
The PAPR Problem in OFDM Transmission: New Directions for a Long-Lasting Problem

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I. ENERGY EFFICIENCY IN MOBILE COMMUNICATION NETWORKS: A DRIVING SOURCE FOR INNOVATION

Energy efficiency particularly matters in future mobile communications networks. Key driving factor is the growing energy cost of network operation which can make up as much as 50% of the total operational cost nowadays [1]. In the context of *green information and communication technology* (ICT) this has led to many global initiatives such as the *Green Touch* consortium².

A major source for reducing energy costs is to increase the efficiency of the high power amplifier (HPA) in the radio frequency (RF) front end of the base stations [1]. However, efficiency of the HPA is directly related to the *peak-to-average power ratio* (PAPR) of the input signal. The problem especially becomes serious in orthogonal frequency-division multiplexing (OFDM) multicarrier transmission which is applied in many important wireless standards such as the 3GPP Long Term Evolution Advanced (LTE-A). In the sequel of this article we refer to it simply as the *PAPR problem*. The PAPR problem still prevents OFDM from being adopted in the uplink of mobile communication standards, and, besides from power efficiency, it can also place severe constraints on output power and therefore coverage in the downlink.

In the past, there have been many efforts to deal with the PAPR problem resulting in numerous papers and several overview articles, e.g., [2], [3], [4]. However, with the upcoming of novel systems, new challenges emerge which have been rarely addressed so far: 1.) the envisioned boost in network energy efficiency (e.g. at least by a factor of 1000 in the Green Touch consortium) will tighten the requirements on component level so that the efficiency gap with respect to single-carrier transmission must considerably diminish 2.) multiple-input/multiple-output (MIMO) multiply the problem due to simultaneously control of parallel transmit signals particularly when considering a huge number of transmit antennas 3.) multiuser (MU) (and multipoint) systems put additional side constraints on the parallel transmit signals which are difficult to implement on top of conventional approaches. Furthermore, many of the existing methods are not either compatible with relevant standards and/or their prospective performance capabilities are not satisfactory. Yet, it is quite safe to say that no standard solution is available.

In this article, we will argue that, in the light of these challenges, the PAPR metric itself has to be carefully reviewed within a much broader scope overthrowing some of the common understanding and results. New metrics become more and more important since they enable the system designer to precisely adjust the algorithms to meet some given performance indicator. It is expected that such design approach will no longer be treated like an isolated problem on physical layer but will affect the design parameters on higher layers as well (e.g. resource allocation). For example, it has been discussed in [1] that from a ICT perspective the system throughput should be related to input power rather than output power. In order to capture this paradigm on HPA power efficiency level, different metrics are currently used such as *total degradation*, *average distortion power* and others. However, with respect to algorithm design all these metrics are solely reflected by the standard PAPR figure of merit. This argument can also be extended to other situations: it has been recently shown in [5] that, if the only concern is average distortion power (instead of peak power), then a much less conservative design is possible compared to conventional design rules in OFDM transmission. Remarkably, such performance limits can be efficiently achieved using *derandomization* algorithms establishing therefore a new powerful tool within the context of the PAPR problem. It is a major aim of this article to review and collect exactly those elements in the current literature of which we believe represent the core of a more general theory.

Besides this point of view, it is interesting to apply new signal processing and mathematical concepts to OFDM. *Compressed sensing* [6], [7] is a new framework capturing sparsity in signals beyond Shannon's sampling theorem and has attracted a lot of attention in recent years. It is based on the observation that a small number of linear projections (measurements) for a sparse signal contain enough information for its recovery. Compressed sensing can be applied to the PAPR problem because sparsity frequently appears in

²see, e.g., the webpage: <http://www.greentouch.org>

the (clipped) OFDM signals. There are currently many research efforts in this direction but some challenges still remain, such as the degraded recovery performance in noisy environment. The adoption of related mathematical concepts such as *Banach space geometry* [8], [9] complement this discussion. It is outlined that these theoretically deeply rooted concepts can help to understand some of the fundamental limits as well as to develop new algorithmic solutions for the PAPR problem.

Summarizing, there is a clear need for a fresh look on the PAPR problem under the general umbrella of the metrics theme discussed before which will open up new research strands not yet explored. In this article we are going to address and discuss some of the fundamentals, challenges, latest trends, and potential solutions which originate from this perspective and which, we believe, are important to come to an innovative breakthrough for this long-lasting problem.

A. Outline and some notations

The outline of the article is as follows: first we motivate and discuss alternative metrics and corresponding methodology for the PAPR problem and present several examples. Then, we propose an appropriate theoretical framework and unified algorithm design principles for these new paradigms by introducing the derandomization principle. In this context, we outline specific challenges imposed by MIMO and MU MIMO systems. Next, we discuss capacity issues which establish fundamental limits. Finally, we discuss some of the future directions the authors believe are the foundations or at least components of emerging solutions.

We recall the following standard notations: the frequency-domain OFDM symbol for each antenna (N_t in total) consists of N subcarriers. The multiplexed transmit symbols $C_{m,n}$ (carrying information and/or control data) are drawn from some common QAM/PSK signal constellation and collected in the *space-frequency codeword* $\mathbf{C} := [\mathbf{C}_1, \dots, \mathbf{C}_{N_t}]$ where $\mathbf{C}_m := [C_{m,1}, \dots, C_{m,N}^T]$ is the transmit sequence of antenna m . In case of a single antenna we write $C_n := C_{1,n}$ and, correspondingly, $\mathbf{C} := \mathbf{C}_1 = [C_1, \dots, C_m]^T$. Given the IDFT matrix $\mathbf{F} := [e^{j2\pi kn/(IN)}]_{0 \leq k < IN, 0 \leq n < N}$ the I -times oversampled discrete-time OFDM transmit symbols in the equivalent complex baseband at antenna m are given by $\mathbf{s}_m = \mathbf{F}\mathbf{C}_m$. The average power of this signal might be normalized to one. We define the PAPR of the transmit signal at antenna m as

$$\text{PAPR}(\mathbf{s}_m) := \|\mathbf{s}_m\|_\infty^2. \quad (1)$$

Comment on oversampling: Please note that PAPR of the continuous-time passband signal differs roughly by 3dB. Clearly, there is also still some overshooting between the samples but due to sufficiently high oversampling the effect is negligible. The trade off between overshooting and oversampling is one of the few subproblems in OFDM transmission that is well understood. The best known results which hold even in the strict band-limited case are given in [10] where overshooting is proved to be below $1/\cos(\frac{\pi}{2T})$.

II. THE DESIGN CHALLENGE

In OFDM transmission many subcarriers (constructively or destructively) add up at a time which causes large fluctuations of the signal envelope; a transmission which is free from any distortion requires linear operation of HPA over a range N times the average power. As practical values of subcarriers are large this high dynamics affords HPA operation well below saturation so that most of the supply power is wasted with deleterious effect on either battery life time in mobile applications (uplink) or energy cost of network operation (downlink). In practice, these values are not tolerable and from a technology viewpoint it is also challenging to provide a large linear range. Hence, the HPA output signal is inevitably cut off at some point relative to the average power (*clipping level*) leading to in-band distortion in the form of intermodulation terms and spectral regrowth into adjacent channels. The effect is illustrated in Fig. 1 where the distorted OFDM signal and corresponding impact on the signal points are depicted.

The PAPR problem brings up several challenges for the system designer: one challenge is to adjust HPA design parameters (HPA backoff, digital predistortion) in some specific way so that power efficiency is traded

against nonlinear distortion which effects the data transmission on a global scale. To capture this trade off by a suitable metric on the level of the HPA is far from clear yet. Special HPA architectures at component level such as Doherty [11] and others can help to improve on this trade off. We also mention that other design constraints such as costs might prevent specific architectures [12].

A second challenge is to process the baseband signal by peak power reduction algorithms in such a way that the key figures of merit in the before mentioned trade off are improved. This alternating procedure makes apparent that the PAPR problem involves joint optimization of HPA, predistortion and signal processing unit. This interplay has only been marginally addressed so far let alone in the context of multiuser systems equipped with multiple antennas such as LTE-A.

