The Over-Extended Kalman Filter - Don't Use It!

David F Bizup
Systems and Information Engineering
University of Virginia
PO Box 400747
151 Engineer's Way
Charlottesville, VA 22904
dfb2ph@virginia.edu

Donald E. Brown
Systems and Information Engineering
University of Virginia
PO Box 400747
151 Engineer's Way
Charlottesville, VA 22904
brown@virginia.edu

Abstract – *Radar range rate measurements are not always used in target tracking filters because they are highly nonlinear in Cartesian space. A linear approximation of range rate composed of its partial derivatives with respect to the track state vector is sometimes used in the measurement equation of an Extended Kalman filter. Unfortunately, this naive linearization can degrade the filter's position estimates. The origins of this phenomenon are investigated and found to lie in the functional relationship induced by the linearization between the position elements of the track state vector and the range rate innovation. An alternative linearization of range rate that is not a function of the position elements is derived. It is shown that the new linearization improves position estimate for some trajectories. An ordinary Kalman filter's gains are compared to those of the usual and alternative extended Kalman filters analytically and via simulation. The results show that the alternative linearization leads to a filter having the same position gains as an ordinary Kalman filter, and an additional gain on the track's radial velocity. This new extended Kalman filter can improve a tracking system's velocity estimates without risk to its position estimates.*

Keywords: Tracking, filtering, extended Kalman filter, range rate.

1 Introduction

Range rate, also known as the radial or Doppler velocity, is highly nonlinear for a radar tracking problem set in a Cartesian frame of reference. Thus, the range rate measurement is unsuited for a Kalman filter (KF) and many tracking systems do not use it to filter the state estimate. Nevertheless, it is reasonable to suppose that proper use of the range rate measurement can improve the filter, particularly its velocity estimates. The usual technique for adding nonlinear functions and measurements to the system is to linearize them by taking their partial derivatives with respect to the state vector. Kalman filters that do this are called extended Kalman filters (EKF).

The range rate measurement has other uses. Association algorithms that consider range rate have lower probability of incorrect report to track associations [4]. When the target is observed by a network of sensors, range rate measurements improve velocity estimates because the target's velocity is measured along multiple axes. With a sensor network, it is possible to estimate position, speed and acceleration using only the range rate measurement and not the position measurements [1]. When the range and range rate measurement errors are negatively correlated, a filter designed to exploit the correlation estimates range better than one that does not [3]. If the tracking frame of reference is rotated so that the target lies on an axis then the range rate measurement is a direct, linear measurement of target velocity along that axis and can be used in an ordinary Kalman filter [5].

Sometimes, EKFs with a linearized range rate measurement are reported to work well [4]. Sometimes, they are reported to diverge. Schutz, etal., [6] state that an experimental EKF tracker designed for the E-2C airborne early warning aircraft diverges for most target trajectories in the sense that its standard position errors grow with time. Reports of divergence are not surprising; Bar-Shalom and Li [2] warn that naive linearizations may introduce biases or errors in the covariance calculations. However, it is surprising that the cause of divergence is not more widely examined given these conflicting reports.

In this paper an alternative linearization of the range rate is proposed. An EKF using the alternative linearization is compared analytically and via simulation to the usual EKF and an ordinary KF that uses only the position measurements. The analytical results show that the naive linearization introduces unreasonable biases in the posterior estimates for certain trajectories. Simulation results show that the usual EKF's position biases can be large even for moderate range rate innovations and prior state estimate errors. It is so overly sensitive to changes in covariance and prior estimated target heading that we call it the over-extended Kalman filter (OEKF) in this paper.

