



The Odd Weibull Inverse Topp–Leone Distribution with Applications to COVID-19 Data

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Abstract

This paper aims at defining an optimal statistical model for the COVID-19 distribution in the United Kingdom, and Canada. A combining the inverted Topp–Leone distribution and the odd Weibull family introduces a new lifetime distribution with a three-parameter to formulate the odd Weibull inverted Topp–Leone (OWITL) distribution. As a simple linear representation, hazard rate function, and moment function, this new distribution has several nice properties. To estimate the unknown parameters of OWITL distribution, maximum likelihood, least-square, weighted least-squares, maximum product spacing, Cramér–von Mises estimators, and Anderson–Darling estimation methods are used. To evaluate the use of estimation techniques, a numerical outcome of the Monte Carlo simulation is obtained.

Keywords Odd Weibull family · Inverted Topp–Leone distribution · Maximum likelihood estimation · Maximum product spacing · COVID-19

1 Introduction

Over the years, statistical lifetime distributions have gained a lot of coverage. Its interest has therefore evolved over time. Distribution theory researchers do this either by adding a new parameter to make the distribution of interest more versatile or even creating a new distribution family or modeling data in a variety of fields, including economics, engineering, reliability, and medical sciences Anake et al. [1]. Because of their applicability in many fields such as biological sciences, life test issues, medical, etc., inverted (or inverse) distributions are of great importance. The density and hazard ratio of the inverted distributions illustrate a distinct structure from the non-inverted distributions of conformation. The applications of inverted distributions have been discussed with many researchers, and the reader can refer

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to Abd AL-Fattah et al. [2], Barco et al. [3], Hassan and Abd-Allah [4], Hassan and Mohamed [5], Muhammed [6], Chesneau et al. [7], Usman and ul Haq [8], Efer-honore et al. [9] among others. The modeling of COVID-19 data have been discussed with many researchers, and the reader can refer to Kumar [10], Khakharia et al. [11], Li et al. [12], Liu et al. [13], Wang [14], Lalmuanawma et al. [15] and Bullock et al. [16].

Hassan et al. [17] suggested the cumulative distribution function (CDF) and probability density function (PDF) of the inverted Topp–Leone distribution (ITL) distribution with form parameter $\delta > 0$ as follows:

$$G(x;\delta) = 1 - \frac{(1 + 2x)^\delta}{(1 + x)^{2\delta}}; \quad x > 0, \delta > 0 \quad (1)$$

and,

$$g(x;\delta) = 2\delta x (1 + x)^{-2\delta-1} (1 + 2x)^{\delta-1}; \quad x, \delta > 0 \quad (2)$$

Kumar and Dharmaja [18] presented the exponentiated Kies distribution and some of its properties for this distribution. Dey et al. [19]. derived the product moments of the modified Kies distribution under Type II progressive censored sample, as well as an approximation of the distribution parameters. Bourguignon et al. [20] introduced the unusual Weibull-G family. Al-Babtain et al. [21] submitted a new distribution family based on the modified Kies (MK) distribution and the T-X family. A special case of the odd Weibull-G (OW) family with one parameter is the MK family. Almetwally et al. [22] introduced modified Kies inverted Topp–Leone distribution. If $G(x;\delta)$ is the baseline CDF depending on a parameter vector δ , then the CDF of the OW family is defined by

$$F(x;\Theta) = 1 - e^{-\lambda \left[\frac{G(x;\delta)}{1-G(x;\delta)} \right]^\alpha}, \quad x > 0, \lambda, \alpha > 0, \quad (3)$$

where Θ is parameter vector $(\alpha, \lambda, \delta)$. The corresponding PDF of (3) is given by

$$f(x;\Theta) = \alpha \lambda \frac{g(x;\delta) G(x;\delta)^{\alpha-1}}{[1 - G(x;\delta)]^{\alpha+1}} e^{-\lambda \left[\frac{G(x;\delta)}{1-G(x;\delta)} \right]^\alpha}, \quad x > 0, \lambda, \alpha > 0, \quad (4)$$

The motivation of the new distribution is modeling the COVID-19. We used the COVID-19 of the United Kingdom and Canada as real data to evaluate the use of the model techniques. The new cases or new deaths of COVID-19 data are discrete data (count). We used the daily mortality rate of COVID-19 for the United Kingdom and Canada. The daily mortality rate is continuous data.

The modulated three-parameter odd Weibull inverted Topp–Leone (OWITL) distribution, which has many desirable properties, which is obtained in this paper. The OWITL distribution has a very flexible PDF, can be positively skewed, symmetrical, and negatively skewed, and can allow tails to be more flexible. It is capable of modeling down, up, bathtub, upside-down bathtub, and reverse-J hazard rates monotonically. Also, it has a closed-form CDF and is very simple to manage, making the distribution a candidate for use in various fields, such as life testing, reliability, biomedical research,

and study of survival. Three real data applications show that certain conventional distributions with scale and shape parameters such as ITL, the Marshall-Olkin exponential, modified Kies exponential, inverse Weibull, inverse exponential, and inverse Rayleigh distributions are very competitive with the suggested distribution.

In alternative estimation methods, the maximum product spacing approach is used to estimate the continuous univariate model parameters as an alternative to the Maximum Likelihood method developed for complete sample by Cheng and Amin [23] and this developed to use under censored sample by Singh et al. [24], Basu et al. [25], Almetwally et al. [26], El-Sherpieny et al. [27], Alshenawy et al. [28, 29]. The least-square and weighted least-square methods are used to estimate the parameters of the beta distribution by Swain et al. [30]. Based on the discrepancy between CDF estimates and the empirical distribution function, the Cramér–von–Mises has been introduced by Cramér [31] and von Mises [32]. Luceño [33], used the Cramér–von–Mises estimators to fit the generalized Pareto distribution.

We plan to make a new extension bivariate OWITL based on copula in future studies, such as done in Almetwally et al. [34], Muhammed and Almetwally [35] and Kim et al. [36]. We plan to discuss a new application for the OWITL distribution quest based on a censored sample such as done in Almetwally et al. [37] and Aslam et al. [38].

The remainder of this paper is structured as follows: We get the OWITL distribution in Sect. 2. We address some of the mathematical properties of the OWITL distribution in Sect. 3. In Sect. 4, we get the OWITL distribution by an estimation process. In Sect. 5, OWITL distribution simulation results are obtained. Three implementations of real data analytics were obtained in Sect. 6. In Sect. 7, the paper is summarized and concluded.

