

## The Observed Moment of a Magnetized Inclusion of high Curie Point within a Titanomagnetite Particle of lower Curie Point

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### Summary

The theoretically observed moment of a permeable magnet embedded in a permeable medium is applied to the case of titanomagnetite particles containing minor quantities of an exsolved magnetite-rich phase of higher Curie point such as are found in basalts. The effect of temperature on the observed moment as the Curie point of the titanomagnetite is reached is discussed, and is shown to qualitatively explain experimental results found by Petherbridge *et al.* who found that an increase in moment occurred on heating synthetic samples in zero field through the titanomagnetite Curie point, and that a corresponding decrease in moment occurred on cooling.

### Introduction

Naturally occurring titanomagnetites which commonly carry the natural remanent magnetization of basalts are not always single-phased because of the 'hoop-shaped' solvus curve (Vincent *et al.* 1957) of the magnetite-ulvöspinel solid solution series. This can result in the exsolution of magnetite-rich phases which may form small isolated inclusions within a matrix of ulvöspinel-rich titanomagnetite of lower Curie point ( $T_c$ ).

In this paper the magnetic moment of such a two phase system is calculated and the variation with temperature around  $T_c$  is discussed. The results are shown to agree qualitatively with those found experimentally by Petherbridge, Campbell & Hauptmann (1974) for synthetic titanomagnetites of composition  $\text{Fe}_{3-x}\text{Ti}_x\text{O}_4$  where  $x$  ranged from 0.4 to 0.6 (see Fig. 4). Magnetite inclusions were produced in these samples during synthesis.

### 2. Theory

Fig. 1 illustrates the starting model where a permanently magnetized spherical inclusion of radius  $r = a$  and composition 1 (magnetite with high Curie point) is at the centre of a spherical particle of radius  $r = b$  and composition 2 (titanomagnetite with lower Curie point). Let the permeabilities of the media be  $\mu_1$ ,  $\mu_2$  and  $\mu_3$ . (In the case under consideration where the moment of the two phase particle is measured in air,  $\mu_3$  is in practice equal to the free space value  $\mu_0$ .) Let  $M^*$  be the moment per unit volume of the inclusion in the absence of any demagnetizing field

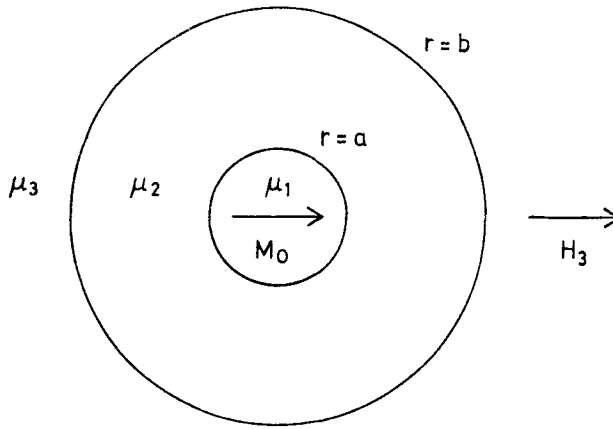


FIG. 1. Model of exsolved phase of permeability  $\mu_1$  and moment  $M_0$  embedded in a permeable medium.  $H_3$  is an applied field.

and let  $H_3$  be an external field (e.g. the applied field in the case of a susceptibility measurement) applied parallel to  $M^*$  as in Fig. 1. Inside the inclusion is a uniform field  $H$  which, provided the demagnetizing field exceeds the applied field  $H_3$ , is opposed to  $M^*$ . This produces an induced magnetization  $M$  such that:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_1 \mathbf{H}^\dagger$$

therefore

$$\mathbf{M} = \mathbf{H} \left( \frac{\mu_1 - \mu_0}{\mu_0} \right) = \chi \mathbf{H}. \quad (1)$$

$M^*$  is thus reduced by an amount  $M$  to  $M_0$  by the field  $H$  which acts on the intrinsic susceptibility  $\chi$  of the inclusion, i.e.

$$\mathbf{M}_0 = \mathbf{M}^* + \mathbf{H} \left( \frac{\mu_1 - \mu_0}{\mu_0} \right). \quad (2)$$

Considering now both the induced and permanent magnetization:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}_0) \quad (3)$$

$$= \mu_0 \mathbf{M}^* + \mu_1 \mathbf{H} \quad \text{from (2) and (3)} \quad (4)$$

