\varepsilon$ derived by Longuet-Higgins [1] for a very narrow band spectrum.

After evaluating $\bar{\xi}$ and $\bar{\zeta}$, the joint probability density can be expressed in terms of the mean wave height and period, $\bar{H}$ and $\bar{T}$. An example of the joint probability density is shown in Fig. 2, which corresponds to the parameter of $\varepsilon_T = 0.51$ and $\nu_T = 0.26$. Asymmetric pattern of the probability density curves is observable even at this level of apparent spectral width parameter.

For comparison with these theoretical joint probability density, a result of compilation of correlation tables of observed sea waves is exhibited in Fig. 3, which represents the data of a group of sea waves with the correlation coefficient r(H,T) between individual wave heights and periods being in the rank of -0.25 to 0.19; the definition of r(H,T) is as follows:

$$r(H,T) = \frac{1}{\sigma_H \sigma_T N_0} \sum_{i=1}^{N_0} (H_i - \bar{H})(T_i - \bar{T}) \qquad (13)$$

where σ_H and σ_T denotes the standard deviations of wave heights and periods, respectively, and N_0 is the number of zero up-crossing waves. The groups of 13 wave records in this rank of correlation coefficient had the mean of $\bar{\nu}_T = 0.26$ for the period bandwidth parameter. The observed density is close to the theory of Longuet-Higgins rather than that of C.N.E.X.O. except for the range of $x = H/\bar{H} < 0.4$.

PRESENTATION OF SEA WAVE DATA

In order to investigate the applicability of the above theories to sea waves, an examination of various wave records was undertaken. The data were taken from the same source with the author's previous analysis of statistical properties [11,12]. Among 171 records analyzed, 89 records were selected under the conditions that each record exhibits a clearly defined single spectral peak and the significant wave height does not exceed about 0.4 times the water depth. The latter condition was introduced to exclude the influence of random wave breaking upon the statistical properties of observed wave records [13]. In total, 10,584 zero up-crossing waves were counted in 89 records.

The stations of wave observations and other data are listed in Table 1 and their locations are shown in Fig. 4. The data at Nagoya were recorded inside and outside a long mole, and they represent deep-water wind waves generated in a short fetch. The other data were recorded at coastal stations, and they mostly represent shallow water waves generated in medium to long fetches; some of them are wind waves and others are young swell.

All the data were recorded on the charts of servo-balanced type pen-writing recorders and were digitized with the aid of a mannually operating X-Y digitizer with a magnifier. The digitized wave records were analyzed for their statistical properties as well as spectral characteristics by a computer program. Examples of wave spectra are shown in Fig. 5, where the spectrum is normalized by means of the frequency at the spectral peak, f_p, and the zeroth moment of spectrum, m_0. The waves observed at Nagoya Port usually show sharp peaks and a few humps at high frequency range, while most of coastal waves have the spectral slope at high frequency range milder than -5. In general, however, the Pierson-Moskowitz type or the Bretschneider-Mitsuyasu type spectral form porvides a fair approximation to the spectra of observed waves.

The analysis of the joint distribution of observed wave heights and periods was proceeded first by classifying 89 records into five groups according to the magnitudes of the correlation coefficient $r(H,T)$, because $r(H,T)$ is considered to represent the pattern of joint distribution best. Table 2 lists the numbers of wave data in five groups of $r(H,T)$ as well as the numbers of wave data in ranked groups of ν_T. The data of Nagoya Port is characterized with low values of $r(H,T)$ and ν_T though its cause is not clarified. There is a possibility that short fetched wind waves may exhibit such characteristics as there is such indication in the data compiled by Bretschneider [13]. The parameters of ν_S and ε_T as listed in Table 1 do not show marked difference between Nagoya data and coastal wave data. Various statistical properties of the data analyzed are listed in groupwise in Table 3 with the mean values and the standard deviations. The correlation coefficient $r_{13}(H,T)$ is the one calculated for the highest one-third waves, which is a measure of correlation among high waves, and Q_p is the spectral peakedness parameter defined as [8]

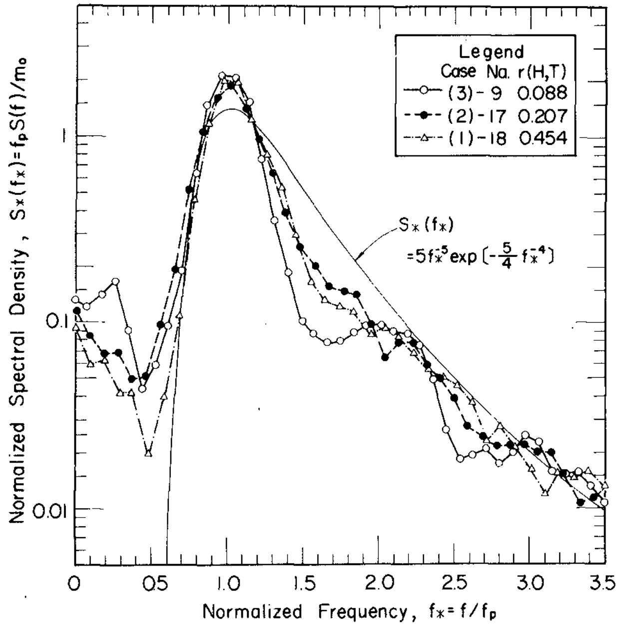
$$Q_p = \frac{2}{m_0} \int_0^\infty f\, S^2(f)\, df \,. \tag{14}$$