2 OWITL Distribution

Consider the ITL distribution of the positive scale factor δ and the CDF of Eqs. (1, 2) given (for $x > 0$). We define the OWITL distribution's CDF by inserting the ITL distribution's CDF into (3), such as:

$$F(x; \Theta) = 1 - e^{-\lambda[(1+x)^{2\delta}(1+2x)^{-\delta}-1]^\alpha}, x > 0, \alpha, \lambda, \delta > 0, \quad (5)$$

$$f(x; \Theta) = 2\alpha\lambda\delta x \frac{[(1+x)^{2\delta} - (1+2x)^\delta]^{\alpha-1}}{(1+2x)^{\alpha\delta+1}(1+x)^{1-2\delta}} e^{-\lambda[(1+x)^{2\delta}(1+2x)^{-\delta}-1]^\alpha}, x > 0, \alpha, \lambda, \delta > 0. \quad (6)$$

where Θ is parameter vector $(\alpha, \lambda, \delta)$. The OWITL distribution's hazard rate (HR) feature is shown as

$$h(x; \Theta) = 2\alpha\lambda\delta x \frac{[(1+x)^{2\delta} - (1+2x)^\delta]^{\alpha-1}}{(1+2x)^{\alpha\delta+1}(1+x)^{1-2\delta}}. \quad (7)$$

Figures 1 and 2 displays some PDF and HR plots of the OWITL distribution for the values specified for the α , λ , and δ functions. The diagrams shown in Fig. 2 indicate

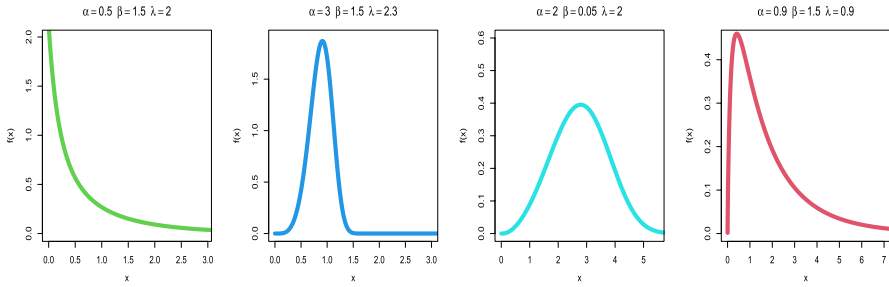


Fig. 1 PDF of the OWITL distribution for certain parameter values

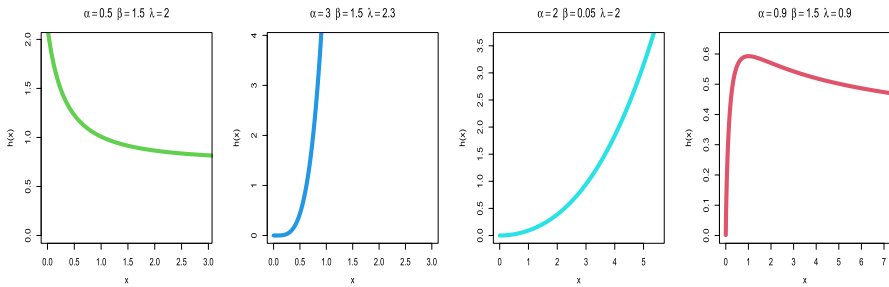


Fig. 2 HF of the OWITL distribution for certain parameter values

that it is possible to increase, decrease and shape a bathtub for the HR feature in the OWITL distribution. One benefit of the distribution of OWITL over an ITL distribution is that the last of them can not model a phenomenon that shows increasing, decreasing shapes, failure rates of the bathtub, and therefore becomes more flexible to analyze data about lifetime.

3 Some Mathematical Properties of the OWITL Distribution

3.1 Linear Representation for the OWITL Distribution

For the OW family, we have a helpful linear representation and use it to provide a useful linear representation for the distribution of OWITL. A combination depiction of the OW family can be given as follows,

$$f(x;\Theta) = \alpha \sum_{j,k=0}^{\infty} \frac{(-1)^{k+j}}{j!} \binom{\alpha(j+1)+k}{k} \lambda^{j+1} g(x;\delta)[G(x;\delta)]^{\alpha(j+1)+k-1}. \quad (8)$$

Using the ITL distribution’s PDF and CDF, the last OWITL distribution equation can be rewritten as

$$f(x; \Theta) = 2\delta(h+1)x \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h} (1+x)^{-2\delta(h+1)-1} (1+2x)^{\delta(h+1)-1}, \quad (9)$$

where $\zeta_{j,k,h} = \frac{\alpha \lambda^{j-1} (-1)^{j+k+h}}{(h+1) j!} \binom{\alpha(j+1)+k}{k} \binom{\alpha(j+1)+k-1}{h}$. Equation (9)

denotes the ITL density with parameter $\delta(h+1)$.

A combination depiction of the CDF of OW family can be given as follows,

$$F(x; \Theta) = 1 - \sum_{j,k=0}^{\infty} \binom{\alpha(j+1)+k}{k} \frac{(-1)^{k+j}}{j!} \lambda^{j+1} [G(x; \delta)]^{\alpha(j+1)+k-1}. \quad (10)$$

By using Eq. 10, the CDF of OWITL distribution equation can be rewritten as

$$F(x; \Theta) = 1 - \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h} (1+x)^{-2\delta(h+1)} (1+2x)^{\delta(h+1)}, \quad (11)$$

3.2 Quantile for the OWITL Distribution

The OWITL distribution's quantile function, i.e. $x = Q(x) = F(x, \Theta)^{-1}(Q)$, is derived as follows by inverting (5):

$$x_Q = \left\{ \left\{ 1 - \left(1 + \left[\frac{-1}{\lambda} \ln(1-Q) \right]^{\frac{1}{\alpha}} \right)^{\frac{-1}{\delta}} \right\}^{-0.5} - 1 \right\}^{-1}; \quad 0 < Q < 1 \quad (12)$$

In particular, the first quartile Q_1 , the second quartile Q_2 , and the third quartile Q_3 are obtained by setting $Q = 0.25, 0.5, 0.75$, respectively, in Eq. (12).

3.3 Moments for the OWITL Distribution

According Hassan et al. [17], the r_{th} moment of X follows simply from Eq. (9) as

$$\begin{aligned} \mu_r &= E(X^r) \\ &= 2\delta \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h} (h+1) \beta(j+r+2, \delta(h+1)-r) \end{aligned} \quad (13)$$

The r_{th} incomplete moment of X can be obtained from (9) as

$$\begin{aligned}
 \omega_r(t) &= \int_0^t x^r f(x, \Theta) dx \\
 &= \sum_{j,k,h=0}^{\infty} \zeta_{j,k,h} 2\delta(h+1) \sum_{b=0}^{\infty} \binom{\delta(h+1)-1}{b} \\
 &\quad B(b+r+2, \delta(h+1)-r, \frac{t}{1+t})
 \end{aligned} \tag{14}$$

where $B(b+r+2, \delta(h+1)-r, \frac{t}{1+t})$ is the incomplete beta function.

4 Estimation Methods

This section uses six different estimation methods called: maximum likelihood, least-square, the maximum product of spacing, weighted least-square, Cramér-von Mises, and Anderson–Darling, to analyze the estimation problem of the OWITL distribution parameters.

4.1 Maximum Likelihood Estimators

Let x_1, \dots, x_n be a random sample with the parameters α, λ and δ from an OWITL distribution. The log-likelihood function for the distribution of OWITL is given by

$$\begin{aligned}
 l(\Theta) &= n[\ln(2) + \ln(\alpha) + \ln(\lambda) + \ln(\delta)] + \sum_{i=1}^n \ln(x_i) - (1-2\delta) \sum_{i=1}^n \ln(1+x_i) \\
 &\quad + (\alpha-1) \sum_{i=1}^n \ln[(1+x_i)^{2\delta} - (1+2x_i)^\delta] - (\alpha\delta+1) \sum_{i=1}^n \ln(1+2x_i) \\
 &\quad - \lambda \sum_{i=1}^n [(1+x_i)^{2\delta}(1+2x_i)^{-\delta} - 1]^\alpha
 \end{aligned} \tag{15}$$

