Mon. Not. R. astr. Soc. (1981) **196**, 13–22

The onset of fragmentation of prestellar clouds by H_2 formation

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Received 1980 July 18; in original form 1980 April 17

Summary. The chemical equation for H_2 formation (in non-equilibrium condition) and the energy equation were solved numerically like a two boundary value problem. The values of the chemical parameter x, the gas temperature T and the mass density ρ, were obtained explicitly as functions of time and they were compared with the change in the mean density produced by the free-fall. The ability of H_2 formation to act as a direct non-linear amplifier of density inhomogeneities in prestellar clouds depends critically on the initial values of the number density of particles n_0, the temperature T_0 and the percentage of chemical heat input to the gas, ϵ. In particular, if the remainder of the binding energy of H_2 formation is radiated by the grains, H_2 formation can induce fragmentation in a contracting cloud providing that $T_0 > 60\,\mathrm{K}$ and $n_0 \gtrsim 10^2\,\mathrm{cm}^{-3}$.

1 Introduction

Observations support the idea that interstellar clouds, constituted mainly of atomic hydrogen, dust and traces of neutral and ionized heavy elements, evolve towards denser stages. These occasionally cross the Jeans limit and start to contract under their own self-gravitation. This last period is rather well understood, at least in the case of a spherical and initially homogeneous cloud, and an abundant literature has appeared since the classical paper of McVittie (1956). However, two fundamental problems remain to be clarified: (a) the onset of the collapse inself and (b) the fragmentation problem.

According to Clayton (1978), the compression produced by the large-scale galactic shock (Roberts 1969), is probably the mechanism by which atomic clouds reach the verge of gravitational collapse. According to Larson (1977), the bulk of the fragmentation would have to occur as a one shot process and during the initial collapse of the cloud.

There are reasons to believe that the fragmentation process of prestellar clouds is not simply a consequence of the dynamical (gravitational) properties. Many of the physical processes and fields involved, like turbulence, chemistry, radiation, magnetic and gravitational fields, probably play their own role. A theory that involves many of these aspects, fulfils the Larson's requirements, and in addition predicts remarkably well the Salpeter (1955) initial mass function and other observational properties of star formation, has been

outlined by Reddish (1975, 1977, 1978), in which the formation of the hydrogen molecule plays a crucial role as the instigator of the fragmentation process.

Reddish appealed to an early paper of Schatzman (1958) who proposed a kind of instability related to the change in number of particles in an isothermal gas as a result of recombination. On the other hand, Giaretta (1977) has proved that no instability takes place, at least under those particular conditions assumed. However, in the Reddish model the dust temperature is the critical parameter in determining the rate at which H_2 can form on dust grain and in the Schatzman problem the recombination is controlled by the gas parameters which also control the UV field. Therefore, the two physical situations are different.

The starting point of Reddish's scenario is a cloud, initially of pure atomic hydrogen and dust particles, at the verge of gravitational contraction, in a turbulent (subsonic) state which provides the mechanism generator of density fluctuations*. The mean diffuse galactic radiation field incident on the boundary surface of the cloud and the 2.7 K background radiation field is also present. Initially, the particle number density is probably $n \gtrsim 10^2\ \mathrm{cm}^{-3}$ and the gas temperature $T \simeq 10^2$ K.

As shown by Solomon (1969), Solomon & Wickramasinghe (1969) and Federman, Glassgold & Kwan (1979) in clouds with n and T in the above ranges, whenever the dust temperature T_d is lower than a critical temperature T_{cri}, hydrogen atoms recombine on dust grains to form H_2 and UV dissociating radiation is completely ineffective. Therefore, the only factor governing the recombination is T_{cri}. Graphite, for instance, according to the experimental results of Lee (1975) has $T_{cri} = 25$ K.

The dust temperature at any point in a cloud is determined by the local energy balance equation. At $T \lesssim 150$ K and $n < 10^6\ \mathrm{cm}^{-3}$ the transfer of energy by dust gas collision is negligible in comparison with the radiation heating (Hayashi & Nakano 1965). For $n \lesssim 10^4\ \mathrm{cm}^{-3}$ the energy by H_2 formation input can also be neglected, if the local radiation field can maintain the dust temperature T_d above about 9 K (Solomon & Wickramasinghe 1969). This temperature would represent an upper limit because it was obtained assuming that all the binding energy can be absorbed by the grain. However, according to Hunter & Watson (1978), H_2 probably recombines into high rotational states and new molecules return to the gas in rotational states $J \gtrsim 7$. Therefore, the dust temperature in Reddish's cloud would be determined by the common radiative heating equation.

A random density distribution is probably a common characteristic in real interstellar clouds (Zuckerman & Palmer 1974; Zuckerman & Evans 1974). Thus, 'tongues' of radiation penetrate into the clouds producing an irregular T_d distribution. This means that patches in the cloud become cold enough ($T_d < T_{cri}$) and H_2 formation starts there first.

