The Pass-Through of Sovereign Risk[*]

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Abstract

This paper examines the aggregate implications of sovereign credit risk in a business cycle model in which banks are exposed to risky government debt. An increase in the probability of a future sovereign default leads to a reduction in credit to firms because of two channels. First, it lowers the value of government debt on the balance sheet of banks, tightening their funding constraints and leaving them with fewer resources to lend to firms. Second, it raises the required premia demanded by banks for lending to firms because this activity has become riskier: if the sovereign default occurs, the economy falls in a major recession and claims to the productive sector pay out little. I estimate the nonlinear model with Italian data using Bayesian techniques. I find that sovereign credit risk led to a rise in the financing premia of firms that peaked 100 basis points, and cumulative output losses of 4.75% by the end of 2011. Both channels were quantitatively important drivers of the propagation of sovereign credit risk to the real economy. I then use the model to evaluate the effects of subsidized long term loans to banks, calibrated to the ECB's Longer Term Refinancing Operations. The presence of a significant risk channel at the policy enactment explains the limited stimulative effects of these interventions.

JEL: E32, E44, G01, G21

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1 Introduction

At the end of 2009, holdings of domestic government debt by banks in European peripheral countries - Greece, Italy, Portugal and Spain - were equivalent to 93% of banks' total equity. At the same time, these banks provided roughly three-quarters of external financing to domestic firms. Prior research has established that the sovereign debt crisis in these economies resulted in a substantial increase in the borrowing costs for domestic firms.[1] One proposed explanation of these findings is that the exposure to distressed government bonds hurts the ability of banks to raise funds in financial markets, leading to a pass-through of their increased financing costs into the lending rates payed by firms.[2] This view was at the core of policy discussions in Europe and was a motive for major interventions by the European Central Bank (ECB).

I argue, however, that this view is incomplete. A sovereign default triggers a severe macroeconomic downturn and adversely affects the performance of firms. Consequently, as an economy approaches a sovereign default, banks perceive firms to be more risky. Because banks require fair compensation for holding this additional risk, firms' borrowing costs rise. If this mechanism is quantitatively important, policies that address the heightened liquidity problems of banks but do not reduce the increased riskiness of firms may prove ineffective in encouraging bank lending.

I formalize this mechanism in a quantitative model with financial intermediation and sovereign default risk. In the model, an increase in the probability of a sovereign default both tightens the funding constraints of banks (*leverage-constraint channel*) and raises the risks associated with lending to the productive sector (*risk channel*). I structurally estimate the model on Italian data with Bayesian methods. I find that the risk channel is indeed quantitatively important: it explains up to 47% of the impact of the sovereign debt crisis on the borrowing costs of firms. I then use the estimated model to assess the consequences of credit market interventions adopted by the ECB and to propose and evaluate alternative policies that are more effective in mitigating the implications of increased sovereign default risk.

My framework builds on a business cycle model with financial intermediation, in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013). In the model, banks collect savings from households and use these funds, along with their own wealth (net worth), to buy long-term government bonds and to lend to firms. This intermediation is important because firms need external finance to buy capital goods. The model has three main ingredients. First, an agency problem between households and banks generates

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[1]See, for example, the evidence in Klein and Stellner (2013) and Bedendo and Colla (2013) using corporate bond data, the analysis of Bofondi et al. (2013) using Italian firm level data and Neri (2013) and Neri and Ropele (2013) for evidence using aggregate time series. See also ECB (2011).

[2]The report by CGFS (2011) discusses the transmission channels through which sovereign risk affected bank funding during the European debt crisis. For example, banks in the Euro area extensively use government bonds as collateral, and the decline in the value of these securities during the sovereign debt crisis reduced their ability to access wholesale liquidity. See also Zole (2013) and Albertazzi et al. (2012).

constraints in the borrowing ability of these latter. These constraints on bank leverage bind only occasionally, and typically when bank net worth is low. Second, financial intermediation is risky: bank net worth varies over time mainly because banks finance long-term risky assets with short-term risk-free debt. Third, the probability that the government defaults on its bonds and imposes losses on banks is time-varying and follows an exogenous stochastic process.

To understand the key mechanisms of the model, consider a scenario in which the probability of a future sovereign default rises. The anticipation of a "haircut" on government bonds depresses their market value and lowers the net worth of banks.[3] This tightens their leverage constraints and has adverse consequences for financial intermediation: banks' ability to collect funds from households decline, lending to the productive sector declines and so does aggregate investment. This is the conventional leverage-constraint channel in the literature.

However, even when the leverage constraints are *currently* not binding, a higher probability of a future sovereign default induces banks to demand higher compensation when lending to firms. This is the case because the sovereign default triggers a deep recession characterized by a severe decline in the payouts of firms. Thus, when the probability of a future sovereign default increases, banks have an incentive to deleverage in order to avoid these losses. More specifically, if the sovereign default happens in the future, bank leverage constraints tighten because of the government haircut. This forces banks to liquidate their holdings of firms assets. The associated decline in their market value leads to a further deterioration in bank net worth, feeding a vicious loop. *Ex-ante*, forward-looking banks demand a premium for holding these claims because they anticipate that they will pay out little precisely when banks are mostly in need of wealth. The resulting risk premium is increasing in the probability of a sovereign default. This constitutes the risk channel.