The partial derivatives of $l(\Theta)$ with respect to the model parameters α, λ and δ are

$$\begin{aligned}
 \frac{\partial l(\Theta)}{\partial \alpha} &= \frac{n}{\alpha} + \sum_{i=1}^n \ln[(1+x_i)^{2\delta} - (1+2x_i)^\delta] - \delta \sum_{i=1}^n \ln(1+2x_i) \\
 &\quad - \lambda \sum_{i=1}^n [(1+x_i)^{2\delta}(1+2x_i)^{-\delta} - 1]^\alpha \ln[(1+x_i)^{2\delta}(1+2x_i)^{-\delta} - 1]
 \end{aligned} \tag{16}$$

$$\frac{\partial l(\Theta)}{\partial \lambda} = \frac{n}{\lambda} + - \sum_{i=1}^n [(1+x_i)^{2\delta}(1+2x_i)^{-\delta} - 1]^\alpha \tag{17}$$

and

$$\begin{aligned} \frac{\partial l(\Theta)}{\partial \delta} &= \frac{n}{\delta} + 2 \sum_{i=1}^n \ln(1+x_i) - \alpha \sum_{i=1}^n \ln(1+2x_i) \\ &+ (\alpha-1) \sum_{i=1}^n \frac{2(1+x_i)^{2\delta} \ln(1+x_i) - (1+2x_i)^\delta \ln(1+2x_i)}{(1+x_i)^{2\delta} - (1+2x_i)^\delta} \\ &- \alpha \sum_{i=1}^n \left[\frac{(1+x_i)^{2\delta}}{(1+2x_i)^\delta} - 1 \right]^{\alpha-1} \left[\frac{(1+x_i)^2}{1+2x_i} \right]^\delta \ln \left[\frac{(1+x_i)^2}{1+2x_i} \right] \end{aligned} \quad (18)$$

It is possible to obtain the maximum likelihood estimation (MLE) of α , λ , and δ by maximizing the last equation for α , λ , and δ . By using the Newton-Raphson method, the R packages can be used to optimize the log-likelihood function for obtaining the MLE.

4.2 Least-Squares and Weighted Least-Squares Methods

To estimate the parameters of various distributions, the least-squares (LS) and weighted least-square (WLS) methods are used. Let $x_{(1)} < \dots < x_{(n)}$ be a random sample with the α , λ and δ parameters from the OWITL distribution. LS estimators (LSE) and WLS estimators (WLSE) of the α , λ and δ distribution parameters of OWITL can be obtained by minimizing the following:

$$V(\Theta) = \sum_{i=1}^n v_i \left[1 - e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} - \frac{i}{n+1} \right]^2 \quad (19)$$

$v_i = 1$ for LSE and $v_i = \frac{(n+1)^2(n+2)}{[i(n-i+1)]}$ for WLSE with respect to α , λ and δ . Furthermore, by resolving the nonlinear equations, the LSE and WLSE follow:

$$\begin{aligned} \frac{\partial V(\Theta)}{\partial \alpha} &= 2\lambda \sum_{i=1}^n v_i \left[1 - e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} - \frac{i}{n+1} \right]^\alpha \\ &- \frac{i}{n+1} \left[e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} - \frac{i}{n+1} \right]^\alpha \\ &\quad \left[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta} - 1 \right]^\alpha \ln \left[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta} - 1 \right], \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial V(\Theta)}{\partial \lambda} &= 2 \sum_{i=1}^n v_i \left[1 - e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} - \frac{i}{n+1} \right]^\alpha \\ &- \frac{i}{n+1} \left[e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} - \frac{i}{n+1} \right]^\alpha \\ &\quad \left[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta} - 1 \right]^\alpha, \end{aligned} \quad (21)$$

and

$$\begin{aligned} \frac{\partial V(\Theta)}{\partial \delta} &= 2\alpha\lambda \sum_{i=1}^n v_i \left[1 - e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} \right]^\alpha \\ &\quad - \frac{i}{n+1} \left[e^{-\lambda[(1+x_{(i)})^{2\delta}(1+2x_{(i)})^{-\delta}-1]} \right]^\alpha \\ &\quad \left[\frac{(1+x_{(i)})^{2\delta}}{(1+2x_{(i)})^\delta} - 1 \right]^{\alpha-1} \left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^\delta \ln \left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]. \end{aligned} \quad (22)$$

4.3 Maximum Product of Spacings Method

If $x_{(1)} < \dots < x_{(n)}$ is a random sample of the size n , you can describe the uniform spacing of the OWITL distribution as:

$$D_i(\Theta) = F(x_{(i)}, \Theta) - F(x_{(i-1)}, \Theta); i = 1, \dots, n+1 \quad (23)$$

where $D_i(\Theta)$ denotes to the uniform spacings, $F(x_{(0)}, \Theta) = 0$, $F(x_{(n+1)}, \Theta) = 1$ and $\sum_{i=1}^{n+1} D_i(\Theta) = 1$. The maximum product of spacing (MPS) estimators (MPSE) of the OWITL parameters can be obtained by maximizing

$$G(\Theta) = \frac{1}{n+1} \sum_{i=1}^{n+1} \ln \left[e^{-\lambda \left(\left[\frac{(1+x_{(i-1)})^2}{1+2x_{(i-1)}} \right]^\delta - 1 \right)^\alpha} - e^{-\lambda \left(\left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^\delta - 1 \right)^\alpha} \right] \quad (24)$$

with respect to α , λ and δ . Further, the MPSE of the OWITL parameters can also be obtained by solving

$$\begin{aligned} \frac{\partial G(\Theta)}{\partial \alpha} &= \frac{\lambda}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda([\tau_i]^\delta - 1)^\alpha} ([\tau_i]^\delta - 1)^\alpha \ln([\tau_i]^\delta - 1)}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}} \\ &\quad - \frac{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} ([\tau_{i-1}]^\delta - 1)^\alpha \ln([\tau_{i-1}]^\delta - 1)}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}}, \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{\partial G(\Theta)}{\partial \lambda} &= \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda([\tau_i]^\delta - 1)^\alpha} ([\tau_i]^\delta - 1)^\alpha}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}} \\ &\quad - \frac{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} ([\tau_{i-1}]^\delta - 1)^\alpha}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}}, \end{aligned} \quad (26)$$

and

$$\frac{\partial G(\Theta)}{\partial \delta} = \frac{\alpha \lambda}{n+1} \sum_{i=1}^{n+1} \frac{e^{-\lambda([\tau_i]^\delta - 1)^\alpha} [(\tau_i)^\delta - 1]^{\alpha-1} [\tau_i]^\delta \ln [\tau_i]}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}} - \frac{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} [(\tau_{i-1})^\delta - 1]^{\alpha-1} [\tau_{i-1}]^\delta \ln [\tau_{i-1}]}{e^{-\lambda([\tau_{i-1}]^\delta - 1)^\alpha} - e^{-\lambda([\tau_i]^\delta - 1)^\alpha}}. \quad (27)$$

where $\tau_i = \frac{(1+x_{(i)})^2}{1+2x_{(i)}}$.