In the following we discuss some potential metrics that can be used in the optimization.

III. THE RIGHT PARADIGM? ALTERNATIVE METRICS FOR PAPR

Classically, in OFDM transmission the PAPR of the transmit signal is analyzed and minimized by applying transmitter-side algorithms. Meanwhile it has been recognized that it may be reasonable to study other parameters as well. Especially when aiming at minimizing the energy consumption of the transmitter including the analog front end or when operating low-cost, low-precision power amplifier—sometimes referred to as “dirty RF” [12]—potentially other signal properties need to be controlled.

Let us present some illustrative example first. Suppose, we are interested in the clipped energy instead of the PAPR (we give some justification in terms of capacity below for this). Naturally, since the total energy is approximately one the clipped energy is finite as well but when N increases the required clipping level for asymptotically zero clipping energy might be of interest for design purposes. Clearly, no clipping at all is trivially sufficient but, surprisingly, it is actually *not* necessary: it is proved in [5] that clipping level can be adjusted along the $\log \log(N)$ law so that it is practically almost constant. This stands in clear contrast to the $\log(N)$ PAPR scaling discussed in Sec. IV.

Subsequently, some alternative metrics replacing the PAPR value in specific situations are briefly summarized.

- Of course, the *PAPR* has still its justification. As it relates peak and mean power the PAPR is the adequate metric for quantifying the required input power backoff of the power amplifier. When using higher-order modulation per carrier the energy per OFDM frame is no longer constant and hence average power fluctuates. In this case, the *peak power* is as suited metric. Both metrics are well-suited measures if purely limiting effects (modeled, e.g., as soft limiter) should be characterized. Since PAPR is random, we are also interested in the complementary cumulative distribution function (CCDF) $F(x)$ and other characteristic figures such as the mean $\mathbb{E}\{\cdot\}$ etc.
- Besides looking at the transmit signal itself, in many situations the impact of the nonlinear power amplifier to system performance is of interest. One possible approach is to quantify the nonlinear distortions caused by a particular power amplifier model. The *signal-to-distortion ratio (SDR)* [15], [16] or *error vector magnitude (EVM)* [13], [14] capture the in-band distortion of the OFDM signal and are immediately related to error rate of uncoded transmission. Both, SDR and EVM and their interdependence are illustrated in Fig. 1.
- Within 3GPP, a power amplifier model which cause non-linear distortions according to the third power of the RF transmit signal has become popular (so-called cubic polynomial model) [17]. In this case, the *cubic metric (CM)* measuring the mean of time domain sample energy to the third power is well adapted to the specific scenario [18], [19]. However, any additional clipping is not included here.
- CM metric is a special case of the *amplifier oriented metric (AOM)* defined in [20], [19]. AOM measures the mean squared absolute difference of desired and distorted HPA output. Here, the HPA output is calculated based upon some model such as the mentioned cubic polynomial model (or the well-known Rapp model etc.).

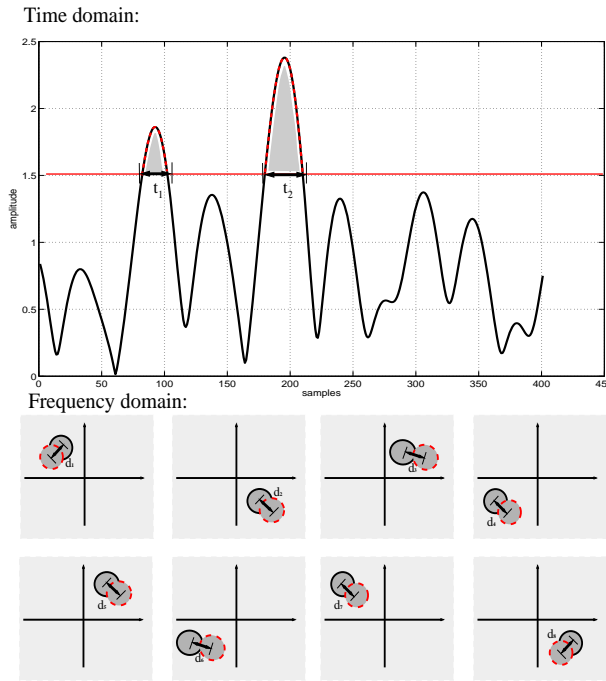


Fig. 1. Illustration of distorted OFDM signal (in time domain, clipping level=3.5 dB) and corresponding impact on the subcarrier signalling points (in frequency domain): the distortion signal is typically a sequence of *clips* with clip duration t_1, t_2, \dots . The SDR metric then relates the mean useful signal energy to the mean distortion energy while EVM collects the mean of sum of squared errors d_1, d_2, \dots in the data sequence (due to the very same time domain distortion). In case of Nyquist-sampling both are actually equal (subject to a scaling factor) [13], [14].

- A much severe problem in communication systems is that nonlinear devices cause a spectral widening and hence generate out-of-band radiation. In order not to violate spectral masks imposed by the regulatory body, a metric quantifying the out-of-band power or the shoulder attenuation is desirable [20]. In this field, significant work has to be done.
- When applying strong channel coding schemes—which nowadays is the state of the art in OFDM transmission—SDR or EVM are no longer suited performance characteristics. Instead, the end-to-end *capacity* of the entire OFDM scheme including the nonlinear devices matters. Unfortunately, neither the capacity of the continuous-time peak-power limited additive white Gaussian noise (AWGN) channel itself, nor the capacity of OFDM over such channels are known. Moreover, when using (as often done) a statistical model for the behavior of the nonlinear device, only lower bounds on the capacity are obtained as the statistical dependencies within one OFDM frame are not exploited. However, recent work [21], [22] indicates that clipped OFDM performs (almost) the same as unclipped OFDM with signal-to-noise ratio reduced according to the clipping power loss. Hence, the main source of the loss are not the introduced distortions or errors but simply the reduced output power. This, in turn, leads to the conclusion that a suited metric for capacity maximization is simply the average power of the power amplifier output signal. In this case, unfortunately, the generation of out-of-band radiation is not penalized.
- The symbol error rate (SER) is a related measure which has been directly applied to peak power control algorithms in [23].
- In future applications, more than a single signal parameter will have to be controlled. E.g., the capacity should be maximized but at the same time the out-of-band power should be minimized. Consequently, suited combinations of metrics capturing the desired trade off are requested.

Comment on Gaussian approximation: Noteworthy, many metrics have been analyzed in the past with the help of the Gaussian approximation. This, however, is not in all cases a feasible path. It is true that as N

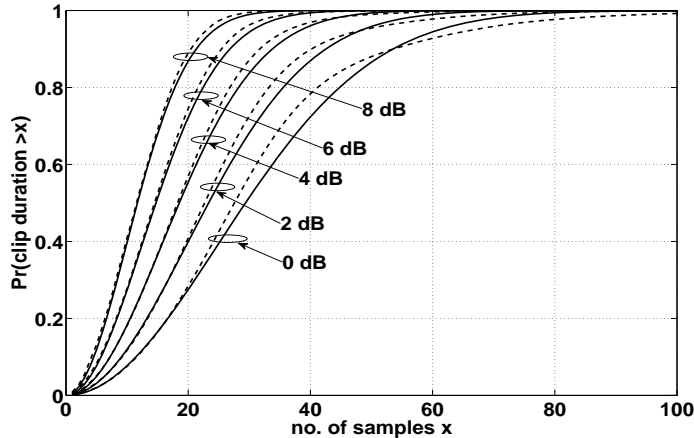


Fig. 2. Palm distribution of clip duration of an OFDM signal with $N = 2048$ and different clipping levels in [dB]. Note that the theory (dotted) is valid only for large clipping levels but fits relatively well also for low clipping levels.

gets large the finite-dimensional distributions converge and since the signals are band-limited also the process itself. However, we mention that it is a local property, valid only for any finite interval. For example in Fig. 2 the empirical CCDF of the clip duration (see the illustration in Fig. 1) is shown for OFDM signals and compared to a widely unknown result for envelopes of Gaussian processes (and their Hilbert transforms) for large clipping levels [24]³. It is seen that simulation and analysis match very well [23]. On the other hand, metrics such as EVM [14] and SER [25] have been shown not to match well.