Table 1 Description of Wave Data

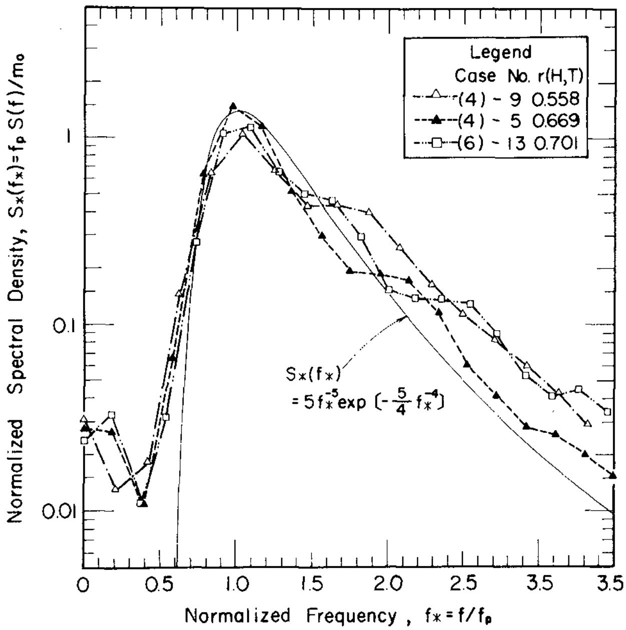
Station	Nos. of Data	Water Depth (m)	Fetch	Range of $H_{1/3}$ (m)	Range of $T_{1/3}$ (s)	ν_S	ε_T	Wave Recorder	Sampling Time, Δt (s)
Nagoya	42	10	short	0.20∿1.22	2.1 ∿ 3.7	0.22∿0.63	0.47∿0.77	capacitance gage	0.25
Rumoi	24	11	long	2.2 ∿ 4.5	5.9 ∿ 9.6	0.28∿0.66	0.65∿0.90	step-resist. gage	0.5
Yamasedomari	6	13	long	1.9 ∿ 4.9	7.0∿13.7	0.54∿0.89	0.79∿0.95	step-resist. gage	0.5
Tomakomai	3 5	11 14	long long	2.9 ∿ 4.1 2.4 ∿ 2.8	6.9 ∿ 8.0 6.7 ∿ 7.5	0.46∿0.53 0.34∿0.69	0.64∿0.65 0.63∿0.77	step-resist. gage step-resist. gage	1.0 0.5
Kanazawa	9	20	long	1.0 ∿ 6.8	4.7 ∿12.4	0.34∿0.60	0.47∿0.82	invt. echo-sounder	1.0

Table 2 Numbers of Wave Data in Respective Ranks of r(H,T) and ν_T

Station	Rank of r(H,T)					Total	Rank of ν_T					Total
	-0.25 ∿ 0.19	0.20 ∿ 0.39	0.40 ∿ 0.59	0.60 ∿ 0.69	0.70 ∿ 0.81		0.19 ∿ 0.29	0.30 ∿ 0.39	0.40 ∿ 0.49	0.50 ∿ 0.59	0.60 ∿ 0.67	
Nagoya	13	17	12	-	-	42	25	14	2	-	1	42
Rumoi	-	-	7	15	2	24	-	2	11	9	2	24
Yamasedomari	-	-	2	2	2	6	-	1	1	2	2	6
Tomakomai	-	-	1	6	1	8	-	1	5	1	1	8
Kanazawa	-	1	1	4	3	9	2	1	2	3	1	9
Total	13	18	23	27	8	89	27	19	21	15	7	89



(1) Wave data of Nagyoya Port



(2) Wave data of coastal stations

Fig. 5 Examples of Spectra of Observed Waves

Table 3 Compilation of Statistical Data in Five Ranks of r(H,T)