4.4 Cramér–von–Mises Method

The Cramér–von–Mises (CVM) can be obtained for OWITL by minimizing the following function with respect to α , λ , and δ , the CVM estimators (CVME) of the OWITL parameters α , λ , and δ are obtained.

$$C(\Theta) = \frac{1}{12} + \sum_{i=1}^n \left(1 - e^{-\lambda \left(\left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^\delta - 1 \right)^\alpha} - \frac{2i-1}{2n} \right)^2 \quad (28)$$

In addition, by resolving the nonlinear equations, the CVME as follows

$$\frac{\partial C(\Theta)}{\partial \alpha} = 2\lambda \sum_{i=1}^n \left(1 - e^{-\lambda([\tau_i]^\delta - 1)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda([\tau_i]^\delta - 1)^\alpha} \ln \left([\tau_i]^\delta - 1 \right), \quad (29)$$

$$\frac{\partial C(\Theta)}{\partial \lambda} = 2 \sum_{i=1}^n \left(1 - e^{-\lambda([\tau_i]^\delta - 1)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda([\tau_i]^\delta - 1)^\alpha} \left([\tau_i]^\delta - 1 \right)^\alpha, \quad (30)$$

and

$$\frac{\partial C(\Theta)}{\partial \delta} = \alpha 2 \sum_{i=1}^n \left(1 - e^{-\lambda([\tau_i]^\delta - 1)^\alpha} - \frac{2i-1}{2n} \right) e^{-\lambda([\tau_i]^\delta - 1)^\alpha} \left([\tau_i]^\delta - 1 \right)^{\alpha-1} [\tau_i]^\delta \ln [\tau_i]. \quad (31)$$

4.5 Anderson–Darling Method

In Anderson–Darling (AD), other forms of minimum distance estimators are the AD estimators (ADE). The ADE of the parameters of the OWITL is acquired by minimizing

$$A(\Theta) = -n - \frac{1}{n} \sum_{i=1}^n (2i-1) \left(\ln \left(1 - e^{-\lambda \left(\left[\frac{(1+x_{(i)})^2}{1+2x_{(i)}} \right]^\delta - 1\right)^\alpha} \right) - \left(\left[\frac{(1+x_{(n+1-i)})^2}{1+2x_{(n+1-i)}} \right]^\delta - 1\right)^\alpha \right)^2 \quad (32)$$

Regarding α , λ and δ , respectively. It is also possible to obtain the ADE by resolving the nonlinear equations.

5 Simulation Results

In this portion, a simulation analysis evaluates the output of six different estimators of the OWITL parameters. For the different values of parameter $\alpha = (0.5, 3)$, $\lambda = (0.5, 3)$, and $\delta = (0.5, 3)$, we consider the different sample sizes $n = 50, 100, 150$. We create 10000 iteration of random samples for OWITL distribution. We get the average values of relative bias (RB) and their corresponding mean square error (MSE) for each calculation.

The output of the different estimators is evaluated in terms of RB and MSE, i.e. those whose MSE values are closer to zero would be the most effective method of estimating. Simulation results are obtained via the R program. Tables 1 and 2 show the RB and MSE values for MLE, LS, WLS, MPS, CVM, and AD. Also, as the sample size increases in all situations, the mean depending on all estimation methods tends towards the true parameter values, suggesting that all estimators are asymptotically unbiased. If $\delta < 1$, and $\alpha > 1$, the MPS is the best estimation methods in most times. If $\delta > 1$, and $\alpha > 1$, the AD method is the best estimation methods in most times. If $\delta < 1$, $\lambda < 1$ and $\alpha < 1$, the LS is the best estimation methods in most times. If $\delta < 1$, $\lambda > 1$ and $\alpha < 1$, the LS is the best estimation methods in most times. If δ , and λ increases and $\alpha < 1$, the LS is the best estimation methods in most times (Figs. 3, 4).

6 Application of Real Data Analysis

This section is dedicated to demonstrate the potential of two real data sets for the OWITL distribution. Compared with other competitive models, OWITL delivery, namely: extended odd Weibull inverse Rayleigh (EOWIR) which is introduced by Almetwally [39], generalized inverse Weibull (GIW) distribution which is introduced

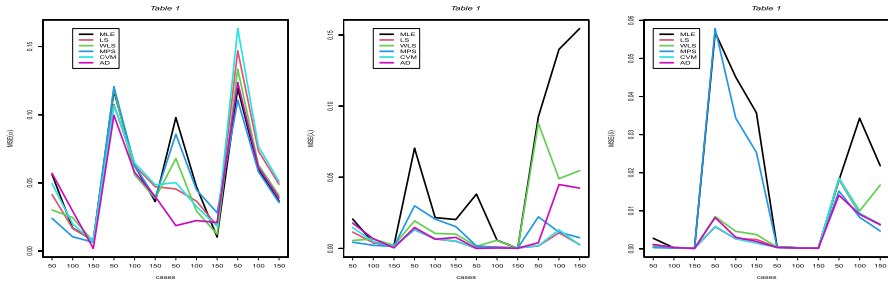


Fig. 3 MSE for different cases in Table 1

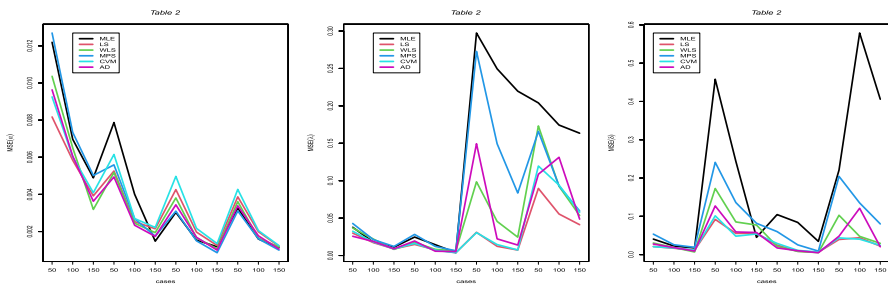


Fig. 4 MSE for different cases in Table 2

by De Gusmao et al. [40], exponential Lomax (ELo) distribution which is introduced by El-Bassiouny et al. [41], modified Kies exponential (MKEx) which is introduced by Al-Babtain et al. [21], and power Lomax (PL) distribution by Rady et al. [42].

For both models fitted on the basis of two real data sets, Tables 3 and 4 include Cramér-von Mises (W^*), Anderson–Darling (A^*) and Kolmogorov–Smirnov (KS) statistic values along with its P value. Furthermore, these tables include the parameters MLE and Standard Errors (SE) for the models considered (Fig. 5).

The OWITL distribution has the highest P value and the lowest distance of the Kolmogorov–Smirnov(KS), W^* and A^* values in Tables 3 and 4 when compared to all other models used here to suit the COVID-19 results. Figures 6 and 7 show the empirical, histogram, QQ-plot, and PP-plot fit for the OWITL distribution of Canadian and United Kingdom COVID-19 results. These applications show that the OWITL model can yield better fit than some other distribution.

In order to classify the possible shapes behind these data of the unknown hrf, we plot the total time on test (TTT) plot in Fig. 5 (see Aarset [43] for further details on the use of TTT plots in data analysis). In Fig. 5, since the blue line is convex, then concave, the unknown hrf probably presents a bathtub shape. Therefore, the OWITL distribution is appropriate to fit the data, where probTV is the accumulated probability distribution for time value and sorted_time_value is the vector time Value sorted of data. If data include outliers, we can use the Robust methods as least trimmed square, least median square, M, S, and MM, see Almongy and Almetwally [44, 45]. Artificial intelligence techniques can also be used see Olson et al. [46], Shi et al. [47] and Tien [48].

