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ABSTRACT

The plant parameter variation problem in multivariable linear systems described by state vector equations is formulated using a new sensitivity measure. This formulation involves a direct comparison of open-loop and state feedback performance in the presence of parameter variations, and provides a basis for guaranteeing the superiority of the feedback design. Results are obtained for both continuous and discrete multi-input multi-output systems. Furthermore, it is shown for single-input multi-output plants that a low sensitivity design is also an optimal feedback control design with respect to a quadratic performance index. This provides a new interpretation of a similar result previously obtained by Kalman.

I. INTRODUCTION

There are two basic configurations used for control systems, the open-loop configuration and the closed-loop configuration. Both structures can be used equally well in the realization of input-output (or filter) transmission specifications. However, the feedback configuration offers two additional possibilities:

i.) The feedback configuration may exhibit smaller effects due to parameter variations than an open-loop configuration having the same transfer characteristics.

ii.) The feedback configuration may exhibit smaller effects due to disturbance or unwanted inputs than does the open-loop configuration.

It has been demonstrated[1] [1] that these possible benefits of feedback do not occur automatically in every case; they must be sought after as basic objectives of the design.

This paper is concerned with the first of these benefits, the possibility of reduced parameter variation effects. The systems to be studied will be linear and time-invariant, and will be described by state-variable differential equations. The systems may have several inputs and several outputs. The parameter variations are assumed to occur in the plant, or fixed elements. Furthermore, the variations are such that the system is always linear and time-invariant.

The effects of parameter variations will be studied quantitatively through the use of a sensitivity matrix and a scalar sensitivity measure depending on this matrix. The matrix and its interpretation derive from a new formulation of

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[1] Numbers in brackets designate references at end of paper.

the sensitivity problem recently proposed by the authors [2]. This approach involves direct comparison of errors due to parameter variations for the open-loop and the state feedback configurations. This approach leads to a satisfactory formulation of the sensitivity problem in the multivariable case. The percentage change formulation, commonly used for single-input single-output systems, does not generalize meaningfully to the multivariable case.

This approach will be used to study continuous and discrete state variable feedback designs. In particular, the optimal design for linear, time-invariant systems with quadratic performance index will be evaluated from the point-of-view of parameter variations.

II. THE SENSITIVITY MATRIX

Figure 1 shows a matrix block diagram representation of a continuous plant described by the state vector differential equation

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u}, \tag{1}$$

where $\underline{x}$ is the plant state vector, and $\underline{u}$ is the plant input (control) vector. For simplicity, we assume the state variables are available as plant outputs. The plant input vector $\underline{u}$ can be generated using either an open-loop or closed-loop control, in general. Figure 2 shows the open-loop configuration. Figure 3 shows a closed-loop configuration using state-variable feedback that produces the same plant input $\underline{u}$ as the open-loop structure, in the absence of parameter variations.

Because of plant parameter variations, the outputs of the systems differ from their nominal (or designed) values. The same plant variations will result in different actual outputs for the open and closed-loop configurations. Define the open and closed-loop errors by

$$\underline{e}_o = \underline{x} - \underline{x}'_o = -\underline{\Delta x}_o \tag{2}$$

$$\underline{e}_c = \underline{x} - \underline{x}'_c = -\underline{\Delta x}_c \tag{3}$$

where $\underline{x}'_o$ is the actual open-loop output in the presence of parameter variations, and $\underline{x}'_c$ is the actual closed-loop output in the presence of parameter variations. We shall obtain an expression relating these two errors.

With no parameter variations, we have

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{4}$$

$$\dot{\underline{w}} = C\underline{w} + D\underline{r} \tag{5}$$

$$\underline{u} = N\underline{w} \tag{6}$$

in the open-loop case. When plant parameters vary, $\underline{u}$ remains unchanged. We then have

$$\dot{\underline{x}}'_o = A'\underline{x}_o + B'\underline{u} \tag{7}$$

where

$$A' = A + \Delta A \tag{8}$$

$$B' = B + \Delta B \tag{9}$$

$$\underline{x}_o' = \underline{x} + \underline{\Delta x}_o \tag{10}$$

Taking the Laplace transform of Equation (7), and using the identities of Equations (8-10), we obtain

$$(sI - A')\, \underline{\Delta X_o}(s) + \Delta A\underline{X}(s) + \Delta B\underline{U}(s). \tag{11}$$