I measure the quantitative importance of the leverage constraint channel and the risk channel by estimating the structural parameters of the model with Italian data from 1999:Q1 to 2011:Q4 using Bayesian techniques. The major empirical challenge is to separate these two propagation mechanisms since they have qualitatively similar implications for indicators of financial stress commonly used in the literature (e.g., credit spreads). I demonstrate that the Lagrange multiplier on bank leverage constraints is a function of observable variables, specifically of the TED spread (spread between the prime interbank rate and the risk free rate) and of the leverage of banks. I construct a time series for this multiplier and use it in estimation, along with output growth, to measure the cyclical behavior of the leverage constraint. In addition, I use credit default swap (CDS) spreads on Italian government bonds and data on holdings of domestic government debt by Italian banks to measure the time-varying nature of sovereign risk and the exposure of banks to that risk.[4] The

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[3]In the model, a haircut is the fraction of the principal that is reneged by the government in the event of a default.

[4]A CDS is a derivative used to hedge the credit risk of an underlying reference asset. CDS spreads on government securities

structural estimation is complicated by the fact that the model features time-variation in risk premia and occasionally binding financial constraints. I develop an algorithm for its global solution based on projections and sparse collocation, and I combine it with the particle filter to evaluate the likelihood function.

Having established the good fit of the model using posterior predictive analysis, I use it to answer two applied questions. First, I quantify the importance of the leverage-constraint channel and the risk channel for the propagation of sovereign credit risk to the financing premia of firms and output. I estimate that the increase in the probability of a sovereign default in Italy during the 2010:Q1-2011:Q4 period raised substantially firms' financing premia, with a peak of 100 basis points in 2011:Q4. This increase reflects both tighter constraints on bank leverage and increased riskiness of firms, with the risk channel explaining up to 47% of the overall effects. Moreover, the rise in the probability of a sovereign default had severe adverse consequences for the Italian economy: cumulative output losses were 4.75% at the end of 2011.

In the second set of quantitative experiments, I evaluate the effectiveness of a major unconventional policy adopted by the ECB in the first quarter of 2012 to address the crisis, the Longer Term Refinancing Operations (LTROs). I model the policy as a subsidized long-term loan offered to banks. Because of the inherent nonlinearities of the model, initial conditions matter for policy evaluation. Thus, I implement this intervention conditioning on the state of the Italian economy in 2011:Q4. I find that the effects of LTROs on credit to firms and output vary over the 2012:Q1-2014:Q4 window, but they are small and not significantly different from zero when we average over this time period. This is due to the fact that risk premia were sizable when the policy was enacted. Banks, thus, have little incentives to increase their exposure to firms and they mainly use LTROs to cheaply refinance their liabilities.

The lesson from the policy evaluation is that the success of unconventional policies, such as LTROs, crucially depends on current economic conditions, in particular on the relative importance of binding leverage constraints versus risk premia. The former prevents banks from undertaking otherwise profitable investment. Policies that relax these constraints have sizable effects on bank lending and capital accumulation. The latter, instead, signal that firms are forecasted to be a "bad asset" in the future and bank lending is less responsive to refinancing operations. In these circumstances, policies that insure banks from the downside risk of a sovereign default (for example through a large injection of equity or a floor on the price of government bonds) can achieve stimulative effects. These interventions lower the risk associated with lending to the private sector because they limit the contagion effects that occur in the event of a sovereign default through banks' balance sheets. However, these stimulative effects should be weighed against the increased risk taking behavior that these policies are likely to bring and that I do not capture in my analysis.

Related Literature. This paper is related to several strands of the literature. Empirical studies docu-

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are typically used in the literature as a proxy of sovereign risk, see Pan and Singleton (2008).

ment a strong link between sovereign spreads and private sector interest rates, both in emerging economies and more recently in southern European countries.[5] Several authors recognize the importance of this relationship in different settings. For example, Neumeyer and Perri (2005) and Uribe and Yue (2006) suggest that sovereign spreads are a major driver of business cycles in emerging markets, while Corsetti et al. (2013) study the implications of the sovereign risk pass-through for fiscal policy. However, in these and related papers, the reasons underlying the connection between sovereign spreads and private sector interest rate are not modeled. Part of the contribution of this paper to the literature is to microfound this link in a fully specified dynamic equilibrium model.