Table 1 Mean and MSE of the MLE, LS, MPS, WLS, CVM and AD estimates for OWITL distribution when $\alpha = 3$

β	λ	n	MLE		LS		WLS		MPS		CVM		AD		
			RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	
0.5	0.5	50	α	-0.02041	0.05612	0.01571	0.04136	-0.01411	0.03010	-0.02038	0.02395	0.02738	0.04941	0.00774	0.05697
			β	0.03888	0.02066	-0.06825	0.01152	0.03552	0.00558	0.03882	0.00438	-0.10389	0.01468	-0.04320	0.01784
			λ	-0.00829	0.00273	0.01926	0.00077	-0.00357	0.00035	-0.00835	0.00031	0.02911	0.00103	0.01841	0.00111
100	0.5	50	α	-0.01339	0.01675	0.00437	0.01660	0.00382	0.02450	-0.01338	0.01039	0.01156	0.01981	-0.00590	0.02924
			β	0.03074	0.00412	-0.02187	0.00403	-0.01950	0.00669	0.03073	0.00223	-0.04199	0.00480	0.00923	0.00681
			λ	-0.00718	0.00022	0.00658	0.00027	0.00657	0.00029	-0.00721	0.00016	0.01113	0.00031	0.00165	0.00030
150	0.5	50	α	-0.00892	0.00773	-0.00975	0.00769	0.00267	0.00685	-0.00890	0.00640	-0.00742	0.00673	-0.00100	0.00187
			β	0.01931	0.00194	0.02515	0.00150	-0.01238	0.00220	0.01926	0.00139	0.01921	0.00135	0.00212	0.00041
			λ	-0.00433	0.00011	-0.00389	0.00010	0.00370	0.00014	-0.00433	0.00009	-0.00248	0.00010	0.00026	0.00007
3	50	50	α	-0.04226	0.11728	-0.01304	0.10754	0.00370	0.11935	-0.04216	0.12054	0.01241	0.10678	0.00143	0.09947
			β	0.03994	0.07046	0.02221	0.01317	0.00131	0.01940	0.03996	0.02997	-0.00648	0.01310	0.00569	0.01472
			λ	0.01190	0.05683	-0.00029	0.00572	0.00435	0.00844	0.01180	0.05786	0.00432	0.00588	0.00362	0.00818
100	0.5	50	α	-0.02897	0.06284	-0.00519	0.06256	-0.00011	0.05594	-0.02895	0.06268	0.00783	0.06459	0.00093	0.05772
			β	0.05447	0.02168	0.00892	0.00665	0.01908	0.01056	0.05449	0.02061	-0.00625	0.00683	0.00346	0.00648
			λ	0.00131	0.04507	0.00057	0.00259	-0.00109	0.00455	0.00126	0.03430	0.00303	0.00261	0.00144	0.00299
150	0.5	50	α	-0.01986	0.03623	-0.00209	0.04718	0.00444	0.03856	-0.01981	0.03901	0.00716	0.04849	0.00322	0.03994
			β	0.03670	0.02031	-0.00082	0.00507	0.02338	0.01010	0.03662	0.01534	-0.01130	0.00520	0.01585	0.00777
			λ	0.00207	0.03572	0.00024	0.00242	-0.00236	0.00371	0.00207	0.02530	0.00409	0.00143	-0.00027	0.00190

Table 1 (continued)

β	λ	n	MLE		LS		WLS		MPS		CVM		AD		
			RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	
3	0.5	50	α	0.09784	-0.00583	0.04548	-0.00094	0.06783	-0.04409	0.08560	0.00386	0.05016	0.00045	0.01860	
			β	0.00088	0.03817	0.00025	0.00014	0.00006	0.00149	0.00088	0.00170	-0.00024	0.00016	-0.00009	0.00007
			λ	-0.01541	0.00046	-0.00310	0.00035	-0.00169	0.00039	-0.01546	0.00043	0.00130	0.00035	0.00053	0.00032
100	0.5	50	α	0.04711	-0.01938	0.03654	-0.01164	0.02927	-0.03088	0.04488	-0.01209	0.03260	-0.00171	0.02228	
			β	0.00130	0.00575	0.00121	0.00087	0.00298	0.00573	0.00130	0.00086	0.00073	0.00047	0.00034	0.00038
			λ	-0.01061	0.00022	-0.00709	0.00022	-0.00407	0.00022	-0.01065	0.00023	-0.00381	0.00021	-0.00022	0.00017
150	0.5	50	α	0.01019	-0.00612	0.01858	-0.01044	0.01231	-0.02022	0.02799	0.00061	0.01816	-0.00658	0.02089	
			β	0.00090	0.00003	0.00060	0.00041	0.00067	0.00006	0.00089	0.00028	0.00001	0.00046	0.00027	0.00016
			λ	-0.00682	0.00010	-0.00223	0.00017	-0.00305	0.00011	-0.00681	0.00013	0.00053	0.00017	-0.00195	0.00013
3	50	50	α	0.11941	-0.00116	0.14687	0.00741	0.13336	-0.03711	0.11054	0.02698	0.16317	0.00937	0.12355	
			β	-0.00908	0.09253	-0.00087	0.00175	0.00866	0.08792	-0.00902	0.02222	0.00032	0.00182	-0.00024	0.00400
			λ	-0.01163	0.01801	-0.00417	0.01817	-0.00129	0.01859	-0.01170	0.01514	0.00570	0.01844	0.00061	0.01404
100	0.5	50	α	0.05961	-0.00364	0.07334	0.00088	0.06315	-0.02730	0.05821	0.01042	0.07671	0.00154	0.06154	
			β	-0.00228	0.13994	-0.00166	0.01102	-0.00709	0.04903	-0.00225	0.01145	0.00026	0.01313	-0.00072	0.04493
			λ	-0.00913	0.03429	-0.00201	0.00916	0.00212	0.00991	-0.00917	0.00827	0.00267	0.00931	0.00072	0.00903
150	0.5	50	α	0.03656	-0.00020	0.04892	0.00386	0.04068	-0.01851	0.03546	0.00912	0.05086	0.00307	0.03881	
			β	-0.00280	0.15454	-0.00124	0.00272	0.00461	0.05472	-0.00279	0.00756	-0.00086	0.00243	0.00065	0.04241
			λ	-0.00559	0.02180	-0.00076	0.00634	0.00109	0.01672	-0.00559	0.00463	0.00254	0.00635	0.00102	0.00629

Table 2 Mean and MSE of the MLE, LS, MPS, WLS, CVM and AD estimates for OWITL distribution when $\alpha = 0.5$