Combining Equations (11) and (2), we obtain

$$\underline{E}_o(s) = (sI - A')^{-1}\, [\Delta A\underline{X}(s) + \Delta B\underline{U}(s)]. \tag{12}$$

We now consider the closed-loop configuration. With no parameter variations, the equations are:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{13}$$

$$\underline{u} = F\underline{x} + G\underline{r} \tag{14}$$

As the plant parameters vary, $\underline{u}$ varies, in contrast to the open-loop case. This is the mechanism that allows the possible reduction of parameter variation effects. For the actual case,

$$\dot{\underline{x}}_c' = A'\underline{x}_c + B'\underline{u}' \tag{15}$$

$$\underline{u}' = F\underline{x}_c' + G\underline{r} \tag{16}$$

where A' and B' are as in Equations (8-10). Equations (15) and (16), together with Equation (3), yield an expression for the transform of the closed-loop error:

$$\underline{E}_c(s) = (sI - A' - B'F)^{-1} [\Delta A\underline{X}(s) + \Delta B\underline{U}(s)]. \tag{17}$$

The $\underline{X}$ in Equation (17) is the same as the $\underline{X}$ in Equation 12, being the transform of the plant state vector in the absence of parameter variations. Eliminating $\underline{X}$ from these equations, the following relation between transforms of $\underline{e}_c$ and $\underline{e}_o$ is obtained:

$$\underline{E}_c(s) = [sI - A' - B'F]^{-1} [sI - A'] \underline{E}_o(s) \tag{18}$$

A sensitivity matrix S(s) is now defined by the relation

$$\underline{E}_c(s) \triangleq S(s) \underline{E}_o(s) \tag{19}$$

We see that, in this case,

$$S(s) = [sI - A' - B'F]^{-1} [sI - A']. \tag{20}$$

This can be rewritten in several useful forms. Noting that the Laplace transform of the state transition matrix of the actual plant is

$$\Phi_p'(s) \triangleq [sI - A']^{-1}, \tag{21}$$

and that corresponding to the system composed of the actual plant with state feedback we have

$$\Phi_f'(s) \triangleq [sI - A' - B'F]^{-1}, \tag{22}$$

we then obtain

$$S(s) = \Phi_f'(s)\,[\Phi_p'(s)]^{-1}. \tag{23}$$

Another very useful expression for S(s) can be obtained. Equation (20) can be rewritten as

$$S(s) = [(sI - A')\,(I - \Phi_p' B'F)]^{-1}\,\Phi_p'^{\,-1} \tag{24}$$

Expanding Equation (24), we obtain

$$S(s) = [I - \Phi_p'(s)\,B'F]^{-1} \tag{25}$$

But $\Phi_p' B'$ is the transfer function matrix relating the transforms of $\underline{u}'$ and $\underline{x}'$:

$$\underline{X}'(s) = \Phi_p'(s)\,B'\underline{U}'(s) \triangleq H'(s)\,\underline{U}'(s) \tag{26}$$

The matrix product $H'(s)F$ can be interpreted as a matrix loop gain (or return ratio) for the closed-loop configuration. With the loop broken at $\underline{x}$,

$$H'(s)\,\underline{U}'(s) = \underline{X}'(s), \tag{27}$$

but

$$\underline{U}'(s) = F\,\underline{X}'(s). \tag{28}$$

Thus, we define

$$L'(s) \triangleq H'(s)\,F. \tag{29}$$

So we obtain the important relationship

$$S(s) = [I - L'(s)]^{-1}. \tag{30}$$

It is important to observe that since S(s) depends only on transfer function matrices, it is invariant with respect to choice of states. Also, note that the sensitivity matrix for a system described by state variables can be written in terms of transfer function matrices. This provides an interesting connection between current state-variable methods and the older transfer function methods.