In doing so, my paper also relates to the literature covering the output costs of sovereign debt defaults, more precisely to papers studying the effects of defaults on domestic bondholders. Motivated by robust empirical evidence, Gennaioli et al. (2013b) and Sosa Padilla (2013) study the effects of sovereign defaults on domestic banks, and the impact that the associated output losses have on the government's incentives to default.[6] My research is complementary to theirs: I take sovereign default risk as exogenous, but I explicitly model the behavior of private credit markets when sovereign risk increases. The novel insight of my paper is that the mere *anticipation* of a sovereign default can be recessionary because of its impact on the perceived riskiness of firms and on the funding constraints of exposed banks.[7] While this exogeneity of sovereign default risk rules out important feed-back effects between banks and sovereigns (Uhlig, 2013; Acharya et al., 2013), it does allow for a transparent analysis of these transmission channels.[8]

This paper contributes to a growing literature on the aggregate implications of shocks to the balance sheet of financial intermediaries. In particular, I build on the modeling framework developed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011, 2013), where the limited enforcement of debt contracts generates endogenous constraints on intermediaries' leverage. Differently from these papers, my analysis studies how changes in the expectation of these constraints being binding in the future influence the choices of financial intermediaries regarding their lending behavior today.[9] In my application, these phenomena arise because of shocks to the the default probability of government bonds, but the same logic could be applied

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[5]For emerging market economies, Durbin and Ng (2005) and Borensztein et al. (2006) provide an empirical analysis of the "sovereign ceiling", the practice of agencies to rate corporations below their sovereign. Cavallo and Valenzuela (2007) document the effects of sovereign spreads on corporate bonds spreads. See footnote 1 and 2 for evidence on southern European economies.

[6]Kumhof and Tanner (2005) and Gennaioli et al. (2013a) document that banks are highly exposed to domestic government debt in a large set of countries. Reinhart and Rogoff (2011) and Borensztein and Panizza (2009) show that sovereign defaults typically occur simultaneously, or in close proximity, to banking crises.

[7]In an empirical study, Yeyati and Panizza (2011) point out that anticipation effects are key to understand the unfolding of sovereign debt crises. See also Aguiar et al. (2009) and Dovis (2013) for models where anticipation effects arise because of debt overhang problems.

[8]Pancrazi et al. (2013) and Mallucci (2013) are two contemporaneous papers studying the effects of sovereign credit risk on the funding costs of firms. Even though these authors model explicitly the incentives of the government to default on its debt, their production sector is static. As such, their analysis abstract from the effects that a sovereign default has on the perceived riskiness of firms, the key novel mechanism of this paper.

[9]Technically, I capture these effects because I study the full nonlinear model rather than local approximation around a deterministic steady state.

to the study of other assets. My analysis uncovers two important phenomena. First, these changes in expectations can induce quantitatively sizable variation in risk premia. Brunnermeier and Sannikov (2013), He and Krishnamurthy (2012a,b) and Bianchi and Mendoza (2012) study related effects, but in the present context they emerge because of shocks to the volatility of an unproductive assets' payoff. Productive assets are affected because the balance sheet of banks generates contagion (e.g., produces correlation among the payoffs of different assets held by banks). Second, stabilization policies are state and size dependent in this environment.[10] As explained earlier, these nonlinearities depend on the relative importance of currently binding leverage constraints and risk premia.

The measurement of these two latter components is therefore a key aspect of this paper. The construction of a model consistent indicator for the Lagrange multiplier on bank leverage constraints is novel, and it is related to the measurement of financial shocks in Jermann and Quadrini (2012). Methodologically, I draw from the literature on the Bayesian estimation and validation of dynamic equilibrium economies (Del Negro and Schorfheide, 2011), more specifically of models where nonlinearities feature prominently (Fernández-Villaverde and Rubio-Ramírez, 2007). The decision rules of the model are derived numerically using a projection algorithm. I use a Smolyak sparse grid (Krueger and Kubler, 2003), which sensibly reduces the curse of dimensionality.[11] I evaluate the likelihood function tailoring the auxiliary particle filter of Pitt and Shephard (1999) to the present application. To my knowledge, this is the first paper to estimate a model with occasionally binding financial constraints using global methods and nonlinear filters. However, there are other papers using related techniques for different applications (see Gust et al., 2013; Bi and Traum, 2012, 2013).

Finally, the shock to sovereign default probabilities considered in this paper is a form of time-varying volatility. As such, my research is related to the literature that studies how different types of volatility shocks influence real economic activity (Bloom, 2009; Bloom et al., 2012; Fernández-Villaverde et al., 2011). In particular, Rietz (1988) and Barro (2006) emphasize the role of large macroeconomic disasters in accounting for asset prices and Gourio (2012) studies how changes in the probability of these events affect risk premia and capital accumulation. The sovereign default studied in this paper can be seen as a potential source of macroeconomic disasters.[12]

Layout. The paper is organized as follows. Section 2 presents the model, while Section 3 discusses its main mechanisms using two simplified examples. Section 4 presents the estimation and an analysis of the

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[10]There are a number of papers that study unconventional monetary policy in related environments. See, for example, Curdia and Woodford (2010), Curdia and Woodford (2011), Del Negro et al. (2012) and Bianchi and Bigio (2013).