β	λ	n	MLE		LS		WLS		MPS		CVM		AD		
			RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	
0.5	0.5	50	α	-0.04867	0.01219	0.00189	0.00817	0.00471	0.01036	-0.04826	0.01269	0.03848	0.00925	0.00822	0.00963
			β	0.08763	0.03776	0.05293	0.02998	0.06975	0.03687	0.08956	0.04283	0.05268	0.03244	0.03595	0.02575
			λ	0.11416	0.04050	0.03572	0.02112	0.05382	0.03020	0.11242	0.05351	0.02407	0.02073	0.06526	0.02760
100	0.5	50	α	-0.02960	0.00697	-0.00568	0.00584	-0.00104	0.00651	-0.02889	0.00734	0.01149	0.00602	0.00282	0.00606
			β	0.04788	0.01791	0.01561	0.01723	0.03086	0.02202	0.05036	0.02121	0.01428	0.01808	0.02295	0.01880
			λ	0.06096	0.02235	0.05465	0.01665	0.04867	0.01846	0.05879	0.02565	0.05080	0.01729	0.04960	0.01838
150	0.5	50	α	-0.02542	0.00489	-0.00633	0.00392	0.00890	0.00319	-0.02487	0.00502	0.00543	0.00409	0.00719	0.00363
			β	0.03298	0.01117	-0.00381	0.00888	0.01781	0.00822	0.03175	0.01209	-0.00496	0.00896	0.01502	0.00897
			λ	0.04789	0.01750	0.04803	0.01145	0.00904	0.00686	0.04738	0.01893	0.04507	0.01183	0.02104	0.00937
3	50	50	α	-0.01846	0.00787	0.01257	0.00526	0.01144	0.00518	-0.01826	0.00559	0.04447	0.00615	0.02625	0.00494
			β	0.09194	0.02467	0.03070	0.01485	0.02619	0.01752	0.09198	0.02820	0.02243	0.01617	0.03971	0.01941
			λ	-0.02033	0.45807	-0.01133	0.09195	0.00421	0.17277	-0.02055	0.24099	-0.01263	0.10126	-0.01495	0.12711
100	50	50	α	-0.00678	0.00401	0.00459	0.00253	0.00912	0.00245	-0.00655	0.00269	0.02005	0.00270	0.00731	0.00234
			β	0.06590	0.01448	0.02348	0.00768	0.02529	0.00825	0.06689	0.01219	0.01729	0.00703	0.01145	0.00579
			λ	-0.02393	0.24358	-0.00367	0.05540	-0.00304	0.08529	-0.02439	0.13591	-0.00420	0.04809	0.00641	0.05908
150	50	50	α	-0.00685	0.00148	0.00129	0.00215	0.01140	0.00194	-0.00677	0.00175	0.01198	0.00222	0.01386	0.00174
			β	0.03757	0.00385	-0.00002	0.00379	0.01704	0.00625	0.03768	0.00629	-0.00307	0.00375	0.01599	0.00527
			λ	-0.01991	0.04503	0.00568	0.05564	-0.01122	0.07723	-0.01996	0.08180	0.00430	0.05388	-0.01407	0.05773

Table 2 (continued)

β	λ	n	MLE		LS		WLS		MPS		CVM		AD		
			RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	RB	MSE	
3	0.5	50	α	-0.02950	0.00303	0.01069	0.00426	0.01427	0.00381	-0.02937	0.00310	0.03928	0.00497	0.01753	0.00344
			β	0.00422	0.29736	0.00406	0.03102	-0.00436	0.09851	-0.01702	0.27278	0.00536	0.03061	0.01234	0.14940
			λ	0.04629	0.10453	0.01975	0.02492	0.04551	0.02152	0.04527	0.06005	0.08555	0.02908	0.02836	0.01743
100	0.5	50	α	-0.01355	0.00154	0.01109	0.00197	0.01452	0.00166	-0.01328	0.00150	0.02497	0.00217	0.01389	0.00169
			β	-0.01736	0.24951	-0.00221	0.01235	0.01687	0.04560	0.00496	0.14935	0.00256	0.01465	0.00160	0.02226
			λ	-0.01096	0.08405	0.00167	0.00875	-0.00942	0.00835	-0.01235	0.02543	0.03781	0.01095	0.01408	0.01015
150	0.5	50	α	-0.00616	0.00118	0.01142	0.00125	0.01308	0.00107	-0.00594	0.00086	0.02071	0.00135	0.01311	0.00100
			β	0.01367	0.21963	0.00208	0.00735	0.00502	0.02452	0.01459	0.08353	0.00372	0.00738	0.00528	0.01399
			λ	-0.03757	0.03456	0.00283	0.00632	0.00490	0.00526	-0.03942	0.00915	0.02396	0.00679	0.00281	0.00492
3	0.5	50	α	-0.03478	0.00327	-0.01459	0.00387	0.00337	0.00361	-0.03468	0.00311	0.00935	0.00427	0.00046	0.00339
			β	-0.00249	0.20393	0.00183	0.08968	0.01124	0.17315	-0.01247	0.16593	0.00207	0.11958	-0.00247	0.10862
			λ	-0.03852	0.22066	-0.00733	0.03950	-0.00248	0.10276	-0.03876	0.20435	0.00770	0.04365	0.00078	0.04806
100	0.5	50	α	-0.02500	0.00160	-0.00363	0.00199	0.00132	0.00165	-0.02498	0.00161	0.00988	0.00205	0.00190	0.00177
			β	-0.00035	0.17435	-0.00680	0.05548	0.00651	0.09257	-0.00255	0.09308	0.01554	0.09410	0.01220	0.13127
			λ	-0.04053	0.57885	-0.00553	0.04404	-0.00931	0.04760	-0.04065	0.13495	0.00392	0.04027	-0.00016	0.12101
150	0.5	50	α	-0.01705	0.00114	-0.00455	0.00121	0.00301	0.00116	-0.01705	0.00100	0.00366	0.00124	0.00119	0.00106
			β	-0.01242	0.16346	-0.02066	0.04118	0.00596	0.05344	-0.00035	0.05952	0.00177	0.05779	0.00150	0.04860
			λ	-0.02932	0.40590	0.00404	0.02209	-0.00063	0.02942	-0.02932	0.08014	0.00912	0.02416	0.00360	0.02043

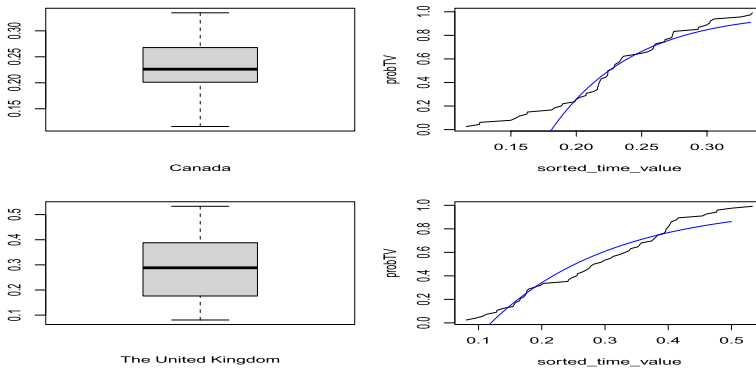


Fig. 5 Boxplot and TTT plot of these data

Table 3 MLE, SE, KS test and *P* values for COVID-19 data of Canada

	α	λ	δ	KS	<i>P</i> Value	W*	A*
OWITL							
Estimate	2.7178	29.9649	6.1262	0.1030	0.5576	0.0699	0.3791
SE	0.4372	116.9170	6.8681				
EOWIR							
Estimate	1.8700	0.0337	0.0420	0.1176	0.5475	0.0707	0.3919
SE	0.4922	0.3806	0.0052				
GIW							
Estimate	0.3850	0.0850	3.6875	0.1703	0.0688	0.3963	2.3662
SE	0.8440	0.6864	0.3405				
Elo							
Estimate	55.9357	111.6019	5.5394	0.1346	0.2396	0.1790	1.1295
SE	22.7480	172.2316	8.8683				
MKEx							
Estimate	3.6641		2.7451	0.1139	0.4300	0.0853	0.4615
SE	0.3845		0.0759				
PL							
Estimate	15.0575	4.1503	0.0433	0.1293	0.2811	0.1867	1.1323
SE	12.6272	0.4413	0.0477				

Firstly: The data represent a COVID-19 data belong to Canada of 56 days, from 1 November to 26 December 2020 [<https://covid19.who.int/>]. these data formed of drought mortality rate. The data are as follows: 0.1622 0.1159 0.1897 0.1260 0.3025 0.2190 0.2075 0.2241 0.2163 0.1262 0.1627 0.2591 0.1989 0.3053 0.2170 0.2241 0.2174 0.2541 0.1997 0.3333 0.2594 0.2230 0.2290 0.1536 0.2024 0.2931 0.2739 0.2607 0.2736 0.2323 0.1563 0.2677 0.2181 0.3019 0.2136 0.2281 0.2346 0.1888 0.2729 0.2162 0.2746 0.2936 0.3259 0.2242 0.1810 0.2679 0.2296 0.2992 0.2464 0.2576 0.2338 0.1499 0.2075 0.1834 0.3347 0.2362.