IV. APPROACHING THE $\log(N)$ BARRIER: DERANDOMIZATION

In this section we discuss several fundamental principles for the peak power control problem. We believe that all of them actually connect to a broader theory general enough to capture alternative metrics as well and will open the door for new, provably more efficient algorithms.

A. The LDP

By analyzing PAPR of multicarrier signals one faces a fundamental barrier which to overcome seems quite challenging: the $\log(N)$ barrier (recall: N is the number of subcarriers). In fact, it is an exercise in large deviations to show that multicarrier signals with statistically independent subcarriers have PAPR of $\log(N)$ in a probabilistic sense [26], [27], [28]. This means that with very high probability the PAPR lies in an open, arbitrarily small interval containing $\log(N)$: this is what we call the *large deviation principle* or in short *LDP*.

Implicitly, LDP affects the performance of many peak power control schemes. The LDP has been known since long in the context of random polynomials but in the OFDM context the most general form is due to [29] where it is shown that as long as N is large enough and the subcarriers are independent that the following inequality is true:

$$\Pr \left\{ \left| \sqrt{\text{PAPR}} - \sqrt{\log(N)} \right| > \gamma \frac{\log[\log(N)]}{\sqrt{\log(N)}} \right\} \leq \frac{1}{[\log(N)]^{2\gamma - \frac{1}{2}}} \quad (2)$$

Here, $\gamma > \frac{1}{4}$ is some design parameter which trades off probability decay over deviation from $\log(N)$. While the analysis is tricky when it comes to show that PAPR is not below $\log(N)$ with high probability, it is a

³To be specific, Ref. [24] derives the so-called *Palm distribution* of the clip duration which describes the statistical average after an upcrossing of the Gaussian process.

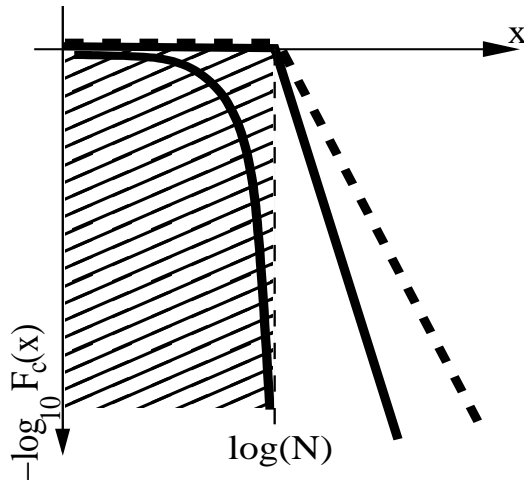


Fig. 3. Illustration of the virtue of the LDP: [from right to left] CCDF of PAPR of a.) uncoded data b.) SLM and c.) derandomization.

surprisingly easy task to show the converse: standard inequalities (such as Chernoff bounds) or any other Markov-style bound do the job. Some can be exploited for algorithm design.

The inequality states that PAPR concentrates more and more around the value $\log(N)$ which establishes therefore an important theoretical scaling law. The proof is technical but the result might be surprising since 1) the factor before the logarithmic term is exactly unity and 2) the scaling law differs from the well-known law of iterated logarithm which would suggest only doubly logarithmic scaling.

The LDP contains some valuable illustrative aspects which we are going to reveal now. The LDP in eqn. (2) is somewhat inaccessible and shall be rewritten in the more convenient form:

$$\log(F_c(x)) = [\log(N) + O(\log[\log(N)]) - x]^- \quad (3)$$

where we used the order notation $O(\cdot)$ and the definition $[x]^- := \min(0, x)$. Disregarding the order term $O(\log[\log(N)])$ we have the interpretation that the probability decreases linearly on a logarithmic scale from some cut-off point $\log(N)$ on which is illustrated in Fig. 3. The proximity to filter design terminology is intended and it makes obviously sense to speak of a pass band and a stop band in the figure. Comparing this to the standard analysis where statistical independence and Nyquist sampling is assumed gives

$$\log(F_c(x)) = [\log(N) - x]^- \quad (4)$$

where the order term is missing. Hence, we conclude that a careful non-Gaussian analysis for continuous-time OFDM signals entails an error of at most $O(\log[\log(N)])$.

The LDP is very useful for assessing the performance of peak-power control schemes. Before we show this we might ask why this concentration happens? Let C_1, C_2, \dots, C_N be a (data) sequence of independent random variables; when estimating PAPR without a priori information the expectation is the best possible choice. Using successive knowledge of already fixed data we have the following estimations:

$$y_0 = \mathbb{E}\{\text{PAPR}(\mathbf{s})\} \quad (5)$$

$$y_1 = \mathbb{E}\{\text{PAPR}(\mathbf{s}) | C_1\} \quad (6)$$

$$y_2 = \mathbb{E}\{\text{PAPR}(\mathbf{s}) | C_1, C_2\} \quad (7)$$

...

$$y_N = \mathbb{E}\{\text{PAPR}(\mathbf{s}) | C_1, C_2, \dots, C_N\} \quad (8)$$

It can be shown that this process establishes a *Martingale* with bounded increments $|y_i - y_{i-1}|$ from which it follows (see [30]) measure concentration of the PAPR around its average via the *Azuma-Hoeffding* inequality

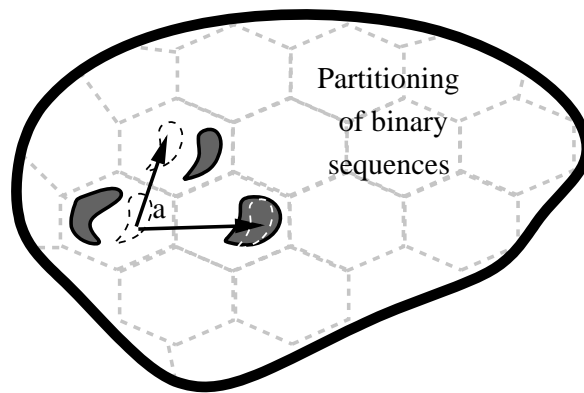


Fig. 4. A general model for peak power control: each cell contains the same information; sets with large PAPR (dotted areas) are mapped to sets where PAPR is below some threshold (grey-shaded areas).

or *McDiarmid's* inequality. Another approach used in [30] for proving measure concentration of the PAPR around its median is based on the convex-hull distance inequality of *Talagrand*. The tails of the concentration inequalities are even exponential then. Let us now apply the LDP within the context of peak power control.

B. Multiple signal representation and partitioning

The basic principle for most of the peak power control schemes is *multiple signal representation* which roots in the classical methods *selected mapping* (SLM) [31] and *partial transmit sequences* (PTS) [32], [33]. The idea is simple: instead of transmitting the original OFDM data frame multiple redundant candidates are generated and the “best” candidate is singled out for transmission. By using suitable transforms or mappings the main goal is to achieve statistical independence between the candidates’ metrics. Clearly, instead of PAPR, also alternative metrics can be used in the selection process [20], [19]. SLM and PTS are similar, the difference between SLM and PTS is that the mappings are applied to a subset of the data frame.

For SLM many transforms have been proposed: in the original approach the data frame is element-wise multiplied by random phases; other popular approaches include binary random scrambling and permutation of the data (see ref. in [19]; here is it also where side transmission is discussed). Similar for PTS, random phases have been used as well. While typically a full search is carried out efficient algorithms to find the phases have been proposed. An exhaustive list can be found in [2].

SLM can be analyzed within the context of LDP. The transforms define U alternatives each assumed with independent PAPR. Clearly this independence assumption is crucial: it might be argued that it has to hold for the PAPR only but the model clearly fails when the number of alternatives is large. By exploiting the LDP we have simply then

$$\log(F_c(x)) = U [\log(N) - x]^- \quad (9)$$

so that the decay is U times faster as depicted in Fig. 3. A similar analysis can be carried out when the selection is done directed or over extended time [19]. Note that in principle PTS can be analyzed as well; however since the transform is on subsets of the data the independence assumption is far more critical. Another main problem so far is that side information is treated separately and *not* within the same communication model.