Items	Rank of r(H,T)					Whole Data
	-0.25 ∿ 0.19	0.20 ∿ 0.39	0.40 ∿ 0.59	0.60 ∿ 0.69	0.70 ∿ 0.81	
Nos. of Records	13	18	23	27	8	89
Nos. of Waves, N_o	129.7(21.1)	148.6(14.6)	112.7(31.6)	105.5(39.4)	97.6(31.7)	118.9(35.0)
[Wave Height Ratio]						
$H_{max}/H_{1/3}$	1.702(0.210)	1.752(0.272)	1.693(0.256)	1.597(0.164)	1.536(0.148)	1.663(0.231)
$H_{1/10}/H_{1/3}$	1.281(0.063)	1.276(0.037)	1.285(0.039)	1.284(0.051)	1.267(0.039)	1.281(0.046)
$H_{1/3}/\bar{H}$	1.610(0.058)	1.570(0.042)	1.594(0.074)	1.619(0.053)	1.614(0.057)	1.601(0.061)
[Wave Period Ratio]						
$T_{max}/T_{1/3}$	0.943(0.131)	0.963(0.083)	1.025(0.112)	1.018(0.105)	1.060(0.105)	1.002(0.113)
$T_{1/10}/T_{1/3}$	0.994(0.022)	0.999(0.028)	1.012(0.043)	1.023(0.041)	1.029(0.030)	1.012(0.038)
$T_{1/3}/\bar{T}$	0.983(0.047)	1.061(0.025)	1.156(0.047)	1.252(0.041)	1.331(0.042)	1.156(0.116)
$T_{1/3}/T_p$	0.966(0.035)	0.950(0.020)	0.915(0.062)	0.901(0.046)	0.890(0.045)	0.923(0.053)
$\bar{T}/T_p$	0.983(0.061)	0.895(0.030)	0.794(0.070)	0.720(0.044)	0.669(0.035)	0.808(0.116)
[Correl. Coef.]						
r(H,T)	0.014(0.137)	0.295(0.064)	0.511(0.048)	0.650(0.034)	0.730(0.033)	0.457(0.239)
$r_{13}(H,T)$	-0.068(0.134)	-0.032(0.104)	0.010(0.163)	0.078(0.160)	0.179(0.098)	0.026(0.159)
[Spectr. Parameter]						
ε_S	0.832(0.035)	0.803(0.059)	0.865(0.044)	0.878(0.042)	0.880(0.042)	0.853(0.054)
ε_T	0.584(0.074)	0.570(0.054)	0.716(0.100)	0.747(0.075)	0.781(0.078)	0.683(0.113)
ν_S	0.515(0.061)	0.487(0.059)	0.489(0.108)	0.562(0.081)	0.657(0.109)	0.526(0.102)
ν_T	0.268(0.048)	0.287(0.028)	0.384(0.086)	0.500(0.062)	0.582(0.054)	0.400(0.123)
Q_p	2.412(0.698)	2.369(0.356)	2.116(0.566)	1.740(0.228)	1.758(0.210)	2.064(0.526)
$\Delta t/T_p$	0.092(0.014)	0.105(0.026)	0.069(0.019)	0.060(0.015)	0.063(0.015)	0.076(0.026)

RESULTS OF DATA ANALYSIS

Marginal Distribution of Wave Heights

As reported by many researchers, the marginal distribution of wave heights does not show any significant deviation from the Rayleighan. An indication is the mean values of three height ratios listed in Table 3, which are close to the theoretical values of the Rayleigh distribution. The chi-square test for the goodness-of-fitness to the Rayleighan was made for the present data with 14 classes of wave heights. The probability that the total data has come from the population of the Rayleigh distribution is calculated as about 0.30, and the probability that the data in the rank of $r(H,T) = 0.70 \sim 0.81$ has come from the Rayleigh distribution is about 0.10. Thus the hypothesis of the Rayleigh distribution cannot be discarded for the present data.

Marginal Distribution of Wave Periods

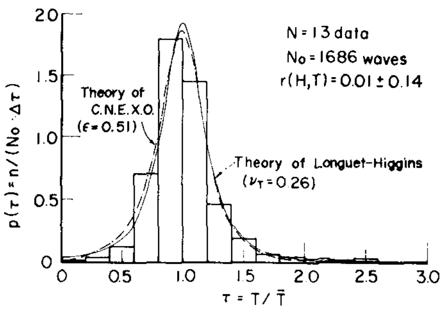
Figure 6 shows the marginal distribution of observed wave periods normalized by the mean wave period of respective wave records. For the data with r(H,T) less than 0.4, both of the theoretical distributions of Eqs. 4 and 7 provide good approximation so long as the period bandwidth parameter obtained from the data is employed in theoretical estimation. The two theories do not yield much difference for small values of ν_T. As ν_T increases, the observed marginal distribution deviates gradually from the theoretical ones, which become unapplicable for $r(H,T) \gtrsim 0.6$. It should be noted that the maximum value of r(H,T) and ν_T predicted by the theory of C.N.E.X.O. is about 0.69 and 0.55, respectively, both of which correspond to the case of $\varepsilon = 0.99$.

Joint Distribution of Wave Heights and Periods

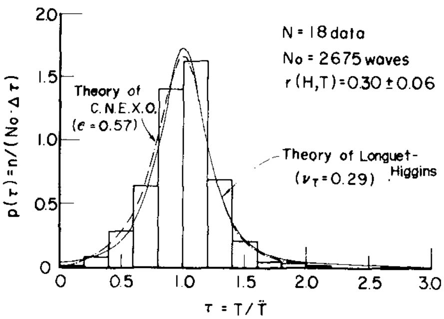
The joint distribution of wave heights and periods were analyzed with the rank of $\Delta H/\bar{H} = \Delta T/\bar{T} = 0.2$ after having been normalized with $\bar{H}$ and $\bar{T}$. The correlation table with the curves of probability density for the group of $r(H,T) = -0.25$ to 0.19 has been presented as Fig. 3 for comparison with theoretical ones. The correlation tables for the other four groups of r(H,T) are shown in Fig. 7. As r(H,T) increases, the asymmetry of joint distribution becomes conspicuous and the position of maximum probability density moves toward the origin. The change of the pattern is qualitatively in accordance with the theory of C.N.E.X.O.