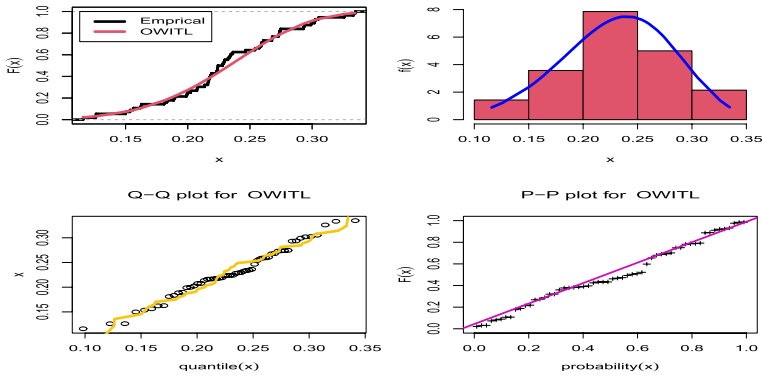


Fig. 6 Cumulative function and empirical cdf, histogram, Q-Q plot and P-P plot for the OWITL distribution for COVID-19 data of Canada

Table 4 MLE, SE, KS test and *P* values for COVID-19 data of The United Kingdom

	α	λ	δ	KS	<i>P</i> Value	W*	A*
OWITL							
Estimate	1.2330	0.7076	13.1360	0.1021	0.5262	0.0817	0.5149
SE	0.3265	0.9580	9.8608				
EOWIR							
Estimate	0.3763	- 0.7678	0.1165	0.1323	0.2240	0.1181	0.6706
SE	0.0541	0.0436	0.0052				
GIW							
Estimate	0.4682	0.1871	2.0434	0.1509	0.1171	0.3966	2.2908
SE	4.1902	3.4222	0.1887				
Elo							
Estimate	7.3456	64.4861	6.9706	0.1029	0.5128	0.2072	1.1858
SE	1.9085	95.1558	10.6752				
MKE_x							
Estimate	2.0555		2.0902	0.1042	0.4994	0.0866	0.5468
SE	0.2206		0.0982				
PL							
Estimate	278.7158	2.7716	12.3695	0.1089	0.4441	0.1050	0.6327
SE	1479.239	0.2891	65.4890				

Secondly: The data represent a COVID-19 data belong to The United Kingdom of 60 days, from 1 December 2020 to 29 January 2021 [https://covid19.who.int/]. These data formed of drought mortality rate. The data are as follows: 0.1292 0.3805 0.4049 0.2564 0.3091 0.2413 0.1390 0.1127 0.3547 0.3126 0.2991 0.2428 0.2942 0.0807 0.1285 0.2775 0.3311 0.2825 0.2559 0.2756 0.1652 0.1072 0.3383 0.3575 0.2708 0.2649 0.0961 0.1565 0.1580 0.1981 0.4154 0.3990 0.2483 0.1762 0.1760 0.1543 0.3238 0.3771 0.4132 0.4602 0.3523 0.1882 0.1742 0.4033 0.4999 0.3930

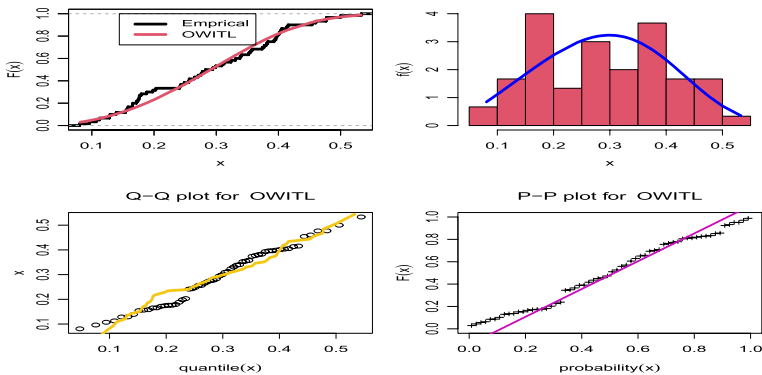


Fig. 7 Cumulative function and empirical cdf, histogram, Q–Q plot and P–P plot for the OWITL distribution for COVID-19 data of The United Kingdom

0.3963 0.3960 0.2029 0.1791 0.4768 0.5331 0.3739 0.4015 0.3828 0.1718 0.1657
0.4542 0.4772 0.3402.

7 Summary

We suggest a new three-parameter model in this paper, called the odd Weibull inverted Topp–Leone distribution, which can be denoted as an OWITL distribution. The distribution of OWITL is motivated by the wide use in life testing of the ITL model and provides more flexibility for evaluate lifetime data. Survival function, hazard function, linear, quantile representation, and OWITL distribution moments are given. We compare the methods of MLE, LSE, MPSE, WLSE, CVME, and ADE and conclude that the alternative methods of MLE are better than the MLE method. In the sense of statistics, we have two implementations of the OWITL distribution for COVID-19 data. The OWITL distribution estimation parameters are derived from MLE, LSE, MPSE, WLSE, CVME, and ADE. Estimation methods are used to estimate the parameters of the model and results of the simulation are given to test the performance of the model. The proposed model of two real-life data offers a consistently better fit than the distributions EOWIR, GIW, WLo, MKEx, and PL.

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Declarations

Funding The author received no specific funding for this study.

Availability of data and material The data is included in Section 6. Application of Real Data Analysis.

Code availability Function “mle2” of “bbml” package in the R program has been used.

Conflict of interest The author declares that they have no conflicts of interest to report regarding the present study.

Authors' contributions Modeling for the COVID-19 distribution in the United Kingdom and Canada is studied. A new lifetime distribution with a three-parameter of odd Weibull inverted Topp–Leone is introduced. Important properties are studied. Parameter estimation was obtained by using different estimation methods.

Ethical statements All of the followed procedures were in accordance with the ethical and scientific standards. This article does not contain any studies with human participants performed by the author.

