The Optimization of Flat Plate Solar Collectors

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Abstract

Due to shortages in renewable energy resources, solar photovoltaic panels and solar thermal collectors have been widely researched and continue to be installed on commercial and residential buildings in order to reduce greenhouse gas emissions. Improvements in the efficiencies of photovoltaic panels are on the rise, making their use more feasible in more geographic locations. However, the angle at which the panels are mounted (angle of inclination or slope angle) can also significantly affect the solar irradiation they receive. The theoretical optimal angle of inclination of these panels has been determined based on the extraterrestrial solar constant, the effect of air mass, and assumption of clear skies. However, the assumption of clear skies may lead to significant discrepancies between theoretical model predictions and otherwise available meteorological data. Therefore, this paper aims to determine whether the optimal angle of inclination of solar photovoltaic panels will differ when the meteorologically accurate irradiation values are used. In this paper, 20 years of solar irradiation data was examined at three different geographic locations of the continental United States to determine if the optimal mounting angle of solar photovoltaic panels was different when comparing theoretical clear sky data to meteorologically accurate data. The data analyzed was from the National Solar Radiation Database from years 1990-2010. This irradiation data is a combination of meteorological statistical models, actual solar irradiation values measured by pyranometers from over 1400 locations in the United States, and cloud data using satellite imagery and a ceilometer. This data set is called METSTAT and is one of the two data sets provided by the National Solar Radiation Database. However, it is the only data set which provided consistent data over all of the twenty years analyzed at all three locations. The locations at which the clear sky and meteorologically accurate data were analyzed were dispersed in latitude and longitude across the United States and included: San Antonio, TX, San Francisco, CA, and Grand Rapids, MI. Using the generally accepted rule of thumb that the optimal angle of inclination is between -15˚ and 15˚ of the latitude of the location, both the clear sky and meteorologically accurate data were analyzed at each of the angles of inclination in this range to determine the optimal angle. From this analysis, it was determined that the optimal angle of inclination did vary between the two datasets. The results of this study show that the optimal angle of inclination of solar photovoltaic/solar thermal panels decreases as the cloudiness of the geographic location at which they are mounted increases. These results will be presented and discussed in detail in this paper.

Introduction

Solar photovoltaic (PV) panels are one of the most widely researched alternative energy solutions in today's world and are continuing to increase in efficiency and capability [1]. In fact, the United States government is funding $102 million dollars in the hopes of having 30% of the electricity used by the US population come from solar energy by 2050[2]. PV cells convert photons into electricity and have become widely used at both small and large scales in items from watches to space satellites [1]. Flat plate solar cells made from silicon are the most common type of solar panel and are generally mounted at a specific slope facing south in order to be exposed to the maximum amount of irradiation possible [1]. This research investigated the differences in the optimal mounting angles (slope) of solar panels at three locations based on data accounting for weather conditions and as separate data set assuming clear skies.

Method

Instead of using a theoretical calculation for the solar irradiation present at each location, data from the National Solar Radiation Database (NSRD) was used for calculations; this data uses a combination of actual measurements, observations, and statistical models to predict the actual, meteorologically accurate, solar irradiation present at that location (as opposed to assuming clear skies)[1, 3]. The data used for this research was METSTAT data and was compiled and developed by MetStat Inc. In the early 1990's METSTAT started replacing their manual cloud observations with an automated system called ASOS (Automated Surface Observing System) [5]. This model used a ceilometer with a 12,000ft reach in altitude to detect opacity in the atmosphere, however, its resolution and ability was subpar [5]. In order to compensate for the ASOS system, the Supplemental Cloud Product (SCP) was used in the METSTAT model to record accurate cloud cover [5]. The SCP uses satellite imagery to determine the height and opacity of the clouds in order to more accurately estimate irradiation at various locations [5]. It should also be noted that hourly uncertainties are present in the data, for example, a cloud cover amount of 50% does not clearly identify whether the sun is fully visible between scattered clouds or if it is entirely blocked by an opaque cloud [5]. While the data is one of the best examples of meteorologically accurate data on the NSRD database, the error in the models that were used were, at times, as high as 16% [1, 3, 5].

The data compiled represents the irradiation measurements for each hour of each day from years 1990 – 2010. The irradiation for each hour was averaged over the 20 year span. This yielded two columns of useful irradiation data: one comprised of the averaged theoretical clear sky irradiation, and one comprised of the averaged meteorologically accurate irradiation. Averaging the data individually for each hour over a 20 year span gives a good estimate of the amount of irradiation that will be present at each hour during this year.

