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The Optimal Currency Composition of External Debt: Theory and Applications to Mexico and Brazil

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The changes in exchange rates, interest rates, and commodity prices during the past decades have had large impacts on developing countries. Many developing countries have limited access to already incomplete international long-term hedging markets. Thus the question arises whether the currency composition of external debt can be used to minimize exposure to external price risk. Using a utility-maximizing framework, this article shows that, by choosing the optimal currency composition, a country can indeed manage its external exposure. The optimal, risk-minimizing currency composition depends on the relation between export receipts and the costs of borrowings in each currency and on the relations among the costs of borrowings in different currencies. A simple methodology can be used to derive the optimal shares of individual currencies and is applied to Mexico and Brazil. The results show that Mexico and Brazil could have lowered their external exposure to a limited degree by continuously altering the currency composition of their debts. The low correlations between the costs of borrowings and export and import prices make the currency composition of debt a very imperfect hedging tool, and it is likely that hedging instruments directly linked to prices are preferable.

> Other things being equal, a strengthening of the dollar will worsen the terms of trade of net commodity exporters and hence reduce their welfare. For net commodity importers the reverse pattern will hold (Dornbusch 1985, p. 335).
>
> . . . for some developing countries, the fall in the dollar increased the burden of debt relative to their economies (World Bank 1987, p. 49).

Which of these statements about the effect of exchange rate changes on the welfare of developing countries is correct? Even though placed out of context, both quotations illustrate some of the unresolved issues regarding the effect of exchange rate changes. This article aims to clarify some of these issues and to

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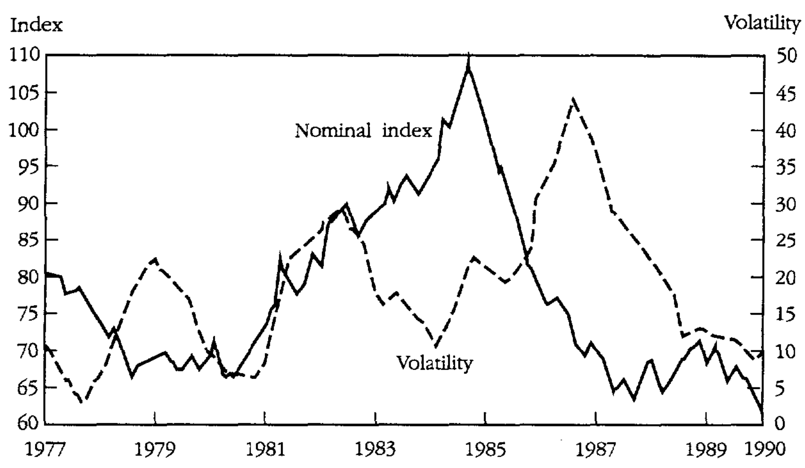
present conceptual and practical guidelines that may help with external debt management in general.

The issue of the optimal currency composition of external debt can be approached from a narrow or a broad perspective. Given the existence of external debt, the narrow perspective attempts to determine its optimal composition. Given not only the presence of debt but also the availability of a range of other international financial contracts, the broader perspective attempts to identify the welfare-maximizing liability structure and the subsequent optimal currency composition. Here we take the broader perspective.

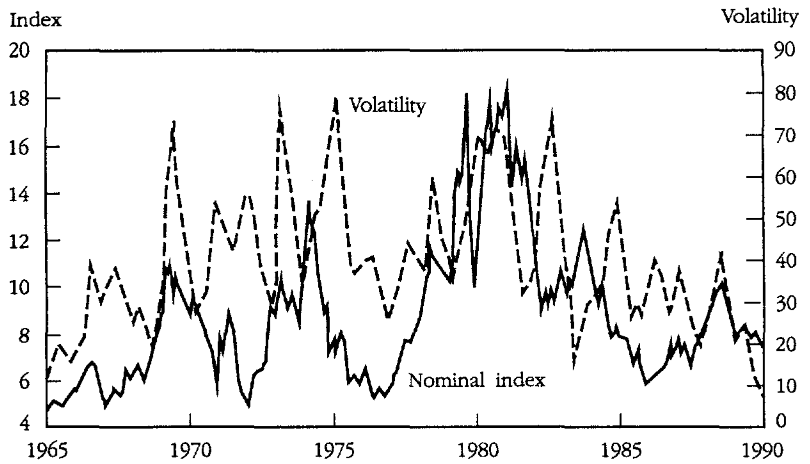
Developing countries in particular have been affected by large changes in exchange rates, interest rates, and prices of international goods during the past decades. The degree of uncertainty in these international variables is illustrated in figures 1, 2, and 3. Figure 1 plots the nominal effective U.S. dollar exchange rate and a measure of the volatility of this rate for 1977–90. Exchange rate volatility, which had increased after the movement to floating exchange rates in the early 1970s, did not decline in the 1980s. Figure 2 plots the nominal interest rate most relevant for developing countries from 1965 to 1990 as well as the coefficient of variation (CV) of the interest rate over the preceding 24 months at each point in time. Nominal interest rates have experienced large fluctuations, and there have been few periods of tranquillity. Figure 3 plots measures of commodity prices and commodity price volatility during 1962–90. Even though part of the price movements can be explained by shifts in the demand for commodities and supply factors, the large fluctuations in commodity prices resulted in large risks to both producers and consumers.

Figure 1. *Nominal Effective U.S. Dollar Exchange Rate: Index and Volatility, 1977-90*



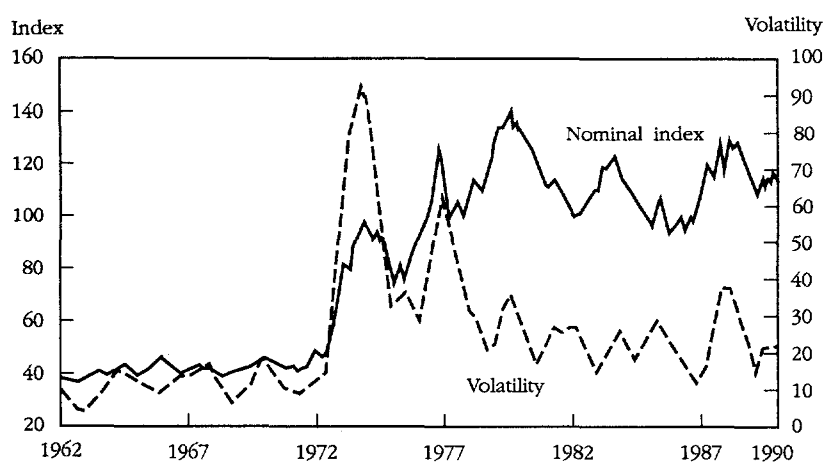
Note: The weights used to create the index of effective exchange rates are the IMF weights. Exchange rate volatility is calculated as the coefficient of variation of the effective exchange rates over the preceding 24-month period. Real effective exchange rates show a very similar pattern for level as well as volatility.
Source: IMF, *International Financial Statistics* (various years).

Figure 2. *Nominal Interest Rate: Index and Volatility, 1965-90*



Note: The nominal rate is the London interbank offer rate (LIBOR) on six-month U.S. dollar deposits (period averages in percent per annum) on which much commercial bank debt is indexed. Interest rate volatility is measured as the coefficient of variation over the preceding 24 months. No distinction is made between expected movements and actual deviations from these expectations; overall variability is considered as risk.
Source: IMF, *International Financial Statistics* (various years).