For single-input single-output systems and differentially small plant parameter variations, $L' \approx L$, and Equation (25)(or (30))reduces to the usual scalar percentage change sensitivity function. However, for large parameter variations, instead of the return difference $[1 - L(s)]$, we have the return difference $[1 - L'(s)]$ using the perturbed loop gain. For the multivariable case, we may call $[I - L'(s)]$ a matrix return difference. If the plant parameter variations are differentially small, then $L' \approx L$ and our matrix return difference is identical to Sandberg's matrix return difference [3]. So the matrix sensitivity function as defined in Equation (19) has an interpretation as the inverse of a generalized matrix return difference.

III. A SCALAR SENSITIVITY INDEX

In this section a scalar performance index involving the sensitivity matrix S(s) is given. Choosing the weighted sum of integrated squared errors as a performance index, the condition

$$\int_{0}^{\infty} \underline{e}_c^T(t)\, Q\, \underline{e}_c(t)dt < \int_{0}^{\infty} \underline{e}_o^T(t)\, Q\, \underline{e}_o(t)\, dt, \tag{31}$$

where Q is a positive definite weighting matrix, guarantees that the open-loop design is worse than the closed-loop design, provided the integrals exist. Using Parseval's theorem, Equation (31) can be expressed as

$$\int_{-\infty}^{\infty} \underline{E}_c^T(-j\omega) Q \underline{E}_c(j\omega) d\omega < \int_{-\infty}^{\infty} \underline{E}_o^T(-j\omega)\, Q\underline{E}_o(j\omega) d\omega. \tag{32}$$

Replacing $E_c(j\omega)$ by $S(j\omega)\ E_o(j\omega)$, and transposing,

$$\int_{-\infty}^{\infty} \underline{E}_o(-j\omega)\ [S^T(-j\omega)\ Q\ S(j\omega) - Q]\ \underline{E}_o(j\omega)\ d\omega < 0. \tag{33}$$

If $S^TQS - Q$ is positive definite for all frequencies, Equation (31) is never satisfied, and the open-loop realization is better than any closed-loop realization. On the other hand, the condition

$$S^T(-j\omega)\ Q\ S(j\omega) - Q < 0 \tag{34}$$

guarantees the superiority of the closed-loop design for any system inputs for which the integrals in Equation (31) exist. That is, if in spite of perturbations in plant matrices A and B, as described by Equations (8) and (9), Equation (34) is satisifed, then Equation (31) is satisfied, provided the integrals exist.

Since the sensitivity matrix is the inverse of $[I - L']$, (Equation (30)), a different but equivalent form of Equation (34) is useful. Substituting $\underline{E}_o = S^{-1}\underline{E}_c$ in Equation (32), we obtain

$$[S^{-1}(-j\omega)]^T Q\, S^{-1}(j\omega) - Q > 0 \tag{35}$$

as a sufficient condition equivalent to Equation (34). Using Equation (30), Equation (35) becomes

$$[I - L'^T(-j\omega)]\, Q\, [I - L'(j\omega)] - Q > 0. \tag{36}$$

where $L'(j\omega) = \Phi_p'(j\omega)\, B'F$.

In choosing the integrated square errors as a performance index, it is assumed that the integrals exist, of course. In cases where the integrals do not exist, another performance index must be chosen. For example, for stochastic inputs one might use the average power as a criterion. Then, analogous to Equation (31), we have

$$\lim_{T \to \infty} \frac{1}{T} \int_0^T \underline{e}_c^T(t)\, Q\, \underline{e}_c(t)\, dt \leq \lim_{T \to \infty} \frac{1}{T} \int_0^T \underline{e}_o^T(t)\, Q\, \underline{e}_o(t) dt \tag{37}$$

as the condition guaranteeing the superiority of the closed-loop design, provided the limits exist.

IV. PARAMETER VARIATIONS IN DISCRETE SYSTEMS

The foregoing discussion has involved the sensitivity problem in continuous-time, linear, time-invariant systems described by state vector equations. A similar development of the sensitivity problem in discrete-time, linear, time-invariant systems can be made. Much of the procedure is analogous to the continuous-time case.

The plant is described by the difference equations

$$\underline{x}_{k+1} = A\,\underline{x}_k + B\,\underline{u}_k, \tag{38}$$

where the subscripts denote the discrete time instant involved.