For each of the three locations evaluated in this study, Grand Rapids, MI, San Francisco, CA, and San Antonio, TX, it was necessary to find the angle of incidence of the beam radiation on the solar panel plane. In order to maximize the output from the solar PV cells, the angle of incidence should be as close to normal as possible, or in other words, its cosine should be as close to 1 as possible. The equation for calculating the cosine of the angle of incidence is below:

$$\cos(\theta) = \sin(\delta)\sin(\varphi)\cos(\beta) - \sin(\delta)\cos(\varphi)\sin(\beta)\cos(\gamma) + \cos(\delta)\cos(\varphi)\cos(\beta)\cos(\omega) + \cos(\delta)\sin(\varphi)\sin(\beta)\cos(\gamma)\cos(\omega) + \cos(\delta)\sin(\beta)\sin(\gamma)\sin(\omega)$$

where δ is the declination angle, φ is the latitude, ω is the hour angle, γ is the azimuth angle, and β is the slope of the panel [4]. In this analysis, it was assumed that the solar PV cells were facing south which would make the solar azimuth angle 0˚. Therefore, the equation can be simplified to the following:

$$cos(\theta) = sin(\delta)sin(\varphi)cos(\beta) - sin(\delta)cos(\varphi)\sin(\beta)(1) + cos(\delta)cos(\varphi)cos(\beta)cos(\omega) + cos(\delta)sin(\varphi)sin(\beta)(1)cos(\omega) + 0$$

The declination angle, δ, is the angular position of the sun at solar noon with respect to the plane of the equator [4]. This angle varies between -23.45˚ and 23.45˚ depending on the time of the year [4]. It was assumed that the declination angle was the same for each hour of the same day. The following equation was used for the calculation of declination, where n is the day of the year:

$$\delta = 23.45^o \sin\left(360 * \frac{284 + n}{365}\right)$$

The latitude, φ, varied based on the location of each city. Table 1 shows the angles of latitude and longitude for each city that were considered in this study.

Table 1: Latitudes and longitudes for each city considered in this study

City	Latitude (φ)	Longitude	Reference Longitude
Grand Rapids, MI	42.96˚	85.65˚	75˚
San Francisco, CA	37.78˚	122.42˚	120˚
San Antonio, TX	29.41˚	98.50˚	90˚

The hour angle, ω, was calculated by multiplying the number of hours away from solar noon by 15˚ (the rotation of the Earth per hour). These values are negative before the solar noon (morning) and positive after solar noon (evening). This calculation, however required the conversion of the time given (local time) into solar time and used the data in Table 1. The following equations were used in that conversion, where solar time is:

$$ST = 4\left(Long_{local} - Long_{Ref}\right) * E$$

where E, the equation of time, is:

$$E = 229.2(0.000075 + (0.001868cosB) - (0.032077sinB) - (0.014615cos2B) - (0.04089sin2B))$$

and B is:

$$B = (n - 1) * \left(\frac{360}{365}\right)$$

where n is the numbered day of the year [4]. Once the variance between the local time and the solar time was determined for each day, the hour angle could be calculated for each hour of that day at each location. Each of these calculations was performed in its own column of the excel spreadsheet.

The slope angle, β, was then varied between -15˚ and 15˚ within the respective latitude for each location, and the cosine of the angle of incidence was calculated with each of the different slopes [4]. The given global radiation was divided by the cosine of the average zenith angle for each hour to get the amount of radiation on a surface normal to the sun. Then this amount was multiplied by the cosine of the angle of incidence to get the total amount of irradiation available at each hour. This process was completed for both the theoretical and meteorologically accurate amount of solar radiation to determine the amount of solar radiation that would be intercepted by the panels at each hour at each location and for each respective angle of slope using the following equation:

$$I_A = \left(\frac{I_D}{\cos(z)}\right) * \cos(\theta)$$

where I_A is the actual irradiation (W/m^2) incident on the panel at a specific angle and I_D is the direct irradiation present for that hour [4]. The irradiation intercepted by the panels for each respective angle of slope was then summed over the entire year and the maximum values for both the theoretical and meteorologically accurate models were recorded.

Results and Discussion

The summed totals of irradiation for the entire year were used to determine the slope at which the solar panels should be placed to give the maximum output. By calculating these totals for various slopes, β, the maximum available energy could be determined for both the theoretical and meteorologically accurate data. The slope angles that intercepted the maximum irradiation for both data sets were different for each data set at each of the three locations. Table 2 shows an example of the data for the slope angle of 28˚ for Grand Rapids, MI on January 1st.

Table 2: The input data for the angle of incidence calculation and the irradiation values for hours 1-24 on January 1st in Grand Rapids, MI

