THE ONE-DIMENSIONAL HUBBARD MODEL

The description of a solid at a microscopic level is complex, involving the interaction of a huge number of its constituents, such as ions or electrons. It is impossible to solve the corresponding many-body problems analytically or numerically, although much insight can be gained from the analysis of simplified models. An important example is the Hubbard model, which describes interacting electrons in narrow energy bands, and which has been applied to problems as diverse as high-T_c superconductivity, band magnetism and the metal-insulator transition.

Remarkably, the one-dimensional Hubbard model can be solved exactly using the Bethe ansatz method. The resulting solution has become a laboratory for theoretical studies of non-perturbative effects in strongly correlated electron systems. Many methods devised to analyse such effects have been applied to this model, both to provide complementary insight into what is known from the exact solution and as an ultimate test of their quality.

This book presents a coherent, self-contained account of the exact solution of the Hubbard model in one dimension. The early chapters develop a self-contained introduction to Bethe's ansatz and its application to the one-dimensional Hubbard model, and will be accessible to beginning graduate students with a basic knowledge of quantum mechanics and statistical mechanics. The later chapters address more advanced topics, and are intended as a guide for researchers to some of the more recent scientific results in the field of integrable models.

The authors are distinguished researchers in the field of condensed matter physics and integrable systems, and have contributed significantly to the present understanding of the one-dimensional Hubbard model. FABIAN ESSLER is a University Lecturer in Condensed Matter Theory at Oxford University. HOLGER FRAHM is Professor of Theoretical Physics at the University of Hannover. FRANK GÖHMANN is a Lecturer at Wuppertal University, Germany. ANDREAS KLÜMPER is Professor of Theoretical Physics at Wuppertal University. VLADIMIR KOREPIN is Professor at the Yang Institute for Theoretical Physics, State University of New York at Stony Brook, and author of *Quantum Inverse Scattering Method and Correlation Functions* (Cambridge, 1993).

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Preface

On account of Lieb and Wu's 1968 Bethe ansatz solution, the one-dimensional Hubbard model has become a laboratory for theoretical studies of non-perturbative effects in strongly correlated electron systems. Many of the tools available for the analysis of such systems have been applied to this model, both to provide complementary insights to what is known from the exact solution or as an ultimate test of their quality. In parallel, due to the synthesis of new quasi one-dimensional materials and the refinement of experimental techniques, the one-dimensional Hubbard model has evolved from a toy model to a paradigm of experimental relevance for strongly correlated electron systems.

Due to the ongoing efforts to improve our understanding of one-dimensional correlated electron systems, there exists a large number of review articles and books covering various aspects of the general theory, as well as the Bethe ansatz and field theoretical methods. A collection of these works is listed in the General Bibliography below.

Still we felt – and many of our colleagues shared this view – that there would be a need for a coherent account of all of these aspects in a unified framework and from the perspective of the one-dimensional Hubbard model, which, moreover, would be accessible to beginners in the field. This motivated us to write this volume. It is intended to serve both as a textbook and as a monograph. The first chapters are supposed to provide a self-contained introduction to Bethe's ansatz and its application to the one-dimensional Hubbard model, accessible to beginning graduate students with only a basic knowledge of Quantum Mechanics and Statistical Mechanics. The later chapters address more advanced issues and are intended to guide the interested researcher to some of the more recent scientific developments in the field of integrable models.

Although this book concentrates on the one-dimensional Hubbard model, we would like to stress that the methods used in its solution are general in the sense that they apply equally well to other integrable models, some of which we actually deal with in passing. In fact, the application of Bethe's ansatz to the Hubbard model is more involved than in other cases. We expect the reader who has mastered the solution of the Hubbard model to be able to apply his/her knowledge readily to other integrable theories.

This volume does not pretend to cover its subject completely. Rather, we attempted to find a balance between being didactic and being comprehensive. Our selection of material was necessarily governed by our predispositions. We apologize if we have failed to cover important issues adequately.

Ultimately this book originates in the many collaborations between the authors over the last ten years, which are documented in the reference section at the end of the book. Although the material presented has matured in the discussions between us, it is not difficult to infer from our different styles which author bears primary responsibility for which chapter, namely FG for chapters 2, 3, 11, 12, 14, 15, FHLE for chapters 4–7, 10 and 17, HF for chapters 8 and 9, AK for chapter 13, VEK for chapter 16, and FG and FHLE jointly for chapter 1.