In the open-loop configuration, the control vector is given by

$$\underline{w}_{k+1} = C\,\underline{w}_k + D\underline{r}_k \tag{39}$$

$$\underline{u}_k = N\,\underline{w}_k \tag{40}$$

See Figure 4. In the closed-loop case with state-variable feedback, the control is given by

$$\underline{u}_k = F\,\underline{x}_k + G\,\underline{r}_k. \tag{41}$$

See Figure 5.

If the plant parameters vary, the open-loop and closed-loop system outputs will differ from their desired or nominal values. We define the errors as

$$\underline{e}_o(t_k) = \underline{x}(t_k) - \underline{x}_o{}'(t_k) \tag{42}$$

$$\underline{e}_c(t_k) = \underline{x}(t_k) - \underline{x}_c'(t_k) \tag{43}$$

analogous to Equations (2) and (3) in the continuous case. We then derive a relation involving the z-transforms of the errors. The steps are similar to Equations (11-25) above, except that the z-transform, rather than the Laplace transform, is used. The result is

$$\underline{E}_c(z) = S(z)\,\underline{E}_o(z), \tag{44}$$

where

$$S(z) = [I - z^{-1}\,\Phi_p'(z)\,B'F]^{-1}. \tag{45}$$

Observing that $z^{-1}\Phi_p' B'$ is the input-output transfer matrix for the plant,

$$\underline{X}'(z) \triangleq H'(z)\,\underline{U}'(z) = z^{-1}\,\Phi_p'(z)\,B'\underline{U}'(z), \tag{46}$$

the sensitivity matrix can be written as

$$S(z) = [\,I - H'(z)\,F\,]^{-1}. \tag{47}$$

Denoting $H'(z)\,F$ as a loop gain $L'(z)$, the sensitivity matrix becomes

$$S(z) = [I - L'(z)]^{-1}. \tag{48}$$

As with the continuous case, the discrete sensitivity matrix is invariant with respect to choice of state variables for the discrete plant.

We select as scalar performance index the weighted sum squared error. Then the condition guaranteeing the superiority of the closed-loop system is

$$\sum_{k=o}^{\infty} \underline{e}_c^T(t_k)\, Q\, \underline{e}_c(t_k) < \sum_{k=o}^{\infty} \underline{e}_o^T(t_k)\, Q\, \underline{e}_o(t_k), \tag{49}$$

provided the two series converge.

Using the discrete analog of Parseval's Theorem, Equation (48) can be replaced by

$$\frac{1}{2\pi j} \oint_c \underline{E}_c^T \left(\frac{1}{z}\right) Q\, \underline{E}_c(z)\, \frac{dz}{z} < \frac{1}{2\pi j} \oint_c \underline{E}_o^T \left(\frac{1}{z}\right) Q\, \underline{E}_o(z)\, \frac{dz}{z} \tag{50}$$

where the integrals are evaluated on the unit circle in the z plane. Substituting $\underline{E}_c(z) = S(z)\, \underline{E}_o(z)$, we obtain

$$S^T\left(\frac{1}{z}\right) Q\, S(z) - Q < 0 \tag{51}$$

as a sufficient condition for the satisfaction of the requirement expressed in Equation (50). Since z is on the contour $|z| = 1$, and the S matrix has real coefficients, $S^T(\frac{1}{z}) = S^{*T}(z)$, where * denotes complex conjugate, then Equation (50) may be replaced by

$$S^{*T}(z) Q S(z) - Q < 0 \tag{52}$$

for all $z = 1$.

Alternatively, $\underline{E}_o(z)$ could be replaced by $S^{-1}(z)E_c(z)$ in Equation (50).

Then, using Equation (47), we obtain

$$[I - L'^{*T}(z)]\ [I - L'(z)] - I > 0 \qquad (53)$$

for all $|z| = 1$ as a sufficient condition for the guaranteed superiority of the closed loop system.

Observe that for the single-input single-output plant the condition of Equation (53) becomes

$$|\ 1 - L(z)|\ > 1 \text{ for all } |z| = 1, \qquad (54)$$

a condition previously obtained using the percentage change definition of sensitivity [4,5]. The method presented herein has the advantage of being applicable in the multivariable case, however, which the percentage change approach does not have.