2 Alternative Linearization

Consider a two dimensional radar at the origin measuring range ρ, bearing η, and range rate $\dot{r}$. Two dimensions suffice from demonstrating the differences between the ordinary and alternative linearization, and it is easy to extend the problem to three dimensions. A tracking system converts the position measurements to Cartesian coordinates in the usual way [2]

$$\begin{bmatrix} x \\ y \end{bmatrix} = \rho \begin{bmatrix} \cos\eta \\ \sin\eta \end{bmatrix} \tag{1}$$

so the k^{th} measurement is then $z_k = \begin{bmatrix} x & y^m & \dot{r} \end{bmatrix}_k^{(m)}$, where superscript (m) denotes a measured value. Target location and velocity are represented with the state vector $\theta = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T$. In this system range rate is a nonlinear function of the state vector (2).

$$\dot{r} = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2+y^2}} \tag{2}$$

Track state estimates are updated with a Kalman filter having process and measurement equations (3) and (4), respectively.

$$\begin{aligned} \theta_k &= F\theta_{k-1} + w_k & (3) \\ z_k &= H_k\theta_k + v_k & (4) \end{aligned}$$

F is linear motion, w_k is a white noise acceleration with covariance W_k, v_k is white measurement noise with covariance V_k, and

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\partial \dot{r}}{\partial x} & \frac{\partial \dot{r}}{\partial y} & \frac{\partial \dot{r}}{\partial \dot{x}} & \frac{\partial \dot{r}}{\partial \dot{y}} \end{bmatrix} \tag{5}$$

with the last row evaluated at the current state estimate. This is the usual extended Kalman filter [4]. The ordinary Kalman filter has the same process equation, but lacks the third row of the measurement equation; it uses only the position measurements.

Writing the partial derivatives explicitly, multiplying them by the state vector, and combining terms makes it clear that the expected range rate measurement equals the true range rate.

$$\begin{aligned} E\left[\dot{r}_k^{(m)}\right] &= x_k\left(\frac{\dot{x}_k}{\sqrt{x_k^2+y_k^2}} - \frac{x_k\left(x_k\dot{x}_k + y_k\dot{y}_k\right)}{\left(x_k^2+y_k^2\right)^{3/2}}\right) + \\ &\quad y_k\left(\frac{\dot{y}_k}{\sqrt{x_k^2+y_k^2}} - \frac{y_k\left(x_k\dot{x}_k + y_k\dot{y}_k\right)}{\left(x_k^2+y_k^2\right)^{3/2}}\right) + \\ &\quad \dot{x}_k\left(\frac{x_k}{\sqrt{x_k^2+y_k^2}}\right) + \\ &\quad \dot{y}_k\left(\frac{y_k}{\sqrt{x_k^2+y_k^2}}\right) & (6) \\ E\left[\dot{r}_k^{(m)}\right] &= \frac{x_k\dot{x}_k + y_k\dot{y}_k}{\sqrt{x_k^2+y_k^2}} & (7) \end{aligned}$$

Notice that the first two elements of (6) sum to exactly zero; $x\frac{\partial \dot{r}}{\partial x} + y\frac{\partial \dot{r}}{\partial y} = 0$. Thus, an alternative linearization with measurement matrix (8) exists

$$H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\partial \dot{r}}{\partial \dot{x}} & \frac{\partial \dot{r}}{\partial \dot{y}} \end{bmatrix} \tag{8}$$

We call this filter the alternative extended Kalman filter (AEKF).

3 Gain Differences

The differences in the third row of (5) and (8) significantly impact filter gain. This is demonstrated by solving the recursive, Kalman filter equations for each filter. To simplify the presentation, let a, b, c, d denote the partial derivatives $\frac{\partial \dot{r}_k}{\partial x_k}, \frac{\partial \dot{r}_k}{\partial y_k}, \frac{\partial \dot{r}_k}{\partial \dot{x}_k}, \frac{\partial \dot{r}_k}{\partial \dot{y}_k}$, let P denote the 4x4 prior state covariance matrix. Denoting the measurement matrices of the KF, OEKF, and AEKF H_0, H_{oekf}, and H_{alt}, respectively,

$$\begin{aligned} H_0 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & (9) \\ H_{oekf} &= \begin{bmatrix} & H_0 & & \\ a & b & c & d \end{bmatrix} & (10) \\ H_{alt} &= \begin{bmatrix} & H_0 & & \\ 0 & 0 & c & d \end{bmatrix} & (11) \end{aligned}$$

In the OEKF range rate is functionally related to the position and velocity estimates. In the AEKF it is only related to the velocity estimates.