Characteristics of Heightwise Ranked Wave Periods

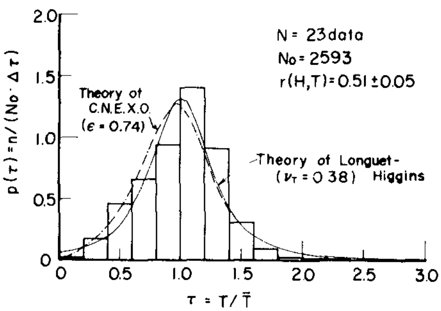
One feature of Fig. 3 and Fig. 7 is that the center of wave periods at each level of wave height does not vary in the upper portion of correlation table and the distribution is symmetric there. This feature is clearly shown in Fig. 8, where the mean of heightwise ranked wave periods is plotted against the wave height level. The ranked mean $\bar{T}_H$ of high waves relative $\bar{T}$ shifts toward large values as r(H,T) increases, but it holds a common value among high waves as seen in Fig. 8(a). The shift of $\bar{T}_H$ is an apparent phenomenon, however, as demonstrated in Fig. 8(b), where the ranked mean of wave period is normalized with the period corresponding to the spectral peak, T_p. Figure 8(b) shows that $\bar{T}_H$ of high waves remains in the range of $(0.87 \sim 0.98)T_p$ irrespective of r(H,T). The mean of wave periods with $H > 1.4\bar{H}$, for example, is calculated to be $(0.91 \sim 0.97)T_p$. The figure also indicates that the increase