				GIVEN	DIV ZEN	GIVEN	DIV ZEN							
HH:MM (LST)		Zenith (deg)	Cos ZEN	Glo (Wh/m^2)		CSKY Glo (Wh/m^2)		declination	hour angle	(latitude)	B (beta - slope)	Cos θ	I - Clear (W/m2)	I - Cloudy (W/m2)
1:00	1.00	99	-0.15643	0	0	0	0	-23.01164	-176.37	43	28	-0.98845	0	0
2:00	2.00	99	-0.15643	0	0	0	0	-23.01164	-161.37	43	28	-0.94363	0	0
3:00	3.00	99	-0.15643	0	0	0	0	-23.01164	-146.37	43	28	-0.84141	0	0
4:00	4.00	99	-0.15643	0	0	0	0	-23.01164	-131.37	43	28	-0.68873	0	0
5:00	5.00	99	-0.15643	0	0	0	0	-23.01164	-116.37	43	28	-0.49601	0	0
6:00	6.00	99	-0.15643	0	0	0	0	-23.01164	-101.37	43	28	-0.27639	0	0
7:00	7.00	99	-0.15643	0	0	0	0	-23.01164	-86.37	43	28	-0.04483	0	0
8:00	8.00	99	-0.15643	0	0	0	0	-23.01164	-71.37	43	28	0.182896	0	0
9:00	9.00	86.9	0.054079	12.35	228.37	19.4	358.7357	-23.01164	-56.37	43	28	0.391261	140.359192	89.352372
10:00	10.00	79.72	0.178459	72.55	406.536	129.25	724.257	-23.01164	-41.37	43	28	0.566066	409.977565	230.12667
11:00	11.00	72.905	0.293957	136.95	465.885	253.4	862.0311	-23.01164	-26.37	43	28	0.695401	599.456955	323.97644
12:00	12.00	68.2	0.371368	179.3	482.81	345.55	930.4791	-23.01164	-11.37	43	28	0.770449	716.886986	371.98043
13:00	13.00	66.05	0.405939	191.65	472.115	388.15	956.1775	-23.01164	3.63	43	28	0.786098	751.649377	371.1287
14:00	14.00	66.725	0.395145	172	435.284	374.35	947.3744	-23.01164	18.63	43	28	0.741281	702.270317	322.66728
15:00	15.00	70.155	0.339477	156.6	461.298	306.9	904.0383	-23.01164	33.63	43	28	0.639051	577.72657	294.79303
16:00	16.00	75.95	0.242769	116.4	479.469	196.2	808.1772	-23.01164	48.63	43	28	0.486376	393.078049	233.20227
17:00	17.00	83.545	0.112423	39.2	348.684	69.35	616.8676	-23.01164	63.63	43	28	0.29366	181.149563	102.39456
18:00	18.00	88.88	0.019546	1.15	58.8342	1.5	76.74031	-23.01164	78.63	43	28	0.074037	5.6816326	4.3559183
19:00	19.00	99	-0.15643	0	0	0	0	-23.01164	93.63	43	28	-0.15753	0	0
20:00	20.00	99	-0.15643	0	0	0	0	-23.01164	108.63	43	28	-0.38525	0	0
21:00	21.00	99	-0.15643	0	0	0	0	-23.01164	123.63	43	28	-0.59361	0	0
22:00	22.00	99	-0.15643	0	0	0	0	-23.01164	138.63	43	28	-0.76842	0	0
23:00	23.00	99	-0.15643	0	0	0	0	-23.01164	153.63	43	28	-0.89775	0	0
24:00:00	24.00	99	-0.15643	0	0	0	0	-23.01164	168.63	43	28	-0.9728	0	0

The same calculation was completed for all of the slope angles, β, for every hour of the year to determine which angles yielded the highest harnessable amount of solar energy for both the "clear" and "cloudy" sky data. Due to the fact that the irradiation was integrated over each hour, the irradiation units become Wh/m^2. For Grand Rapids, MI it was determined that a 38.25° slope gave the best outcome when considering the clear sky data, but a 35° slope gave the best outcome when considering the cloudy sky data. Figures 1, 2, and 3 illustrate the total yearly irradiance for each of the angles considered for both clear and cloudy sky data in Grand Rapids, MI.

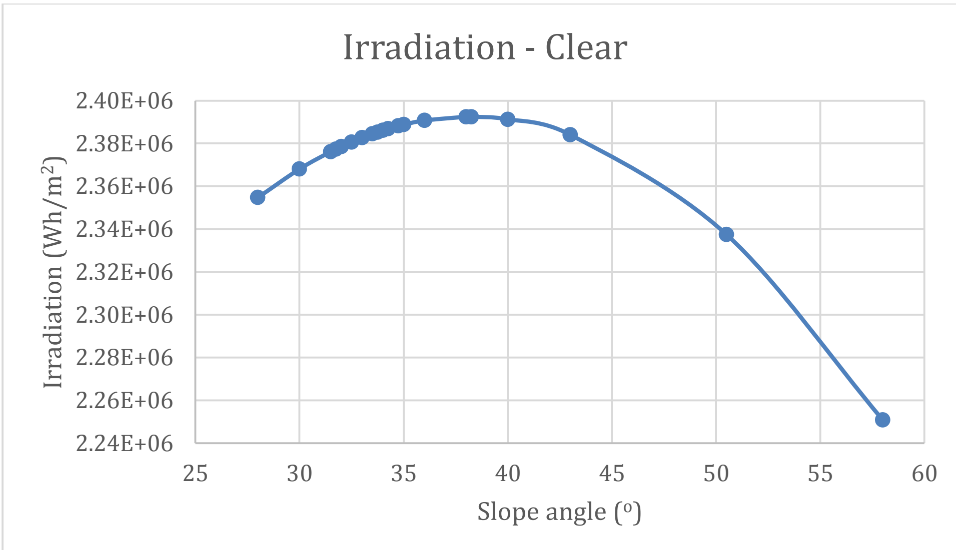

Figure 1: Yearly irradiation based on clear sky data for multiple slope angles in Grand Rapids