Figure 3. *Nominal Price Index for Developing Countries' Non-Oil Commodity Exports, 1962-90*



Note: The figure plots the commodity price volatility as the coefficient of variation of the IMF index of 34 nonfuel commodity prices for developing countries in the preceding 24 months.
Source: IMF, *International Financial Statistics* (various years).

For industrial countries, external exposures are small, partly because the export and import patterns of these economies are often diversified. In addition, exposures are largely private, and volatilities are thus not an issue for public-budget management. These volatilities have of course been important issues for industrial countries in the larger context of macroeconomic management, but seldom in the narrow context of their financial impact on the public sector. Only a few industrial countries, such as Norway and the Netherlands, have felt a direct financial impact of commodity price and exchange rate movements on their public sector's budget.

But for many developing countries external exposures are large because of a more concentrated export and import pattern. They are also primarily public or quasi-public (as in the case of public external debts and the exports and imports of state enterprises). The high volatilities have had serious implications for the government budgets of these countries and other costs through their disruption of the economy and associated resource misallocations. This is demonstrated by Indonesia, in which the ratio of debt service to exports rose from 8.2 percent in 1981 to 27.8 percent in 1987. More than 65 percent of the increase could be explained by the depreciation of the U.S. dollar after 1985 and the fall in oil and other commodity prices in 1986 and 1987. If commodity prices and cross-currency exchange rates had remained at their end-of-1982 values, the ratio of debt service to exports for all developing countries would have been approximately 17 percent as opposed to the actual ratio of 24 percent in 1987.

The fact that we observe large volatilities in several external prices (exchange rates, interest rates, and commodity prices) and corresponding large changes in measures of debt service burden draws attention to the importance of properly measuring external exposures. Large volatilities of external variables do not necessarily have to affect a firm or a country adversely. The issue is whether and to what extent the interaction between external price movements and overall external transactions affects the firm or country adversely. In order to determine this, a framework for measuring a country's overall (net) exposure to external price uncertainty is needed; the magnitude of external exposures can then be measured.

Once external exposure is quantified, one can try to manage it by using financial instruments to transfer exposures to other parties more able to absorb them. What type of financial instruments are available to manage external risks? Firms in industrial countries have access to and use many financial instruments to reduce and transfer risks. Examples are futures, options, and swaps on currency and interest rates, and more recently on commodities. In many developing countries, however, neither the private sector nor the public sector has used these instruments to the same degree as firms in industrial countries have to shift risks to (international) financial markets in line with comparative advantage. Some hedging instruments—such as currency, interest, and commodity futures and options—are in principle available to most developing countries. However, these have short maturities and, even when rolled over, provide limited hedging value over longer periods. Furthermore, the problem with rolling-over coverage based on short-term maturing instruments is that this usually implies a large exposure in these instruments. This can imply large margin calls or large option premiums for these instruments, which makes them less attractive for foreign-exchange-constrained countries and reduces them largely to self-insurance instruments. The markets for longer-term hedging instruments—such as currency, commodity, and interest swaps, and especially commodity price-linked instruments—are relatively thin. And these longer-term hedging instruments tend to be unavailable to most developing countries due to institutional, credit, and other constraints.

The presence of large exposures, for which limited or potentially expensive hedging tools are available, raises the question whether a developing country can use alternative means to manage its external exposure. One possibility, of course, is that the country actually diversifies through the sourcing, producing, and exporting of a broader mix of products. However, many developing countries depend heavily on primary exports and may have little room to diversify into other export products as a means to hedge risks. And self-insurance through diversification may not be the most efficient option (because it may not be in line with the country's comparative advantage) and may take a long time to achieve.

A possibly more effective mechanism is to use the currency composition of existing and new external long-term liabilities to reduce external exposure. For example, Kahn (1988) mentions that a number of rescheduling agreements between developing countries and commercial banks have provided currency conversion options for non-U.S. banks. This indicates that, in addition to the possibility of incurring new loans in the currencies of choice, developing countries may be able to alter the currency composition of their existing external debt in rescheduling negotiations, possibly to their own and the commercial banks' advantage. The currency composition of external debt may be able to achieve hedging advantages similar to those of a portfolio of short-term hedging instruments. Changes in external earnings may be offset by simultaneous shifts in the costs of borrowings, but without possible adverse margin calls or premiums. The effectiveness of the currency composition strategy compared with, say, a rolling-over of futures strategy, will depend on the relation among the different external risks. Thus exchange, interest, and commodity price risk management and determination of the currency composition must be integrated.

Several questions follow if a country wants to use the currency composition of external liabilities to manage external exposures. What is the overall objective function to be managed (maximized), and what is the definition of risk that follows from the chosen objective? Which factors play a role in choosing the optimal currency? What kind of rules follow for managing the external liability of a country? The existing literature provides little guidance on these questions. Some objective functions and rules have been suggested for determining the currency denomination of borrowings by a developing country: matching the currency composition of debt with the trade direction or currency composition of export revenues, matching it with the composition of the basket of currencies with which the domestic currency is pegged or managed, and matching it with the currency composition of the foreign exchange flows into the country (see further Lessard and Williamson 1985). However, most of these rules have not explicitly been related to a specific goal or objective, and the risks have not been explicitly defined. In general, most decision rules proposed have been of an ad hoc nature, and detailed guidelines for implementation have not been developed.

This article develops therefore an integrated model for deriving the optimal currency composition for a small open economy facing external uncertainties. The model uses international portfolio theory and identifies the factors that determine the optimal currency decision. The optimal currency composition for a country to hedge itself against commodity price, interest, and exchange rate movements—and not to speculate on relative exchange rate movements—depends only on the covariances between the effective costs of borrowings and the country's exports and on the covariances among the effective costs of borrowings. A simple operational procedure, which relies on the coefficients of appropriately specified ordinary least squares regressions for calculating the optimal currency shares, is developed and applied to Mexico and Brazil.

I. The Portfolio Model

The optimal currency composition of external debt is determined for a small open economy that acts as a price taker in international goods markets and that faces perfect world capital markets. We will use here a simple, two-period model to determine the optimal portfolio investment and consumption decisions. Alternatively, an intertemporal capital asset pricing model—as in Merton (1971), Breeden (1979), and Stulz (1981)—could be used. In that case the model would have features similar to Krugman (1981), Macedo (1982 and 1983), Fraga (1986), and Stulz (1984). Such a model is derived in Claessens (1988).

It is assumed that the country can be represented by one domestic individual who lives two periods, or, alternatively, the government acts benevolently in choosing the optimal liability composition for its own citizens. This approach is appropriate if the private citizens have limited access to foreign capital markets. For many developing countries this is the case and is confirmed by the small amount of private, nonguaranteed debt compared with total public and publicly guaranteed debt (about 15 percent). In cases where the private sector has more access to foreign capital markets, it may be more appropriate for the government to concentrate on managing its own external liabilities optimally with respect to its fiscal revenues and expenditures.

The country has a fixed first-period endowment and can invest in activities that produce goods for export. In the first period the consumer makes investment, consumption, and borrowing decisions based on its expectations of second-period variables. In the second period, the consumer receives payments on its exports, which it uses to finance debt service payments and imports of goods. Export receipts and import payments are uncertain because international commodity prices are uncertain. The consumer maximizes its welfare function defined over this and next period's consumption of each good, c_i, $i = 1, \ldots K$.