First, compute the measurement covariances, $S = HPH^T + V$, just prior to taking the measurement. The 2x2 matrix S_0 for the KF is

$$S_0 = H_0PH_0^T + \begin{bmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{bmatrix} \tag{12}$$

The upper left elements of the 3x3 matrices S_{oekf} and S_{alt} equal S_0. Their third columns, and the corresponding elements of their third rows, are

$$S_{ekf}(:,3) = \begin{bmatrix} [abcd]\, P_{\cdot 1} \\ [abcd]\, P_{\cdot 2} \\ [abcd]\, P\, [abcd]^T + v_{33} \end{bmatrix} \quad (13)$$

$$S_{alt}(:,3) = \begin{bmatrix} [00cd]\, P_{\cdot 1} \\ [00cd]\, P_{\cdot 2} \\ [00cd]\, P\, [00bcd]^T + v_{33} \end{bmatrix} \quad (14)$$

where $P_{\cdot j}$ is the j^{th} column of P.

Expressions for the gain matrices, $K = PH^T S^{-1}$, as functions of the matrix elements are easy to obtain but algebraically cumbersome. Letting $\{k\}_{ij}$ denote the elements of K_0, $\{\alpha\}_{ij}$ and $\{\epsilon\}_{ij}$ denote the elements of K_{alt} and K_{oekf}, respectively,

$$K_0 = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \\ k_{41} & k_{42} \end{bmatrix} \quad (15)$$

$$K_{alt} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \end{bmatrix} \quad (16)$$

$$K_{oekf} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \\ \epsilon_{41} & \epsilon_{42} & \epsilon_{43} \end{bmatrix} \quad (17)$$

The posterior state estimate equals the prior estimate plus the measurement innovations premultiplied by the gain Innovations are the difference between the actual and expected measurements. The first two columns of K are gains on the position innovations, and the third column are the gains on the range rate innovation. Of course, the gain matrix for the ordinary KF tracker has no third column because it only uses the position measurements. Given the prior state estimate θ_{prior}, the measurement z, and the matrix H, the posterior state estimate θ_{post} is

$$\theta_{post} = \theta_{prior} + K\left(z - H\theta_{prior}\right) \quad (18)$$

In the general case all three filters have different gains. The ϵ's and α's are nonzero, cumbersome, and difficult to interpret directly. In the following sections simplifying assumptions that ease interpretation of the gains and highlight the nature of their differences are made.

3.1 Uncorrelated Systems

Kalman filters are linear, so they are invariant under scale and rotation and the coordinate system can be chosen arbitrarily. A coordinate system that simplifies the analysis and interpretation of results places the prior track state estimate exactly on the y-axis at a range of 1. In this system $\dot{y}$ equals the range rate. The prior state vector is $\theta_{prior} = \begin{bmatrix} 0 & 1 & \dot{x} & \dot{y} \end{bmatrix}^T$ and the partial derivatives are

$$\begin{bmatrix} \frac{\partial \dot{r}}{\partial x} & \frac{\partial \dot{r}}{\partial y} & \frac{\partial \dot{r}}{\partial \dot{x}} & \frac{\partial \dot{r}}{\partial \dot{y}} \end{bmatrix} = \begin{bmatrix} \dot{x} & 0 & 0 & 1 \end{bmatrix} \quad (19)$$

Assume that the prior state and measurement vectors are uncorrelated; P and V are diagonal matrices. This assumption is unrealistic in practice because the acceleration noises in 3 always introduce a positive correlation between the positions and velocities. Nevertheless, it is instructive for highlighting the differences between the KF, OEKF and AEKF gains. A more realistic example is given later.