According to Ibañez (1979), density fluctuations with optical thickness $\lambda_d \bar{\kappa}/2 < \pi/2$ (λ_d and $\bar{\kappa}$ being the wavelength of density fluctuation and a mean extinction by dust respectively) are unable to produce appreciable fluctuation $\Delta T_d/T_{cri}$. Fluctuations with $\lambda_d \bar{\kappa}/2 > \pi$ mainly determine the regions where T_d reaches the value T_{cri} first and H_2 formation starts there. Therefore, to a first approximation, one can consider that the H_2 formation occurs discontinuously in cells of linear dimensions $\lambda_d/2 \simeq \pi/2\bar{\kappa}$, towards regions where $\bar{T}_d = T_{cri}$, probably towards the centre of the cloud and towards the peaks of fluctuations at scale $\lambda_d/2 > \pi/\bar{\kappa}$. Hence, the crucial point in the Reddish scenario for fragmentation would be whether or not, regions with $T_d < T_{cri}$ can become effectively denser and cooler than regions where $T_d > T_{cri}$, and in a time shorter than the free-fall time. This problem is the aim of this paper.

* This seems to be the state in which H I clouds are left by the compression of the stationary spiral density wave of the galaxy (Roberts 1969; Clayton 1978).

2 Basic equations and their solution

The mean density of the cloud, at least at the early stage of the contraction, is determined by the background contraction of the cloud as a whole. The shortest limit for this process is the free-fall which is determined by the equation

$$\frac{1}{\bar{\rho}}\frac{d\bar{\rho}}{dt} = [24G\bar{\rho}_0]^{1/2}\{\bar{\rho}/\bar{\rho}_0[1-(\bar{\rho}_0/\bar{\rho})^{1/3}]\}^{1/2}; \tag{1}$$

where $\bar{\rho}$ is the mean density at any time t and $\bar{\rho}_0$ is the mean density at time $t = 0$, i.e. when the contraction starts.

Subsonic turbulence provides density fluctuations superimposed on the mean value $\bar{\rho}$, with a certain spectrum ranging from the size of the cloud to the threshold imposed by viscosity.

To determine whether or not the Reddish condition for fragmentation is satisfied, one would compare the change of the mean density $\bar{\rho}(t)$ with the change of the local values of ρ and T produced by H_2 formation in cells of dimensions $\lambda_d/2 \simeq \pi/2\bar{\kappa}$. It will be assumed that at $t = 0$, the chemical reaction starts simultaneously with the gravitational contraction.

The state equation of the gas is

$$P = \frac{R}{2}(1+x)\rho T, \tag{2}$$

where P, ρ and T are pressure, mass density and temperature of the gas, R the gas constant and $x = n_H/n$ is the chemical parameter indicating the advance of the reaction, n_H and n are the number density of atomic hydrogen and total number of atoms respectively.

The rate of hydrogen recombination on the grain surface is given by

$$\frac{dx}{dt} = -\gamma\langle\sigma_d\rangle(3k/m_H^3)^{1/2}(n_d/n)T^{1/2}x, \tag{3}$$

γ being the total recombination efficiency which depends on the sticking probability, the recombination efficiency (Hollenbach, Werner & Salpeter 1971) and probably on the surface contamination with H_2 (Marenco *et al.* 1972). In the range of interest $T \simeq 10^2$ K and $n \gtrsim 10^2$ cm^{-3}, $\gamma = 0.0$ if $T_d > T_{cri}$ and $\gamma = 0.5$ if $T_d < T_{cri}$ (Solomon & Wickramasinghe 1969); $\langle\sigma_d\rangle$ is the mean geometrical cross-section of a single grain, n_d is number density per cm^{-3} of dust particles, k and m_H are the Boltzmann constant and hydrogen mass respectively.

In addition to equations (2) and (3), the energy equation is introduced, i.e.

$$\rho\frac{du}{dt} = \Gamma - \Lambda + \frac{P}{\rho}\frac{d\rho}{dt}, \tag{4}$$

Γ and Λ being the heating and cooling rates respectively and u being the internal energy per unit mass given by

$$u(x,\rho,T) = \frac{5+x}{4}RT + \epsilon N_0\chi x, \tag{5}$$

where ϵ is the fraction of the binding energy $\chi(= 4.477$ eV) going to heat the gas and N_0 the Avogadro number.

In order to evaluate only the effects of the H_2 formation in a cell of dimensions $\lambda_d/2 = \pi/2\bar{\kappa}$, the gas will be considered at $t < 0$, with $\Gamma - \Lambda = 0$. This adjusts the gas temperature at some value $T_0(\simeq 10^2$ K). At times $t \geqslant 0$, $\Gamma - \Lambda = \Lambda_{H_2}$ and from equations (4) and (5) one

obtains the relation

$$\frac{R}{4}\rho(5+x)\frac{dT}{dt}+\rho\left(\frac{R}{4}T+\epsilon N_0\chi\right)\frac{dx}{dt}=\Lambda_{H_2}+\frac{P}{\rho}\frac{d\rho}{dt}. \tag{6}$$

Physically, equation (6) means that only the compressional heating $(P/\rho)(d\rho/dt)$ and the net cooling (or heating) due to H_2 molecules, i.e. $\Lambda_{H_2}-\epsilon\rho\chi N_0(dx/dt)$ are under consideration.