Introducing a magnetic potential  $\phi$ ,  $H(r) = -(\partial\phi/\partial r)$ , and considering the magnetic potentials in the system:

For

$$0 \leq r \leq a : \phi = -H_1 r \cos \theta \quad (5)$$

where  $H_1$  is a uniform field inside the inclusion in the direction of the applied field  $H_3$ . For

$$a \leq r \leq b : \phi = -H_2 r \cos \theta + a^3 H_2' \frac{\cos \theta}{r^2} \quad (6)$$

<sup>†</sup>Note that all equations may be converted to emu by putting  $\mu_0 = 1$ , replacing  $\chi$ ,  $M$ , and  $m$  by  $4\pi\chi$ ,  $4\pi M$  and  $4\pi m$  respectively, and replacing  $N$  by  $N/4\pi$ .

and for

$$b \leq r : \phi = -H_3 r \cos \theta + b^3 H_3' \frac{\cos \theta}{r^2}. \quad (7)$$

In the latter two equations the first term arises from a uniform field and the second from a dipole field. By noting that  $\phi$  and  $B$  are continuous at  $r = a$  and  $r = b$ , four simultaneous equations are derived from which  $H_3'$  can be found.  $H_3'$  is related to the observed dipole moment  $m$  by the equation  $m = 4\pi b^3 H_3'$ , since

$$H(r) = -\frac{\partial \phi}{\partial r} = \frac{2b^3}{r^3} H_3' \cos \theta = \frac{2m \cos \theta}{4\pi r^3} \quad (8)$$

and is as follows:

$$H_3' = \frac{3\sigma\mu_0\mu_2 M^* + H_3[(\mu_1 + 2\mu_2)(\mu_2 - \mu_3) + \sigma(\mu_1 - \mu_2)(2\mu_2 + \mu_3)]}{(\mu_1 + 2\mu_2)(\mu_2 + 2\mu_3) + 2\sigma(\mu_1 - \mu_2)(\mu_2 - \mu_3)} \quad (9)$$

where  $\sigma = (a/b)^3$ . The first term, which contains  $M^*$ , represents the contribution from the permanent magnetization and the second term comes from the induced magnetization and is proportional to the applied field  $H_3$ . This latter term (with  $\mu_3 = \mu_0$ ) is the same as that derived by Parkin, Dyal & Daily (1973) for the induced moment of the moon assuming it to consist of two concentric spherical shells containing material of permeability  $\mu_1$  and  $\mu_2$ , respectively. Thus the observed susceptibility  $\chi_0$  of the whole system is given by  $(4\pi b^3/3)^{-1}(\partial m/\partial H_3)$ , i.e.,

$$\chi_0 = 3 \frac{(\mu_1 + 2\mu_2)(\mu_2 - \mu_0) + \sigma(\mu_1 - \mu_2)(2\mu_2 + \mu_0)}{(\mu_1 + 2\mu_2)(\mu_2 + 2\mu_0) + 2\sigma(\mu_1 - \mu_2)(\mu_2 - \mu_0)}. \quad (10)$$

Note that for  $\mu_1 = \mu_2$ , the susceptibility of the whole sphere of uniform permeability is given by the usual equation:

$$\chi_0 = \frac{\chi}{1 + \frac{1}{3}\chi} \quad (11)$$

where  $\chi = (\mu_1 - \mu_0)/\mu_0$  and the factor  $\frac{1}{3}$  in the denominator is the demagnetizing factor.

In the case of the two-phase particle, only the observed permanent magnetic moment is now considered (e.g. in practice by using an astatic magnetometer in zero applied field) and it is also assumed that the inclusions of the phase 1 composition (magnetite) are spherical and of total volume very much less than that of the spherical phase 2 matrix (titanomagnetite). For simplicity, a distribution of non-interacting phase 1 inclusions is approximated to a single inclusion of the same total volume  $v$  concentrated at the centre of the sphere of composition 2, so that the above theory is applicable with  $\sigma \ll 1$ . The observed moment thus becomes (for  $\mu_3 = \mu_0$ ):

$$m = m^* \frac{9\mu_{2r}}{(\mu_{1r} + 2\mu_{2r})(\mu_{2r} + 2\mu_{3r})} \quad (12)$$

where  $m^* = \frac{4}{3}\pi a^3 M^*$ , and the relative permeability  $\mu_{1r} = \mu_1/\mu_0$  etc.  $m^*$  is the observed moment if the inclusion is a 'hard' magnet ( $\mu_{1r} = 1$ ) and the surrounding permeable material is removed ( $\mu_{2r} = \mu_{3r} = 1$ ).