V. APPLICATION TO OPTIMAL LINEAR CONTROL SYSTEMS

The use of the sensitivity conditions derived above in the trial-and-error design of multivariable control systems has been discussed elsewhere[2]. In this section application of these conditions to the parameter variation problem in optimal linear systems will be made. It has been shown [6,7] that the optimal control is linear for a linear plant with performance index quadratic in the state variables and in the control variables. More recently, Kalman has shown [8] that the optimal single-input multi-output system also satisfies the classical percentage change sensitivity requirement

$$|S(j\omega)| \leq 1 \qquad (55)$$

for all ω. However, his scalar return difference corresponds to the loop opened at the input to the plant. His sensitivity is then the sensitivity of the transfer function from external input to plant input, with respect to component variations in the forward loop. If we are interested in the effect of plant parameter variations on the multiple outputs, then it is not clear that optimal feedback control is also better than open loop control in the sense of Equation (31). In this section we will show that if a feedback system with a single-input multi-output plant is better than open loop in the sense of Equation (31), then it must be optimal in the sense that

$$I = \int_{0}^{\infty} (\underline{x}^T \underline{\underline{W}} \underline{x} + u^2) dt \tag{56}$$

is minimized with respect to u.

Let us assume that a feedback system satisfies Equation (36) with Q = I. Then, for small parameter variations we have

$$L^T(-j\omega)\, L(j\omega) - [L(j\omega) + L^T(-j\omega)] > 0 \tag{57}$$

The above relation implies

$$B^T\, \Phi^T(-j\omega) \left\{ L^T(-j\omega) L(j\omega) - [L(j\omega) + L^T(-j\omega)] \right\} \Phi(j\omega) B \geq 0 \tag{58}$$

In general, Equation (58) will have a different dimension than Equation (57). Recalling that

$$L(j\omega) \triangleq \Phi(j\omega)\ BF \tag{59}$$

and defining

$$\Gamma(j\omega) \triangleq F\ \Phi(j\omega)\ B, \tag{60}$$

we have

$$L(j\omega)\quad \Phi(j\omega)\ B = \Phi(j\omega)B\Gamma(j\omega), \tag{61}$$

and Equation (58) can be written as

$$\begin{aligned}\Gamma^T(-j\omega)B^T\Phi^T(-j\omega)\Phi(j\omega)B\Gamma(j\omega) - B^T\Phi^T(-j\omega)\Phi(j\omega)B\Gamma(j\omega)\\ - \Gamma^T(-j\omega)B^T\Phi^T(-j\omega)\Phi(j\omega)B \geq 0.\end{aligned} \tag{62}$$

Now let

$$\Gamma \triangleq I - K. \tag{63}$$

Then Equation (62) becomes

$$K^T(-j\omega)B^T\Phi^T(-j\omega)\Phi(j\omega)B\ K(j\omega) - B^T\Phi^T(-j\omega)\Phi(j\omega)\ B \geq 0. \tag{64}$$

This is of the form

$$K^T(-j\omega)\ Q_1K(j\omega) - Q_1 \geq 0. \tag{65}$$

Note that

$$Q_1 \triangleq B^T\Phi^T(-j\omega)\ \Phi(j\omega)B \geq 0 \tag{66}$$

and that from Equations (60) and (63)

$$K = I - F\Phi(j\omega)B. \tag{67}$$

From Figure 3, we observe that $F\Phi(j\omega)B$ is a loop transmission matrix with the loop opened at the input to the plant. The matrix K has an interpretation as a return difference. If the perturbations in the plant matrices A and B are not differentially small, then $\Phi(j\omega)$ and B have to be replaced by the primed quantities. Equation (65) is analogous to Equation (36). However, Q_1 is a function of frequency whereas Q is a constant matrix. Furthermore, Equation (67) is a necessary condition for the satisfaction of Equation (36) with $Q \doteq I$.

For the single input case, K and Q_1 reduce to scalars and hence

$$|K(j\omega)|^2 - 1 \geq 0 \tag{68}$$

or

$$|1 - F\Phi(j\omega)B|^2 \geq 1 \tag{69}$$

From Theorem 6 of Reference 8, Equation (69) implies optimality in the sense of Equation (56). That is, a low sensitivity design in the sense of Equation (31). is <u>necessarily</u> an optimal design in the sense of Equation (56), if the plant has a single input only. The plant may have several outputs. This result connecting sensitivity and optimal control is similar to that obtained by Kalman in reference 8. However, the results differ because the interpretation of the notion of sensitivity in reference 8 is different from ours.

