Geophys. J. R. astr. Soc. (1976) **45**, 707–712.

Research Note

The Origin of Outer Topographic Rises Associated with Trenches

Barry Parsons and Peter Molnar

Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

(Received 1976 January 23; in original form 1975 November 7)

Summary

The outer topographic rise seaward of many island arcs can be explained in terms of a bending moment applied to the oceanic lithosphere either by the downgoing slab to which it is attached or by modest shear stresses acting along the underthrusting plate boundary. The existence of the outer rise does not require large horizontal compressive stresses to maintain it.

In order to explain the outer topographic rises and associated gravity highs seaward of trenches Hanks (1971) and Watts & Talwani (1974) suggested that very large horizontal compressive stresses (5–10 kbar) exist across island arcs. Hanks (1971) showed that the outer topographic rise south-east of the Kurile trench could be modelled as the response of a thin ($\sim$ 30 km thick) elastic plate to vertical and horizontal forces at the trench axis. Watts & Talwani (1974) demonstrated that such an outer rise existed continuously seaward of the Pacific trench system from the Aleutians to the Marianas arc, and that in many instances, a positive gravity anomaly was associated with the outer rise. This gravity anomaly could be interpreted as that produced by the excess mass at the deformed upper and lower surfaces of the thin elastic plate model. To match the amplitudes of the topographic rises and gravity highs by the model, compressive stresses, up to 12 kbar, acting at the trench were needed. This result presents problems with regard to the material behaviour under such large stresses, and in terms of providing a driving mechanism to maintain these stresses. The purpose of this note is to suggest that such large stresses are not necessarily required across the plate boundary, and to give an alternative explanation within the context of the same model.

The equation describing the bending of a thin elastic plate embedded in a fluid is

$$D\frac{d^4y}{dx^4} - N\frac{d^2y}{dx^2} + \rho g y = 0 \qquad (1)$$

where y is the deflection of the plate at a distance x from the origin (Fig. 1), N is the horizontal force per unit length (positive for tension, negative for compression) applied at the origin, ρ is the density difference between the material below and above the plate, and g is the gravitational acceleration (Le Pichon, Francheteau & Bonnin 1973). The flexural rigidity, D, is given by

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

where E is Young's modulus, h is the effective elastic thickness of the plate, and ν is Poisson's ratio. The boundary conditions are that the solution is bounded as $x \to \infty$, and that at $x = 0$ the deflection $y = -y_0$ is specified and there is an applied moment, $M = D(d^2y/dx^2)$ (positive for a clockwise applied moment in the plane of Fig. 1). The solution can be written:

$$y = \left\{\frac{2}{\alpha\beta N_{cr}}\left(M + \frac{Ny_0}{2}\right)\sin\beta(\lambda x) - y_0\cos\beta(\lambda x)\right\}\exp\{-\alpha(\lambda x)\} \tag{3}$$

with

$$\alpha = \left(1 - \frac{N}{N_{cr}}\right)^{1/2}, \quad \beta = \left(1 + \frac{N}{N_{cr}}\right)^{1/2}, \quad \lambda = \left(\frac{\rho g}{4D}\right)^{1/4},$$

and $N_{cr} = -4D\lambda^2$ is the critical compressive buckling stress at which $\alpha \to 0$ and the solution blows up. Horizontal distances can be scaled by λ as in equation (3); from the values used by Hanks (1971), $1/\lambda \sim 70$ km.