A better model is complete *partitioning* of the set of transmit sequences. The idea is illustrated in Fig. 4. Suppose that the transmit sequence belong to some set which is partitioned into many cells all of them containing the same information. Note that if the actual cell selection is required at the receiver for decoding, side information is generated. This side information belongs in our general model to the transmit sequence itself and must be specially protected. This can be done via an embedded code which is decoded before or after the actual information decoding procedure [2]. Let us mark the subsets where PAPR is below some

threshold: the reasoning is that by the mapping of codewords from one cell into another, sets with larger PAPR should be mapped to a marked subset by at least one mapping which will ensure peak power below the threshold. Obviously, the definition of such a mapping will determine the performance of the scheme.

One of the simplest examples is when the data is over some constellation and side information is encoded into a sequence of BPSK symbols: each sequence defines a specific BPSK vector determining the sign vector. Both modified information and side information sequence define the transmitted codeword. This method is called *sequence balancing* [34]. It is characteristic for this method that correlation is inserted in the stream by using suitable binary codes. We will call this the *binary correlation model*. Noteworthy, if the side information is purely redundant the method reduces to *tone reservation* [4]. Moreover, if the selection defines phase relations between partial sequences then it is a version of PTS [32]. Related approach is also Trellis shaping [35], [2].

Sequence balancing using binary codes can achieve (even though easily generalized) already a sufficient fraction of the theoretically possible performance gain: the main required property of the set of binary vectors is their ability of as many sign changes as possible over any subvector which is called the *strength of the code* [34]. Many binary codes have this property and are thus suited for this procedure. The strength is related to the dual distance. It can be shown that if the strength grows as $\log(N)$ then PAPR is below $\log(N)$ for large N . Unfortunately, similar to SLM and PTS, the number of candidates grows as well.

There are other methods which use partitioning as well such as *tone injection* [4] where the constellation is artificially extended or *translates of codes* [36]. Schemes such as *active constellation extension* [37] introduce redundancy as well but can be continuously formulated so that other methods such as convex optimization can be applied.

All discussed approaches assume to run a full cell selection search which is too complex in many situations. A better approach is discussed next.

C. Derandomization of choices

The LDP provides a method to circumvent full search by assuming a suitable underlying probability model for the cell selection. By derandomizing the cell selection one can easily devise suitable algorithms guaranteeing a PAPR reduction very close to the $\log(N)$ barrier [38], [29], [39]. The basis algorithm goes back to Spencer [40] who called it the *probabilistic method*.

The derandomization method is best explained along an example: consider again the binary correlation model where any possible sign change for some information sequence \mathbf{C} is allowed. Denote this sign vector by $\mathbf{A} := [A_0, \dots, A_{N-1}]$ and the resulting transmit sequence by $\mathbf{A} \circ \mathbf{C}$ (respectively $\mathbf{s}_{\mathbf{A} \circ \mathbf{C}}$). Suppose that all the sign changes happens at random with equal probability and each sign change is independent. As for the LDP, define the random variables $y_i(A_0^*, \dots, A_{i-1}^*) := \mathbb{E} \{ \text{PAPR}(\mathbf{s}_{\mathbf{A} \circ \mathbf{C}}) | A_0^*, \dots, A_{i-1}^* \}$. Then we can mimic the steps (5)-(8) and successively reduce randomness by applying:

$$A_i^* := \arg \min_{a_i} y_i(A_0^*, \dots, A_{i-1}^*, a_i)$$

By the properties of (conditional) expectations

$$y_i(A_0^*, \dots, A_{i-1}^*, A_i^*) \leq y_i(A_0^*, \dots, A_{i-1}^*)$$

and finally $\text{PAPR}(\mathbf{s}) \leq y_0$ since $y_N(A_0^*, \dots, A_{N-1}^*)$ is simply a non-random quantity. Finally, by the LDP $y_0 \leq \log(N)$ for N large enough. Since the expectation are somewhat difficult to handle instead of the $\text{PAPR}(\mathbf{s})$ typically the set function and corresponding bounds have been used. For example, Chernoff bounds have been used in [38], [29], [41] showing good performance and low complexity. Moment bounds with better tail properties have been used but the complexity is higher [42]. Performance results of the derandomization method are reported in Fig. 5 comparing sequence balancing (Sec. IV-B) with and without derandomization. The benefit of the derandomization method is clearly observed and corresponds to more than 4 dB gain in

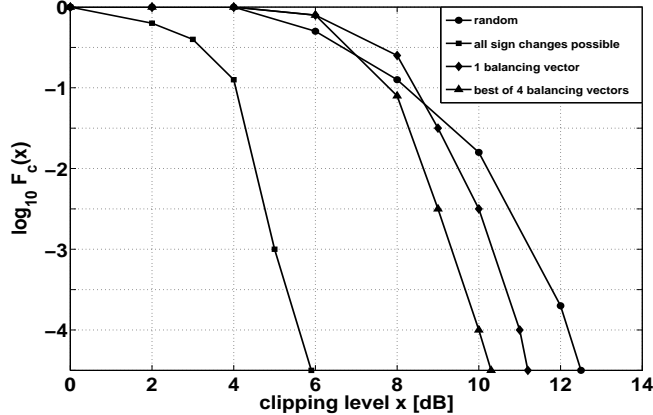


Fig. 5. The figure compares the sequence balancing method of Sec. IV-B in terms of the CCDF of PAPR for a 128 subcarrier OFDM system using a.) derandomization algorithm where all possible subcarrier sign changes are allowed and b.) codes of given strength with randomly chosen balancing vectors. For the code a strength 10 dual BCH code with only 18 redundant subcarrier is applied. Please note that the simulation matches very well the theory predicting cut off point $\log(128) \approx 6.8$ dB.

HPA backoff (at 10^{-3} outage probability) which mimics exactly the results of the LDP analysis (Sec. IV-A). However, it comes at the cost of 1 bit/dimension rate loss. The relevant trade off between rate and PAPR have been rarely investigated so far.

Combining the derandomization method with partitioning yields several improved algorithms for standard problems. For example the PTS method has been applied in [43]. It is proved that with derandomization method PTS can achieve $r \log(N)$ where r is the percentage of partial transmit sequences related to N . The tone reservation method has been treated using derandomization in [29] (see also Sec. VII-B4). Implicitly, derandomization has been used in [36] to show that PAPR of some translate of a code \mathcal{C} is below $|\mathcal{C}| \log(N)$. Related derandomization algorithms have been used in [39] adopting the so-called pusher-chooser game from [40]. The idea is to choose l_p -norms and prove a recursive formula similar to the Chernoff method. The approach can be generalized to alternative metrics if appropriate bounds are available: in [23] the SER has been reduced using derandomization showing that $\frac{\log(N)}{2}$ clipping level is sufficient asymptotically for zero error probability (instead of $\log(N)$). Furthermore, in the recent paper [5] zero clipped energy is asymptotically achieved with clipping level $\log \log(N)$.

There is still plenty of room for improvements, e.g. by considering correlations between different samplings points and incorporating other metrics as well [5], [23]. It has not been noticed yet that this field is particularly underdeveloped and bears great potential for significant improvements of current systems. Another point to be improved is the rate loss imposed by the current methods.

V. ADDITIONAL RESOURCES: MIMO AND MULTIUSER SYSTEMS

While utmost beneficial in terms of spectral efficiency MIMO systems complicate the PAPR problem: in single-antenna systems the PAPR (or other metrics) of only one transmit antenna has to be controlled. In the MIMO setting a large number of OFDM signals are transmitted in parallel and typically the worst-case candidate dictates the PAPR metric (e.g., due to out-of-band power) [19].