Equations (2), (3) and (6) provide the necessary relations to find x, ρ and T as functions of t with initial conditions

$$x(0)=1.0; \quad \rho(0)=\rho_0 \quad \text{and} \quad T(0)=T_0. \tag{7}$$

Defining the dimensionless density $\tilde{\rho}$, temperature $\tilde{T}$ and time $\tilde{t}$ by the equations

$$\tilde{\rho}(t)=\rho(t)/\rho_0, \quad \tilde{T}(t)=T(t)/T_0, \quad \tilde{t}=t/t_{ff}, \tag{8}$$

where t_{ff} is the free-fall time given by

$$t_{ff}=[3\pi/(32G\bar{\rho}_0)]^{1/2} \tag{9}$$

from equations (2), (3) and (6) one obtains the following relations

$$\frac{dx}{dt}=-\theta_0\tilde{T}^{1/2}\tilde{\rho}x, \tag{10a}$$

$$\frac{d\tilde{\rho}}{dt}=\theta_0\left\{\frac{1-x}{(1+\xi)(7+3x)}\frac{\tilde{\rho}}{\tilde{T}}g(\tilde{T})\exp(-\alpha_0/\tilde{T})+\left(\frac{4}{1+x}-\frac{\gamma_0}{\tilde{T}}\right)\frac{x\tilde{T}^{1/2}\tilde{\rho}^2}{7+3x}\right\}, \tag{10b}$$

$$\frac{d\tilde{T}}{d\tilde{t}}=\theta_0\left\{-c_0\frac{1-x}{(1+\xi)(7+3x)}g(\tilde{T})\exp(-\alpha_0/\tilde{T})+(3+\gamma_0/\tilde{T})\frac{x\tilde{T}^{3/2}\tilde{\rho}}{7+3x}\right\}, \tag{10c}$$

where

$$\theta_0=\gamma\langle\sigma_d\rangle(3k/m_H^3)^{1/2}(n_d/n)T_0^{1/2}\rho_0 t_{ff}, \tag{11a}$$

$$c_0=10.0t_{ff}/(RT_0\theta_0), \tag{11b}$$

$$\alpha_0=512.0/T_0, \tag{11c}$$

$$\gamma_0=4\chi N_0\epsilon/(RT_0), \tag{11d}$$

$$\xi=\frac{\beta_0}{(1+19x)\tilde{T}^{1/2}\tilde{\rho}}, \tag{11e}$$

$$\beta_0=840.0/(n_0T_0^{1/2}) \tag{11f}$$

and $g(\tilde{T})$ is a correction factor for the cooling function of the H_2 molecule, which is greater than 1.0 when $T\geqslant 150$ K, due to excitation of rotational levels greater than $J=2$ (Hattori, Nakano & Hayashi 1969).

In view of the tendency towards pressure equilibrium, the condition of isobaricity will be introduced in this first approximation. One must recognize that although this condition is not very realistic (mainly because the effect of both reduction of the number of particles and the presence of the new coolant produces reduction in pressure with respect to the gravitational force. This results in an inwards motion) it permits one to gain insight into the effects of H_2 formation. It also avoids the difficult gas dynamical problem which appears if

one removes this condition for a contracting, reacting and turbulent dusty medium; where the reaction rate is controlled by the radiative field through the dust temperature.

The system of equations (10) with initial conditions (7) has been solved numerically like a two boundary value problem, using Merson's method and Newton iteration.

3 Results

Runs were done for a mean dust cross-section $\langle \sigma_d \rangle = 7.1 \times 10^{-10}\,\mathrm{cm}^2$. (Solomon & Wickramasinghe 1969; Greenberg 1979) and for different values of the following parameters: fraction of the binding energy going to heat the gas and dust to gas ratio n_d/n. The range of variation of these parameters and the initial values of gas temperature T_0 and gas particle density n_0 is shown in Table 1.

Table 1.

$x(0) = 1.0$
$T_0 = 120, 100, 60\,\mathrm{K}$
$n_0 = 10^2, 10^3, 5\times10^3, 10^4\,\mathrm{cm}^{-3}$
$10^{-12} < n_d/n < 10^{-11}$ $\quad 0.00 < \epsilon < 0.04^{\star}$

* Upper limit calculated by Hunter & Watson (1978).

Generally speaking, from the numerical solutions one sees that the capacity of the H_2 formation as direct amplifier of density inhomogeneities depends sensitively on: the fraction of chemical heating ϵ, the initial mean density and temperature of the gas at the onset of contraction.