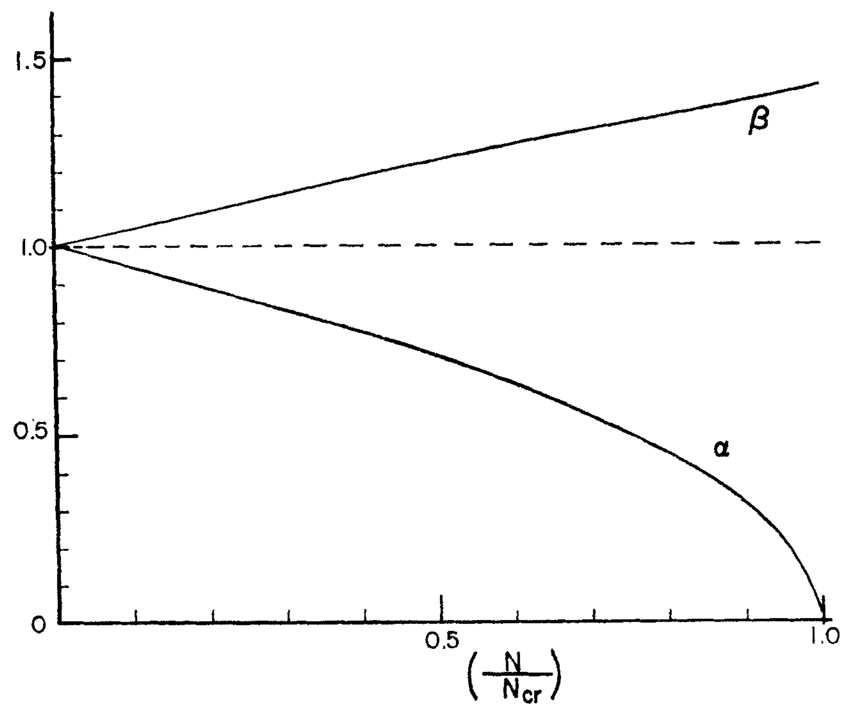

FIG. 2. Variation of the attenuation coefficient α and wavenumber β as a function of N/N_{cr}. For $h = 30$ km N_{cr} corresponds to a compressive stress of 44 kbar.