Given the coordinate system and covariance matrices, the gains are

$$K_0 = \begin{bmatrix} \frac{p_{11}}{(p_{11}+v_{11})} & 0 \\ 0 & \frac{p_{22}}{(p_{22}+v_{22})} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (20)$$

$$K_{alt} = \begin{bmatrix} K_0 & \begin{matrix} 0 \\ 0 \\ 0 \\ \frac{p_{44}}{(p_{44}+v_{33})} \end{matrix} \end{bmatrix} \quad (21)$$

$$K_{oekf} = \begin{bmatrix} \epsilon_{11} & 0 & \epsilon_{13} \\ 0 & \frac{p_{22}}{(p_{22}+v_{22})} & 0 \\ 0 & 0 & 0 \\ \epsilon_{41} & 0 & \epsilon_{43} \end{bmatrix} \quad (22)$$

The AEKF's position gains identically equal the KF's. The gain on $\dot{y}$ equals the ratio of the prior state and measurement uncertainties. This is an intuitively satisfying situation for the uncorrelated system. The position measurements update the position estimates and the velocity measurement updates the velocity estimates, whereas in the OEKF the velocity measurement also updates the position, and the position measurements also update the velocity. These oddities arise from the functional relation between range rate and position estimates in (5)

The OEKF's posterior solution has a different cross radial position than the KF, and a different range rate than the AEKF. Unfortunately, the $\{\epsilon\}_{ij}$ do not have a simple, easily interpretable form. They are functions of, for example, $\left(\frac{\partial \dot{r}}{\partial x}\right)^2$ and $p_{11} \cdot p_{44}$.

3.2 OEKF Biases

The sensitivity of the OEKF's posterior estimate to P, V, θ_{prior} and range rate innovation is evaluated

Parameter	Parameter change	Posterior Position Difference
p_{11}	$\uparrow$	$\uparrow$
p_{44}	$\downarrow$	$\uparrow$
v_{11}	$\uparrow$	$\uparrow$
v_{33}	$\downarrow$	$\uparrow$
range rate innovation	$\uparrow$	$\uparrow$
heading	cross radial	$\uparrow$

Table 1: Relation between covariances, measurements, and OEKF posterior position biases

numerically. Table 1 shows how these parameters effect the OEKF's posterior position biases.

When the prior velocity variances are large and the range rate innovation is small, then the biases are moderate. However, the range rate innovation can be substantially larger than its measurement's standard deviation, especially when the target is maneuvering. Range rate is hard to estimate well because the estimates of all four state elements must be accurate. During maneuvers, even if the position estimates are good, the velocity estimates are not. In the cases of misassociation due to clutter or multiple targets the range rate innovation may be quite large.

OEKF biases are sensitive to target heading. They are larger for off-radial headings than for radial headings. When the prior estimated heading is exactly radial the OEKF's posterior position estimates exactly equal the AEKF's. This is obvious from (19) because $\dot{x} = 0$.

An interesting characteristic of the OEKF is that its biases increase as its prior state velocity variance decreases. This leads to a paradox in which *stronger prior beliefs increase the dispersion of the posterior estimate*. This is a direct result of the functional relation between range rate and the position estimates in H_{oekf}.. Low prior velocity variance reduces the sensitivity of the posterior velocity estimates to the range rate innovation, so the filter compensates by increasing the sensitivity of the position estimates.

4 Simulation Results

It is easy to show via simulation that the gains can be so large that the OEKF posterior position estimates may be several standard deviations away from both the prior and the measurement. It is reasonable to expect filter gains to lie in $(0, 1)$, so the posterior estimate should move from the prior toward the measurement and lie somewhere in between. In the OEKF the posterior can actually move away from the measurement. The net effect can be interpreted as a gain less than 0 or greater than 1.

Figures 1 through 4 compare KF, AEKF and OEKF posterior position estimates for the uncorrelated system. The prior track state estimate always lies exactly on the y-axis at a range of 50 miles. The speed is always 500 knots but the heading changes. The covariance matrices are $P = diag([2, 2, 50, 50])$ and $V = ([1, 1, 5])$. In these figures, the large, filled dot in the center is the prior position and the line extending from the dot is in the direction of the heading. The measurements are denoted '+'. KF and AEKF posterior positions are denoted by large, empty circles. OEKF posterior positions are denoted by small, filled dots. Measurements are shown every 45 degrees, relative the prior. They are not to be interpreted as several measurements updating the estimate sequentially. Each posterior estimate is the prior updated with only one measurement.