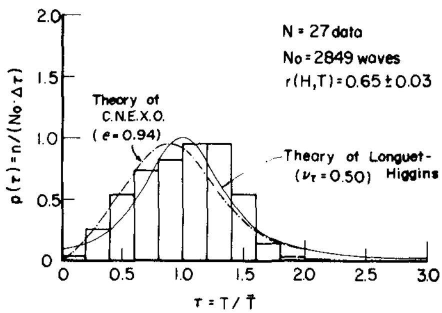
(1) r(H,T) = -0.25 ∿ 0.19



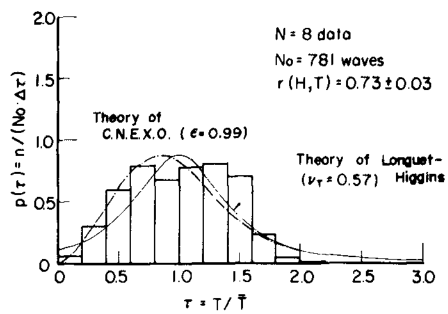
(2) r(H,T) = 0.20 ∿ 0.39



(3) r(H,T) = 0.40 ∿ 0.59

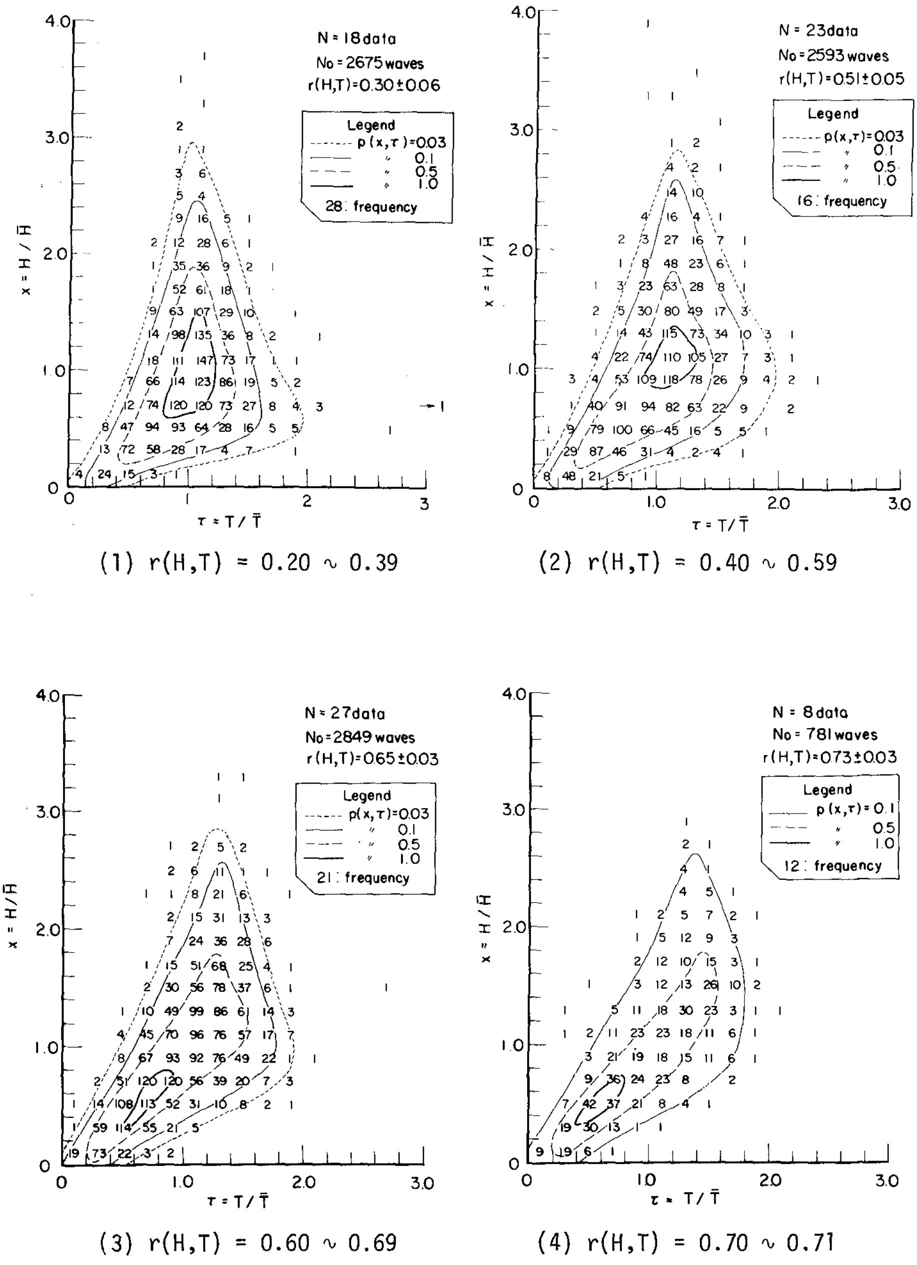


(4) r(H,T) = 0.60 ∿ 0.69



(5) r(H,T) = 0.70 ∿ 0.81

Fig. 6 Marginal Distribution of Wave Periods Ranked in Five Groups of Correlation Coefficient, r(H,T)



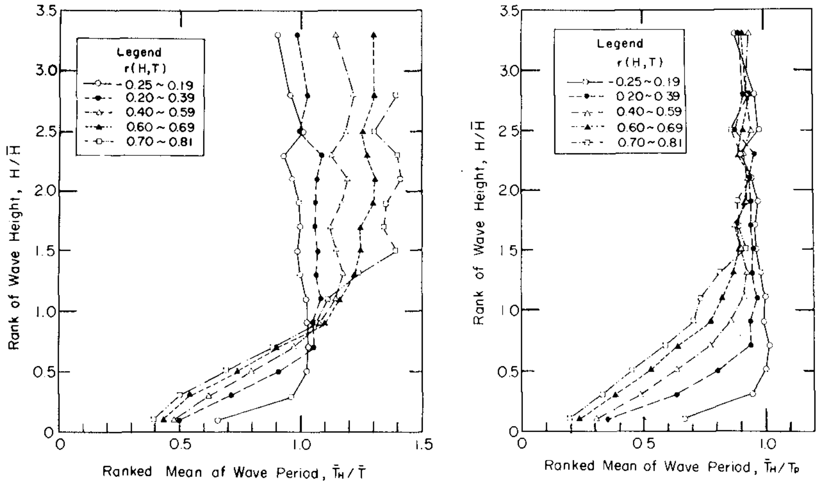
(1) r(H,T) = 0.20 ∿ 0.39

(2) r(H,T) = 0.40 ∿ 0.59

(3) r(H,T) = 0.60 ∿ 0.69

(4) r(H,T) = 0.70 ∿ 0.71

Fig. 7 Joint Distribution of Observed Wave Heights and Periods Grouped in the Rank of Correlation Coefficient, r(H,T)



(a) reference period of $\bar{T}$ (b) reference period of T_p

Fig. 8 Mean Period of Heightwise Ranked Waves

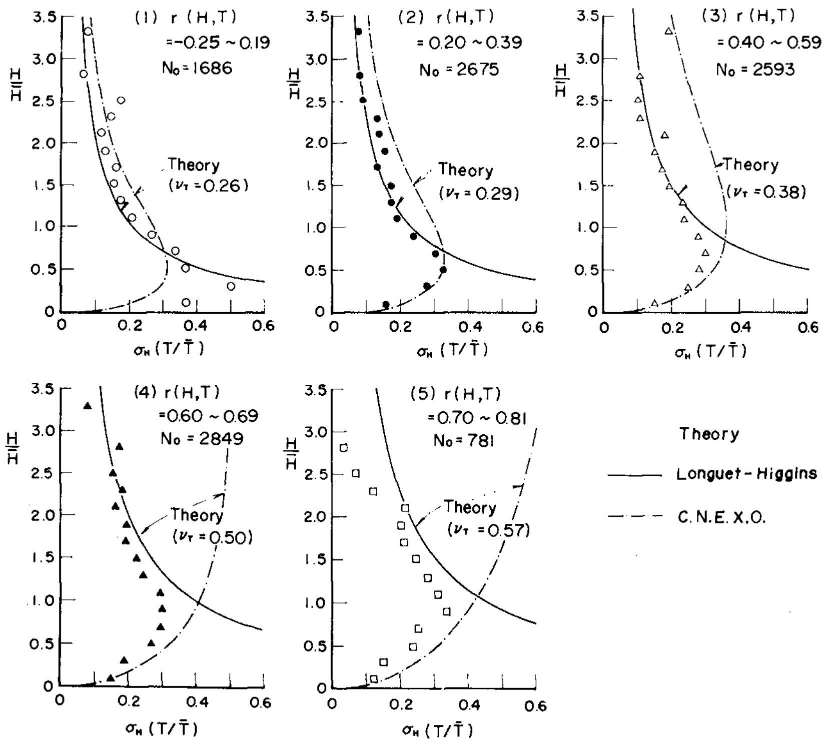


Fig. 9 Standard Deviation of Heightwise Ranked Wave Periods

of r(H,T) is owing to the appearance of small waves with very short periods.