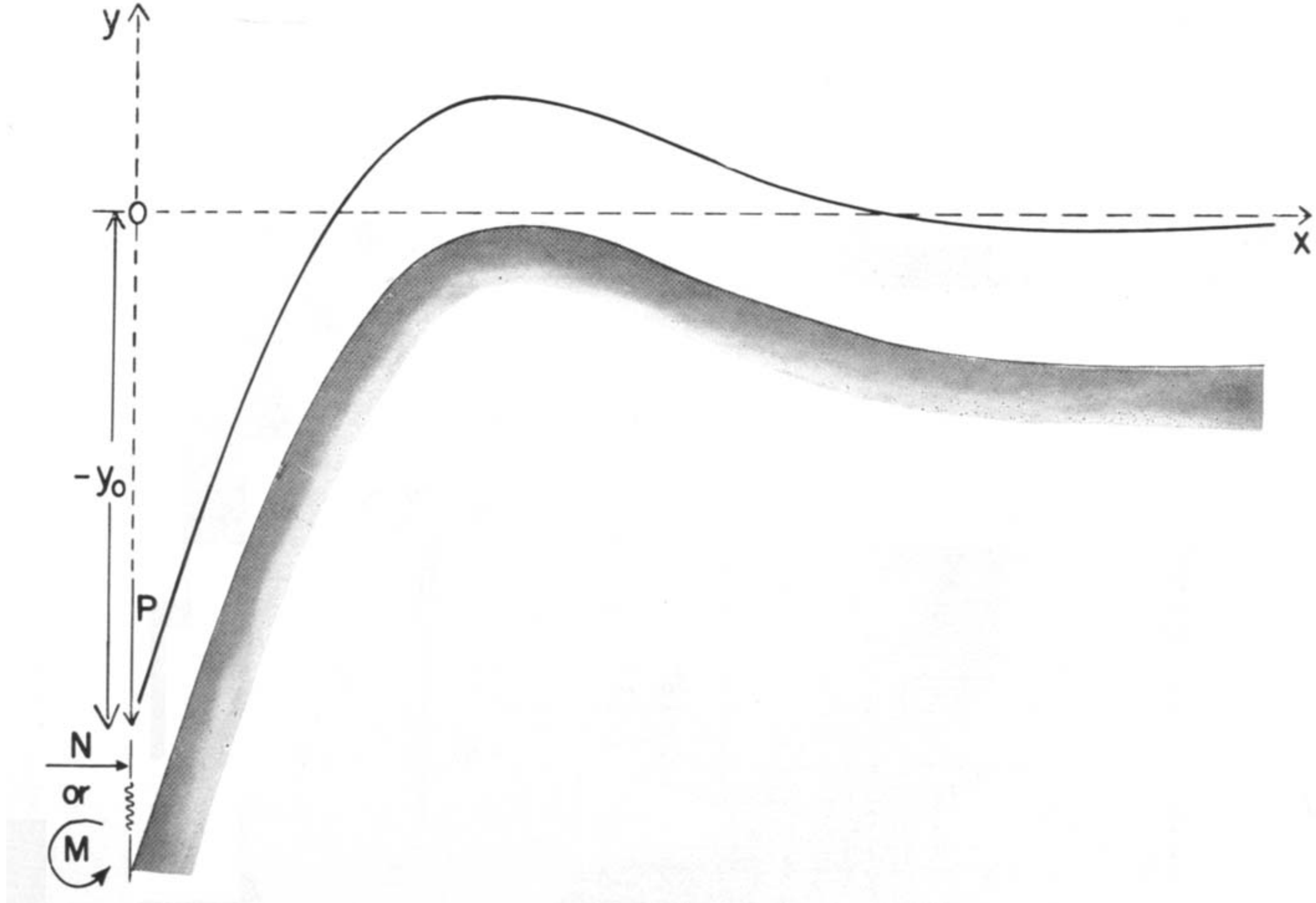

FIG. 1. Simple geometry of the thin elastic plate model showing the boundary conditions applied at the origin, assumed to be the trench axis. The thickness of the plate is not to scale.