Figures 1 and 2 show posterior estimates for prior headings of 0 and 90 degrees and a range rate innovation of 20 knots. When the heading is radial all posterior estimates are identical. When the heading is cross radial the posterior OEKF estimates are pushed in the direction of the heading. Some of the net gains are greater than one, and some are negative. For example, measurements to the left of the prior estimate induce posteriors to the right.

Figure 3 shows the effect of decreasing prior velocity variance. All parameters are the same as in figures 1 and 2, except that $P = diag([2, 2, 10, 10])$ and heading equals 30 degrees. The posterior position estimates are about three miles from the measurements and two miles from the prior. Again, the net effective position gains are outside $(0, 1)$.

Figure 4 shows the effect of increasing range rate innovation. All parameters are the same as in figures 1 and 2, except that range rate innovation equals 50 knots and heading equals 75 degrees. The posterior position estimates are more than three standard deviations from the measurements.

The difference between the OEKF's posterior velocity estimates and those of the other filters are minor. Posterior $\dot{x}$ estimates are identical, as expected given the gains (20) to (22). Posterior $\dot{y}$ estimates differ by no more than one half the range rate innovation for the examples in figures 1 to 4. In all cases, the net effective range rate gain is in $(0, 1)$.

The examples in this section show that the OEKF's biases are not trivial. They are most severe when target heading is off-radial, cross radial position variance is large, the radial velocity variance is small, and the range rate innovation is large. In practice, these conditions arise when the radar's bearing error is much

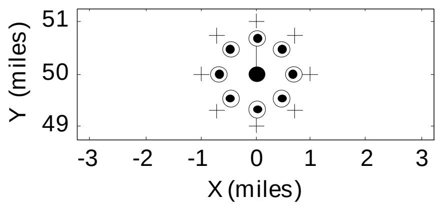

Figure 1: Baseline for comparing OEKF and AEKF trackers, with radial prior heading.

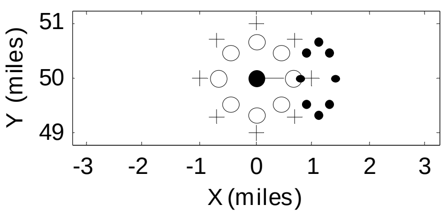

Figure 2: Comparison of OEKF and AEKF trackers, with cross radial prior heading.

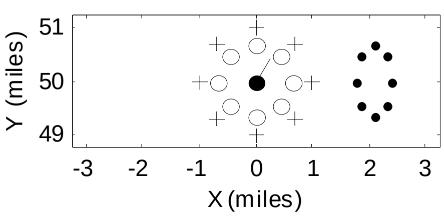

Figure 3: Comparison of OEKF and AEKF trackers, with off radial prior heading.and small prior velocity variance.

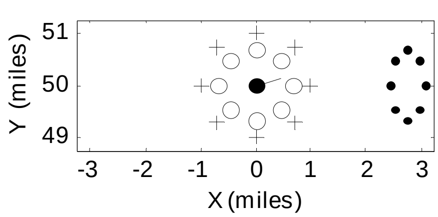

Figure 4: Comparison of OEKF and AEKF trackers, with off radial prior heading and large range rate innovation.

greater than its range error, and a target that has been moving at a constant velocity for a long time suddenly maneuvers when its trajectory is off radial with respect to radar position. Divergence is possible with the OEKF, but only likely under these conditions. However, even though divergence is infrequent, it can be avoided altogether with the alternate linearization.

4.1 Realistic Example 1

The covariance assumptions in the previous section are useful for highlighting the OEKF's posterior biases, but unrealistic. In this section more realistic covariances are determined via simulation, and an example showing the worst case biases is presented. AEKF position gains do not necessarily identically equal to those of the KF because of non-zero prior state covariance. However, in the examples in this section, the differences are so small that no visual distinction can be made.