VI. CONCLUSION

In this paper it has been shown that the parameter variation problem for linear time-invariant multivariable systems described by state vector equations can be formulated using a direct comparison of open-loop control and state feedback control. This comparison leads to a new sensitivity criterion. Sufficient conditions for low sensitivity derived from this criterion guarantee superiority of the state feedback control design over the open loop design for a large class of input signals and parameter variations. These sensitivity conditions are obtained for both continuous and discrete systems. These results extend those recently obtained by the authors for systems described by square nonsingular transfer function matrices.

It has also been shown that a multivariable system having a single-input plant satisfying the frequency domain sensitivity conditions is necessarily optimal in the sense of minimizing the integral of the sum of some positive definite quadratic form in the states and the square of the control.

VII. ACKNOWLEDGMENT

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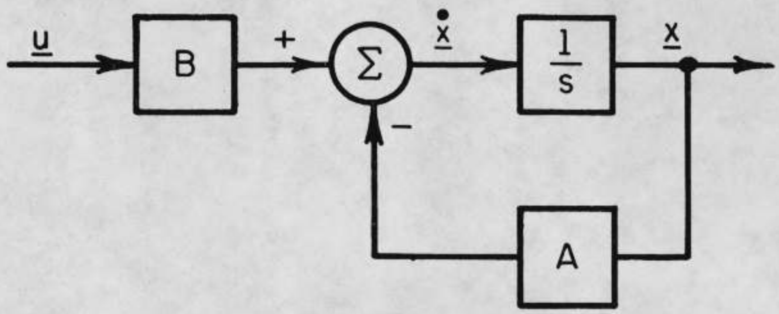