facing page 708

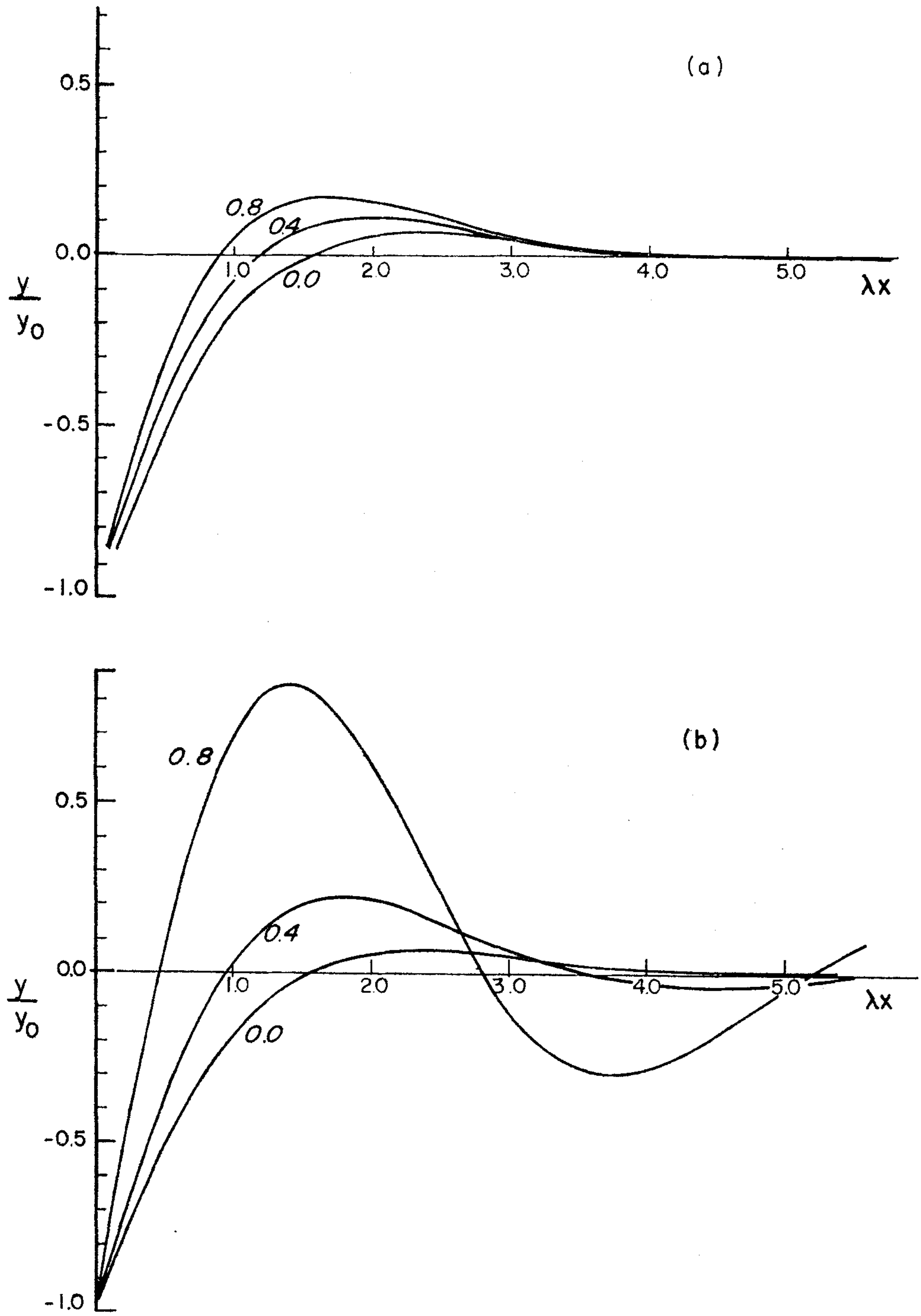

FIG. 3. (a) Deflection produced by a moment M with $N = 0$. The numbers labelling the curves are the equivalent non-dimensional compressive stress $2M/y_0 N_{cr}$. (b) Deflection produced by compressive stresses N with $M = 0$. Numbers by curves give the non-dimensional compressive stress N/N_{cr}. Note that for very large values of N/N_{cr} a compressive stress is more efficient than an equivalent moment because of its effect on α.

The value of N enters into the solution in two ways. First it affects the values of α and β (Fig. 2). However, for reasonable values of the compressive stress this effect is small. Following Hanks (1971) we take $\lambda = 1{\cdot}4 \times 10^{-7}\ \mathrm{cm}^{-1}$ and $D = 1{\cdot}7 \times 10^{30}$ dyne cm so that $N_{cr} = -1{\cdot}33 \times 10^{17}$ dyne cm^{-1}. A value of $N/N_{cr} = 0{\cdot}23$ corresponds approximately to a compressive stress of 10 kbar, and hence α and β will not vary by more than 10 per cent for values less than this. The width of the outer rise, defined as the distance between the first two points of zero deflection, is $\pi/\beta\lambda$. As β varies little with N this value should be approximately constant. However, the width of the outer rise mapped by Watts & Talwani (1974) increases considerably further east along the Aleutian trench, and cannot be accounted for by the elastic plate model. They attributed this to uneven sediment cover seaward of the trench. An alternative explanation of the discrepancy lies in the variation of the age of the ocean floor being subducted around the trench system. The ocean floor east of the western Pacific trenches is relatively old (> 110 My) and therefore closer to thermal equilibrium, so that the depth is not far from the equilibrium depth (Parsons & Sclater 1976) and provides a good reference level for zero deflection. The ocean floor being subducted in the eastern Aleutian trench was originally formed at the now extinct Kula–Pacific rise and is much younger. Profiles of Watts & Talwani (1971) cross anomalies 24–32 approximately, and hence a change in depth away from the trench of 300–400 m can be accounted for by the continued cooling of the lithosphere (Sclater, Anderson & Bell 1971). This would produce the long tail observed on the topographic profiles. Although they apparently observe the same width in the gravity field the values further from the trench are small and from their gravity profiles it would appear possible to choose a smaller width.