[11]Christiano and Fisher (2000) is an early paper documenting the performance of projections in models with occasionally binding constraints. See also Fernández-Villaverde et al. (2012) for an application of the Smolyak sparse grid in a model where the zero lower bound constraint on nominal interest rate bind occasionally.

[12]Arellano et al. (2012), Gilchrist et al. (2013) and Christiano et al. (2013) study the real effects of a different form of time-varying volatility in models with financial frictions.

model's fit. Section 5 presents key properties of the estimated model that are useful to interpret the two quantitative experiments, which are reported in Section 6. Section 7 concludes.

2 Model

I consider a neoclassical growth model enriched with a financial sector as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In this setting, I introduce long term government bonds to which financial intermediaries are exposed. These securities pay in every state of nature unless the economy is in a sovereign default- an event that can occur every period according to an exogenous and time-varying probability.

The model economy is populated by households, final good producers, capital good producers and a government. Each household is composed of two types of members: *workers* and *bankers*. Workers supply labor to final good firms in a competitive factor market, and their wages are made available to the household. Bankers manage the savings of other households and use these funds to buy government bonds and claims on firms. Bankers offer a risk free rate on households' savings. The perfectly competitive non-financial corporate sector produces a final good using a constant return to scale technology that aggregates capital and labor. Firms rent labor from households and buy capital from perfectly competitive capital good producers. Their capital expenses are financed by bankers. Finally, the government issues bonds and taxes households in order to finance government spending. In every period the government can default on its debt. This event is modeled as an exogenous stochastic process.

The key friction of the model is the limited enforcement of debt contracts between households and bankers: bankers can walk away with the assets of their franchise, and households can recover only a fraction of their savings when this event occurs. This friction gives rise to constraints on the leverage of banks, with bank net worth being the key determinant of their borrowing capacity. When these incentive constraints bind, or are expected to bind in the future, credit to the productive sector declines. This has adverse consequences for capital accumulation. An increase in the probability of a future government default is recessionary because it adversely impacts the *current* and *expected* level of bank net worth, thereby influencing their lending behavior.

In the remainder of this section I describe the agents' decision problems, derive the conditions characterizing a competitive equilibrium, and sketch the algorithm used for the numerical solution of the model. In Section 3, I discuss the key mechanisms of interest. I denote by $\mathbf{S}$ the vector collecting the current value for the state variables and by $\mathbf{S}'$ the future state of the economy.

2.1 Agents and their Decision Problems

2.1.1 Households

A household is composed of a fraction f of workers and a fraction $1 - f$ of bankers. There is perfect consumption insurance between its members. Let $\Pi(\mathbf{S})$ be the net payments that bankers make to their own household, and let $W(\mathbf{S})$ be the wage that workers receive from supplying labor to final good firms. Households value consumption c and dislike labor l according to the flow utility $u(c, l)$, and they discount the future at the rate β. The problem for the household is that of making contingent plans for consumption, labor supply and savings b' so as to maximize lifetime utility. Savings are deposited into financial intermediaries that are managed by bankers belonging to other households, and they earn the risk free return $R(\mathbf{S})$. Taking prices as given, a household solves

$$v_h(b; \mathbf{S}) = \max_{b' \geq 0, c \geq 0, l \in [0,1]} \left\{ u(c, l) + \beta \mathbb{E}_{\mathbf{S}}[v_h(b'; \mathbf{S}')] \right\},$$

$$c + \frac{1}{R(\mathbf{S})} b' \leq W(\mathbf{S})l + \Pi(\mathbf{S}) + b - \tau(\mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

$\tau(\mathbf{S})$ denotes the level of lump sum taxes while $\mathbf{\Gamma}(.)$ describes the law of motion for the aggregate state variables. Optimality is governed, at an interior solution, by the intra-temporal and inter-temporal Euler equations

$$u_l(c, l) = u_c(c, l)W(\mathbf{S}), \tag{1}$$

$$\mathbb{E}_{\mathbf{S}}[\Lambda(\mathbf{S}', \mathbf{S})R(\mathbf{S})] = 1, \tag{2}$$

where $\Lambda(\mathbf{S}', \mathbf{S}) = \beta \frac{u_c(c', l')}{u_c(c, l)}$. For the empirical analysis I will use preferences that are consistent with balanced growth, $u(c, l) = \log(c) - \chi \frac{l^{1+\nu^{-1}}}{1+\nu^{-1}}$, where ν parameterizes the Frisch elasticity of labor supply.