The country does not face any borrowing or lending constraints and can denominate its liabilities (borrow) and invest its wealth (lend) in N currencies. The amount borrowed or lent in currency j is denoted by B_j, which is a representative element of **B**, the $N \times 1$ vector of demand for foreign bonds. The effective cost (or returns) of B_j in terms of the domestic currency depends on the foreign interest rate and the exchange rate. Each of the N foreign interest rates, r_j^*, is assumed to be uncertain. Similarly, each of the N exchange rates, e_j, defined in terms of units of domestic currency per unit of foreign currency (for example, pesos per dollar), is also uncertain. The effective (gross) interest rate on B_j is defined as R_j^*, and $\mathbf{R}^*$ is the $N \times 1$ vector of the expected effective costs on the N foreign loans. R_j^* is thus given by the (gross) foreign interest rate r_j^* times (one plus) the rate of depreciation of the domestic currency in relation to currency j, that is, $(1 + r_{j,1}^*)\ (e_{j,2}/e_{j,1})$. It is assumed that the effective costs of liabilities denominated in the currency of each foreign country do not necessarily equal each other. And the standard finance assumption of no transaction costs is made.

Due to certain barriers, domestic investors are prevented from investing in foreign firms and stocks. This restriction prevents domestic investors from using foreign equities as hedging instruments against unanticipated changes in external prices. Foreign investors cannot own domestic firms. We assume that there are no nontraded domestic assets (such as human capital). (See Svensson [1989] for the case of nontraded assets.)

Domestic assets that are traded are domestic bonds, denoted by D, which are in zero net supply, with a gross interest rate of R, and shares of export firms. Export goods are produced by domestic, profit-maximizing firms whose shares are not traded abroad. In the first period investment levels, denoted by I_h, $h = 1, \ldots L$ are determined for production of each of the L export goods in the second period. The production functions, denoted by $Q(I_h)$, exhibit decreasing returns to scale, and **Q**(**I**) denotes the vector of second-period outputs. First-period prices of all goods are normalized at one, and second-period prices in the domestic currency for the export goods are denoted by P_h^*. In the second period the country will receive payments per unit of exports in domestic currency equal to P_h^*. The word "price" should be interpreted broadly here and includes all factors that determine the value of one unit of an export good. The exact currency in which the payments are received is immaterial to our results: what matters is that the payments have a stochastic unit value in terms of domestic currency. Second-period import good prices are P_i.

The first- and second-period asset flow constraints for the representative consumer are defined in equations 1 and 2, respectively:

$$(1) \qquad \sum_i^K c_{1,i} = \sum_j^N B_j - \sum_h^L I_h + D$$

$$\sum_{i}^{K} p_i c_{2,i} = -\sum_{j}^{N} R_j^* B_j + \sum_{h}^{L} P_h^* Q(I_h) - RD \tag{2}$$

where first-period exchange rates are normalized to one.

It is assumed that the (representative) domestic investor maximizes an expected utility function, which is von Neuman-Morgenstern and depends only on the consumption of the K commodities. The expected utility function of the investor is given by

$$U\left(\prod_{i=1}^{k} c_{1,i}^{\alpha_i}\right) + \beta E\left[U\left(\prod_{i=1}^{k} c_{2,i}^{\alpha_i}\right)\right] \tag{3}$$

where Π is the standard multiplication function, E is the expectation operator conditional on all information available at time 1, and β is the factor of time preference. The utility function is characterized by constant expenditure shares: the investor will always spend a share α_i of total expenditures on good i. This allows us to represent the objective function in terms of one composite good for which we will use the notation C_i, $i = 1,2$, with average price P (geometrically weighted with weights α_i). The investor will maximize utility subject to the constraints on income.

$$\max_{B_j, D, I_h} \; U(C_1) + \beta E[U(C_2)] \tag{4}$$

$$\text{subject to } C_1 - \sum_{j}^{N} B_j + \sum_{h}^{L} I_h - D = 0$$

$$\text{and } PC_2 + \sum_{j}^{N} R_j^* B_j - \sum_{h}^{L} P_h^* Q(I_h) + RD = 0.$$

To simplify the solution for the optimal amounts borrowed in each foreign currency, invested in domestic bonds, and held in first-period investment levels, we assume that the utility function U is quadratic, that is, $U = aC - (b/2)C^2$. For the more general case, see Claessens (1988) or Svensson (1988). We define $\mathbf{V}_{\mathbf{rr}}$ as the $N \times N$ variance-covariance matrix of the effective costs of foreign borrowings deflated by the consumer price index and $\mathbf{V}_{\mathbf{rp^*}}$ as the variance-covariance matrix of those costs with the changes in the unit values of the export goods (again expressed in domestic currency and deflated by the consumer price index). After imposing that the domestic market for bonds clears, the optimal borrowings are, where $\mathbf{1}$ is a $N \times 1$ vector of ones (see further the appendix):

$$\mathbf{B} = \mathbf{V}_{\mathbf{rr}}^{-1} \left\{ \frac{E[(1/P)\mathbf{R}^*] - \mathbf{1}\dfrac{U_1'}{\beta E(U_2')}}{-\dfrac{b}{E(U_2')}} \right\} + \mathbf{V}_{\mathbf{rr}}^{-1}\mathbf{V}_{\mathbf{rp^*}}\,\mathbf{Q}(\mathbf{I}). \tag{5}$$

Equation 5 implies that the demand for foreign bonds consists of two mutual funds. The first fund is the speculative portfolio:

(6) $$\mathbf{B}^s = \mathbf{V}_{rr}^{-1}E[(1/P)\mathbf{R}^* - 1\tau]$$

where $\tau = U_1'/(\beta E(U_2'))$ is the effective rate of intertemporal substitution. The composition of the speculative portfolio of foreign bonds depends on the expected effective real costs of excess borrowings. In equilibrium these costs will depend on the real rate of return on investment, adjusted for risks, and the inverse of the variance-covariance matrix of effective real costs. The demand for the fund depends on a risk aversion parameter, γ, where $\gamma = -b/E(U_2')$. The higher the aversion against risks, the higher the γ, and the lower the amount borrowed on account of the speculative fund.

The second portfolio is the minimum variance hedge:

(7) $$\mathbf{B}^m = \mathbf{V}_{rr}^{-1}\mathbf{V}_{rp^*}\mathbf{Q}(\mathbf{I})$$

and is independent of the level of risk aversion. The investor will borrow in foreign currencies to insulate against changes in the domestic currency value of future receipts from exports. These changes are caused not only by movements in the domestic exchange rate at which export receipts are converted into domestic currency, but also by movements in the unit values of export goods. Since cross-currency movements can be related to unit value movements, foreign borrowings can serve as a hedge, and the stronger the correlation between domestic currency values and the cost of borrowings the larger the demand for foreign borrowings.

The equation for the optimal amount of loans is specified with respect to the effective, real cost of foreign liabilities, relative to the domestic borrowing costs. Alternatively, the portfolio can be written in nominal terms and then split into three funds: a nominal speculative fund, a minimum variance fund, and a price hedge (see the appendix). In addition, we need to assume that the distribution of all prices is lognormal. The nominal fund will be $\mathbf{V}_{rr}^{-1}R$, and the minimum variance fund will be $\mathbf{V}_{rr}^{-1}\mathbf{V}_{rp^*}\mathbf{Q}(\mathbf{I})$, and the matrices are now based on the nominal costs of borrowings and the nominal unit values of exports. The price hedge fund will be $-\mathbf{V}_{rr}^{-1}\mathbf{V}_{rp}$, where $\mathbf{V}_{rp}$ is the vector of covariances between the effective costs of borrowings and the consumer (import) price index. The demand for the price level hedge will be $-(1 - 1/\gamma)$ (see the appendix).