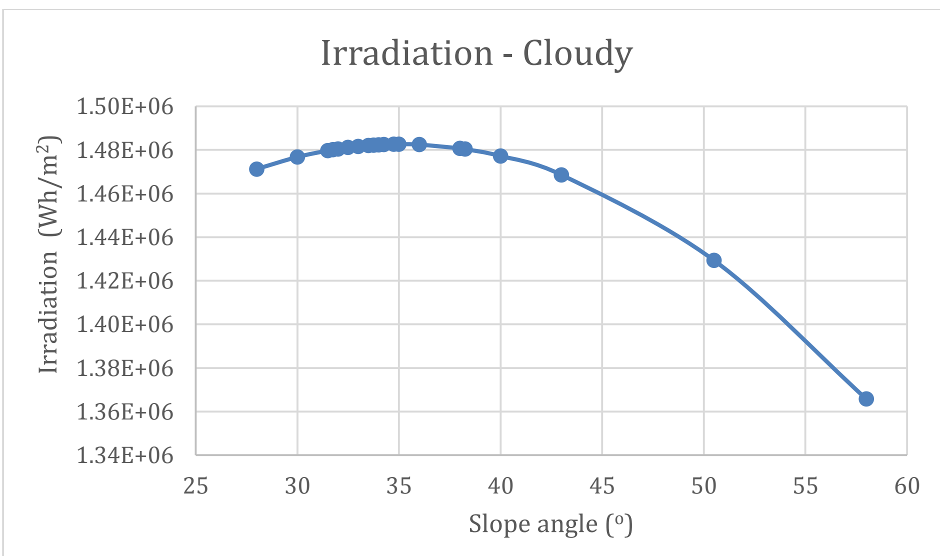

Figure 2: Yearly irradiation based on cloudy sky data for multiple slope angles in Grand Rapids

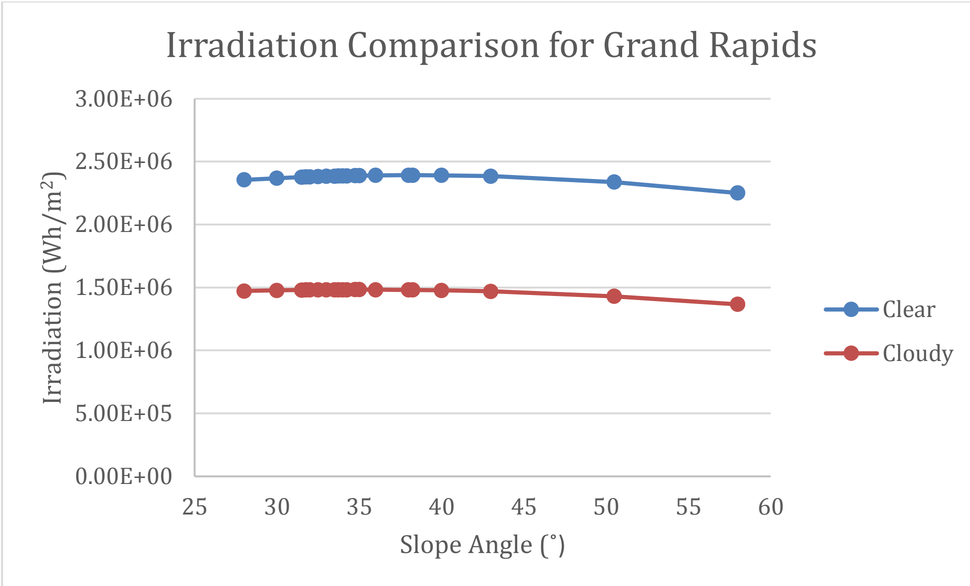

Figure 3: Yearly irradiation for both clear and cloudy sky data for multiple slope angles in Grand Rapids

Figure 1 shows that the maximum yearly irradiation using clear sky data is 2,392,530 Wh/m^2 while Figure 2 shows the maximum yearly irradiation using cloudy sky data is 1,482,681 Wh/m^2. On average, the meteorologically accurate irradiation data was approximately 61.5% of the clear sky irradiation values for Grand Rapids. This process was repeated for San Francisco, CA and San Antonio, TX. Figures 4, 5, and 6 show the clear and cloudy sky irradiation present in San Francisco, CA. The maximum irradiation occurred at a slope of 33.75˚ for clear sky data and 32.5˚ for the cloudy sky data.

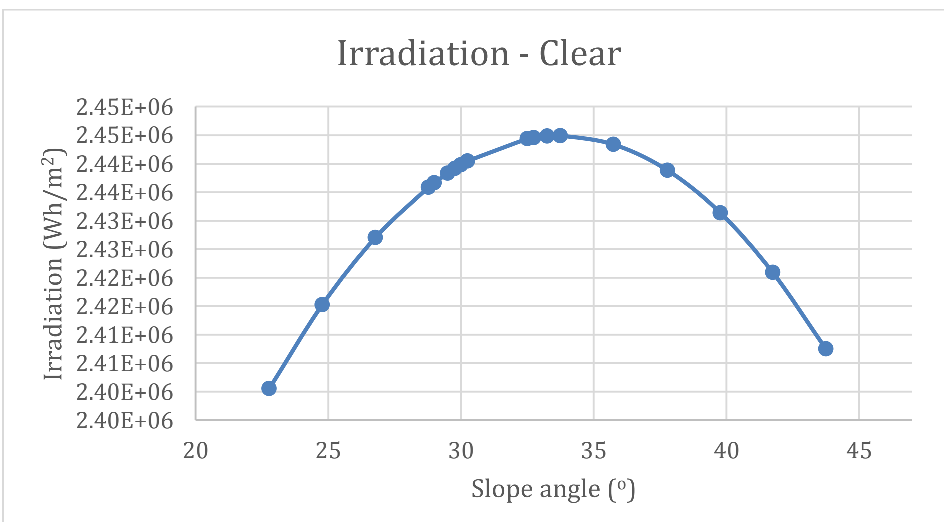

Figure 4: Yearly irradiation based on clear sky data for multiple slope angles in San Francisco