The covariance matrix below for a target at $(x, y) = (0, 50)$ was determined via simulation. Admittedly, the results in this section are extreme; we ran the simulation until it produce a covariance matrix that would cause large position biases. The intent was to provide a worst case example. In the next section, we show position error averages and interval estimates with the intent of showing expected performance.

The simulated target travels 500 miles per hour for a long time in the positive x direction along the line defined by $y = 50$ miles. The radar scans once every 10 seconds and has range, bearing, and range rate standard errors of 100 feet, 0.6 degrees and 2.5 knots, respectively. Simulation results show that (23) is a reasonable prior state covariance matrix.

$$P = \begin{bmatrix} 0.04 & 0.0008 & 2.09 & 0.278 \\ 0.0008 & 0.001 & .057 & .0.148 \\ 2.09 & .057 & 290.0 & -8.48 \\ 0.278 & 0.148 & -8.48 & 67.0 \end{bmatrix} \quad (23)$$

$$V = \begin{bmatrix} 6.75 & 0 & 0 \\ 0 & .0004 & 0 \\ 0 & 0 & 5.25 \end{bmatrix} \quad (24)$$

If the target continues along its path at constant velocity, then the simulated range rate innovations are almost always less than 20 knots. If the target makes a $2g$ coordinated turn, its radial velocity can change by as much as 180 knots between scans. Posterior position estimates given prior state estimate $\theta_{prior} = \begin{bmatrix} 0 & 50 & 500 & 0 \end{bmatrix}^T$, covariance (23), and range rate innovations of 20 and 180 knots are shown in figures 5 and 6, respectively. (24) is the covariance of a measurement having range 50 miles and bearing 0 degrees.

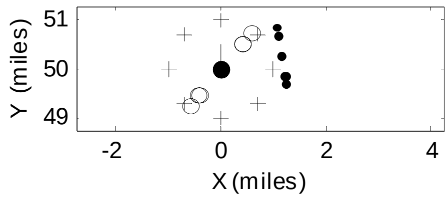

Figure 5: Comparison of OEKF and AEKF trackers with realistic covariances and radial prior heading.

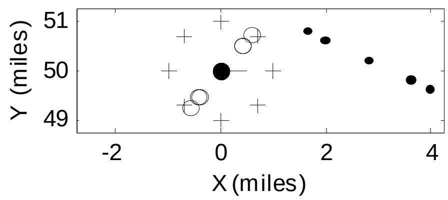

Figure 6: Comparison of OEKF and AEKF trackers with realistic covariances and cross radial prior heading.

Figure 5 shows that small range rate innovations, such as might be expected for a nonmaneuvering target, produce small OEKF position biases. Figure 6 shows that OEKF biases are significant in some realistic situations The bias in the x direction, roughly 2.5 miles, is about 1 standard deviation of the x measurement error but more than 25 standard deviations of the prior x variance. It is obvious that the net effective position gains are outside $(0, 1)$.

4.2 Realistice Example 2

In this section, we simulate 500 knot target on a 90 degree heading that makes a 2G turn when its path is nearly cross radial with respect to a radar fixed at the origin, as in figure 7. The simulation is run 100 times. Position error means and 95% interval are shown in figure 8.

Notice that the AEKF's errors are higher just before the turn than the OEKF's, but lower during the turn. The AEKF actually has lower position errors during the turn than before or after. The designer choosing the linearization must trade off non-maneuvering accuracy for maneuvering. The results in figure 8 suggest that one possible, attractive design choice is to pick a linearization based on a maneuver detection test. If the

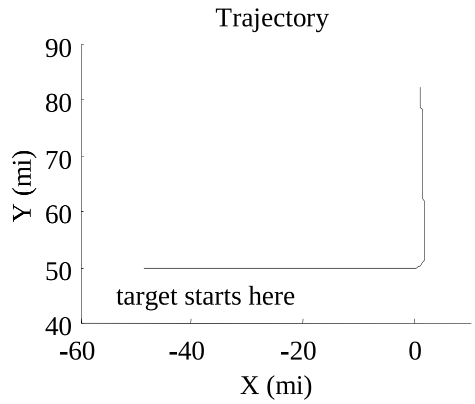

Figure 7: Simulated trajectory

target is not maneuver, choose the usual linearization. If it is maneuver, choose the alternative.