It is useful to look at the price level hedge when we assume that the foreign interest rates are constant and thus that the uncertainty in the effective foreign interest rates is only a result of exchange rate uncertainty. This implies that the variance-covariance matrix of effective costs ($\mathbf{V}_{rr}$) would be equal to the variance-covariance matrix of exchange rates, and the vector of covariances between the effective costs of borrowings and the consumer price index ($\mathbf{V}_{rp}$) is equal to the vector of covariances between exchange rates and consumer prices. Here we look at the price level hedge in two cases. In the first case changes in the prices of imported goods are perfectly correlated with the exchange rate, that is, the law of one price holds for imported goods. Holding foreign bonds provides then a perfect hedge against unanticipated changes in the domestic price of foreign goods. The demand for each foreign bond will be determined by the share of total consumption expenditures spent on the imports from each currency area. In the second case purchasing power parity holds. Foreign bonds are perfect hedges against unanticipated domestic price movements as exchange rate movements offset perfectly relative price movements. With no unanticipated inflation abroad there will be no demand for foreign bonds on account of the price level hedge.

To summarize, the model outlined above indicates that the optimal external liability composition of a country depends on the following factors:

- The expected costs of borrowing in each of the foreign currencies relative to the domestic cost of funds
- The variance-covariance matrix of the expected costs of borrowing in each foreign currency
- The variance-covariance matrix of domestic goods prices and expected costs of borrowings in each foreign currency
- The shares of consumption expenditures spent on the different goods in the country
- The export receipts of the country
- The vector of covariances between export receipts and expected costs of borrowings in each foreign currency
- The level of risk tolerance in the country.

When purchasing power parity holds and foreign prices are (relatively) stable, the rules for the optimal currency composition of external liabilities can be further simplified. A country may be very risk averse and want to hedge itself against commodity prices and interest and exchange rate movements. Or it may take the view that the expected real costs of borrowing in each of the foreign currencies are equal to the domestic real cost of funds (that is, it may not want to speculate on the relative costs of different instruments). In these cases the optimal composition will depend only on the vector of covariances between export receipts and the expected costs of borrowing in each foreign currency, and the variance-covariance matrix of the expected costs of borrowing in each foreign currency.

II. Practical Rules for the Currency Composition of External Debt

From the theoretical analysis it follows that if a country does not want to take an active view on exchange and interest rate movements—and associated costs of borrowings—or if it is relatively risk averse, then its optimal *net* borrowing portfolio (gross borrowings minus foreign assets, that is, reserves) will be the risk-minimizing hedge portfolio. This is a very familiar result from simple one-period mean-variance hedging models such as those used to determine the optimal amounts of futures to buy or sell to hedge an exposure (see, for example, Gemmill 1985). Techniques introduced by Adler and Dumas (1980) and later on refined by others for the operational measurement of the economic exposure of a firm to external risks are also similar. We will pursue the latter similarity further.

Economic exposure to external risks for a firm has been defined in terms of the sensitivity of its objective function with respect to unanticipated changes in external variables. The firm's objective function may be defined in terms of the net present value of future expected cash flows or in terms of near-term cash flows or profits. When the hedging instruments available are foreign borrowings, the operational way of measuring exposure as a cash flow sensitivity can then be reduced to measuring the covariance between cash flows and the effective costs of borrowings relative to the variance of total cash flows. In other words the measure is the ordinary least squares (OLS) regression coefficient of cash flows on the relevant effective costs of borrowings: $\text{cov}(CF_t,R_t)/\text{Var}(CF_t)$, where CF_t is the cash flow in terms of domestic currency in period t and R_t the relevant effective cost of borrowings at time t. The absolute values of these estimated coefficients provide then the minimum variance hedging quantities. These are the amounts of foreign borrowings (multiple exposure measures follow if one regresses the cash flows on multiple cost of borrowings). A related exposure measure is discussed by Oxelheim and Wihlborg (1987).

The portfolio model above is consistent with this approach because it indicates that—when the expected foreign and domestic real costs of funds are equal and purchasing power parity holds between foreign and domestic goods—the optimal composition is simply determined by the minimum variance fund $(\mathbf{V}_{rr}^{-1}\mathbf{V}_{rp^*}\mathbf{Q}(\mathbf{I}))$. In the practical approach we consider neither the relative costs of funds in different currencies nor the benefits of using foreign borrowings to hedge against domestic price uncertainty. It is unlikely that a developing country will be able to successfully exploit (speculate) at an acceptable risk the small differences between costs of funds. The international financial capital markets at large are better equipped for this and will assure that these differences remain small.

Domestic price uncertainty is more often the result of endogenous government policy choices than of exogenous influences; consequently it is unlikely that it can be hedged using foreign borrowings. It is not assumed that the law of one price holds among foreign goods (that is, P_i is not necessarily equal to $e_h P^*_{i,h}$ for all i and h, where $P^*_{i,h}$ is the price of traded good i in terms of foreign currency h) and foreign exchange rates can influence the prices of foreign goods. We leave the precise mechanisms through which nominal foreign exchange rate movements influence prices of foreign goods unspecified. For the effects of currency movements on the behavior of absolute and relative prices, see Giovannini (1986), Dornbusch (1987), and Varangis and Duncan (1987).

More generally, the risk-minimizing borrowing portfolio would be based on the relation between the ability of a country to service its debt and the effective cost of debt service in each of the relevant borrowing currencies. The main problem with this approach is measuring the ability of a country to pay, something that has been the subject of extensive research in the sovereign debt literature. In addition, ability and willingness to pay may differ, thus resulting in actual payments that may be determined by a bargaining game between a debtor and its creditors (see Eaton [1990] for a survey). In that case the factors determining the payment behavior belong on the right side of the OLS equation.

We suggest here that total exports of a country is the relevant measure to hedge. (Healy [1981] uses a similar simple regression technique for the optimal diversification of foreign exchange reserves.) Total exports measured in domestic currency are thus regressed on the effective costs of foreign funds (both expressed as percentage changes), and the coefficients are used to calculate the optimal liability portfolio. The equation to be estimated becomes then

$$\ln(P^*Q)_{t+1} - \ln(P^*Q)_t = \sum_i \beta_i \left\{ \ln\left[(1 + r_{i,t})\left(\frac{e_{i,t+1}}{e_{i,t}}\right)\right] - \ln\left[(1 + r_{i,t-1})\left(\frac{e_{i,t}}{e_{i,t-1}}\right)\right]\right\} + u_t. \tag{8}$$

Further disaggregation, for example by commodity groups or direction of trade, can also be used to calculate the optimal portfolio shares if one expects that the disaggregated relationships are more stable. For instance, prices of some commodities tend to have a close relationship with a single currency because the supply (or demand) tends to come from (or go to) that currency area. For example, the price of coniferous timber products has been closely associated with the Nordic countries' currencies because they are large suppliers and influence the dollar price of timber. Similar relationships may exist for other commodities (see further Lessard and Williamson 1985).

III. Applications to Mexico and Brazil

In applying the model, the objective is to find the currency composition of net liabilities (gross liabilities minus reserves) that minimizes the variability in domestic currency of export earnings net of foreign liability debt service. We assume that at the beginning of each period new net liabilities are incurred, which are then paid off at the end of the period. We have thus a rolling portfolio of liabilities. If the composition of the optimal portfolios obtained is now stable over time, then little rebalancing of the portfolio would be required, and the rolling portfolio would essentially mimic a portfolio of long-term liabilities that has debt service payments falling due each period. We would then be justified to use the currency composition of long-term liabilities (which is inherently difficult to change over short periods) to hedge the short-term exposure arising from changes in export earnings. Large reductions in overall variability would indicate that the currency composition of external debt can be an effective tool for risk management, especially since it would avoid the drawbacks of rolling over short-dated instruments (margin calls and premiums).