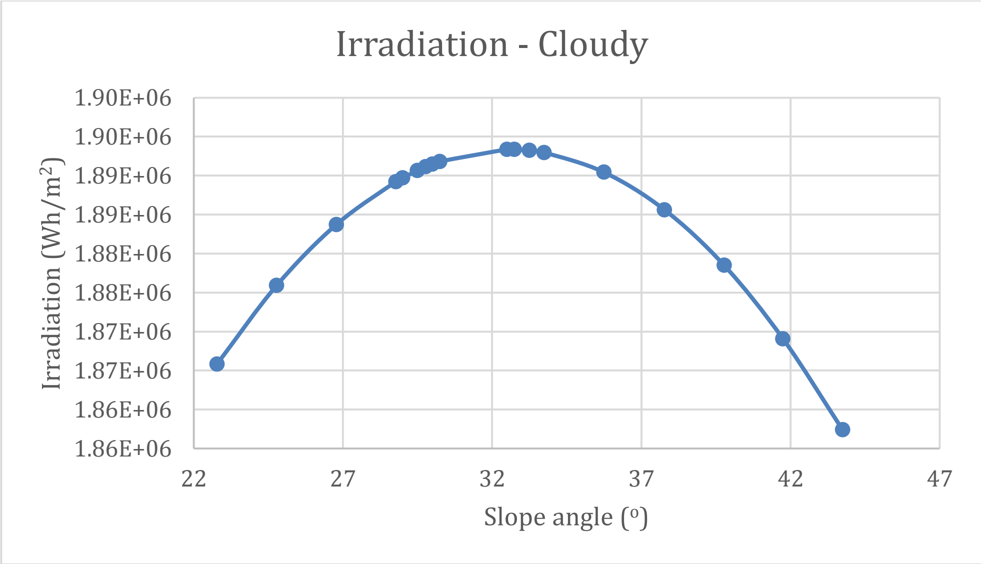

Figure 5: Yearly irradiation based on cloudy sky data for multiple slope angles in San Francisco

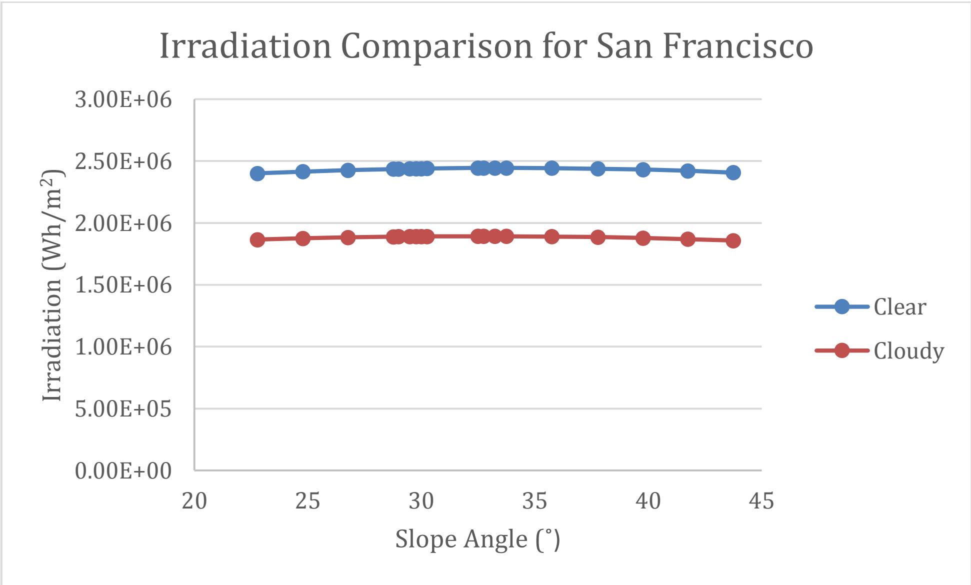

Figure 6: Yearly irradiation for both clear and cloudy sky data for multiple slope angles in San Francisco

These figures show that the maximum yearly irradiation using clear sky data is 2,444,939 Wh/m^2 and the maximum yearly irradiation using cloudy sky data is 1,893,378 Wh/m^2. On average, the meteorologically accurate irradiation data was approximately 77.4% of the clear sky irradiation values for San Francisco. Figures 7, 8, and 9 show the clear and cloudy sky irradiation present in San Antonio, TX. The maximum irradiation occurred at a slope of 27.25˚ for clear sky data and 26˚ for the cloudy sky data. On average, the cloudy sky data was 70.4% of the clear sky data in San Antonio.