If the H_2 molecule is formed in highly excited rotational levels and the remainder of the binding energy is radiated by the grains $\epsilon = 0$, i.e. no heat input to the gas occurs, the corresponding solutions of equations (10) are those plotted in Figs 1–4. The time-scale for spontaneous radiative decay from these levels to the ground one is of the order of 10^{10} s, which is three orders of magnitude smaller than the time-scales under consideration and therefore the H_2 cooling function adopted in equation (10) is correct to a first approximation.

For $n_d/n = 10^{-12}$ the following results emerge: for $n_0 = 10^2\,\mathrm{cm}^{-3}$ effective density amplification by H_2 formation would require a rather high initial gas temperature, probably $T_0 > 80\,\mathrm{K}$. For example, for $T_0 = 120\,\mathrm{K}$ (Fig. 1b) the amplification produced by H_2 formation would be a factor $\simeq 2.0$ at the time $t = 0.4\,t_{ff}$.

It is necessary to emphasize that the factors of density amplification obtained here would underestimate the true values because the free-fall time is the shortest limit to the gravitational contraction and clouds do not start to collapse free of pressure. In addition, the density amplification by H_2 formation is coupled with that produced by the contraction of the cloud as a whole. This non-trivial and strongly non-linear coupling will be explored in future research.

The situation looks much more favourable in the case $n_0 = 10^3\,\mathrm{cm}^{-3}$ (Fig. 2a and b). For $T_0 > 60\,\mathrm{K}$ the effect of the new coolant becomes important and an effective enhancement in density occurs.

An interesting aspect to note is that the temperature exhibits a maximum at a time depending on the initial values T_0 and n_0. This is because at the start of the H_2 reaction there is not enough H_2 to cool effectively. This maximum scarcely appears at $T_0 = 120\,\mathrm{K}$ (for $n_0 = 10^3\,\mathrm{cm}^{-3}$) (Fig. 2b). At this initial temperature, the enhancement of density appears very strong.

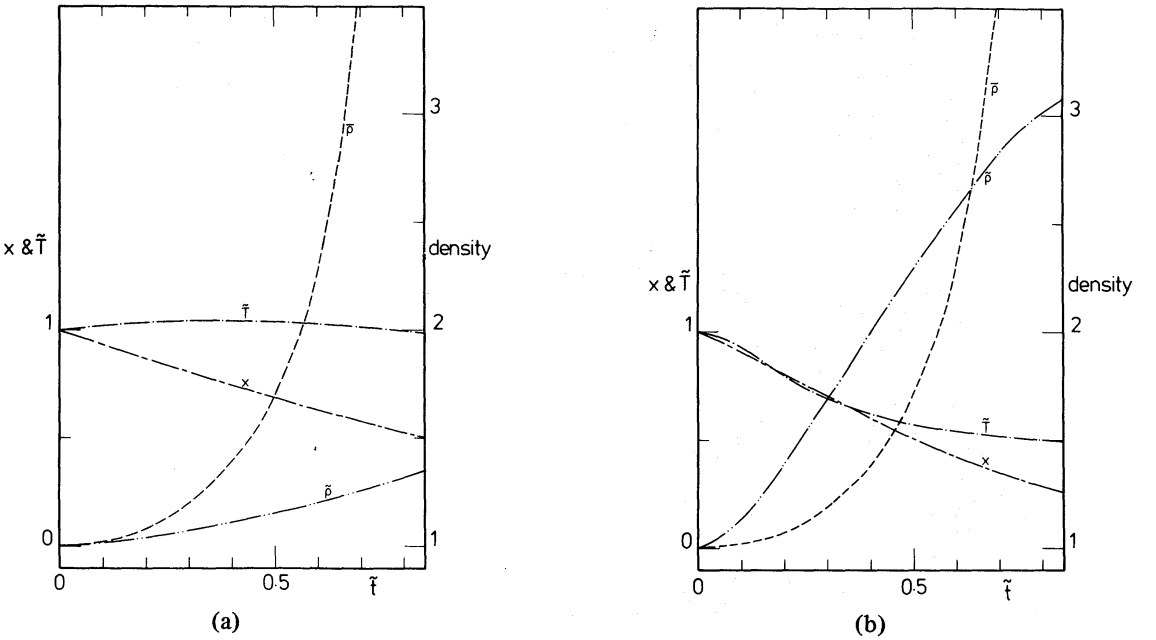

Figure 1. (a) The functions $x(\tilde{t})$, $\tilde{T}(\tilde{t})$, $\tilde{\rho}(\tilde{t})$ and $\bar{\rho}(\tilde{t})$ for $T_0 = 60\,\mathrm{K}$, $n_0 = 10^2\,\mathrm{cm}^{-3}$, $x_0 = 1.0$, $\epsilon = 0.00$ and $n_d/n = 10^{-12}$. (b) The functions $x(\tilde{t})$, $\tilde{T}(\tilde{t})$, $\tilde{\rho}(\tilde{t})$ and $\bar{\rho}(\tilde{t})$ for $T_0 = 1.2 \times 10^2\,\mathrm{K}$, $n_0 = 10^2\,\mathrm{cm}^{-3}$, $x_0 = 1.0$, $\epsilon = 0.00$ and $n_d/n = 10^{-12}$.