The constancy of mean ranked wave periods is inherent in the theory of Longuet-Higgins although its shift with respect to $\bar{T}$ cannot be dealt with. The theory of C.N.E.X.O., on the other hand, can yield the increase of mean wave periods of high waves with the increase of ε, but the mean wave period steadily elongates itself as the level of wave height rises.

The spread of wave periods in a particular rank of wave height can be represented with their standard deviation. Figure 9 shows the results of the calculation of standard deviations of heightwise ranked wave periods for five groups of wave data classified by the rank of r(H,T). The theory of Longuet-Higgins is seen to predict the spread of wave periods of high waves when the mean period bandwidth parameter of respective wave group is employed in calculation of Eq. 5, although the decrease of standard deviations in the region of small waves cannot be predicted. The theory of C.N.E.X.O., on the other hand, can present the standard deviation decreasing in the lower portion of wave heights, but it yields the deviation much larger than the observed ones in the upper portion of wave heights especially for groups with high correlation coefficients.

DISCUSSIONS ON THE GOVERNING PARAMETERS OF JOINT DISTRIBUTION

It has been demonstrated that the joint distribution of the heights and periods of sea waves exhibit quite large variations. One of the questions may be what the parameter is governing the joint distribution. There are two spectral width parameters of ν_S and ε_S defined by Eqs. 2 and 9, respectively. Statistical analysis of wave records yields the apparent spectral width parameter of ε_T by Eq. 10, the period bandwidth parameter of ν_T by Eq. 6, and the correlation coefficient between individual wave heights and periods by Eq. 13. Among these parameters, ε_S is not qualified for describing the statistical properties of sea waves, because ε_S is essentially 1.0 for wind-generated water waves and becomes less than 1.0 owing to incompetence in the high frequency response of a wave recorder [11,12]. The parameter ν_S may need further examination, but the present data at least reject the effectiveness of ν_S because of large scatter of ν_S without associating itself with the statistical properties of waves analyzed.

The question is thus focussed on the selection among three parameters of ε_T, ν_T, and r(H,T). The relationships among them are first examined as shown in Figs. 10 to 12. The relationship between ν_T and r(H,T) is most conspicuous with the correlation coefficient between them amounting to 0.80. If one makes a polynomial regression analysis instead of linear regression, a much higher correlation will be obtained. Among the whole data, those in the range of r(H,T) $\gtrsim$ 0.4 closely follow the trend of the theory of C.N.E.X.O. as well as the result of numerically simulated wave analysis [8], even though both the theory and the simulation data cannot explain the existence of data with r(H,T) > 0.7 or ν_T > 0.55. The relationship between ν_T and ε_T is obscure although the means of rankwise data indicate the existence of interrelation, which is close to the data of simulation study. The correlation coef-

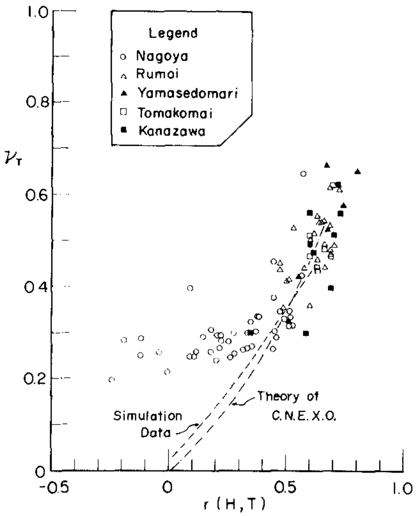


Fig. 10 Correlation between ν_T and R(H,T)