We use monthly data for 1973–89 for Mexico and for 1973–86 for Brazil. In accordance with the model, we use as the dependent variable Mexico's and Brazil's total exports, expressed in domestic currency. (All data are from the International Monetary Fund's *International Financial Statistics.*) As possible borrowing currencies we choose the U.S. dollar, Japanese yen, and German deutsche mark (DM). These three currencies were selected for the borrowing portfolio because Mexico and Brazil's current debt is largely denominated in these currencies (respectively, 70 and 72 percent U.S. dollar, 15 and 11 percent DM, and 8 and 10 percent yen); the DM and other European currencies are highly correlated, and the three chosen currencies are therefore good proxies for the diversification available in international currency markets; and these countries' future access to financial markets is likely concentrated in these currencies (or currency blocks).

The estimations were based on the (monthly) percentage change in the costs of borrowing in the different currencies expressed in local currency. The foreign interest rates used were the monthly quotes of the respective six-month LIBOR rates, which were then multiplied with the exchange rate depreciation over the next month. Table 1 presents some data on the average annualized costs in pesos during 1973–89 and in cruzados during 1973–86. The variability in the local cost of foreign borrowings was quite high. The variability of foreign costs can be compared with the monthly coefficient of variation of export earnings in local currency: 62 percent for Mexico and 233 percent for Brazil.

The estimate for the optimal portfolio for the whole sample period was based on regression equation 6. The estimates for the optimal amount borrowed in each currency lead to the portfolio compositions for Mexico and Brazil given in table 2. The absolute amount to be borrowed as a percentage of export earnings is 0.25 percent for Mexico and 5.62 percent for Brazil. The small fractions reflect the fact that the variability in export earnings is much less than the variability in effective costs of borrowings and that the correlation between the two is low. The results indicate that, of the absolute amount to be borrowed,

Table 1. *Average Annual Costs of Borrowings*
(percent)

	Cost in domestic currency			
Currency of debt	*Mean*	*Coefficient of variation*	*Minimum*	*Maximum*
Mexico, 1973–89 (pesos)				
U.S. dollar	46	256	−156	1,133
DM	46	266	−138	1,059
Yen	48	249	−144	1,100
Brazil, 1973–86 (cruzados)				
U.S. dollar	70	90	6	475
DM	70	106	−92	467
Yen	72	104	−105	444

Source: Author's calculations.

Table 2. *Optimal Portfolio Compositions for Mexico and Brazil*

	Percentage of total amount borrowed in			*Amount borrowed as a percentage of export earnings*	R^2	*Durbin-Watson statistic*
	U.S. dollars	*DM*	*Yen*			
Mexico, 1973–89	111.0	16.3	−27.4	0.25	0.068	2.00
Brazil, 1973–86	98.0	3.4	−1.5	5.62	0.053	2.04

Source: Author's calculations.

Mexico should borrow more than 100 percent in U.S. dollars, borrow some additional DM, and hold some of the foreign funds in yen reserves. Brazil should also borrow U.S. dollars and DM and hold yen in reserves.

If Mexico and Brazil had borrowed and invested their reserves in this fashion during 1973–89, the variance of monthly exports net of debt service would have been slightly lower compared with no borrowings and no hedging. The small reduction in uncertainty reflects the fact that the fits of the estimated equations are relatively poor (R^2 of only 0.068 and 0.053 for Mexico and Brazil, respectively) and that the underlying relationships between export earnings and costs of borrowings are not stable over time. The latter implies that portfolios should be more frequently adjusted and thus that the hedging effectiveness of a constant portfolio is limited.

The results so far were calculated using historical data and then applied over the previous sample period. The results therefore amount to an in-sample test. However, such historical comparisons cannot indicate the effectiveness of the strategy as a planning tool. After all, in reality, the country would only have had information up to time t with which to determine the composition for time $t + 1$. To show whether or not this technique is effective in terms of risk minimization, we perform a number of "out-of-sample" tests. We construct portfolios for period $t + 1$ that are based on a sample of observations up to time t. The sample is then updated to include time $t + 1$ information, portfolios are constructed for period $t + 2$, and so forth. The relative effectiveness of this strategy of rolling hedges is then calculated as the risk reduction realized over the whole hedging period compared with the risk reduction realized by the other strategies.

Specifically, we calculated optimal portfolios for Mexico and Brazil for each quarter using the last 48 monthly observations up to that quarter. This results in 51 portfolios for Mexico and 39 for Brazil. Table 3 summarizes the results. Annex tables A-1 and A-2 list the individual portfolios, indicating the mean square residual, the (adjusted) R^2, and the Durbin-Watson statistic. For some individual portfolios the total borrowing amount as a percentage of export earnings is negative, thus implying that a net lending strategy would have been preferred at some points in time.

The summary table indicates that the standard deviation of the portfolio shares is quite large. Portfolio shares vary between −3,622 and 2,997 percent for Mexico and between −123 and 202 percent for Brazil. Some of the large individual portfolio shares are caused by the very low absolute amount to be borrowed in a specific quarter, which inflates the individual portfolio shares. For example, the total borrowing amount as a percentage of export earnings of portfolio 37 for Mexico is only 0.000087.

Some of the outliers in the case of Mexico are also caused by the two large discreet valuations that occurred in 1982, which distort the estimation for all portfolios that include these data points. This is for instance reflected in the amounts to be borrowed for portfolios 23 through 39 (1982–87 through 1986–87, which thus include the 1982 data), which are all negative. To control for this, we have also excluded from the sample the negative sums. We also solved for the optimal shares while imposing some constraint on the absolute amounts to be borrowed, for example, a certain ratio of debt service to exports. The results for both approaches were, however, not significantly better. Similar explanations exist for Brazil for portfolios after 1983-10. Excluding these periods, the estimates for the portfolios indicate a consistent strategy of borrowing in U.S. dollars combined with some minor DM or yen borrowings or reserve holdings. The absolute amounts to be borrowed are lower than the actual borrowings by these two countries and are closer to the net transfers these countries pay on their debts, expressed as a fraction of exports, consistent with the idea that the portfolios are rolled over.

With these individual portfolios, we performed the following out-of-sample test. For any portfolio one can calculate the residual in each of the next three months that was not hedged as $u_t = \Delta E_t - \sum_i \beta_i \, \Delta R_{i,t}$, where ΔE_t is the realized monthly percentage change in export earnings, $\Delta R_{i,t}$ is the realized monthly

Table 3. *Summary Statistics of Optimal Portfolios for Mexico and Brazil*
(percent)

Variable	Mean	Standard deviation	Value: Minimum	Value: Maximum
Mexico				
Share of debt in				
U.S. dollars	257	499	−89	2,997
DM	63	174	−72	940
Yen	−309	679	−3,622	29
Total borrowing amount as a percentage of export earnings	2	3	−1	8
Brazil				
Share of debt in				
U.S. dollars	56	93	−97	202
DM	−15	23	−123	7
Yen	2	10	−8	38
Total borrowing amount as a percentage of export earnings	2	11	−20	17

Note: For Mexico there were 51 observations; for Brazil there were 39.
Source: Author's calculations.

percentage change in the costs of borrowing (for i = U.S. dollar, DM, and yen, all expressed in local currency), and β_i are the shares, which change every three months. A new portfolio is then used for the next three months to calculate the residual, and so forth. This allows us to compare the effectiveness of a particular borrowing strategy with any other borrowing strategy.