As a consequence, PAPR reduction methods tailored to this situation should be utilized, instead of performing single-antenna PAPR reduction in parallel. Multi-antenna transmitter provide additional degrees of freedom which can be utilized beneficially for PAPR reduction—the full potential has not yet been explored in literature. Basically, the peak power can be redistributed over the antennas. By this, MIMO PAPR reduction may lead to an increased slope of the CCDF curves (cf. Fig 3), i.e., the probability of occurrence of large signal peaks can significantly lowered compared to single-antenna schemes. This effect is similar to that of achieving some diversity gain.

When studying MIMO PAPR reduction schemes two basic scenarios have to be distinguished: on the one hand, in point-to-point MIMO transmission joint processing of the signals at both ends (transmitter and receiver) is possible. On the other hand, in point-to-multipoint situations, i.e., multi-user downlink transmission joint signal processing is only possible at the transmitter side⁴. This fact heavily restricts the applicability of PAPR reduction schemes.

For the point-to-point setting, a number of PAPR reduction schemes have been designed, particularly extension of SLM. Besides *ordinary SLM* (conventional SLM is simply applied in parallel) *simplified SLM* (the selection is coupled over the antennas) has been proposed in [44]. *Directed SLM* [45] is tailored to the MIMO situation and successively invests complexity (test of candidates) only where PAR reduction is really needed.

It might be sufficient that the PAPR stays below a tolerable limit, determined by the actual radio frontend. Here, complexity can be saved if candidate generation and assessment is done successively and stopped if the tolerable value is reached. Interestingly, the average number of assessed candidates is simply given by the inverse of the cdf of PAPR of the underlying original OFDM scheme. Noteworthy, for $\text{PAPR} = \log(N)$ and reasonably large number N of carriers, average complexity per antenna is in the order of $e = 2.718 \dots$ (Euler's number) [46]. Alternative metrics have been used in [47].

Compared to point-to-point MIMO systems, PAPR reduction schemes applicable in point-to-multipoint scenarios (multi-user downlink) are a much more challenging task. Since no joint receiver-side signal processing is possible, at the transmitter side in candidate generation only operations are allowed which can individually be reversed at each of the receivers. Among the SLM family, only simplified SLM can be used here. However, in this situation the usually present transmitter side multi-user pre-equalization can be utilized for PAPR reduction. Applying Tomlinson-Harashima precoding the sorting in each carrier can be optimized to lower PAPR at almost no cost in (uncoded) error rate [19]. The same is true when applying lattice-reduction-aided pre-equalization. Here the unimodular matrices (describing a change of basis) can be optimized to control the properties of the transmit signals [48]. There are also links from MIMO PAPR reduction and derandomization to code design (cf. Sec. VII-A).

VI. GOING BEYOND: OFDM CAPACITY FUNDAMENTALS

While the capacity of the discrete-time peak-power-constraint channel is known and computable, the capacity of the OFDM peak-power-constraint channel is still an open problem [49], [50]. The problem is indeed intricate as it has been unknown until very recently that there are exponentially many OFDM signals with constant PAPR (cf. Sec. VII-A). However, no practical encoding scheme is known which comes even close to this merely theoretical result. From this perspective the capacity problem awaits a more thorough theoretical solution.

Recent work [21], [22] on practical schemes indicates that clipped OFDM performs (almost) the same as unclipped OFDM with signal-to-noise ratio reduced according to the clipping power loss. The main source of the loss are not the introduced distortions or errors but simply the reduced output power. Given the OFDM frame in frequency domain $\mathbf{C} = [C_1, \dots, C_N]$, via IDFT the time-domain samples $s[k]$ are calculated. These samples then undergo clipping in the amplifier frontend. As usually the clipping behavior can be described by a nonlinear, memoryless point symmetric function $g(x)$ (with $g(x) \leq x$, $x > 0$, applied element-wise to vectors). In frequency domain, the clipped signal is given by $\mathbf{Z} = \text{DFT}\{g(\text{IDFT}\{\mathbf{C}\})\}$. Note that clipping is a deterministic function and a one-to-one relation between the vector \mathbf{C} of unclipped symbols and the vector $\mathbf{Z} = [Z_1, \dots, Z_N]$ of clipped ones exist. Assuming an AWGN channel, at the receiver side the vector \mathbf{Z} , disturbed by additive white Gaussian noise is present. In case of intersymbol-interference channels, the symbols Z_n are additionally individually scaled by the fading gain at the respective carrier.

⁴In the *multipoint-to-point scenario* (multiple-access channel) no joint optimization of the transmit signals can be performed, hence this case is not amenable for MIMO approaches.

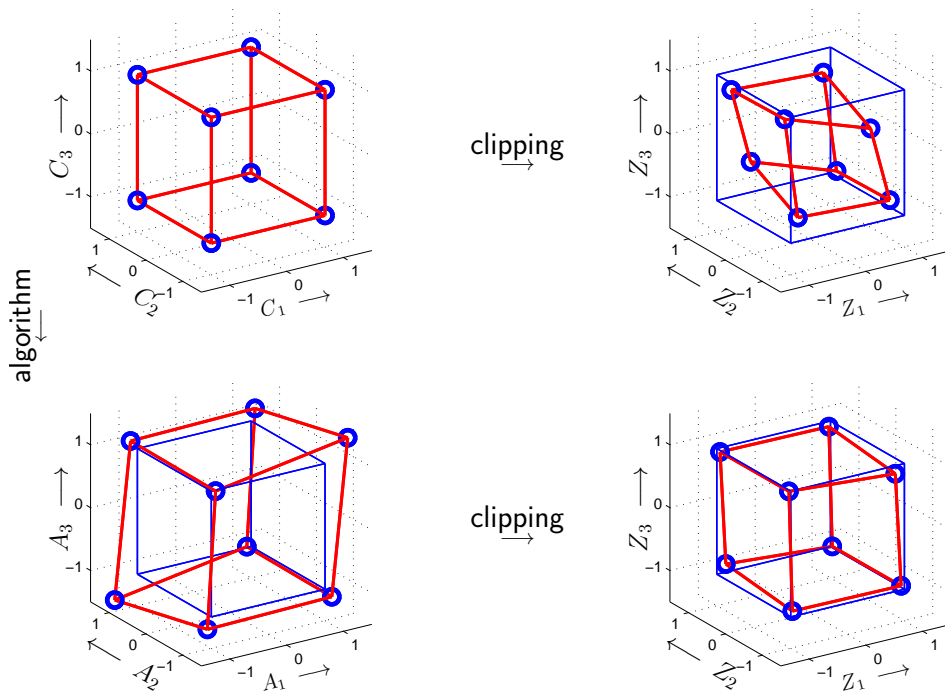


Fig. 6. Top row: visualization of the effect of clipping on the set of possible OFDM frames (here: $N = 3$, 2-PAM per carrier). Bottom row: visualization of predistortion via an algorithm for maximizing the power of the signal after clipping.

This clipping behavior can be visualized for $N = 3$ and 2-PAM per carrier, see top of Fig. 6. The initial hypercube with vertices given by all possible vectors \mathbf{C} is distorted. However, the attenuation of the useful signal (the vector \mathbf{Z} has lower energy) will be the dominating effect over deformation. This, in turn, leads to the conclusion that a suited metric for capacity maximization is simply the average power of the power amplifier output signal.

A possible strategy is shown on the bottom of Fig. 6. A signal shaping algorithm may adjust the signal points in $2N$ -dimensional real-valued space such that after clipping the set of all possible OFDM vectors in frequency domain forms (approximately) an hypercube with energy close to that of the initial constellation. First work on using the strategy of *active constellation extension* for achieving is goal has been presented [51].

VII. EMERGING SOLUTIONS: AN OPEN FIELD

A. New Trends in Code Design

Jones et al. [52] were the first to describe block coding schemes in the present context. This framework has been put in systematic form by observing the connection of cosets of Reed-Muller codes and complementary sequences [53], [54]. Unfortunately, these approaches have limited potential for modern OFDM systems due to their limited coding rate. The fundamental trade off between different code key properties such as rate, PAPR etc. was explored and discussed in [55]. More recent ideas use the idea of sequence balancing and code extensions in form of erasure coding in other domains (e.g. MIMO [56]) to tackle the PAPR problem with an inner code, while error correction still is done via an outer code [34].