### 3. Extension to ellipsoidal inclusions within an ellipsoidal particle

It is now possible to deduce the form of the equation for the case of ellipsoidal titanomagnetite particles containing ellipsoidal inclusions if it is assumed that the exsolved material grows anisotropically with the axes of the identical exsolved ellipsoids parallel, and with demagnetizing factors  $N_1$  along the axes of the permanent moments. The demagnetizing factor of the surrounding phase 2 material along this same axis is  $N_2$  (i.e. the principal axes of the inner and outer ellipsoids, which are of different shape, coincide).

In the spherical case, equation (12) above can be written:

$$m = m^* \frac{\mu_{2r}}{[\mu_{2r} + \frac{1}{3}(\mu_{1r} - \mu_{2r})][\mu_{3r} + \frac{1}{3}(\mu_{2r} - \mu_{3r})]} \quad (13)$$

Noting that  $N$  for a sphere is  $\frac{1}{3}$  enables the following analogous equation to be deduced for the ellipsoidal case:

$$m = m^* \frac{\mu_{2r}}{[\mu_{2r} + N_1(\mu_{1r} - \mu_{2r})][\mu_{3r} + N_2(\mu_{2r} - \mu_{3r})]} \quad (14)$$

where  $m^*$  is now  $vM^*$ . Note that if  $\mu_{2r} = \mu_{3r}$ , the above equation for the observed moment becomes:

$$m = \frac{m^*}{\mu_{2r} + N_1(\mu_{1r} - \mu_{2r})} \quad (15)$$

This equation is identical to that derived rigorously and given by Lowes (1974, equation (14)). Thus equation (14) above enables various properties of magnetic particles containing small amounts of an exsolved phase to be deduced.

### 4. Dependence of observed moment on $\mu_{2r}$

In equation (14) the observed moment of the two phase particle is not greatest when  $\mu_{2r} = 1$  since  $m$  has a maximum value obtained for the condition  $(\partial m / \partial \mu_{2r}) = 0$ . This gives the result:

$$\mu_{2m}^2 = \mu_{1r} \frac{N_1(1 - N_2)}{N_2(1 - N_1)} \quad (16)$$

Thus the observed moment of a permeable ellipsoidal magnet is enhanced by some factor  $\alpha$  if it is embedded in a body of permeability  $\mu_{2m}$  given by equation (16) (note that if  $N_1 = N_2$ ,  $\mu_{2m} = \sqrt{(\mu_{1r})}$ ). This result may be explained qualitatively by noting that the demagnetizing field inside the magnet will be reduced by surrounding it with a permeable material. Thus the volume magnetization of the 'soft' magnet will increase and, provided that this increase outweighs the flux trapped by the surrounding medium, the observed moment will increase. If, however, the permeability of the surrounding medium is very high, all the flux from the magnet will be trapped and the observed moment will be zero (equation (14) with  $\mu_{2r} = \infty$ ). Note that this result is independent of how much material surrounds the magnet since there is no dependence on  $\sigma$  in equation (14) ( $\sigma \ll 1$ ).

The value of the maximum enhancement ratio  $\alpha_m$  may be obtained by substituting (16) in (14) to give:

$$\alpha_m = \frac{m_m}{m_0} = \frac{1 + N_1(\mu_{1r} - 1)}{[\sqrt{(\mu_{1r} N_1 N_2)} + \sqrt{(1 - N_1)(1 - N_2)}]^2} \quad (17)$$