This was done here by comparing the strategy where portfolio shares are adjusted every quarter according to the estimates derived from the observations during the previous 48 months—here called the rolling strategy—with three alternative strategies. The first alternative strategy was Mexico's and Brazil's actual debt composition in 1988 (reported above); the second one was the composition that was historically optimal for 1977–89 and 1977–86 (89.8 and 126.6 percent U.S. dollar, 11.5 and −25.0 percent DM, and −1.4 and −1.6 percent yen, respectively, for Mexico and Brazil); and the third one was the composition that was historically optimal for 1973–89 and 1973–86 (reported in table 2). For all three alternatives only the composition of debt was changed. The total amount to be borrowed (or loaned) was assumed to be equal to the rolling strategy (otherwise using the optimal amounts of the historically optimal portfolios would amount to an in-sample test).

For both countries the residuals of all four portfolios were calculated; in total there were 152 residuals for Mexico and 117 for Brazil. The results are shown in table 4. For Mexico the results are very encouraging. The rolling strategy achieved approximately a 46 percent lower coefficient of variation of residuals compared with the actual borrowing strategy. The compositions that were historically optimal for 1977–89 and 1973–89 achieved less or no risk reduction compared with the actual portfolio, and the rolling portfolios outperformed all three portfolios. For Brazil, the results are less encouraging. The rolling strategy achieved only a 2 percent lower coefficient of variation than the actual borrowing strategy. Much risk reduction was achieved with the rolling portfolio compared with the 1977–86 historically optimal strategy (a 69 percent reduction), but not compared with the 1973–86 historically optimal strategy, which per-

Table 4. *Risk Reduction Achieved with Optimal Portfolios*
(compared with the actual portfolio)

	Mexico		*Brazil*	
Portfolio	*Coefficient of variation*	*Reduction (percent)*	*Coefficient of variation*	*Reduction (percent)*
Actual	686.0	n.a.	367.0	n.a.
Rolling strategy	371.9	46	358.9	2
Optimal composition for				
1977–89	565.8	18	n.a.	n.a.
1973–89	916.2	−33	n.a.	n.a.
1977–86	n.a.	n.a.	1,153.0	−214
1973–86	n.a.	n.a.	356.0	3

n.a. Not applicable.
Source: Author's calculations.

formed overall the best. For both countries, all strategies did much better than a strategy using the actual amounts as well as the actual shares in which the countries borrowed (statistical results are not reported here).

None of the strategies performed better than a strategy where both composition and amounts were determined in an historically optimal way. This may be expected since historical portfolios will always lead to risk reduction given the benefits of hindsight. The instability in the covariances between changes in costs of borrowings and exports implies that planned portfolios will always result in less hedging. Only with more stability can better results be expected from planned portfolios compared with historical ones. This is, however, not a fair comparison, because, as noted above, in-sample tests of effectiveness of hedging are misleading. Other out-of-sample tests were also performed. One of these was based on a rolling hedge strategy using the past 24 months of observations. This led to much larger standard deviations in borrowing shares, a result of the fact that the sample period for the estimation was halved. Compared with the actual and historical optimal composition this strategy led to little or no risk reduction.

IV. Conclusions

This article shows that an integrated analysis of the external risks an economy is facing is necessary to measure external exposures correctly. The article uses international portfolio theory for such an analysis and finds that in particular exchange and commodity price exposure need to be integrated. A developing country wishing to implement more active management with respect to the currency composition of its net liabilities will need to consider its objective function very carefully. Nominal dimensions (such as direction of trade flows and currency composition of cash-flows receipts) do not necessarily provide correct indications for the optimal currency composition of debt. Determining the optimal currency composition requires a careful empirical investigation of relationships between cross-currency exchange rates and indicators of the country's external account, such as terms of trade, exports, and the non-interest current account. Determining the optimal liability structure requires facts such as the historically observed inverse relationship between the value of the dollar and dollar commodity prices. This relationship indicates for instance that nondollar currencies can perform a hedging function for (primary) commodity exporters because commodity prices, and thus export earnings, increase when nondollar currencies rise in value and vice-versa.

The article applies a reduced form of the analytical model developed to Mexico and Brazil and finds some, albeit limited, reduction in external exposures from constructing optimal, rolling portfolios. The results indicate, however, that portfolios are unstable over time as a result of time-varying covariances. Thus transactions costs can significantly reduce the benefits of adjusting portfolios frequently. Results obtained elsewhere (Kroner and Claessens 1991) indicate that even when using Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) to deal with time-varying covariances—and to achieve less time-varying portfolios—instability remains an issue and that using the currency composition of long-term debt in a cost-effective way to hedge against real shocks will be difficult.

In general, the results indicate that the currency composition of debt is an imperfect hedging tool against external price uncertainty. Other types of contingent contracts (such as commodity price linked instruments) are likely better able to transfer risk from developing countries to the international capital markets. Unfortunately, the longer-term spectrum of commodity risk management instruments is not yet well developed for many commodities of interest to developing countries and will in any case be difficult to access because of credit constraints. In principle, developing countries should then also be using the short-dated commodity-linked markets, such as options, forwards, and futures, to hedge directly against price risks. These will provide the countries with some risk reduction benefits when longer-term markets become available or the countries achieve some real diversification. Unless strong relationships are found between the prices of export and import products, the currency choice of external debt may for many developing countries be largely immaterial as it relates to reducing overall exposure to external factors.

Appendix. Derivation of the Optimal Portfolio Model

For simplicity we will derive first the optimal portfolio amounts in case only one foreign bond and one domestic investment opportunity are available. The generalization to N foreign bonds and L investment opportunities will then follow easily.

The Simplified Case of One Foreign Bond and One Domestic Investment Opportunity

From the maximization problem in equation 4 the following first-order conditions with respect to the amount of investment, foreign bonds, and domestic bonds can be derived.

$$\text{(A-1)} \qquad 0 = -U_1' + \beta E\left(U_2' \frac{P^* Q'}{P}\right) \qquad \text{(w.r.t. I)}$$

$$\text{(A-2)} \qquad 0 = -U_1' + \beta E\left(U_2' \frac{R^*}{P}\right) \qquad \text{(w.r.t. B)}$$

$$\text{(A-3)} \qquad 0 = -U_1' + \beta E\left(U_2' \frac{R}{P}\right) \qquad \text{(w.r.t. D)}$$

Given the assumption of a quadratic utility function, we can rewrite the first-order conditions as

$$\text{(A-1}'\text{)} \qquad 0 = -U_1' + \beta\Big[E(U_2')E\Big(\frac{P^*Q'}{P}\Big) - b\text{cov}\Big(\frac{P^*Q'}{P}, \frac{P^*Q}{P}\Big)$$

$$+ b\text{cov}\Big(\frac{P^*Q'}{P}, \frac{R^*}{P}\Big)B + b\text{cov}\Big(\frac{P^*Q'}{P}, \frac{R}{P}\Big)D\Big]$$

$$\text{(A-2}'\text{)} \qquad 0 = -U_1' + \beta\Big[E(U_2')E\Big(\frac{R^*}{P}\Big) - b\text{cov}\Big(\frac{R^*}{P}, \frac{P^*Q}{P}\Big)$$

$$+ b\text{cov}\Big(\frac{R^*}{P}, \frac{R^*}{P}\Big)B + b\text{cov}\Big(\frac{R^*}{P}, \frac{R}{P}\Big)D\Big]$$

$$\text{(A-3}'\text{)} \qquad 0 = -U_1' + \beta\Big[E(U_2')E\Big(\frac{R}{P}\Big) - b\text{cov}\Big(\frac{R}{P}, \frac{P^*Q}{P}\Big)$$

$$+ b\text{cov}\Big(\frac{R}{P}, \frac{R^*}{P}\Big)B + b\text{cov}\Big(\frac{R}{P}, \frac{R}{P}\Big)D\Big].$$