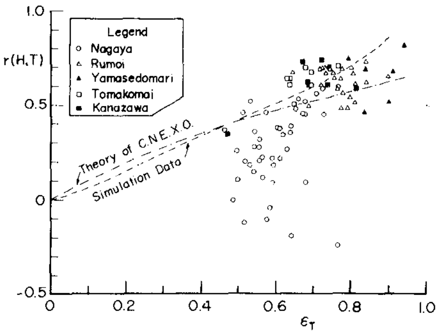


Fig. 11 Correlation between r(H,T) and ε_T

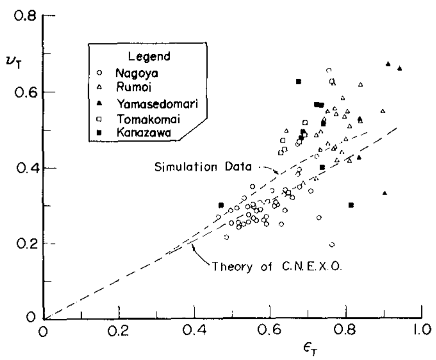


Fig. 12 Correlation between ν_T and ε_T

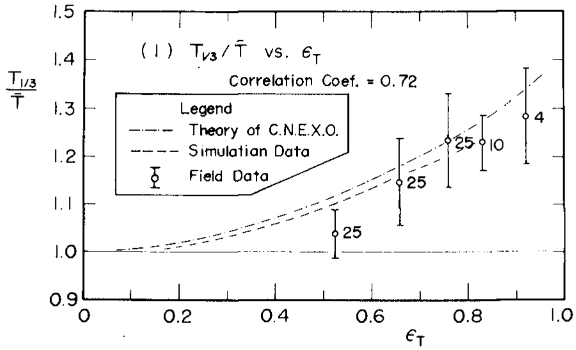
ficient between ν_T and ε_T is calculated as 0.72. The third set of relationship, that is the one between r(H,T) and ε_T, is clouded with the presence of the data with r(H,T) $\lesssim$ 0.3, which come from the Nagoya Port data. The correlation coefficient between r(H,T) and ε_T nevertheless has the value of 0.66 for the present data.

A criterion for the selection of governing parameter will be a high level of correlation with statistical properties of the joint distribution of wave heights and periods. One of the appropriate properties is the ratio of significant to mean wave period, $T_{1/3}/\bar{T}$, as employed by the group of C.N.E.X.O. for demonstration of the influence of ε_T. Figure 13 is the result of comparison of the influence of three parameters upon $T_{1/3}/\bar{T}$. Data are shown in the form of ranked mean and standard deviations with the number of records in respective ranks. The wave period ratio, $T_{1/3}/\bar{T}$, is seen to be closely related with the three parameters. The relationships are also close to those derived by the theory of C.N.E.X.O. and the simulation data. Among three parameters, their correlations with $T_{1/3}/\bar{T}$ is lowest for ε_T with the correlation coefficient of 0.72, while ν_T and r(H,T) show the coefficient of 0.92 and 0.94, respectively.

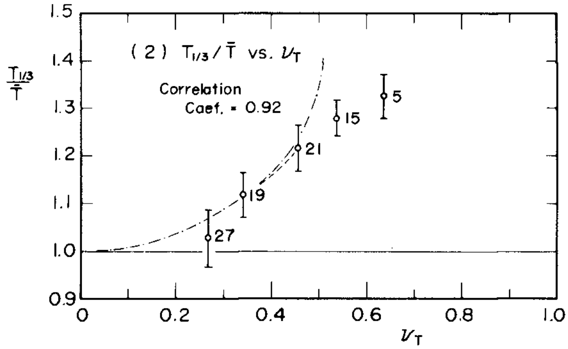
The degree of correlation between $T_{1/3}/\bar{T}$ and ε_T of the present data is about the same with the data presented by the group of C.N.E.X.O., as judged from the magnitude of standard deviations of $T_{1/3}/\bar{T}$. In this sense, ε_T seems inferior to the other two parameters. Between the remaining two, r(H,T) is slightly superior to ν_T on the basis of the former's high correlation coefficient. In the application of the theory of Longuet-Higgins, however, the information of period bandwidth parameter is required, and from this point of view ν_T may be more convenient than r(H,T). The final selection between r(H,T) and ν_T cannot be made at this stage yet, and it will necessitate the anlysis of many more data of sea waves.

A remaining question is the correlation of these parameters with spectral characteristics. Though ν and ε have originally been derived from spectral moments, ν_T and ε_T are the parameters estimated from the statistical analysis of wave profiles; their relationships with ν_S and ε_S cannot be well established at least for the present data. The definition of correlation coefficient r(H,T) is independent of wave spectrum and it has not been related to a wave spectrum (variance) except for a numerical simulation study [8]: the spectrum proposed by Bretschneider [13] with r(H,T) as a parameter is an apparent one and not the variance spectrum analyzed by the spectral theory. As indicated in Figs. 10 to 12, these parameters vary over quite large ranges. A spectral characteristics which seems to be related to these parameters to some extent is the spectral peakedness parameter defined by Eq. 14. As shown in Fig. 14, the period bandwidth parameter ν_T of the present data does indicate an interrelation with Q_p; the correlation coefficient between them is -0.65. The correlations of other two parameters ε_T and r(H,T) with Q_p are not so prominent, however.

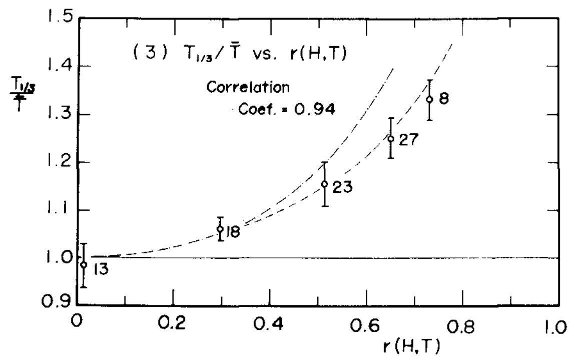
Another factor which may affect the parameter of the joint distribution of wave heights and periods is the sampling interval of wave profile relative to the spectral peak period, or $\Delta t/T_p$. The present data shows the relationship between ν_T and $\Delta t/T_p$ as in Fig. 15.