1) *Codes and sequences with low PAPR*: Though most of multicarrier signals of length N have PAPR close to $\log(N)$, it turns out that signal with constant PAPR are not so rare. Using a remarkable result of Spencer [57] it is possible to show that the number of such BPSK modulated signals is exponential in N . Namely, there are at least $(2 - \delta_K)^N$ such signals with PAPR not exceeding K , where δ_K is a constant depending on K and tending to zero when K grows. It is an open question how to generate many signals for given K .

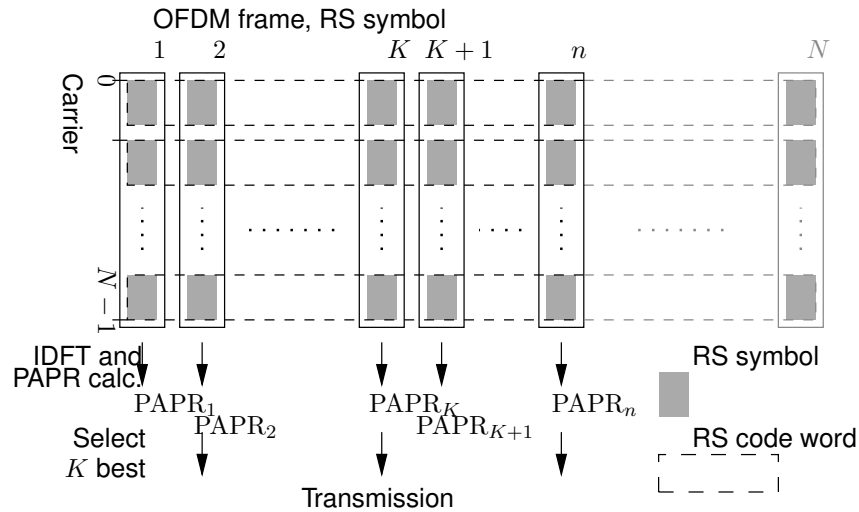


Fig. 7. Illustration of the coding method across symbols using off-the-shelf codes (such as RS codes).

A lot of research was devoted to describing signals with low values of PAPR. For BPSK modulated signals an extreme example is provided by Rudin-Shapiro sequences defined recursively from $P_0 = Q_0 = 1$, and

$$P_{m+1} = (P_m, Q_m), \quad Q_{m+1} = (P_m, -Q_m).$$

These sequences of length being a power of 2 have PAPR at most 2. More general examples of sequences with PAPR at most 2 arise from Golay complementary sequences. Two sequences constitute a complementary pair if the sum of the values of their aperiodic correlation functions sum up to zero. Many methods are known for constructing such sequences, see [2, Section 7.6]. Notice that it is not known if BPSK modulated signals can have PAPR less than 2. However, if one increases the size of multiphase constellations to infinity there exist sequences with PAPR approaching 1 [2, Theorem 7.37]. For constructions of multiphase complementary pairs from cosets of Reed-Muller codes see [58] and references there. PAPR of m -sequences and Legendre sequences is discussed in [2, Sections 7.7 and 7.8].

Often we need to know the biggest PAPR among sequences belonging to a code. Bounds on PAPR of codes on sphere as a function of their sizes and minimum Euclidean distances was studied in [55]. A relation between the distance distribution of codes and PAPR was derived in [29]. This yielded bounds on PAPR of long algebraic codes, such as BCH codes. Analysis of PAPR of codes with iterative decoding, for instance LDPC codes, remains an open problem. PAPR of codes of small size was studied in [55]. In particular, it was shown that PAPR of duals of length N BCH codes are at most $\log^2(N)$. As well bounds on PAPR of Kerdock and Delsarte-Goethals codes were derived. In [34] it was shown that in every coset of a code dual to BCH code with the minimum distance of $\log(N)$ exists a code word with PAPR at most $\log(N)$. At the same time, this leads to a very modest rate loss. Still, constructing codes having low PAPR and high minimum distance seems to be a challenge.

Computing PAPR of a given code is a computationally consuming problem. If a code has a reasonably simple maximum-likelihood decoding algorithm it is possible to determine efficiently its PAPR [59], [60].

In [56] off-the-shelf channel codes, in particular Reed-Solomon (RS) and Simplex codes are employed to create candidates, from which, as in SLM, the best are selected. The codes are thereby arranged over a number of OFDM frames rather than over the carriers. Such an approach is very flexible as due to the selection step any criterion of optimality can be taken into account. Moreover, instead of applying the approach to the MIMO setting, it can also be used if block of temporal consecutive OFDM frames are treated jointly. The method is illustrated in Fig. 7.

2) *Constellation shaping*: In constellation shaping, we have to find a constellation in the N -dimensional frequency domain, such that the resulting shaping region in the time domain has low PAPR. At the same

time we would like to have a simple encoding method for the chosen constellation. Such shaping based on Hadamard transform was considered in [61]. The main challenge in constellation shaping is to find a unique way of mapping (encoding) and its inverse (decoding) of reasonable complexity. The suggested approach in [62], [61] is based on a matrix decomposition. Though the simulation results are quite promising, the implementation complexity still seems to be far from being affordable [63], [62].

B. Banach space geometry

An interesting new approach to the PAPR problem is that of using Banach space geometry. Banach space geometry relates norms and metrics of different Banach spaces to each other. For example, a question that often arises is: assume a Banach space with unit norm ball B_1 and another Banach space with ball B_2 ; both spaces are of finite, possibly different dimension. What is the relation between the norms if the projection of one ball covers the other ball? Furthermore, what is the dependence of this relation on the dimensions?

Interestingly, these relations turn out to be useful for the PAPR problem in several other ways depending on the underlying Banach spaces as the following examples show:

1) *Alternative orthonormal systems: Kashin & Tzafriri's theorem:* In Sec. IV it was shown that OFDM has unfavorable PAPR of order $\log(N)$ if N gets large. One might be inclined to ask if this is an artifact of the underlying orthonormal signaling system. The answer is actually no with the implication that OFDM plays no specific role among all orthonormal systems. Already in [64] it was shown that worst PAPR is of order N regardless of the signaling system (multicarrier CDM etc.). But even if we consider not the worst PAPR but look at the PAPR on average the situation does not get better. In [65], Kashin & Tzafriri proved that for any orthonormal system on a given finite time interval the expectation of PAPR is necessarily of order $\log(N)$. Again, changing the signaling is not beneficial in terms of PAPR. The underlying mathematical problem is that of estimating the supremum norm of a finite linear combination of functions weighted with random coefficients both constrained in the energy norm.

2) *Is PAPR of single-carrier really much better?:* It is common engineering experience that single-carrier has better PAPR than multicarrier. But it might be worth raising this question again within the context of upcoming technological advances (LTE-A etc.) which operate much closer to the Nyquist bandwidth and, moreover, use different modulation and coding schemes. Let us formalize this question.

Suppose, we send a transmit sequence C_1, \dots, C_N and use a band-limited filter to generate the continuous-time signal (bandwidth is set to π for simplicity). The transmit signal can be described by

$$s(t) = \sum_{i=1}^N C_i \frac{\sin \pi(t - t_i)}{\pi(t - t_i)}$$

with sampling points $t_i \in \mathbb{Z}$. Naturally, band-limited signals of this form have very different PAPR behavior compared to OFDM since, obviously, if the coefficients are from some standard modulation alphabet, the signal is nailed down to some finite value at the sampling point independent of N . However, within the sampling intervals (on average) large PAPR could actually occur. Noteworthy, the worst case is growing without bounds linearly in N .

Surprisingly, the exact answer to this problem has not been explored until very recently [66] which is basically a result on large deviations in Banach spaces. It is proved in [66] that such bad PAPR cannot actually happen and that there is a constant $c_0 > 0$ such that:

$$\mathbb{E}(\text{PAPR}) \leq c_0 \log \log(N)$$

But we also see the catch here. Modern communication systems use higher modulation sizes and in that case the influence of the data becomes dominant if the distribution becomes Gaussian like. In that case we approach the $\log(N)$ again.