The other important effect of N is on the amplitude of the rise through the coefficient in equation (3). When this coefficient is zero the outer rise is small; increasing the magnitude of either M or N increases the amplitude of the outer rise (Fig. 3). In this respect, for large values of N, a compressive force is more efficient, because the attenuation coefficient, α, is also reduced. For reasonable values of N, however, this effect is small. Hanks (1971) and Watts & Talwani (1974) assumed that $M = 0$ and also initially took the origin to be at the trench axis with y_0 the deepest part of the trench. Writing the solution in the form (3) shows that the compressive force, in fact, is equivalent to a moment of magnitude $Ny_0/2$. Expression (3) also demonstrates two ways in which the outer rise can be maintained with smaller compressive stresses. In the first case it should be recognized that the choice of origin is arbitrary, and if the origin is chosen landward of the trench so that y_0 is larger, N can be correspondingly reduced. Secondly, we can try to produce the deformation, or at least part of it, with a non-zero couple, M, (Caldwell *et al.* 1976). Hanks recognized these points but did not develop them. In fact both he and Watts & Talwani posed the problem somewhat differently, prescribing the vertical force per unit length at the origin instead of the deflection y_0. They then introduced the horizontal compressive force as a multiple of the vertical force; this changed the deflection at the origin and to rematch y_0 they had to shift the calculated profiles landward. This is equivalent to applying the compressive force at a greater depth as pointed out above. If their problem is solved with the boundary conditions consistently applied at the origin, the values of compressive stress needed are even larger by a factor of 2. That the arbitrary choice of origin at which the horizontal force is applied is a source of the large estimates of compressive stress can be demonstrated by noting that the highest values were obtained for those profiles (Aleutian and Ryukyu arcs) where the maximum depth of the trench, y_0, is smallest (Watts & Talwani 1974), so that N had to be correspondingly increased to produce the necessary moment.

The horizontal forces presumably originate along the zone of underthrusting between the plates and so it should be expected that the forces will be distributed with depth over this zone. At shallow depths (< 70–100 km) the inclined seismic zones, and hence also the underthrust lithosphere, at island arcs appear to dip at shallow angles (~ 10°). For the shallowest depths near the trench axis, earthquakes showing underthrusting are rare, presumably because the downgoing plate is only in contact with weak sediments. Thus it is likely that the horizontal forces are distributed deeper and applied for deflections larger than the deflection at the trench axis. It might then be objected that the thin plate theory would not apply, as this theory is assumed to be valid for $y \ll h$. The general equations of deformation of a thin plate due to Föppl (Landau & Lifshitz 1970) for deflections large compared to h (although small compared to the areal dimensions) are non-linear and difficult to solve. In the special case, however, where the problem is restricted to two dimensions (independent of z) and shear stresses applied at the base of the plate are neglected, the equations reduce to (2). Hence we can assume that the form of (3) is a good approximation even for large deformations, and that the above comments on the distribution of forces hold.

A horizontal compressive stress of 10 kbar, with $h = 30$ km (Hanks 1971), applied at the origin with $y_0 = 3$ km is equivalent to a moment per unit length of $Ny_0/2 = -4{\cdot}5{\cdot}10^{21}$ dynes. Consider a shear stress distributed along a zone of underthrusting between 15 and 75 km in depth over a horizontal extent of 200 km. We suppose that the vertical component of this force is balanced by part of the vertical forces causing the deflection. The horizontal component can then provide a moment equal to the above value for a shear stress of 125 bar. In a similar way the vertical stresses needed in deflecting the descending lithosphere can be reduced by distributing them over a larger area.

The horizontal stresses needed can also be reduced by a mechanism that would directly apply a moment M at the trench axis. The downgoing slab is colder than the surrounding medium and produces negative buoyancy forces that generate a circulation in the mantle. The sinking lithosphere and the effect of the mantle motions surrounding it could produce the required couple. In order to crudely estimate the magnitude of this couple we use the estimate of McKenzie (1969) that the total buoyancy force (B) of the slab per unit length acting downward is about $1{\cdot}3 \times 10^{16}$ dyne cm^{-1}. Because of the small dip at shallow depths the downward pull of the heavy downgoing slab is not applied directly beneath the trench but landward of the trench. A rough value for the bending moment per unit length is given by $-1/2\, Bh \sin\theta$ where θ is the dip of the main part of the slab. Using $h = 30$ km gives $M = -2{\cdot}10^{22} \sin\theta$ dyne. Although this estimate is very crude, and probably overestimated having neglected the reducing effect of viscous stresses on the slab, it indicates that the forces associated with the motion of the downgoing slab could provide a sufficiently large moment. Hence for either of the above reasons there seems no need to invoke large horizontal compressive stresses across island arcs in order to maintain the outer rise.