(1) influence of ε_T on $T_{1/3}/\bar{T}$



(2) influence of ν_T on $T_{1/3}/\bar{T}$



(3) influence of r(H,T) on $T_{1/3}/\bar{T}$

Fig. 13 Comparison of the Parameters of ε_T, ν_T, and r(H,T) for Their Influences upon the Wave Period Ratio of $T_{1/3}/\bar{T}$

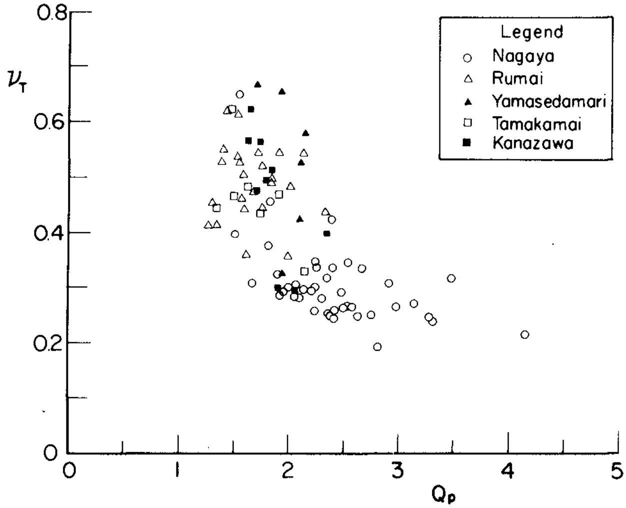


Fig. 14 Correlation between ν_T and Q_p

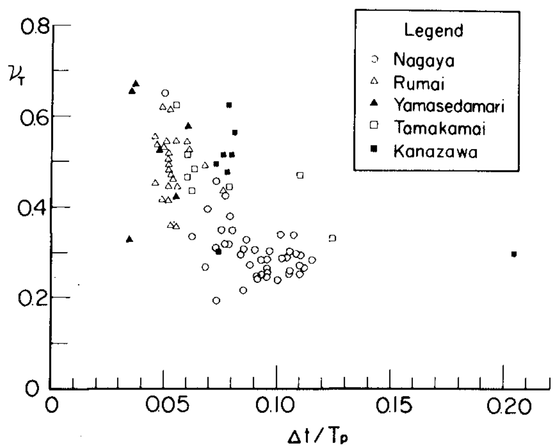


Fig. 15 Correlation between ν_T and $\Delta t/T_p$

A negative correlation between them is somewhat expected, though the quantitative analysis should await the accumulation of a greater number of sea wave data.

SUMMARY

The joint distribution of wave heights and periods exhibits statistical variability greater than the marginal distributions of wave heights and periods. Any analysis of the joint distribution should have a sufficiently large number of wave records as the data base. The present analysis based on 89 records cannot be claimed to be very reliable, it nevertheless covers a variety of wave conditions from short-fetched wind waves to young swell in shallow water. Findings made in this analysis can be summarized as follows:

1. The joint distribution of wave heights and periods of sea waves is characterized with no correlation among high waves and a strong correlation among low waves.

2. The mean period of waves higher than a certain level is independent of the wave height, and it remains at a value slightly less than the period corresponding to the spectral peak.

3. The theory of Longuet-Higgins can explain the characteristics of the joint distribution in its upper portion with high waves if the spectral width parameter is so selected to fit the marginal distribution of wave periods, even though the theory disagrees with the observed joint distribution in its lower portion with low waves.

4. The theory by the group of C.N.E.X.O. can qualitatively predict the change of the joint distribution with the increase of spectral width parameter, but the quantitative agreement is only partial.

5. The parameter governing the joint distribution seems to be the correlation coefficient between individual wave heights and periods and/or the period bandwidth parameter. The apparent spectral width parameter is less influential than the formers.

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LIST OF MAJOR SYMBOLS

H_{max} : height of highest wave
$H_{1/10}$: mean height of highest one-tenth waves
$H_{1/3}$: mean height of highest one-third waves
$\bar{H}$: mean wave height
Q_p : spectral peakedness parameter defined by Eq. 14
$r(H,T)$: correlation coefficient between individual wave heights and periods
$r_{13}(H,T)$: correlation coefficient among highest one-third waves
T_p : wave period corresponding to spectral peak frequency
T_{max} : period of highest wave
$T_{1/10}$: mean period of highest one-tenth waves
$T_{1/3}$: mean period of highest one-third waves
$\bar{T}$: mean wave period
$x = H/\bar{H}$: nondimensional wave height
Δt : time interval between successive sampling of surface elevation
ε_S : spectral width parameter defined by Eq. 9
ε_T : apparent spectral width parameter defined by Eq. 10
$\zeta = \bar{\zeta}\tau$: nondimensional wave period
ν_S : spectral width parameter defined by Eq. 2
ν_T : period bandwidth parameter defined by Eq. 6
$\xi = H/\sqrt{m_0}$: nondimensional wave height
$\tau = T/\bar{T}$: nondimensional wave period