There is some interesting connection of the PAPR problem to the Hilbert transform context: since in many standard communication models, e.g. in Gabor's famous *Theory of Communication* [67], [68], the transmit

signal is a linear combination of a signal and its Hilbert transform, properties such as PAPR in the transform domain become more and more important. Initiated by early works of Logan [69] who investigated the Hilbert transforms of certain bandpass signals it was recognized not until very recently [70], [71] that the results are fragile for wideband signals containing spectral components in an interval around zero frequency. Then, in general, the domain of the Hilbert transform must be suitably extended; further, examples of bandlimited wideband signals are provided where the PAPR grows without bounds in the Hilbert transform domain [72]. Hence, for certain single-carrier analytic modulation schemes the transmit signal has to be shaped very carefully.

3) *Overcomplete expansions with uniformly bounded PAPR*: While the result for arbitrary orthonormal systems appears rather pessimistic there is a possible solution in the form of *frames*. Frames are overcomplete systems of vectors in \mathbb{R}^n , $n < N$. Let us denote this description by $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_N] \in \mathbb{R}^{n \times N}$, $N \geq n$. Then, if the rows are independent there is $\mathbf{x} \in \mathbb{R}^N$ so that

$$\mathbf{y} = \mathbf{U}^T \mathbf{x} \quad (10)$$

for any $\mathbf{y} \in \mathbb{R}^n$ and the elements of \mathbf{U} are a frame. If $\mathbf{U}^T \mathbf{U} = \mathbf{I}_n$, where \mathbf{I}_n is the identity matrix, then it is called a *tight frame*. In seminal work Kashin [8] interpreted the mapping (10) as an embedding of the Banach space with supremum norm l_∞ to the Banach space with standard euclidean norm l_2 and asked for the growth factor $K(\lambda) > 0$, $\lambda := N/n$, between the two norms when the l_2 unit ball in \mathbb{R}^n should be covered by the unit ball l_∞ in \mathbb{R}^N . Such representations are called *Kashin representations* of level λ [73].

Clearly, if $N = n$ then $K(\lambda) = \sqrt{N}$. However, if $N > n$ (overcomplete expansion) then Kashin proved that there is a subspace in \mathbb{R}^N generating by a frame \mathbf{U} such that λ is given by:

$$K(\lambda) := c_1 \left(\frac{\lambda}{\lambda - 1} \log \left(1 + \frac{\lambda}{\lambda - 1} \right) \right)^{1/2}, \quad c_1 > 0$$

Hence, the $K(\lambda)$ is uniformly bounded in n if $\lambda > 1$ is fixed. Good estimates of the constant $c_1 > 0$ are not known [73].

This intriguing result has been applied in the PAPR context in [73] and the implications for peak power reduction are immediate. The matrix \mathbf{U} can be taken as a precoding matrix for classical OFDM transmission and achieve uniformly bounded PAPR. Unfortunately the construction of the optimal subspace is not known [73]. Kashin representations exploiting the *uncertainty principle* of random partial Fourier matrices are presented in [74].

4) *Tone reservation and Szemerédi's theorem*: One of the oldest but still very popular scheme is tone reservation [75]. But, despite its simplicity, many questions involved are still open which does not come by coincidence: recent work in [76] has analyzed the performance of this method and uses an application of the Szemerédi's famous theorem about arithmetic progressions (Abel price 2012).

Recalling the setting where a subset of subcarriers is solely reserved for peak power reduction the challenge is to find for a given set of transmit sequence a subset and corresponding values such that the PAPR is reduced to the most possible gain. Until now, achievability and limits are not known (except for simple cases). Therefore, there is some incentive to look at this problem from a new perspective. Ref. [76] has analyzed the case where the compensation set is arbitrary but fixed. In this typical case it is proved that the efficiency of the system, i.e., the ratio of cardinality of information and compensation sets must decrease to zero if the peak power is constrained independent of the subcarriers. The technique that is used is to show necessary assumptions on the relations of unit spheres in the Banach spaces. This relation is shown not to hold asymptotically for sets with additive structure. However, Szemerédi's theorem states that such sets are included in every subset of cardinality δN where $\delta > 0$. In fact such arithmetic progressions induce signals with bad PAPR behavior naturally to be excluded by the method. The theorem shows that this is not possible.

In extended work [77] also other families of orthogonal signalling such as Walsh sequences are analyzed all of them showing basically the same discouraging result regarding the system's efficiency. This leads to

the conjecture in [77] that all natural orthogonal signalling families have this behavior.

C. Compressed sensing

Compressed sensing [6], [7] is a new sampling method that compresses a signal simultaneously with data acquisition. Each element of the compressed signal or measurements consists of a linear combination of the elements in the original signal and this linear transformation is independent of instantaneous characteristics of each signal. In general, it is not possible to recover an unknown original signal from the measurements in the reduced dimension. Nevertheless, if the original signal has sparsity property, its recovery can be perfectly achieved at the receiver. Since sparsity frequently appears in the PAPR problems of the OFDM systems, compressed sensing can be a powerful tool to solve these problems.

Compressed sensing can be regarded as minimizing the number of measurements while still retaining the information necessary to recover the original signal well (i.e. beyond classical Nyquist sampling). The process can be briefly illustrated as follows. Let \mathbf{f} denote a signal vector of dimension N and \mathbf{g} be a measurement vector of dimension M with $M < N$ obtained by $\mathbf{g} = \Phi \mathbf{f}$, where Φ is called *sensing matrix*. At the transmitter, sampling and compression are performed altogether by simply multiplying Φ by \mathbf{f} to obtain \mathbf{g} . At the receiver, if \mathbf{f} is an S -sparse signal, which means \mathbf{f} has no more than S nonzero elements, it is shown in [7] that the exact \mathbf{f} can be obtained from \mathbf{g} by using l_1 minimization, that is,

$$\min_{\tilde{\mathbf{f}}} \|\tilde{\mathbf{f}}\|_1 \quad \text{subject to} \quad \mathbf{g} = \Phi \tilde{\mathbf{f}} \quad (11)$$

as long as Φ has some good property, which is called *restricted isometry property (RIP)*. For some positive integer S , the isometry constant δ_S of a matrix Φ is defined as the smallest number such that

$$(1 - \delta_S) \|\mathbf{f}\|_2^2 \leq \|\Phi \mathbf{f}\|_2^2 \leq (1 + \delta_S) \|\mathbf{f}\|_2^2$$

holds for all S -sparse vectors \mathbf{f} . Under RIP with $\delta_{2S} < \sqrt{2} - 1$, (11) gives the exact solution for \mathbf{f} [78]. This recovery method using l_1 -minimization is called basis pursuit (BP) [79], which requires high computational complexity. Many greedy algorithms [80], [81], [82], [83] have been developed to reduce the recovery complexity.

In many applications of compressed sensing such as communication systems, it is required to recover \mathbf{f} from the corrupted measurements $\mathbf{g}' = \mathbf{g} + \mathbf{z}$, where \mathbf{z} is a noise vector of dimension M . For this, recovery algorithms such as basis pursuit denoising [79], Lasso [84], and their variants have been developed while the existing recovery algorithms can also be used. However, these algorithms do not still show good performance enough to be adopted in wireless communication systems which usually require very low error rate even in severely noisy environments.

Related to PAPR problems, the properties lying in the compressed sensing such as sparsity, RIP, and recovery algorithms can be utilized in many PAPR reduction schemes. In [85] and [86], a new tone-reservation scheme is proposed, which is different from the existing tone-reservation [4] in that it provides a guaranteed upper bound for PAPR reduction as well as guaranteed rates of convergence. This scheme exploits the RIP of the partial DFT matrix. In [87], a novel convex optimization approach is proposed to numerically determine the near-optimal tone-injection solution. Generally, tone-injection [4] is an effective approach to mitigate PAPR problem without incurring bandwidth loss. However, due to its computational complexity, finding the optimal tone-injection becomes intractable for OFDM systems with a large number of subcarriers. Therefore, a semi-definite relaxation needs to be adopted in the convex optimization [88]. Moreover, based on the observation that only a small number of subcarrier symbols are usually moved, l_0 minimization is required and naturally it can be relaxed to l_1 minimization similar to compressed sensing literature.