2.1.2 Bankers

A banker uses his accumulated net worth n and deposits b to buy government bonds and claims on firms.[13] Let a_j be asset j held by a banker and let $Q_j(\mathbf{S})$ and $R_j(\mathbf{S}', \mathbf{S})$ be, respectively, the price of asset j and its realized returns next period on a unit of numeraire good invested in asset j. The banker's balance sheet

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[13]A worker who becomes a banker this period obtains start-up funds from his households. These transfers will be specified at the end of the section.

equates total assets to total liabilities:

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \leq n + \frac{b'}{R(\mathbf{S})},\tag{3}$$

where subscript B refers to government bonds and K to firms' claims. A banker makes optimal portfolio choices in order to maximize the present discounted value of dividends payed to his own household. At any point in time there is a probability $1 - \psi$ that a banker becomes a worker in the next period. When this happens, the banker pays back a dividend to his own household.[14] Bankers who continue running the business do not pay dividends, and they accumulate net worth. The objective of a banker is that of maximizing the expected discounted value of his terminal wealth. Net worth next period equals the difference between realized returns on assets and the payments promised to households.

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S}',\mathbf{S})Q_j(\mathbf{S})a_j - b'.\tag{4}$$

Note that bad realizations of $R_j(\mathbf{S}',\mathbf{S})$ lead to reductions in bankers' net worth n'. This variation in net worth affects the ability of bankers to obtain funds from the household sector and, ultimately, their supply of credit to the firm. This occurs due to the limited enforcement of contracts between households and banks. At any point in time, a banker can walk away with a fraction λ of the project and transfer it to his own household. If he does, the depositors can force him into bankruptcy and recover a fraction $(1 - \lambda)$ of banks' assets. This friction defines an incentive constraint for the banker: the value of running his franchise must be higher than its outside option, $\lambda[\sum_j Q_j(\mathbf{S})a_j]$.

Taking prices as given, a banker solves the decision problem

$$v_b(n;\mathbf{S}) = \max_{a_B,a_K,b} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}',\mathbf{S}) \left[(1-\psi)n' + \psi v_b(n';\mathbf{S}') \right] \right\},$$

$$n' = \sum_{j=\{B,K\}} R_j(\mathbf{S}',\mathbf{S})Q_j(\mathbf{S})a_j - b',$$

$$\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \leq n + \frac{b'}{R(\mathbf{S})},$$

$$\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \right] \leq v_b(n;\mathbf{S}),$$

$$\mathbf{S}' = \mathbf{\Gamma}(\mathbf{S}).$$

The following result further characterizes this decision problem.[15]

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[14]When a banker exits, a worker replaces him so that their relative proportion does not change over time.

[15]The problem is not well defined for negative values of net worth. When this happens, the government steps in and refinance

Result 1. *A solution to the banker's dynamic program is*

$$v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n,$$

where $\alpha(\mathbf{S})$ solves

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\}}{1 - \mu(\mathbf{S})}, \tag{5}$$

and the multiplier on incentive constraints satisfies

$$\mu(\mathbf{S}) = max\left\{1 - \left[\frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\}N}{\lambda[Q_K(\mathbf{S})A_K + Q_B(\mathbf{S})A_B]}\right], 0\right\}, \tag{6}$$

where N, A_B and A_K are, respectively, aggregate bankers' net worth and aggregate bankers' holdings of government bonds and firms assets.

Proof. See Appendix A. □

This result clarifies that limited enforcement of contracts places an endogenous constraint on the leverage of the banker. Indeed, because of the linearity of the value function, the incentive constraint becomes

$$\frac{\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda}, \tag{7}$$

implying that bank leverage cannot exceed the time-varying threshold $\frac{\alpha(\mathbf{S})}{\lambda}$.[16] Bank net worth is thus a key variable regulating financial intermediation in the model: when net worth is low, the leverage constraint is more likely to bind and this limits the amount of assets that a banker can intermediate.

The implications of this constraint for assets' accumulation can be better understood by looking at the Euler equation for risky asset j

$$\mathbb{E}_{\mathbf{S}}\left[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R_j(\mathbf{S}', \mathbf{S})\right] = \mathbb{E}_{\mathbf{S}}\left[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R(\mathbf{S})\right] + \lambda\mu(\mathbf{S}), \tag{8}$$

where $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ is the economy's pricing kernel, defined as

$$\hat{\Lambda}(\mathbf{S}', \mathbf{S}) = \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]. \tag{9}$$

the bank via lump sum taxation. At the same time, it issues a non-pecuniary punishment to the banker that is equivalent to the net worth losses.

[16]Alternatively, we can interpret equation (7) as a collateral constraint. Indeed, using the balance sheet identity we write the leverage constraint as $b \leq \left[\frac{\alpha(\mathbf{S}) - \lambda}{\alpha(\mathbf{S})}\right] \sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j$. That is, bankers' debt cannot exceed a time varying fraction of the market value of their total assets.