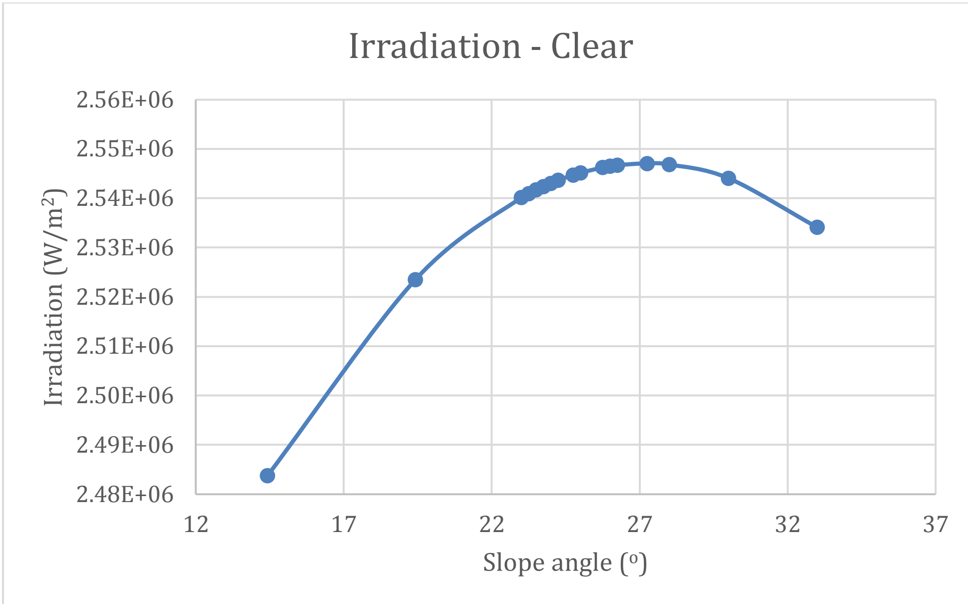

Figure 7: Yearly irradiation based on clear sky data for multiple slope angles in San Antonio

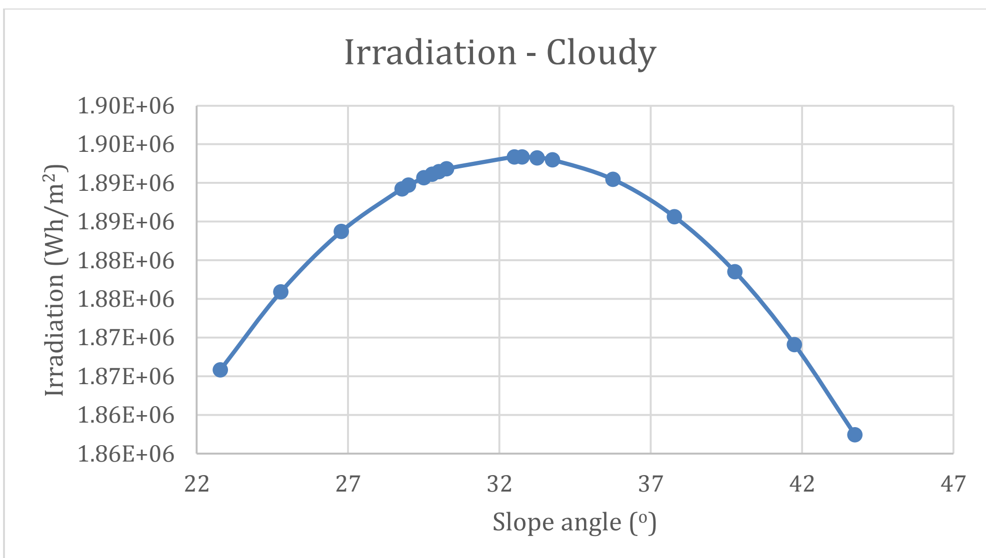

Figure 8: Yearly irradiation based on cloudy sky data for multiple slope angles in San Antonio

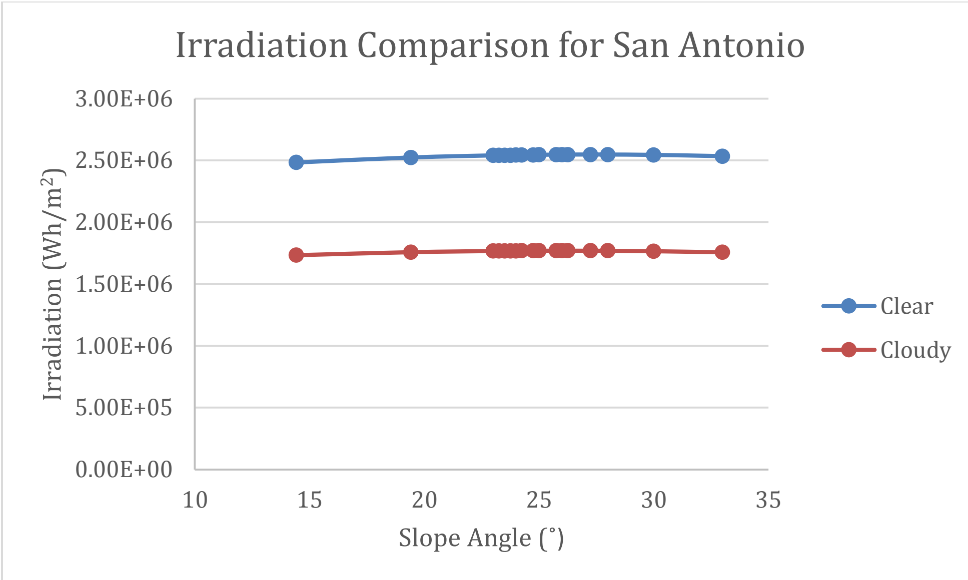

Figure 9: Yearly irradiation for both clear and cloudy sky data for multiple slope angles in San Antonio