Figure 1. Matrix Block Diagram Representation of the Plant

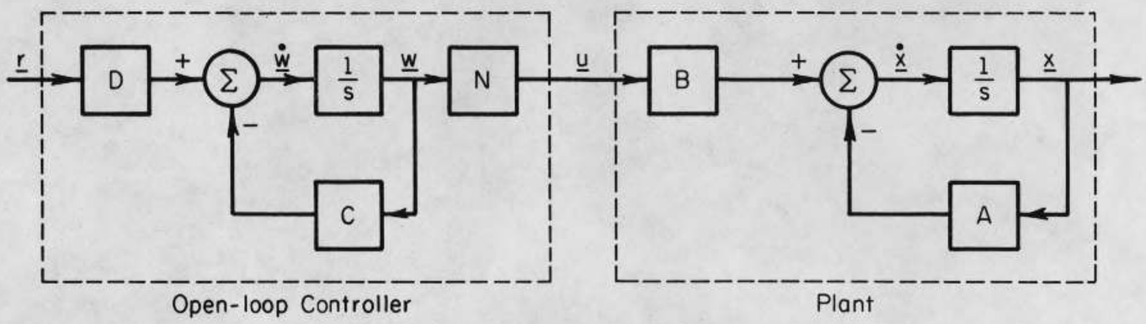

Figure 2. Basic Open-Loop Configuration

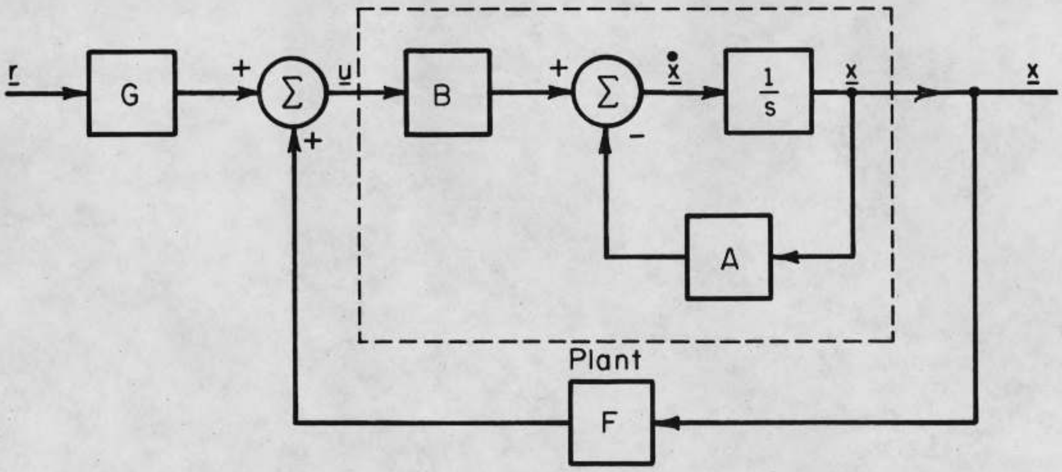

Figure 3. Basic Closed-Loop Configuration

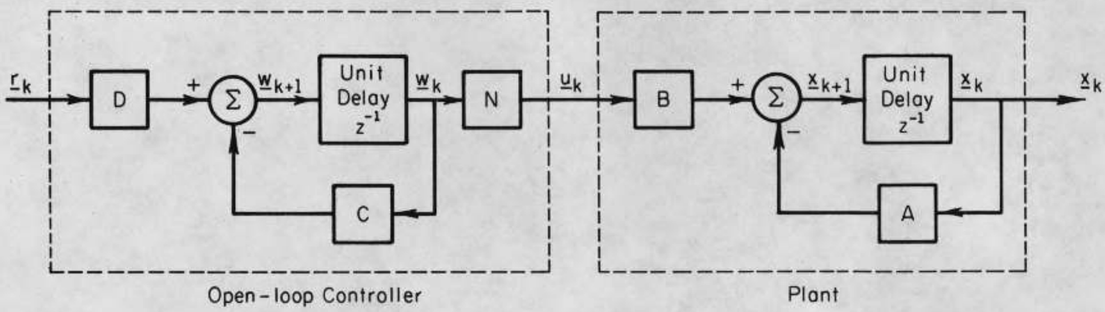

Figure 4. Discrete Open-Loop Configuration

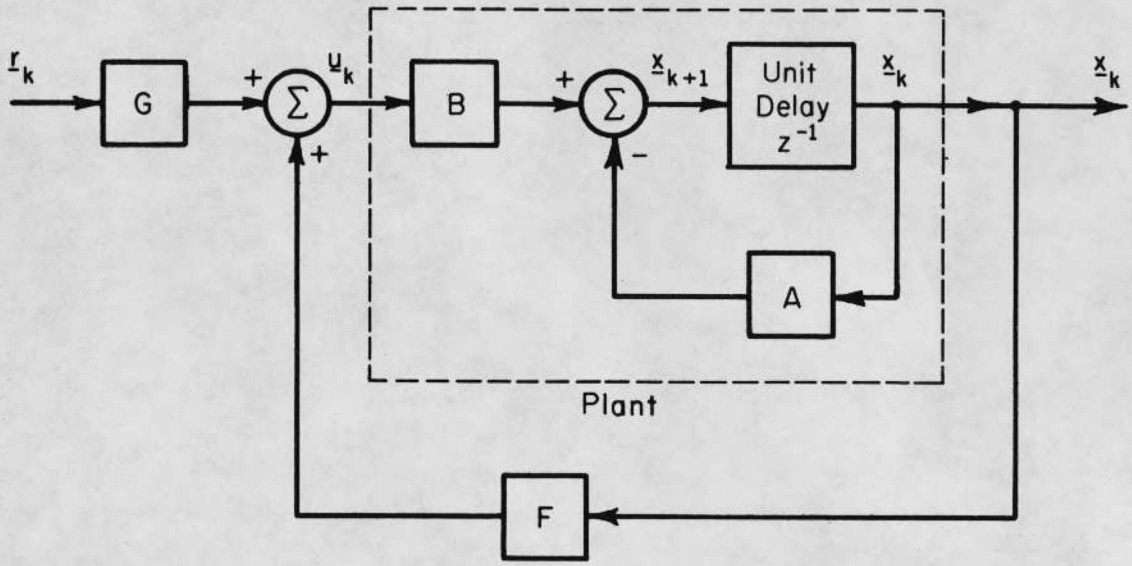

Figure 5. Discrete Closed-Loop Configuration

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