There are two main distinctions between this Euler equation and the one that would arise in a purely neoclassical setting. First, the presence of leverage constraints limits the ability of banks to arbitrage away differences between expected discounted returns on asset j and the risk free rate: this can be seen from equation (8), as the multiplier generates a wedge between these two returns. Second, the pricing kernel in equation (9) is not only a function of consumption growth as in canonical neoclassical models, but also of bank leverage. Indeed, as stated in equation (7), financial leverage is proportional to $\alpha(\mathbf{S})$ when $\mu(\mathbf{S}) > 0$. Adrian et al. (2013) provides empirical evidence in support of leverage-based pricing kernels for the U.S. economy and He and Krishnamurthy (2012b) discuss their asset pricing implications in endowment economies. If the leverage constraint never binds, i.e. $\mu(\mathbf{S}) = 0 \, \forall \, \mathbf{S}$, equation (8) collapses to the neoclassical benchmark.[17]

Result 1 also implies that banks heterogeneity in their net worth and asset holdings does not affect aggregate dynamics. Indeed, equation (8) suggests that assets returns depend on the dynamics of the multiplier $\mu(\mathbf{S})$ which, in turn, is a function of financial leverage (see equation (6)). Since this latter is identical across bankers when the constraint binds, $\mu(\mathbf{S})$ is independent on the cross-sectional distribution of bank net-worth: agents in the economy do not need to know this distribution when forecasting future prices, this making the numerical analysis of the model tractable. For future reference, it is convenient to derive an expression for the law of motion of aggregate net worth

$$N'(\mathbf{S}', \mathbf{S}) = \psi \left\{ \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})]Q_j(\mathbf{S})A_j + R(\mathbf{S})N \right\} + \omega \sum_{j=\{B,K\}} Q_j(\mathbf{S}')A_j. \tag{10}$$

Aggregate net worth equals the sum of the net worth accumulated by bankers who did not switch occupations today and the transfers that households make to newly born bankers. These transfers are assumed to be a fraction ω of the assets intermediated in the previous period, evaluated at current prices. In the empirical analysis, ω has the purpose of pinning down the level of financial leverage in a deterministic balanced growth path of the economy, and it will be a small number.

2.1.3 Capital Good Producers

The capital good producers build new capital goods using the technology $\Phi \left(\frac{i}{K} \right) K$, where K is the aggregate capital stock in the economy and i the inputs used in production. They buy inputs in the final good market, and sell capital goods to final good firms at competitive prices. Taking the price of new capital $Q_i(\mathbf{S})$ as

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[17]Using equation (2), we can see that a solution to equation (5) is $\alpha(\mathbf{S}) = 1 \, \forall \, \mathbf{S}$ whenever $\mu(\mathbf{S}) = 0 \, \forall \, \mathbf{S}$.

given, the decision problem of a capital good producer is

$$\max_{i \geq 0} \left[Q_i(\mathbf{S}) \Phi \left(\frac{i}{K} \right) K - i \right].$$

Anticipating the capital goods market clearing condition, the price for new capital goods is

$$Q_i(\mathbf{S}) = \frac{1}{\Phi' \left(\frac{I(\mathbf{S})}{K} \right)}, \tag{11}$$

where $I(\mathbf{S})$ is equilibrium aggregate investment.

For the empirical analysis, I specify the production function for capital goods as $\Phi(x) = a_1 x^{1-\xi} + a_2$, where ξ parametrizes the elasticity of Tobin's q with respect to the investment-capital ratio.

2.1.4 Final Good Producers

Final output y is produced by perfectly competitive firms that operate a constant returns to scale technology

$$y = k^\alpha (e^z l)^{1-\alpha}, \tag{12}$$

where k is the stock of capital goods, l stands for labor services, and z is a neutral technology shock that follows an AR(1) process in growth

$$\Delta z' = \gamma(1 - \rho_z) + \rho_z \Delta z + \sigma_z \varepsilon'_z, \qquad \varepsilon'_z \sim \mathcal{N}(0,1). \tag{13}$$

Labor is rented in competitive factor markets at the rate $W(\mathbf{S})$. Capital goods depreciate every period at the rate δ. Anticipating the labor market clearing condition, profit maximization implies that equilibrium wages and profits per unit of capital are

$$W(\mathbf{S}) = (1 - \alpha) \frac{Y(\mathbf{S})}{L(\mathbf{S})}, \qquad Z(\mathbf{S}) = \alpha \frac{Y(\mathbf{S})}{K}, \tag{14}$$

where $Y(\mathbf{S})$ and $L(\mathbf{S})$ are equilibrium aggregate output and labor.

To purchase new capital goods, firms need external financing. At the beginning of the period, firms issue claims to bankers in exchange for funds. While these claims are perfectly state contingent and therefore correspond to equity holdings, I interpret them more broadly as privately issued paper such as bank loans. For each claim a_K bankers pay $Q_K(\mathbf{S})$ to firms.[18] In exchange, they receive the realized return on a unit of

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[18]No arbitrage implies that the price of a unit of new capital equals in equilibrium the price of an IOU issued by firms,

the capital stock in the next period:

$$R_K(\mathbf{S}', \mathbf{S}) = \frac{(1 - \delta)Q_K(\mathbf{S}') + Z(\mathbf{S}')}{Q_K(\mathbf{S})}. \tag{15}$$

Realized returns to capital move over time because of two factors: variation in firms' profits and variation in the market value of corporate securities. These movements in $R_K(\mathbf{S}', \mathbf{S})$ induce variation in aggregate net worth, as equation (10) suggests.