These figures show that the maximum yearly irradiation using clear sky data is 2,547,039 Wh/m^2 and the maximum yearly irradiation using cloudy sky data is 1,770,386 Wh/m^2. When comparing the data, the closest values of cloudy and clear sky data were found in San Francisco where the cloudy sky data was 77.4% of the clear sky data. The highest output using clear sky estimates was in San Antonio, and the highest output using cloudy sky estimates was in San Francisco. Table 3 summarizes the data and shows the optimal angles for each location while Figures 10 and 11 show a comparison of the irradiation among the three cities.

Table 3: Summary of findings for each city respectively

	Grand Rapids, MI	San Francisco, CA	San Antonio, TX
Latitude (˚)	44.69	37.78	29.41
Max. Yearly Irradiation - Clear (Wh/m^2)	2,392,530	2,444,939	2,547,039
Max. Yearly Irradiation - Cloudy (Wh/m^2)	1,482,681	1,893,378	1,770,386
Total Cloudy Irradiation as a Percentage of Total Clear Irradiation (%)	61.9	77.4	69.5
Optimal Slope Angle - Clear (˚)	38.25	33.75	27.25
Optimal Slope Angle - Cloudy (˚)	35.0	32.5	26.0
Difference in Angle (˚)	3.25	1.25	1.25