References

1. Anake TA, Oguntunde PE, Odetunmbi OA (2015) On a fractional beta-distribution. *Int J Math Comput* 26(1):26–34
2. Abd AL-Fattah AM, El-Helbawy AA, Al-Dayian GR (2017) Inverted Kumaraswamy distribution: properties and estimation. *Pak J Stat* 33(1):37–61
3. Barco KVP, Mazucheli J, Janeiro V (2017) The inverse power Lindley distribution. *Commun Stat Simul Comput* 46(8):6308–6323
4. Hassan AS, Abd-Allah M (2019) On the inverse power Lomax distribution. *Ann Data Sci* 6(2):259–278
5. Hassan AS, Mohamed RE (2019) Parameter estimation for inverted exponentiated Lomax distribution with right censored data. *Gazi Univ J Sci* 32(4):1370–1386
6. Muhammed HZ (2019) On the inverted Topp Leone distribution. *Int J Reliab Appl* 20(1):17–28
7. Chesneau C, Tomy L, Gillariose J, Jamal F (2020) The inverted modified Lindley distribution. *J Stat Theory Pract* 14(3):1–17
8. Usman RM, ul Haq MA (2020) The Marshall–Olkin extended inverted Kumaraswamy distribution: theory and applications. *J King Saud Univ Sci* 32(1):356–365
9. Eferhonore EE, THOMAS J, ZELIBE SC (2020) Theoretical analysis of the Weibull alpha power inverted exponential distribution: properties and applications. *Gazi Univ J Sci* 33(1):265–277
10. Kumar S (2020) Monitoring novel corona virus (COVID-19) infections in India by cluster analysis. *Ann Data Sci* 7:417–425
11. Khakharia A, Shah V, Jain S, Shah J, Tiwari A, Daphal P, Mehendale N (2021) Outbreak prediction of COVID-19 for dense and populated countries using machine learning. *Ann Data Sci* 8(1):1–19
12. Li J, Guo K, Viedma EH, Lee H, Liu J, Zhong N, Shi Y (2020) Culture versus policy: more global collaboration to effectively combat COVID-19. *The Innovation* 1(2):100023. <https://doi.org/10.1016/j.xinn.2020.100023>
13. Liu Y, Gu Z, Xia S, Shi B, Zhou XN, Shi Y, Liu J (2020) What are the underlying transmission patterns of COVID-19 outbreak? An age-specific social contact characterization. *ClinicalMedicine* 22:100354
14. Wang YXJ (2020) A call for caution in extrapolating chest CT sensitivity for COVID-19 derived from hospital data to patients among general population. *Quant Imaging Med Surg* 10(3):798
15. Lalmuanawma S, Hussain J, Chhakchhuak L (2020) Applications of machine learning and artificial intelligence for Covid-19 (SARS-CoV-2) pandemic: a review. *Chaos, Solitons Fractals*, Amsterdam, p 110059
16. Bullock J, Luccioni A, Pham KH, Lam CSN, Luengo-Oroz M (2020) Mapping the landscape of artificial intelligence applications against COVID-19. *J Artif Intell Res* 69:807–845
17. Hassan AS, Elgarhy M, Ragab R (2020) Statistical properties and estimation of inverted Topp–Leone distribution. *J Stat Appl Probab* (forthcoming)
18. Kumar CS, Dharmaja SHS (2017) The exponentiated reduced Kies distribution: properties and applications. *Commun Stat Theory Methods* 46(17):8778–8790
19. Dey S, Nassar M, Kumar D (2019) Moments and estimation of reduced Kies distribution based on progressive type-II right censored order statistics. *Hacet J Math Stat* 48(1):332–350
20. Bourguignon M, Silva RB, Cordeiro GM (2014) The Weibull-G family of probability distributions. *J Data sci* 12(1):53–68
21. Al-Babtain AA, Shakhatareh MK, Nassar M, Afify AZ (2020) A new modified Kies family: properties, estimation under complete and type-II censored samples, and engineering applications. *Mathematics* 8(8):1345
22. Almetwally EM, Alharbi R, Alnagar D, Hafez EH (2021) A new inverted Topp–Leone distribution: applications to the COVID-19 mortality rate in two different countries. *Axioms* 10(1):25
23. Cheng RCH, Amin NAK (1983) Estimating parameters in continuous univariate distributions with a shifted origin. *J R Stat Soc Ser B (Methodol)* 45(3):394–403

24. Singh RK, Singh SK, Singh U (2016) Maximum product spacings method for the estimation of parameters of generalized inverted exponential distribution under Progressive Type II Censoring. *J Stat Manag Syst* 19(2):219–245
25. Basu S, Singh SK, Singh U (2019) Estimation of inverse Lindley distribution using product of spacings function for hybrid censored data. *Methodol Comput Appl Probab* 21(4):1377–1394
26. Almetwally EM, Almongy HM, ElSherpieny EA (2019) Adaptive type-II progressive censoring schemes based on maximum product spacing with application of generalized Rayleigh distribution. *J Data Sci* 17(4):802–831
27. El-Sherpieny ESA, Almetwally EM, Muhammed HZ (2020) Progressive type-II hybrid censored schemes based on maximum product spacing with application to Power Lomax distribution. *Physica A* 553(1):124251
28. Alshenawy R, Sabry MA, Almetwally EM, Almongy HM (2021) Product spacing of stress-strength under progressive hybrid censored for exponentiated-Gumbel distribution. *Comput Mater Continua* 66(3):2973–2995
29. Alshenawy R, Al-Alwan A, Almetwally EM, Afify AZ, Almongy HM (2020) Progressive type-II censoring schemes of extended odd Weibull exponential distribution with applications in medicine and engineering. *Mathematics* 8(10):1679
30. Swain JJ, Venkatraman S, Wilson JR (1988) Least-squares estimation of distribution functions in Johnson's translation system. *J Stat Comput Simul* 29(4):271–297
31. Cramér H (1928) On the composition of elementary errors: first paper: mathematical deductions. *Scand Actuar J* 1928(1):13–74
32. Von Mises RE (1928) *Wahrscheinlichkeit Statistik und Wahrheit*. Springer, Basel
33. Luceño A (2006) Fitting the generalized Pareto distribution to data using maximum goodness of fit estimators. *Comput Stat Data Anal* 51(2):904–917
34. Almetwally EM, Muhammed HZ, El-Sherpieny ESA (2020) Bivariate Weibull distribution: properties and different methods of estimation. *Ann Data Sci* 7(1):163–193
35. Muhammed HZ, Almetwally EM (2020) Bayesian and non-Bayesian estimation for the bivariate inverse weibull distribution under progressive type-II censoring. *Ann Data Sci* 1–20 (To Appear)
36. Kim JM, Ju H, Jung Y (2020) Copula approach for developing a biomarker panel for prediction of dengue hemorrhagic fever. *Ann Data Sci* 7(4):697–712
37. Almetwally EM, Almongy HM, Rastogi MK, Ibrahim M (2020) Maximum product spacing estimation of Weibull distribution under adaptive type-II progressive censoring schemes. *Ann Data Sci* 7(2):257–279
38. Aslam M, Yousaf R, Ali S (2020) Bayesian estimation of transmuted pareto distribution for complete and censored data. *Ann Data Sci* 7(4):663–695
39. Almetwally EM (2021) Extended odd Weibull inverse Rayleigh distribution with application on carbon fibres. *Math Sci Lett* 10(1):5–14
40. De Gusmao FR, Ortega EM, Cordeiro GM (2011) The generalized inverse Weibull distribution. *Stat Pap* 52(3):591–619
41. El-Bassiouny AH, Abdo NF, Shahan HS (2015) Exponential Lomax distribution. *Int J Comput Appl* 121(13):24–29
42. Rady EHA, Hassanein WA, Elhaddad TA (2016) The power Lomax distribution with an application to bladder cancer data. *SpringerPlus* 5(1):1–22
43. Aarset MV (1987) How to identify a bathtub hazard rate. *IEEE Trans Reliab* 36(1):106–108
44. Almongy HM, Almetwally EM (2020) Robust estimation methods of generalized exponential distribution with outliers. *Pak J Stat Oper Res* 16(3):545–559
45. Almetwally E, Almongy H (2018) Comparison between M estimation, S estimation, and MM estimation methods of robust estimation with application and simulation. *Int J Math Arch* 9(11):1–9
46. Olson DL, Shi Y, Shi Y (2007) *Introduction to business data mining*, vol 10. McGraw-Hill/Irwin, New York, pp 2250–2254
47. Shi Y, Tian Y, Kou G, Peng Y, Li J (2011) *Optimization based data mining: theory and applications*. Springer, Berlin
48. Tien JM (2017) Internet of things, real-time decision making, and artificial intelligence. *Ann Data Sci* 4(2):149–178