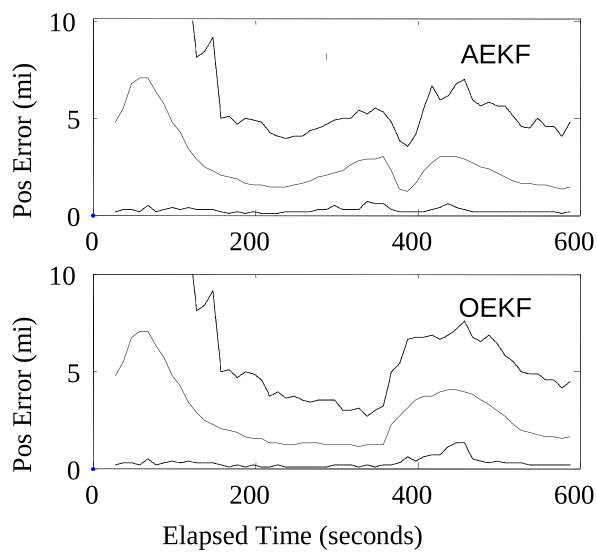

Figure 8: Position errors: averages and 95% intervals

5 Conclusions

The usual extended Kalman filter linearizes the range rate measurement by taking its partial derivatives with respect to the state vector's position and velocity elements. With this linearization, the state position estimates and the range rate measurement are functionally related in the Kalman filter measurement equation. This relation induces such unacceptably large biases in the posterior estimate that the net effective position gains can be negative or greater than one. Because this filter is overly sensitive to reasonable estimation errors during maneuvers that we call it the over-extended Kalman filter.

An alternative linearization of range rate that is a function of the partial derivatives of only the state vector's velocity elements is derived. A filter using this linearization was compared to the over-extended Kalman filter. The alternative Kalman filter was shown to have reasonable gain on the state's velocity estimate, and better position gains during a maneuver. The alternative filter includes the information contained in the range rate measurement in a rational way without subjecting the tracking system to the risk that its position estimate biases during a maneuver. Simulation results suggest that a tracking system designer should choose a linearization based on the target's maneuver state. If it is not turning the usual linearization is preferred. If it is turning the alternative is preferred.

References

[1] Armstrong, B. and Holeman, B., *Target Tracking with a Network of Doppler Radars*, IEEE Transactions on Aerospace and Electronic Systems, vol. 34, no. 1, pp33-48 (January 1998)

[2] Bar-Shalom, Y. and Li, X., *Multitarget-Multisensor Tracking: Principles and Techniques*, YBS Publishers (1995)

[3] Bar-Shalom, Y., *Negative Correlation and Optimal Tracking with Doppler Measurements*, IEEE Transactions on Aerospace and Electronic Systems, vol. 37, no. 3, pp1117-1120 (July 2001)

[4] Kameda, H., Tsujimichi, S. and Kosuge, Y., *Target Tracking Using Range Rate Measurement Under Dense Environments*, Electronics and Communications in Japan, Part I, Communications, vol. 85, pp19-29 (2002)

[5] Kosuge, Y., Kameda, H. and Mano, S. and Kondou, M., *A Cartesian Coordinate Conversion Algorithm for RadarTracking with Range Rate Measurement*, Electronics and Communications in Japan, Part I, Communications, vol. 80, no. 4, pp51-61 (April 1997)

[6] Schutz, R., McAllister, R. and Engleberg., B., *Combined Kalman Filter and JVC Algorithms for AEW Target Tracking Applications*, SPIE Conference on Signal and Data Processing of Small Targets vol. 3163, pp164-175 (July 1997)