Imposing that in equilibrium the amount of domestic bonds will be in zero net supply, equation A-3′ can be solved for the domestic equilibrium nominal interest rate R as

$$\text{(A-4)} \qquad E\Big(\frac{R}{P}\Big) = \frac{U_1'}{\beta E(U_2')} + \frac{b}{E(U_2')}\text{cov}\Big(\frac{R}{P}, \frac{P^*Q}{P}\Big)$$

$$- \frac{b}{E(U_2')}\text{cov}\Big(\frac{R}{P}, \frac{R^*}{P}\Big)B.$$

After some manipulation equation A-2′ can be used to find the expression for the optimal amount of foreign bonds as

$$\text{(A-5)} \qquad B = \frac{E\Big(\frac{R^*}{P}\Big) - \frac{U_1'}{\beta E(U_2')}}{-\frac{b}{E(U_2')}\text{VAR}\Big(\frac{R^*}{P}\Big)} + \frac{\text{cov}\Big(\frac{R^*}{P}, \frac{P^*}{P}\Big)}{\text{VAR}\Big(\frac{R^*}{P}\Big)} Q(I).$$

The equilibrium domestic interest rate—derived in equation A-4—can alternatively be used in equation A-5, in which case the demand for foreign bonds will depend on the real interest differential and the covariance of the real interest differential with foreign interest rates.

The Generalized Case of N Foreign Bonds and L Domestic Investment Opportunities

In case of L domestic investment opportunities and N foreign bonds, there will be $1 + L + N$ first-order conditions: one for domestic bonds, one for each domestic investment opportunity, and one for each foreign bond. The N differ-

Table A-1. *Composition of Rolling Portfolios for Mexico, 1977–89*

Portfolio number	*Year*	*Month*	*U.S. dollar*	*DM*	*Yen*	*Amount to be borrowed as a percentage of export earnings*	*Mean square residual*	*Adjusted R^2*	*Durbin-Watson statistic*
1	1977	1	4.2936	0.8412	−6.135	−0.0018	0.04	0.24	1.97
2	1977	4	4.6652	0.9101	−6.575	−0.0016	0.05	0.23	2.04
3	1977	7	4.6664	0.9008	−6.567	−0.0016	0.05	0.23	2.11
4	1977	10	4.9198	0.9982	−6.918	−0.0015	0.05	0.22	2.12
5	1978	1	13.9847	2.9492	−17.934	−0.0005	0.04	0.21	2.09
6	1978	4	29.9681	7.2515	−36.220	0.0002	0.05	0.20	2.09
7	1978	7	0.8349	0.3283	−0.163	0.0044	0.05	0.17	2.10
8	1978	10	0.8376	0.3278	−0.165	0.0043	0.05	0.16	2.09
9	1979	1	0.8645	0.3724	−0.237	0.0042	0.04	0.14	1.91
10	1979	4	0.8470	0.3726	−0.220	0.0042	0.04	0.16	2.06
11	1979	7	1.4379	−0.0401	−0.398	0.0027	0.03	0.13	2.07
12	1979	10	0.8864	0.3306	−0.217	0.0040	0.03	0.14	2.06
13	1980	1	0.6550	0.2366	0.108	0.0045	0.03	0.13	1.99
14	1980	4	1.0901	−0.2329	0.143	0.0029	0.03	0.14	1.84
15	1980	7	1.0343	−0.1959	0.162	0.0031	0.02	0.16	1.97
16	1980	10	−0.8930	−0.3971	0.290	−0.0015	0.02	0.02	1.97
17	1981	1	1.0446	−0.0808	0.036	0.0078	0.02	0.10	1.91
18	1981	4	1.0866	−0.0834	−0.003	0.0078	0.02	0.15	1.66
19	1981	7	1.0078	−0.0326	0.025	0.0184	0.02	0.11	1.81
20	1981	10	1.0071	−0.0132	0.006	0.0373	0.03	0.06	1.92
21	1982	1	1.0094	−0.0114	0.002	0.0459	0.03	0.10	2.00
22	1982	4	1.0747	−0.0926	0.018	0.0074	0.03	0.06	1.84
23	1982	7	−0.5712	−0.0595	−0.369	−0.0105	0.03	0.06	1.97
24	1982	10	−0.6164	−0.0429	−0.341	−0.0139	0.04	0.06	1.97

25	1983	1	1.1022	−0.4414	−1.661	−0.0013	0.04	0.06	1.97
26	1983	4	1.3839	−0.5198	−1.864	−0.0012	0.04	0.07	1.96
27	1983	7	1.6296	−0.6995	−1.930	−0.0010	0.04	0.06	1.97
28	1983	10	1.6611	−0.7202	−1.941	−0.0010	0.04	0.06	1.92
29	1984	1	1.6979	−0.6765	−2.021	−0.0009	0.04	0.06	1.97
30	1984	4	1.8010	1.0433	−3.844	−0.0008	0.04	0.08	1.97
31	1984	7	1.7718	1.0413	−3.813	−0.0008	0.04	0.08	1.96
32	1984	10	1.8518	1.0763	−3.928	−0.0008	0.04	0.08	1.97
33	1985	1	1.8328	1.0542	−3.887	−0.0008	0.04	0.08	1.95
34	1985	4	3.1815	1.9637	−6.145	−0.0004	0.04	0.07	1.97
35	1985	7	1.4076	0.7766	−3.184	−0.0011	0.04	0.09	1.80
36	1985	10	4.3707	2.1911	−7.562	−0.0004	0.03	0.08	1.97
37	1986	1	17.6909	9.4037	−28.095	−0.0001	0.03	0.06	1.97
38	1986	4	0.4354	0.1885	−1.624	−0.0045	0.03	0.07	2.00
39	1986	7	0.4248	0.1910	−1.616	−0.0047	0.03	0.08	2.01
40	1986	10	2.4271	1.5531	−2.980	0.0006	0.02	0.09	1.94
41	1987	1	0.9899	0.0126	−0.003	0.0740	0.01	0.30	1.88
42	1987	4	1.0161	0.0128	−0.029	0.0754	0.01	0.33	1.82
43	1987	7	1.0080	0.0127	−0.021	0.0758	0.01	0.33	1.97
44	1987	10	1.0047	0.0127	−0.017	0.0752	0.01	0.32	1.95
45	1988	1	1.0116	0.0120	−0.024	0.0773	0.01	0.36	1.98
46	1988	4	1.0376	−0.0060	−0.032	0.0701	0.01	0.32	1.95
47	1988	7	1.0019	−0.0151	0.013	0.0718	0.01	0.32	1.95
48	1988	10	1.0050	−0.0192	0.014	0.0740	0.01	0.32	1.96
49	1989	1	1.0056	−0.0194	0.014	0.0745	0.02	0.32	1.93
50	1989	4	1.0051	−0.0199	0.015	0.0707	0.02	0.32	1.97
51	1989	7	1.0031	−0.0227	0.020	0.0591	0.01	0.31	1.92

Source: Author's calculations.