2.1.5 The Government

In every period, the government engages in public spending. Public spending as a fraction of GDP evolves as follows

$$\log(g)' = (1 - \rho_g)\log(g^*) + \rho_g \log(g) + \sigma_g \varepsilon'_g, \qquad \varepsilon'_g \sim \mathcal{N}(0, 1). \tag{16}$$

The government finances public spending by levying lump sum taxes on households and by issuing long-term government bonds to financial intermediaries. Long term debt is introduced as in Chatterjee and Eyigungor (2013). In every period a fraction π of bonds matures. When this event happens, the government pays back the principal to investors. The remaining fraction $(1 - \pi)$ does not mature: the government pays the coupon ι, and investors retain the right to obtain the principal in the future. The average duration of bonds is therefore $\frac{1}{\pi}$ periods. I introduce risk of sovereign default by assuming that the government can default in every period and write off a fraction $D \in [0, 1]$ of its outstanding debt. The parameter D can be seen as the "haircut" that the government imposes on bondholders in a default. Denoting by $Q_B(\mathbf{S})$ the pricing function for government securities, tomorrow's realized returns on a dollar invested in government bonds are

$$R_B(\mathbf{S}', \mathbf{S}) = \left[1 - d'D\right] \left[\frac{\pi + (1 - \pi)\left[\iota + Q_B(\mathbf{S}')\right]}{Q_B(\mathbf{S})}\right], \tag{17}$$

where d' is an indicator variable equal to 1 if the government defaults next period. Realized returns on government bonds vary over time and they affect the balance sheet of financial intermediaries. First, when the government defaults, it imposes a haircut on bondholders which has a direct negative effect on the net worth of bankers. Second, and to the extent that $\pi < 1$, $R_B(\mathbf{S}', \mathbf{S})$ is sensitive to variation in the price of government securities: a decline in $Q_B(\mathbf{S}')$, for example, lowers the reselling value of government bonds and reduces the returns on holding government debt.

$Q_i(\mathbf{S}) = Q_K(\mathbf{S})$.

Denoting by B' the stock of public debt, the budget constraint of the government is given by

$$Q_B(\mathbf{S}) \left[\underbrace{B' - (1-\pi)B[1-dD]}_{\text{Newly issued bonds}} \right] = \underbrace{[\pi + (1-\pi)\iota]\,B[1-dD]}_{\text{Payments of principals and coupons}} + \underbrace{gY(\mathbf{S}) - \tau(\mathbf{S})}_{\text{Primary Deficit}}. \tag{18}$$

Taxes respond to past debt according to the law of motion

$$\frac{\tau(\mathbf{S})}{Y(\mathbf{S})} = t^* + \gamma_\tau \frac{B}{Y(\mathbf{S})},$$

where $\gamma_\tau > 0$.[19] Finally, I assume that sovereign risk evolves exogenously. In every period the government is hit by a shock ε_d with a standard logistic distribution. The government defaults on its outstanding debt if ε_d is sufficiently large. In particular, d' follows

$$d' = \begin{cases} 1 & \text{if} \quad \varepsilon_d' - s \geq 0 \\ 0 & \text{otherwise,} \end{cases} \tag{19}$$

with s being a Gaussian AR(1) process

$$s' = (1-\rho_s)\log(s^*) + \rho_s s + \sigma_s \varepsilon_s. \tag{20}$$

This formulation allows us to study how the endogenous variables respond to variation in sovereign risk. In fact, the conditional probability of a sovereign default is $p^d(\mathbf{S}) = \frac{e^s}{1+e^s}$: an increase in s is equivalent to an increase in the conditional probability that the government defaults tomorrow.

2.2 Market Clearing

Letting $f(.)$ be the density of net worth across bankers, we can express the market clearing conditions as follows[20]

1. Credit market: $\int a_K(n;\mathbf{S})f(n)dn = K'(\mathbf{S})$.

2. Government bonds market: $\int a_B(n;\mathbf{S})f(n)dn = B'(\mathbf{S})$.

3. Market for households' savings: $\int b'(n;\mathbf{S})f(n)dn = b'(\mathbf{S})$.

4. Market for final goods: $Y(\mathbf{S})(1-g) = C(\mathbf{S}) + I(\mathbf{S})$.

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[19]This formulation guarantees that the government does not run a Ponzi scheme and that its intertemporal budget constraint is satisfied in every state of nature. See Bohn (1995) and Canzoneri et al. (2001).

[20]Note that we have anticipated earlier the market clearing condition for the labor market and for the capital good market.

2.3 Equilibrium Conditions and Numerical Solution

Since the non-stationary technology process induces a stochastic trend in several endogenous variables, it is convenient to express the model in terms of detrended variables. For a given variable x, I define its detrended version as $\tilde{x} = \frac{x}{z}$.[21] The state variables of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. As I detail below, the variable $\tilde{P}$ keeps track of aggregate bank net worth. The control variables $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations (2), (5) and (8) (the last one for both assets).