At initial density $n_0 = 5.0 \times 10^3\,\mathrm{cm}^{-3}$ (Fig. 3a and b), the effects due *only* to the change of number of particles are already appreciable, as can be seen from Fig. 3(a), where the cooling effects of the H_2 are ineffective because the gas temperature is too low (60 K).

It has been found that the H_2 molecule is unable to cool the gas below about 60 K, the exact temperature depending on n, as is expected from simple physical considerations.

The results for the rather high initial density value of $n_0 = 10^4\,\mathrm{cm}^{-3}$ have been plotted in Fig. 4(a) and (b). The behaviour of x, $\tilde{\rho}$ and $\tilde{T}$ are qualitatively the same as the case $n_0 = 5 \times 10^3\,\mathrm{cm}^{-3}$, but the effects of H_2 formation are shifted towards earlier epochs of the contraction. In particular the atomic hydrogen is exhausted at $t < 0.4\,t_{ff}$.

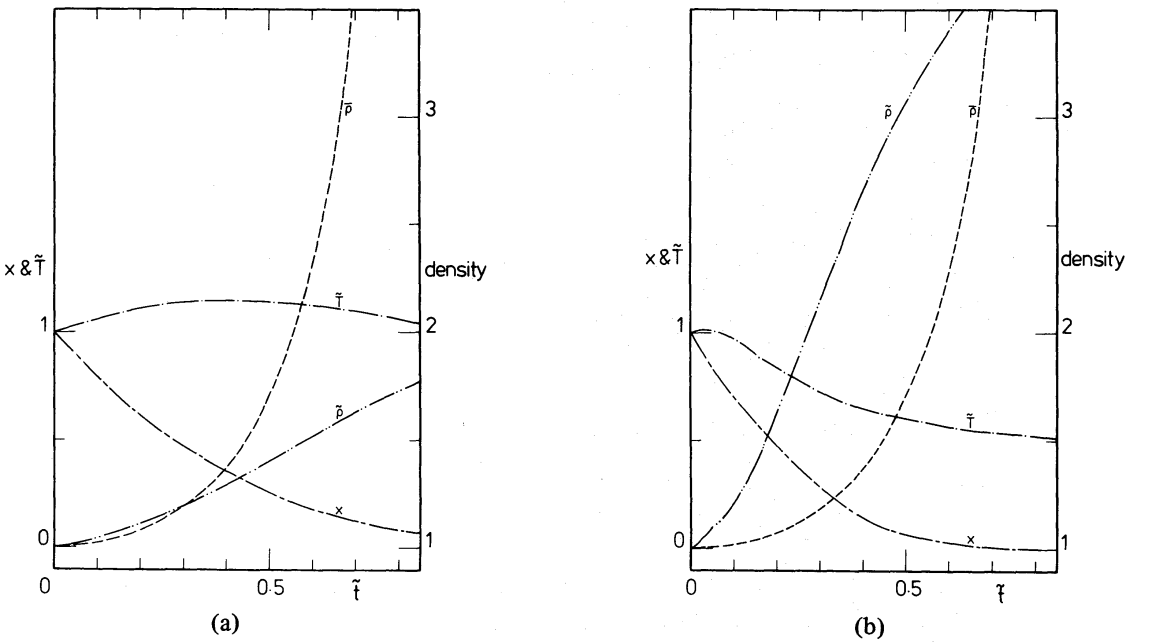

Figure 2. (a) As Fig. 1(a) for $n_0 = 10^3\,\mathrm{cm}^{-3}$. (b) As Fig. 1(b) for $n_0 = 10^3\,\mathrm{cm}^{-3}$.