The above suggestions are particularly relevant in two important areas where work is in progress. In the first case there is an outer rise and gravity high associated with the Puerto Rico trench (Talwani, Sutton & Worzel 1959). Because the fault plane solutions (Molnar & Sykes 1969) and calculated plate motions (Jordan 1975) imply no subduction there, it seems necessary to rely solely on a bending moment in maintaining these features. In this way a short cold slab of Atlantic lithosphere has been warped down at the northern end of the Lesser Antilles arc, moving westward under Puerto Rico and resulting in the trench and outer rise. Secondly the deep Ganges trough and the topographic rise south of the Himalayas are very similar to the trenches and outer rises at island arcs. Although the last ocean floor seems to have been subducted at the Indus suture about 40 My ago, India is still moving northward relative to Eurasia at a fairly high velocity (Molnar & Tapponnier 1975). This and the occurrence of features similar to those at island arcs suggest that perhaps a downward flow of cold material beneath the Himalayas has continued even though no remaining oceanic lithosphere can be detected there. An understanding of the origin of the topography and associated gravity anomalies of this region may provide a key to understanding the forces that maintain the convergence between India and Eurasia 30–40 My after the collision (Molnar & Tapponnier 1975).

Acknowledgments

Molnar acknowledges that A. T. Smith and D. L. Turcotte tried to convince him that the bending moment might be important at a time when he was unprepared to believe it.

The research was supported by the National Science Foundation under Grant DES74-02636, and the Office of Naval Research under Grant N00014-75-C-0291.

References

Caldwell, J. G., Haxby, W. F., Karig, D. E. & Turcotte, D. L., 1976. On the applicability of a universal elastic trench profile, *Earth Planet. Sci. Letters* submitted.

Hanks, T. C., 1971. The Kuril Trench–Hokkaido Rise system: large shallow earthquakes and simple models of deformation, *Geophys. J. R. astr. Soc.*, **23**, 173.

Jordan, T. H., 1975. The present-day motions of the Caribbean Plate, *J. geophys. Res.*, **80**, 4433.

Landau, L. D. & Lifshitz, E. M., 1970. *Theory of elasticity*, 2nd edition, Pergamon Press Ltd, London.

Le Pichon, X., Francheteau, J. & Bonnin, J., 1973. *Plate tectonics*, Elsevier, Amsterdam.

McKenzie, D. P., 1969. Speculations on the consequences and causes of plate motions, *Geophys. J. R. astr. Soc.*, **18**, 1.

Molnar, P. & Sykes, L. R., 1969. Tectonics of the Caribbean and Middle America regions from focal mechanisms and seismicity, *Bull. geol. Soc. Am.*, **80**, 1639.

Molnar, P. & Tapponnier, P., 1975. Cenozoic tectonics of Asia: effects of a continental collision, *Science*, **189**, 419.

Parsons, B. & Sclater, J. G., 1976. An analysis of the variation of ocean floor heat flow and bathymetry with age, *J. Geophys. Res*, in press.

Sclater, J. G., Anderson, R. N. & Bell, M. L., 1971. Elevation of ridges and evolution of the Central Eastern Pacific, *J. geophys. Res.*, **76**, 7888.

Talwani, M., Sutton, G. H. & Worzel, J. L., 1959. A crustal section across the Puerto Rico Trench, *J. geophys. Res.*, **64**, 1545.

Watts, A. B. & Talwani, M., 1974. Gravity anomalies seaward of deep-sea trenches and their tectonic implications, *Geophys. J. R. astr. Soc.*, **36**, 57.