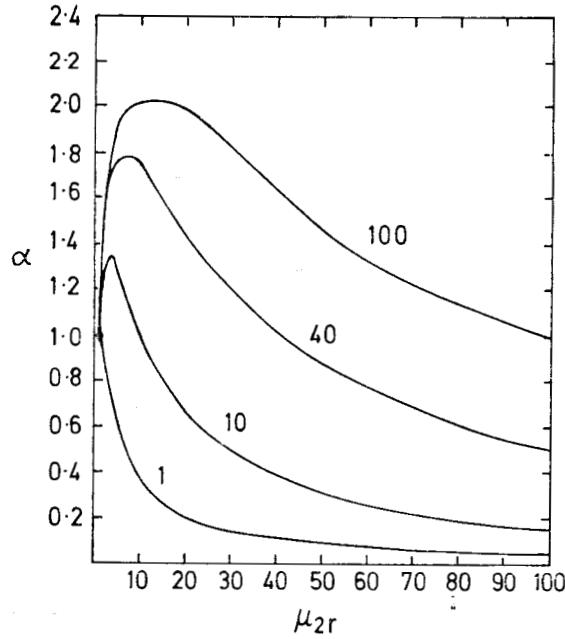


FIG. 2. Factor  $\alpha$  by which the observed moments of spherical permeable magnets of relative permeabilities 1, 10, 40 and 100 change when surrounded by a spherical medium of relative permeability  $\mu_{2r}$  ( $\sigma \ll 1$ ).

where  $m_m$  is the maximum value of the observed moment and  $m_0$  is the moment observed when  $\mu_{2r} = 1$  (equation (19)).

Note that if  $N_1 = N_2$  and if  $\mu_{1r}$  is very large, the factor  $\alpha_m$ , by which the moment is enhanced, is  $1/N_1$ . Thus for spheres ( $N = \frac{1}{3}$ ), an increase in observed moment by a factor of 3 is obtained if the highly permeable magnet ( $\mu_{1r}$  high) is buried inside a spherical medium of permeability  $\sqrt{(\mu_{1r})}$ . In general, it can be shown that if  $N_1 = N_2$ , enhancement of the observed moment occurs provided that  $\mu_{1r} > \mu_{2r} > 1$ . Fig. 2 shows the calculated plot of  $\alpha = m/m_0$  for the spherical case from the equation:

$$\alpha = \frac{3\mu_{2r}(\mu_{1r} + 2)}{(\mu_{1r} + 2\mu_{2r})(\mu_{2r} + 2)}. \quad (18)$$

### 5. Variations of observed moment with temperature

Consider the case where the inclusion of high Curie point has an essentially constant permanent moment  $m^*$  and permeability  $\mu_{1r}$  over a temperature range centred on  $T_c$ , the Curie point of the phase 2 material. If the system is initially above  $T_c$  and is then cooled in zero external field, the observed moment above  $T_c$  is  $m_0$ , since  $\mu_{2r} = 1$ , where

$$m_0 = \frac{m^*}{1 + N_1(\mu_{1r} - 1)}. \quad (19)$$

When the temperature falls through  $T_c$ , however,  $\mu_{2r}$  will rapidly increase from unity to some higher value. If for simplicity it is assumed that the increase is a step

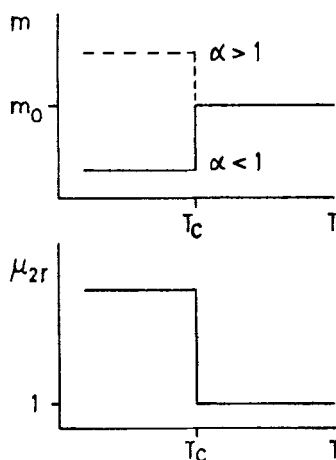


FIG. 3. Change in moment observed at the Curie point  $T_c$  of phase 2, assuming for simplicity that  $\mu_{2r}$  changes as shown. For  $\mu_{1r} > \mu_{2r} > 1$ ,  $\alpha > 1$  and for  $\mu_{1r} < \mu_{2r}$ ,  $\alpha < 1$  provided that  $N_1 = N_2$ .

function, a sudden change in moment at  $T_c$  will be observed by a factor  $\alpha = m/m_0$ , which from equations (14) and (19) is:

$$\alpha = \frac{\mu_{2r}[1 + N_1(\mu_{1r} - 1)]}{[\mu_{2r} + N_1(\mu_{1r} - \mu_{2r})][1 + N_2(\mu_{2r} - 1)]} \quad (20)$$

This may either be an increase if the enhancement conditions in the previous section apply, or if  $\mu_{2r} > \mu_{1r}$  for  $N_1 = N_2$ , a decrease. Fig. 3 shows the result schematically. The ratio  $\alpha$  lies in the range  $0 < \alpha < 3$  for spheres. (Any possible thermoremanence (TRM) acquired by the material of lower Curie point as it cools in the field produced by the magnetized inclusion has been neglected.)