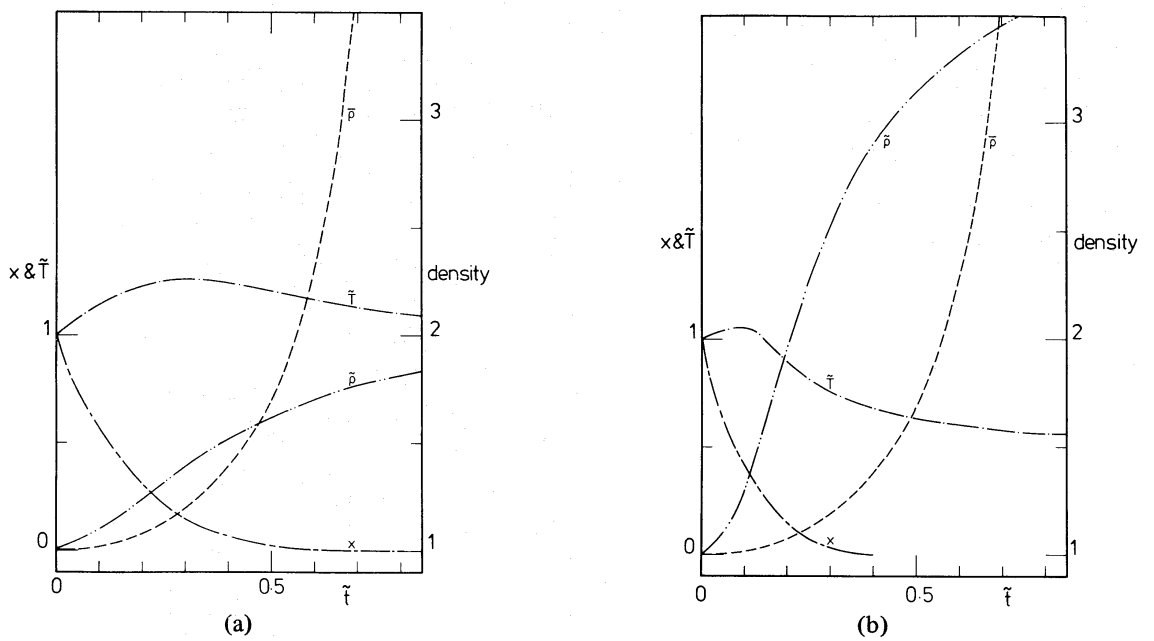

Figure 3. (a) As Fig. 1(a) for $n_0 = 5 \times 10^3$ cm^{-3}. (b) As Fig. 1 (b) for $n_0 = 5 \times 10^3$ cm^{-3}.

For the case $n_d/n = 10^{-11}$ effective enhancement in density due to H_2 formation appears at times $< 0.1\, t_{ff}$, for the whole range of initial density and temperature of the gas given in Table 1. This case could be of particular interest if the clouds reach the verge of gravitational contraction with an irregular dust distribution caused by any of the mechanisms capable of decoupling dust and gas in the interstellar medium, see for instance Harrison (1978) and Flannery & Krook (1978).

The solutions of equations (10) are not altered in essence if $\chi\epsilon \lesssim kT_0/2$, i.e. $\gamma_0 < 2.0$ and therefore, $d\tilde{\rho}/dt > 0$ at $t = 0$. This condition means physically that the chemical heat input

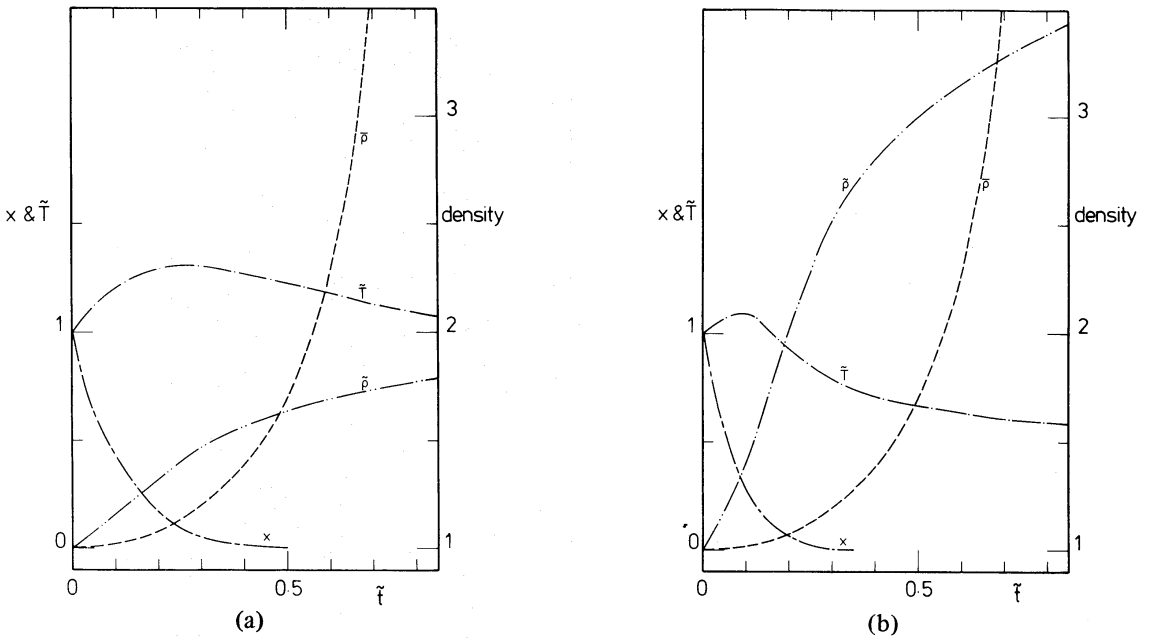

Figure 4. (a) As Fig. 1(a) for $n_0 = 10^4$ cm^{-3}. (b) As Fig. 1(b) for $n_0 = 10^4$ cm^{-3}.

to the gas is less than the thermal energy per particle. For example, for $T_0 \simeq 10^2$ K, $\epsilon \lesssim 1.0 \times 10^{-3}$.