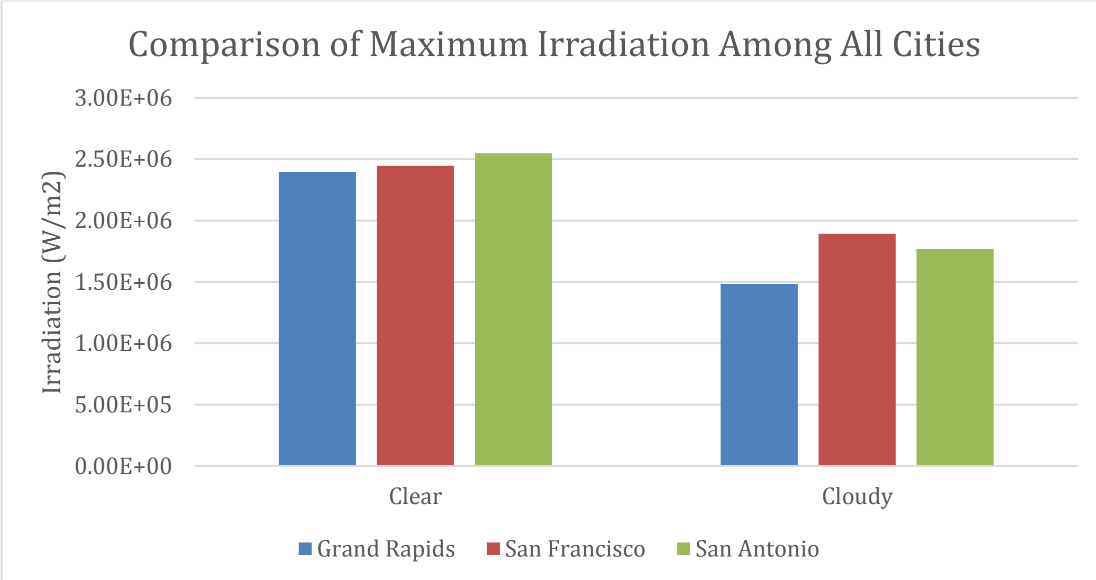

Figure 10: Maximum yearly irradiation comparison among the three cities

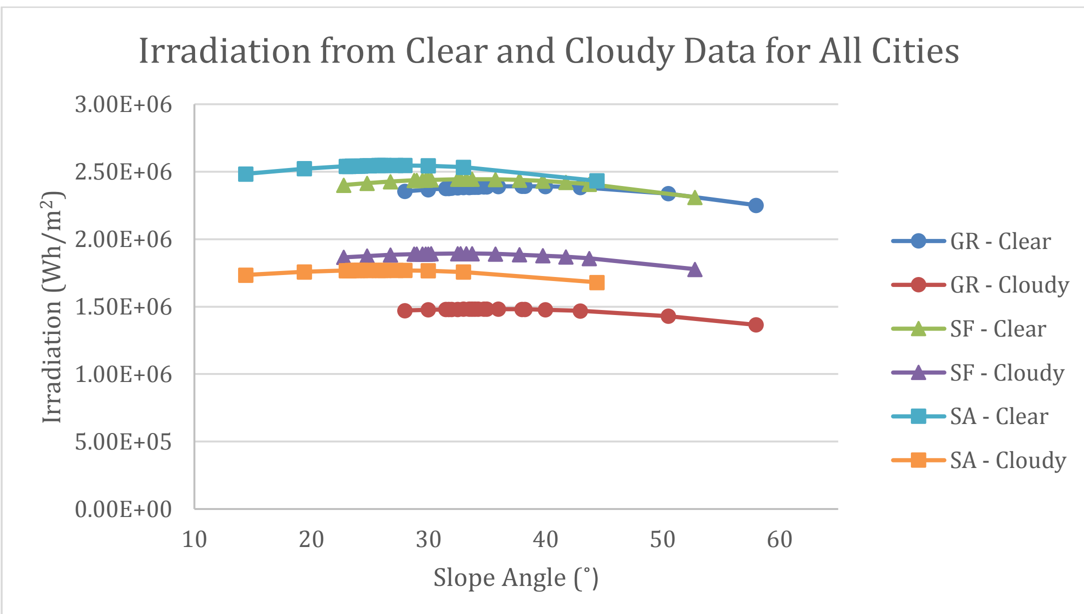

Figure 11: Irradiation comparison between all three cities over the entire span of slope angles

The values for the irradiation on the surface of the solar panels at each location are not surprising. For the clear sky data, the irradiation incident on the surface increased as the latitude of the location decreased. Additionally, when accounting for cloudy sky data, the irradiation present on the surface of a solar panel in Grand Rapids was significantly less due to the cloudy weather. When considering cloudy sky data, San Francisco and San Antonio had very similar solar irradiation present on the surface of the solar panels due to its mild climate and few storms.

The difference in the optimal slope angle for the solar panels when considering the cloudy versus the clear sky data is likely due to the decrease in direct irradiation and the increase in diffuse irradiation. As the direct radiation scatters in the atmosphere due to clouds it does not shine on the solar panel at the same angle as it would with no clouds present. Due to the fact that Grand Rapids has many more cloudy days than San Francisco or San Antonio accounts for the larger discrepancy between the optimal angles for clear or cloudy sky data.

The typical mounting fixture for a fixed solar panel can either remain stationary at all times or involve a manual, step-based adjustment system that allows for the slope angle to be adjusted several times a year. After a review of many commercial and residential mounting systems, the increments by which the slope angle could be adjusted varied between 1˚ and 10˚ with most mounting systems having 5˚ adjustable increments [6, 7]. For a fixed panel system even a 5˚ misalignment would decrease the energy output by a rather insignificant amount [6]. While the accuracy of the slope angle matters significantly in some solar PV applications (i.e. concentrating photovoltaics) the discrepancy between the mounting angle between clear and cloudy sky data in a fixed plane solar PV application is generally not significant.

Both sets of irradiation values were also compared to theoretical calculations with clear sky assumptions that were previously carried out at GVSU. The optimal slopes for flat plate solar PV panels were calculated using theoretical calculations with extraterrestrial irradiation and air mass [6]. The optimal slope of the flat plate solar panels (south facing surface) were determined for several latitudes including the latitude of Grand Rapids [8]. Based on the previous research (theoretical data only), the optimum angle for a solar PV collector is 37˚ and the yearly irradiation is 1778.4 kWh/m^2 [8]. However, using the clear sky data from the METSTAT model, the optimum angle for a solar PV collector is 38.25˚ with a yearly irradiation of 2392.5 kWh/m^2. Additionally, based on the meteorologically accurate data from this analysis, the optimum slope angle for Grand Rapids is 35.0˚ and the yearly irradiation is 1482.7 kWh/m^2. The discrepancies between the optimal angle and yearly radiation are most likely due to the fact that the METSTAT data is a model for global irradiation (diffuse and direct) while the theoretical calculation only considers direct radiation. Tables 4 and 5 show the results from previous research and a summarization of this research respectively.

Table 4: Optimal slope and yearly irradiation values for a flat plate ("South facing surface") and single axis tracking solar PV plate [8]

Latitude (˚N)	South Facing Surface	
	Optimal Slope (˚)	Yearly Irradiation Value at Optimal Slope (kWh/m^2)
0	0	2105.4
20	18	2041.9
43.2	37	1778.4

Table 5: Summary of optimal slope angles and yearly irradiation values for each location

Latitude (Location)	Optimal Slope (˚)		Irradiation (kWh/m^2)	
	Clear	Cloudy	Clear	Cloudy
42.96˚ (Grand Rapids, MI)	38.25	35.0	2392.5	1482.7
37.78˚ (San Francisco, CA)	33.75	32.5	2444.9	1893.4
29.41˚ (San Antonio, TX)	27.25	26	2547.0	1770.4

Conclusion

In a world where energy demands are ever increasing, solar panels are a viable solution to provide useful, renewable energy for individuals and businesses. However, in order to provide maximum output the panels need to be placed at an optimum angle to harness the maximum amount of irradiation possible. In the past, the optimum slope of solar panels has been determined using irradiation estimates based on clear sky data, however, it can now be shown that when using meteorologically accurate data the optimum angles change slightly due to the more diffuse nature of the irradiation. For cloudier locations such as Grand Rapids, MI, the optimum angle changes more than in sunnier locations such as San Francisco, CA. When mounting the solar panels in a fixed plate system, the discrepancies between clear and cloudy mounting angles are not significant enough to encourage manufacturers to provide smaller increments in the mounting brackets. Therefore, the amount of irradiation and ideal slope angle change when considering meteorologically accurate data, for most fixed panel applications the change in the ideal mounting angle is insignificant while the differences in irradiation are significant.

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