One of the popular solutions to PAPR reduction is clipping the amplitude of the OFDM signal although the clipping increases the noise level by inducing a clipping noise. Due to the sparsity of the clipping noise, compressed sensing can be used to recover and cancel the clipping noise. Before the clipping noise

cancellation schemes using compressed sensing appear, some foundations of them have been presented. An impulse noise cancellation system using sparse recovery is firstly proposed in [89]. In practical systems, there exists a set of null tones not used for information transmission, which is exploited as measurements to estimate the impulse noise in time domain at the receiver. As an extension to [89], an alternative recovery algorithm with low complexity is proposed in [90], which exploits the structure of DFT matrix and available a-priori information jointly for sparse signal recovery. In [91], the work in [89] is extended to the case of bursty impulse noise whose recovery is based on the application of block-based compressed sensing. Secondly, a clipping noise cancellation scheme using frame theory is proposed in [92]. Although this scheme uses not compressed sensing but frame expansion, the frame expansion can be viewed as a special case of compressed sensing problem with known positions of nonzero elements. Some additional reserved tones not including data are padded and they are used as the measurements to recover the clipping noise at the receiver.

Motivated by the above works, clipping noise cancellation schemes using compressed sensing have been proposed in [93] and its extended version in [94]. In [93], [94], M reserved tones are allocated before clipping at the transmitter and they cause some data rate loss. These reserved tones can be exploited as measurements instead of null tones in [89], [90], [91], [95]. Let us denote the transceiver model in frequency domain with clipping noise as

$$\mathbf{Y} = \mathbf{H}(\mathbf{C} + \mathbf{D}) + \mathbf{Z} \quad (12)$$

where \mathbf{C} and \mathbf{Y} are $N \times 1$ transmitted and received tone vectors, respectively, \mathbf{H} is a diagonal matrix of the channel frequency response, \mathbf{D} is $N \times 1$ clipping noise vector, and \mathbf{Z} is AWGN vector. Starting from (12), we equalize the channel by multiplying with \mathbf{H}^{-1} and select the rows whose indices correspond to locations of the reserved tones by multiplying with a $M \times N$ row selection matrix \mathbf{S}_r . This results in

$$\underbrace{\mathbf{S}_r \mathbf{H}^{-1} \mathbf{Y}}_g = \underbrace{\mathbf{S}_r \mathbf{F}}_{\Phi} \underbrace{\mathbf{d}}_f + \mathbf{S}_r \mathbf{H}^{-1} \mathbf{Z}, \quad (13)$$

where \mathbf{F} is the DFT matrix and $\mathbf{D} = \mathbf{F}\mathbf{d}$. As seen in (13), the clipping noise on the reserved tones is used as measurements to recover the clipping noise \mathbf{d} in time domain by sparse recovery algorithm. Additionally, in [94], a method exploiting a-priori information together with weighted l_1 minimization for enhanced recovery followed by Bayesian techniques is proposed. However, the performance of [93] and [94] is restricted due to weakness of the compressed sensing against noise.

In [96], more enhanced clipping noise cancellation scheme using compressed sensing is proposed. Different from [93] and [94], the scheme in [96] does not cause data rate loss, because it exploits the clipping noise in frequency domain as measurements underlying in the data tones rather than the reserved tones. In this case, transmitted data and clipping noise are mixed in the data tones. To distinguish the clipping noise from the data tones well, this scheme exploits part of the received data tones with high reliability. To (12), we multiply \mathbf{H}^{-1} and row selection matrix \mathbf{S}_d , selecting the locations of reliable data tones, as

$$\mathbf{S}_d \mathbf{H}^{-1} \mathbf{Y} = \mathbf{S}_d \mathbf{C} + \mathbf{S}_d \mathbf{D} + \mathbf{S}_d \mathbf{H}^{-1} \mathbf{Z}. \quad (14)$$

Then, we estimate the $\mathbf{S}_d \hat{\mathbf{C}}$ and subtract them from (14) as

$$\underbrace{\mathbf{S}_d \mathbf{H}^{-1} \mathbf{Y} - \mathbf{S}_d \hat{\mathbf{C}}}_g = \underbrace{\mathbf{S}_d \mathbf{F}}_{\Phi} \underbrace{\mathbf{d}}_f + \mathbf{S}_d (\mathbf{C} - \hat{\mathbf{C}}) + \mathbf{S}_d \mathbf{H}^{-1} \mathbf{Z}. \quad (15)$$

Then, from partially extracted clipping noise in frequency domain, we can recover the clipping noise \mathbf{d} in time domain via sparse recovery algorithms. Furthermore, this scheme can adjust the number of the measurements M by changing the reliability of received data. Therefore, when there is AWGN noise, we can select the optimal number of measurements corresponding to the noise amount. Consequently, this scheme successfully realizes the clipping noise cancellation scheme by overcoming weakness of the compressed sensing against noise. Additionally, in [96], clipping noise cancellation for orthogonal frequency-division multiple access

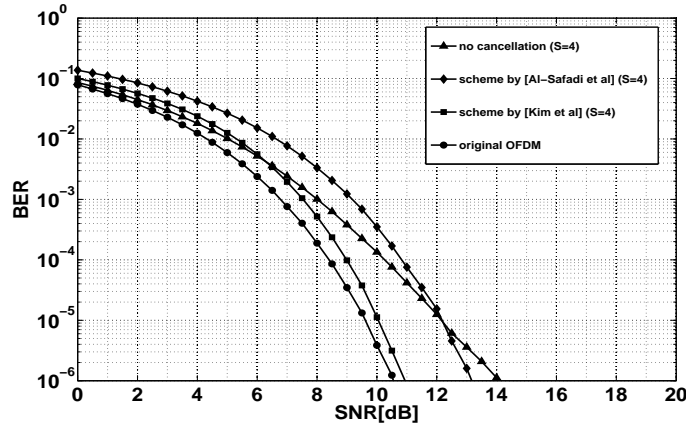


Fig. 8. BER performance in OFDM AWGN channel using compressed sensing based clipping noise cancellation schemes as described in Al-Safadi et al [93] and Kim et al [96]. In [96] 23 out of 64 QPSK subcarriers are used for recovery of the $S = 4$ sparse clipping noise. In [93] 29 tones are selected by a (59, 29, 14) difference set and reserved. Baseline performance is the original OFDM signal with no clipping. It can be seen that provided the AWGN is not too strong, sparsity in the clipping noise can lead to significant performance gains of more than 3 dB given an error probability of 10^{-6} .

(OFDMA) systems is also proposed using compressed sensing. The fast Fourier transform (FFT) block of OFDM systems can be decomposed into the small FFT blocks. And, the subset of rows in the small sized DFT matrix can also be used as a sensing matrix, which can be used to recover the clipping noise for OFDMA systems via sparse recovery algorithm.

Fig. 8 shows the bit error rate (BER) over signal-to-noise ratio (SNR) performance of the clipping noise cancellation schemes based on compressed sensing described in [93] and [96] for OFDM signals over the AWGN channel. The S -sparse clipping noise signal contaminates the original OFDM signal and the case of no clipping noise cancellation shows the worst BER performance among all schemes. In [93], the authors applied the compressed sensing technique to OFDM systems for the first time, but there is a benefit only for the high SNR region due to weakness of compressed sensing recovery against AWGN. The BER performance of the scheme in [96] is better because the number of the measurements can be adjusted corresponding to the AWGN level.

VIII. CONCLUSIONS

Despite two decades of intensive research the PAPR problem remains one of the major problems in multicarrier theory with huge practical impact. This article provides a fresh look on this problem by outlining a new perspective using alternative metrics (including MIMO and multiuser systems as a special case), the corresponding theoretical foundations and related designs. This is followed by thorough discussion of current limits and new future directions.

IX. ACKNOWLEDGEMENTS

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