Unfortunately, Hunter & Watson (1978) in finding the limit for ϵ neglected the most probable mechanism by which the remainder of the binding energy is dissipated (Solomon & Wickramasinghe 1969). Therefore, this limit has to be taken strictly as an upper limit. However, runs were done for $\epsilon = 0.04$.

In all the above cases an initial heating and expansion appear followed by a rebound. For $n_d/n = 10^{-12}$ the whole process occurs in times of the order of $0.5\, t_{ff}$ and no effective enhancement in density occurs for initial densities $n_0 \lesssim 10^3$. For $n_0 \gtrsim 5 \times 10^3$, the heating expansion and rebound occur at shorter times $\simeq 0.3\, t_{ff}$ but effective enhancement in density due to H_2 formation would require temperature higher than 60 K.

For $n_0/n = 10^{-11}$, effective density amplification occurs for the range of density under consideration providing that $T_0 > 60$. For temperature of the order of $T_0 \simeq 60$ (the exact value depends on density) the enhancement in density tends to be quenched; this is because at the start of the reaction there are not enough molecules to cool, and the heat input becomes more important than the reduction in the number of particles.

4 Discussion

Although the isobaric condition is not absolutely correct, it has some degree of validity as can be seen from the following simple considerations: The time-scale for a sound wave to travel across a region of dimensions $\lambda_d/4 \simeq \pi/4\bar{\kappa}$ (radius of the cells where $T_d < T_{cri}$) is $t_s = \pi/(4\bar{\kappa}\, c)$, where $\bar{\kappa} = \langle Q_{ext}\sigma_d \rangle n_d$, Q_{ext} being the extinction efficiency factor and c the sound speed given by $c = (\tilde{\gamma}RT)^{1/2}$.

On the other hand, the time-scale for H_2 formation is $t_{H_2} = (3RT)^{1/2}\gamma^{-1}\langle \sigma_d \rangle^{-1} (n_d/n)^{-1} n^{-1}$. Therefore, $t_{H_2}/t_s = (8\pi)\langle Q_{ext}\rangle(\tilde{\gamma}/3)^{1/2}$, where $\tilde{\gamma}/3 = \sqrt{5/9}$ is for an H I gas and $\tilde{\gamma}/3 = \sqrt{7/15}$ for an H_2 gas. As the ratio t_{H_2}/t_s only depends on the gas parameters through the specific heat ratio of the gas, it can be taken with confidence. With the average number $\langle Q_{ext}\rangle \simeq 2$, (Wickramasinghe 1973; Greenberg 1979) and with the mean value of $\tilde{\gamma}$ between a gas of pure H I and one of pure H_2, one has $t_{H_2}/t_s \simeq 3.6$. That is, the pressure in the bubble with $T_d < T_{cri}$ has time to equalize to the external value while the H_2 reaction proceeds.

The validity of the isobaric condition in the non-steady regime, generated by gravitational contraction, is difficult to envisage. On physical grounds it is expected that such a condition provides rather a lower limit for the density enhancement due to H_2 formation because of the strong non-linear coupling between both processes and the accelerated increase of the background mean density due to the contraction of the cloud as a whole. This aspect can only be estimated quantitatively with the help of a self consistent treatment of the formal multi-dimensional gas dynamic problem.

Subsonic turbulence has been invoked as generator of inhomogeneities, particularly density inhomogeneities. While these fluctuations generated by turbulence are reversible, the production of the H_2 (and of molecules depending on this, mainly CO) introduce irreversibility in turbulent elements at scales $\lambda_d/2 = \pi/2\bar{\kappa}$ and at the early stages of the contraction.

Formal studies of the turbulent generation of density fluctuations in a contracting medium (Sasao 1973), have shown that the amplification of density inhomogeneities resulting only from the turbulence–contraction coupling, requires the presence of initially very strong turbulence and effective amplification could only occur at advanced stages of the collapse, $t > 0.9\, t_{ff}$.

It could be argued that, if at length-scales $\lambda_d/2 \simeq \pi/2\bar{\kappa}$ the time-scale characteristic of turbulence t_t is much shorter than the time necessary to allow appreciable H_2 to form and

their effects dominate the very turbulent ones, inhomogeneities could not survive long enough to permit H_2 production to control the situation.