The decrease of moment on cooling through  $T_c$ , and the corresponding increase on heating observed by Petherbridge *et al.* in their synthetic partially unmixed titanomagnetites ( $T_c$  lay in the range 150 to 350 °C), is thus explained (Fig. 4). For a value of  $\alpha$  of 0.5, which is a typical value which they obtained, it is not possible to calculate from equation (20) unique values of  $\mu_{1r}$  and  $\mu_{2r}$  even if each phase is assumed to be of roughly spherical form. However, it is possible from Fig. 2 to define a range of possible values for  $\mu_{1r}$  and  $\mu_{2r}$ , and because  $\alpha < 1$ ,  $\mu_{2r}$  must be greater than  $\mu_{1r}$ .

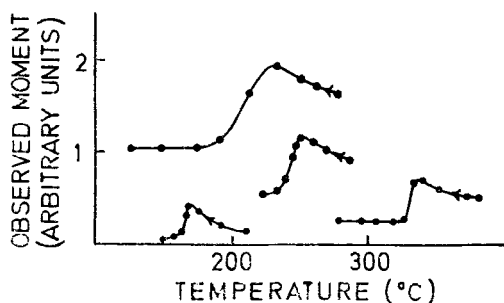


FIG. 4. Some of the results of Petherbridge *et al.* showing the decrease in the moment which they measured as their synthetic titanomagnetite samples were cooled in zero field through their Curie points.

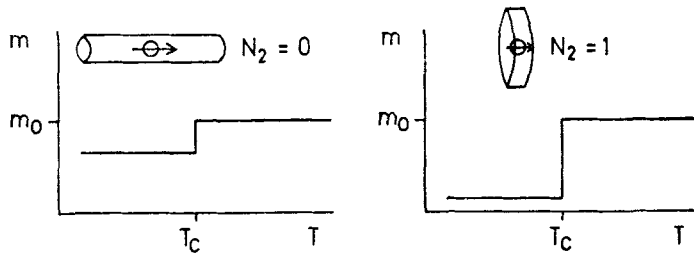


FIG. 5. Schematic plot of observed moment  $m(T)$  for synthetic samples with minimum and maximum possible demagnetizing factors. For factors of 0 and 1 the corresponding values of  $\alpha$  are related by the equation  $\alpha_0 = \mu_{2r} \alpha_1$ .

Values of the permeability of the two phases in synthetic samples can be found in principle by varying the external shape of the pellet and measuring  $\alpha$ , since from equation (20),  $\alpha$  depends on  $N_2$  as well as on  $N_1$ . While  $N_1$  can not be varied, since the small exsolutions are not accessible, the overall shape of the sample can be changed and hence  $N_2$  varied. It is easily shown that

$$\frac{\alpha_0}{\alpha_1} = \mu_{2r} \quad (21)$$

where  $\alpha_0$  and  $\alpha_1$  are the factors for  $N_2 = 0$  and  $N_2 = 1$ , respectively. This situation holds when the axis of the outer body, which is in the form of a thin rod and a disc respectively, coincides with the direction of magnetization of the inclusion (see Fig. 5).

## 6. Conclusion

This approach provides a basis for understanding some aspects of the magnetic behaviour observed experimentally in synthetic titanomagnetite samples containing magnetite inclusions. Naturally occurring titanomagnetites containing exsolved phases are likely to be fairly common, and thus effects such as these may be found in natural samples if suitable experiments are carried out. The permeability of the minor phase will depend very much on the size of the inclusions, and thus titanomagnetites containing exsolved phases may be expected to show different effects depending on the value of  $\mu_{1r}$ , which will vary from unity for inclusions which are hard single-domain grains, to higher values for larger multidomain grains.

If two phase titanomagnetites are given a TRM, there is the possibility that the remanence at ambient temperatures may be less than that just above the Curie point  $T_c$  of phase 2. This will occur if the decrease in moment on cooling through  $T_c$ , of the TRM acquired above  $T_c$  (phase 1), is greater than the TRM gained by the whole system between  $T_c$  and ambient temperature. Such a mechanism, if the geometry is favourable, might explain the larger central magnetic anomalies associated with the mid-ocean ridges where underlying hotter rock could be more strongly magnetized than the cooler material if exsolved magnetite inclusions are present within the titanomagnetite particles in oceanic basalts.

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