Table A-2. *Composition of Rolling Portfolios for Brazil, 1977–86*

Portfolio number	*Year*	*Month*	*U.S. dollar*	*DM*	*Yen*	*Amount to be borrowed as a percentage of export earnings*	*Mean square residual*	*Adjusted* R^2	*Durbin-Watson statistic*
1	1977	1	1.0178	0.0672	−0.0850	0.0762	1.31	0.21	2.05
2	1977	4	1.0267	0.0578	−0.0846	0.0744	1.37	0.20	2.11
3	1977	7	1.0074	0.0508	−0.0582	0.0899	1.30	0.23	2.13
4	1977	10	1.0079	0.0494	−0.0573	0.0932	1.24	0.26	2.12
5	1978	1	1.0588	0.0008	−0.0597	0.0626	1.04	0.14	1.90
6	1978	4	1.0809	−0.0247	−0.0562	0.0703	0.97	0.21	1.98
7	1978	7	1.0551	−0.0278	−0.0274	0.1306	0.88	0.27	1.97
8	1978	10	1.0480	−0.0191	−0.0289	0.1665	0.84	0.26	2.04
9	1979	1	1.0480	−0.0184	−0.0296	0.1571	0.81	0.25	1.96
10	1979	4	1.0388	−0.0226	−0.0162	0.1518	0.73	0.27	1.99
11	1979	7	1.0347	−0.0287	−0.0060	0.1378	0.74	0.28	2.01
12	1979	10	1.0582	−0.0505	−0.0076	0.1079	0.77	0.26	1.99
13	1980	1	1.0674	−0.0591	−0.0083	0.0877	0.77	0.30	1.86
14	1980	4	1.0925	−0.0854	−0.0072	0.0846	0.64	0.31	1.99
15	1980	7	1.1042	−0.0943	−0.0099	0.0820	0.63	0.32	2.02
16	1980	10	1.1144	−0.1047	−0.0097	0.0811	0.63	0.31	2.02
17	1981	1	1.1259	−0.1161	−0.0098	0.0814	0.60	0.32	2.03
18	1981	4	1.0628	−0.0531	−0.0097	0.0828	0.56	0.33	2.03
19	1981	7	1.0632	−0.0528	−0.0104	0.0787	0.60	0.29	1.99
20	1981	10	1.0693	−0.0626	−0.0067	0.0776	0.65	0.25	1.98

21	1982	1	1.1303	−0.1212	−0.0091	0.0744	1.81	0.12	1.91
22	1982	4	1.0617	−0.0521	−0.0095	0.0685	1.91	0.10	2.04
23	1982	7	1.1157	−0.1059	−0.0098	0.0682	1.91	0.11	2.09
24	1982	10	1.1387	−0.1286	−0.0101	0.0687	1.91	0.11	2.09
25	1983	1	1.1494	−0.1395	−0.0098	0.0687	1.95	0.10	2.07
26	1983	4	1.3053	−0.2942	−0.0111	0.0452	3.14	0.08	1.79
27	1983	7	1.4530	−0.5423	0.0893	0.0246	3.36	0.03	2.00
28	1983	10	2.0162	−1.2338	0.2176	0.0097	3.43	0.03	2.00
29	1984	1	−0.8810	−0.1323	0.0133	−0.1614	3.11	0.09	1.91
30	1984	4	−0.8935	−0.1175	0.0109	−0.1747	3.17	0.09	1.99
31	1984	7	−0.8876	−0.1233	0.0109	−0.1923	3.13	0.09	2.00
32	1984	10	−0.9029	−0.1446	0.0475	−0.1877	3.09	0.10	2.01
33	1985	1	−0.9721	−0.0697	0.0417	−0.1994	3.71	0.01	1.88
34	1985	4	−0.8184	−0.2692	0.0876	−0.1253	5.29	0.05	1.76
35	1985	7	−0.8112	−0.2730	0.0842	−0.1349	5.27	0.05	2.00
36	1985	10	−0.7615	−0.2835	0.0450	−0.1330	5.34	0.05	1.95
37	1986	1	−0.9655	−0.4183	0.3838	−0.1192	4.75	0.05	1.84
38	1986	4	−0.9275	−0.3841	0.3116	−0.1233	4.71	0.05	1.98
39	1986	7	−0.7465	−0.4378	0.1843	−0.1279	4.70	0.05	1.97

Source: Author's calculations.

ent first-order equations for the foreign bonds in equation A-2′ can be written in vector notation as one equation. Thus in equation A-6 $\mathbf{V}_{\mathbf{Rr}}$ represents the vector of covariances of the real domestic costs with the real foreign effective costs, $\mathbf{V}_{\mathbf{rp}^*}$ represents the covariance matrix of real domestic returns with the domestic unit value of export receipts, and $\mathbf{V}_{\mathbf{rr}}$ represents the covariance matrix of real foreign returns.

$$\text{(A-6)} \qquad 0 = -U_1'1 + \beta E(U_2')\{E[(1/P)\mathbf{R}^*] - b\mathbf{V}_{\mathbf{rp}^*}\mathbf{Q}(I) + b\mathbf{V}_{\mathbf{rr}}\mathbf{B} + b\mathbf{V}_{\mathbf{Rr}}1D\}.$$

Imposing that the domestic bond market is in zero net supply ($D = 0$), it can easily be shown that this equation can be solved for the optimal amount of foreign bonds as reported in equation 5 in the text.

The Speculative, Minimum Variance, and Price Hedges

When variables are lognormally distributed, we can split the speculative hedge and minimum variance hedge further and derive the price hedge. In the case of one foreign bond and one domestic investment opportunity we first combine equations A-2′ and A-3′, setting $D = 0$, to derive the following expression:

$$\text{(A-7)} \qquad E(U_2')\left[E\left(\frac{R^*}{P}\right) - E\left(\frac{R}{P}\right)\right] = b\,\mathrm{cov}\left(\frac{P^*Q - R^*B}{P}, \frac{R^*}{P}\right) - b\,\mathrm{cov}\left(\frac{P^*Q - R^*B}{P}, \frac{R}{P}\right).$$

Next we redefine variables in terms of percentage changes instead of second-period values. This implies that $E(R/P)$ can be written as $E(r) + E(1/p) + \mathrm{cov}\,(r, 1/p)$, where lower case symbols are percentage changes. This also implies that the covariances can be rewritten. For instance:

$$\text{(A-8)} \qquad \mathrm{cov}\left(\frac{P^*}{P}, \frac{R^*}{P}\right) \approx -\mathrm{cov}(p^*, p) + \mathrm{cov}(p, p) + \mathrm{cov}(p^*, r^*) - \mathrm{cov}(p, r^*).$$

Similar implications can be used for other covariances. Substituting these simplified expressions for the covariances in equation A-7, we see that some of the covariances will drop out. This allows us to rewrite equation A-7 and solve for the optimal amount of foreign bonds as a combination of a speculative, price, and minimum variance hedges, as in equation A-9:

$$\text{(A-9)} \qquad B = \frac{[E(r^*) - E(r)]E(U_2')}{-b\,\mathrm{VAR}(r^*)} + \frac{\mathrm{cov}(r^*, p^*)}{\mathrm{VAR}(r^*)}Q(I) - \frac{\mathrm{cov}(r^*, p)}{\mathrm{VAR}(r^*)}\left[1 - \frac{E(U_2')}{b}\right].$$

For an alternative derivation, see Claessens (1988).

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