With the assumption of isobaricity ($\bar{P}$ = const.) a qualitative criterion to adopt for determining when H_2 formation effects dominate over the turbulent ones would be that the change in density due to the H_2 formation has to be greater than the density fluctuation produced by turbulence, i.e. $(\delta\rho/\bar{\rho})_{H_2} > (\delta\rho/\bar{\rho})_{turb}$. It seems that this criterion is not very far away from the formal one in a non-isobaric model, because to a first approximation $(\delta\rho/\bar{\rho}) \sim (\delta P/\bar{P})$ for either of the above two different mechanisms and therefore, H_2 formation would dominate over the turbulence if $(\delta P/\bar{P})_{H_2} > (\delta P/\bar{P})_{turb}$. For $(\delta\rho/\bar{\rho})_{turb} \simeq 0.1$ for instance, from the numerical solutions of Section 3 one can seen that a rather wide and sufficient condition to ensure that $(\delta\rho/\bar{\rho})_{H_2} > (\delta\rho/\bar{\rho})_{turb}$, is $t_t \simeq t_{H_2}$, t_t being the time-scale characteristic of turbulence at scales $\lambda_d/2 \simeq \pi/2\bar{\kappa}$. That this is the case, can be seen from the following simple consideration.

From the Kolmogorov spectral law, $t_t \simeq l_0^{1/3}\langle v_0\rangle^{-1}(\lambda_d/2)^{2/3} \simeq l_0^{1/3}\pi^{2/3}\langle Q_{ext}\sigma_d\rangle^{-2/3}\langle 3RT\rangle^{-1/2}(n_d/n)^{-2/3}n^{-2/3}$, where l_0 is the linear dimension of the cloud and $\langle v_0\rangle \simeq \langle 3RT\rangle^{1/2}$, in a first approximation. Therefore, the ratio t_{H_2}/t_t is given by $t_{H_2}/t_t \simeq 2^{2/3}\pi^{-2/3}\langle Q_{ext}\rangle^{-1/3}$, $\bar{\tau}_0$ being the mean optical depth to the centre of the cloud. This ratio is rather insensitive to the product $\langle Q_{ext}\rangle\bar{\tau}_0$. For $\langle Q_{ext}\rangle \simeq 2$ and $\bar{\tau}_0 \simeq 3-10$, one obtains $t_{H_2}/t_t \simeq 0.3$, i.e. turbulent elements at scales $\simeq \pi/2\bar{\kappa}$ survive long enough to permit H_2 formation in quantities such that molecule formation dominates turbulent effects.

Another point deserving some discussion is the adopted form of the function $\Lambda - \Gamma$. As mentioned in Section 2, interest was focused only on the direct effects produced by the formation of H_2 molecules on grains, and for that, only a gas of pure H I was considered. In a more realistic model, however, traces of heavy elements have to be allowed for, mainly C, O, Fe and Si and probably some ions of them. The concentrations of these ions depend critically on the detailed radiative transport at ionizing wavelengths. In the density range of interest, $n \gtrsim 10^2$ cm^{-3} the neutral species probably dominate the atomic cooling. If one goes into this more realistic model, one has to consider all chemistry depending on H_2 presence, mainly the CO chemistry because of the effectiveness of CO as a coolant. In addition, the agents producing the heating function have to be considered. It is likely that this aspect is the most puzzling one due to the uncertainty in Γ, (Spitzer 1978).

It is likely that given the power of cooling of the CO molecule, the presence of this new molecule enhances the effects produced by H_2. The CO molecule is probably responsible for cooling the gas from about 60 K to 5–10 K; temperatures which are observed in dense molecular clouds.

Finally, the results show that if H I clouds reach the verge of their gravitational contraction in a subsonic turbulent state, with gas number density $n \gtrsim 10^2$ cm^{-3} and gas temperature $T \simeq 10^2$ K, their fragmentation would be a consequence of the following chain of processes: (i) Subsonic turbulence provides initial density fluctuations with scales of length ranging between the size of the cloud and the threshold imposed by viscosity. (ii) The radiation field imposes a constraint on the spectral range of density fluctuations able to be amplified effectively in times shorter than the contraction time of the cloud as a whole. This constraint is given by the scale length over which fluctuations of the dust temperature occur around the critical value for H_2 formation i.e. $\pi/(2\bar{\kappa}) \lesssim \lambda_d/2 \lesssim \pi/\bar{\kappa}$. (iii) H_2 formation initiates an irreversible process of density amplification at the above scales and provides the contracting cloud with an initial pattern of fragmentation. The mass range of fragments is then given by $M/M_\odot \simeq 4.4\times10^{-58}\langle Q_{ext}\sigma_d\rangle^{-3}(n_d/n)^{-3}n^{-2}(\lambda_d\bar{\kappa}/2)^3$, where $10^2 \lesssim n < 10^4$ cm^{-3} and $\pi/2 \lesssim \lambda_d\bar{\kappa}/2 \lesssim \pi$. For standard values of galactic clouds the above mass range would be $5.9\times10^{-3} \lesssim M/M_\odot \lesssim 4.8\times10^2$.

Acknowledgments

I express my gratitude to Professor V. C. Reddish for his guidance during my stay at the Department of Astronomy of the University of Edinburgh. I am grateful to the Facultad de Ciencias and to C.D.C.H., Universidad de Los Andes, Venezuela, for financial assistance.

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