Throughout this project and in many fruitful collaborations before we have benefitted immeasurably from numerous discussions with our colleagues and friends A. M. Tsvelik, N. d'Ambrumenil, T. Deguchi, H. Fehske, F. Gebhard, F. D. M. Haldane, V. I. Inozemtsev, A. R. Its, E. Jeckelmann, G. Jüttner, N. Kawakami, R. M. Konik, E. H. Lieb, S. Lukyanov, M. J. Martins, S. Murakami, A. A. Nersesyan, K. Schoutens, H. Schulz, M. Shiroishi, F. Smirnov, J. Suzuki, M. Takahashi, M. Wadati, A. Weisse and J. Zittartz. Special thanks are due to Andreas Schadschneider for discussions and his constructive criticism after reading the entire manuscript. We are grateful to M. Bortz, A. Fledderjohann, M. Karbach, P. Boykens, A. Grage, M. Hartung, R. M. Konik and A. Seel for proofreading parts of the manuscript and helpful comments.

Despite the joint efforts of many dear friends we do not expect the first edition of such a thick volume to be free of misprints. We plan to keep a record of all misprints brought to our knowledge on our personal websites.

We thank the Physics Departments at Brookhaven National Laboratory and the Universities of Bayreuth, Dortmund, Hannover, Stony Brook, Warwick and Wuppertal for providing stimulating environments during the course of writing this book.

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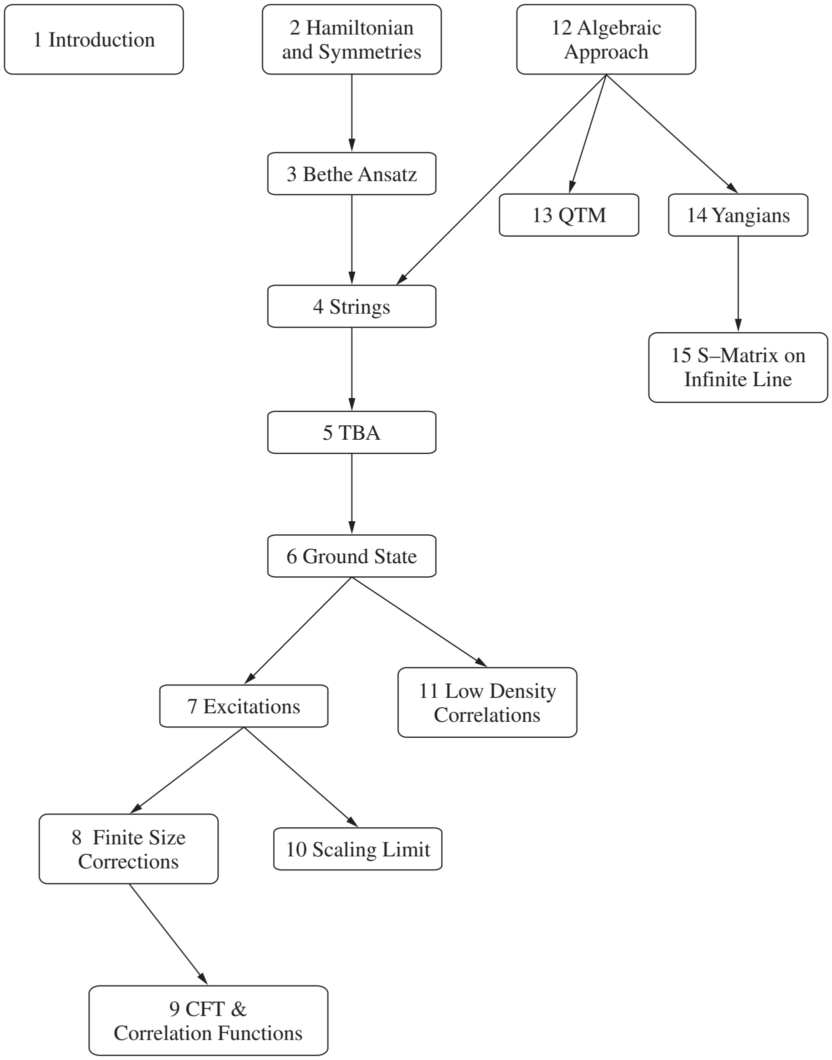
Instead of a reading guide



The figure shows the logical interdependence of the chapters and may serve the reader to find individual paths through this book. Chapters 16 and 17 have the character of appendices and are logically independent from the remaining part of the book.