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows

$$\tilde{K}'(\mathbf{S}) = \left\{ (1-\delta)\tilde{K} + \Phi\left[e^{\Delta z}\left(\frac{\tilde{Y}(\mathbf{S})(1-e^g) - \tilde{C}(\mathbf{S})}{\tilde{K}} \right) \right]\tilde{K} \right\} e^{-\Delta z}, \tag{21}$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1-\pi)[\iota + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S})\left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})} \right) \right]}{Q_B(\mathbf{S})}, \tag{22}$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_K(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \tag{23}$$

The state variable $\tilde{P}$ measures the detrended *cum* interest promised payements of bankers to households at the beginning of the period, and it is necessary to keep track of the evolution of aggregate bankers' net worth. Finally, the exogenous state variables $[\Delta z, \log(g), s]$ follow, respectively, (13), (16) and (20), while d follows

$$d' = \begin{cases} 1 & \text{with probability} \quad \frac{e^s}{1+e^s} \\ 0 & \text{with probability} \quad 1 - \frac{e^s}{1+e^s}. \end{cases} \tag{24}$$

I use numerical methods to solve for the model decision rules. The algorithm for the global numerical solution of the model relies on projection methods (Judd, 1992; Heer and Maussner, 2009). In particular, let $x(\mathbf{S})$ be the function describing the behavior of control variable x. I approximate $x(\mathbf{S})$ using two sets of coefficients, $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}$. The law of motion for x is then described by

$$x(d, \tilde{\mathbf{S}}) = (1-d)\gamma_0^{x\prime}\mathbf{T}(\tilde{\mathbf{S}}) + d\gamma_1^{x\prime}\mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s]$ is the vector of state variables that excludes d, and $\mathbf{T}(.)$ is a vector collecting Chebyshev's polynomials. The coefficients $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}_x$ are such that the residual equations are satisfied for a set of collocation points $(d^i, \tilde{\mathbf{S}}^i) \in \{0,1\} \times \tilde{\mathcal{S}}$. I choose $\tilde{\mathcal{S}}$ and the set of polynomial $\mathbf{T}(.)$ using the

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[21]The endogenous state variables of the model are detrended using the level of technology last period.

Smolyak collocation approach. Krueger and Kubler (2003) and Krueger et al. (2010) provides a detailed description of the methodology. When evaluating the residual equations at the collocation points, I evaluate expectations by "precomputing integrals" as in Judd et al. (2011). Finally, I adopt Newton's method to find the coefficients $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}_x$ satisfying the residual equations. Appendix B provides a detailed description of the algorithm and discusses the accuracy of the numerical solution.

3 Two Simple Examples Illustrating the Mechanisms

Before moving on to the empirical analysis it is useful to describe the mechanisms that ties sovereign risk to the funding costs of firms and real economic activity. An increase in the probability of a future sovereign default lowers capital accumulation via two distinct channels. i) it tightens the leverage constraints of bankers, and ii) it increases the required premia for holding firms' claims.

I illustrate these propagation mechanisms using two stylized versions of the model. We will see that a decline in current net worth tightens bankers' leverage constraints (Section 3.1) and that bad *news* about future net worth leads to an increase in risk premia over firms' assets (Section 3.2). I then discuss how sovereign risk interacts with these two mechanisms in the model described in the previous section. Finally, Section 3.3 explains why disentangling these two mechainisms provides important information for evaluating the effects of credit policies in the model.

3.1 A Decline in Current Net Worth

I consider a deterministic economy with full depreciation ($\delta = 0$), no capital adjustment costs ($\xi = 0$) and no government. Moreover, I assume that the transfers to newly born bankers equal a fraction ω of current output, $N = \omega Y$ and that bankers live only one period ($\psi = 0$).

As in the neoclassical model with full depreciation and log utility, the saving rate is constant in this economy. Specializing equation (6) to this particular parametrization, we obtain an expression for the multiplier on incentives constraints

$$\mu = \frac{\lambda\sigma - \omega}{\lambda\sigma},$$

where σ is the saving rate. Using equation (8), we can solve for σ

$$\sigma = \min\left\{\frac{\alpha\beta + \omega}{1 + \lambda}, \alpha\beta\right\}.$$

I assume that the leverage constraints are currently binding ($\lambda\beta\alpha > \omega$), and I analyze the implications of

an unexpected transitory decline in the transfers to bankers. More specifically, I assume that at time $t = 1$ the transfer to bankers ω declines, then goes back to its previous level at $t = 2$, and no further changes occur at future dates. Agents do not expect such a change, but are perfectly informed about the path of the transfer from period $t = 1$ onward and they make rational choices based on this path. While analytical solutions for this example can be easily derived, I illustrate the transition to steady state using a numerical example.[22]

Figure 1: **A Decline in Current Net Worth**
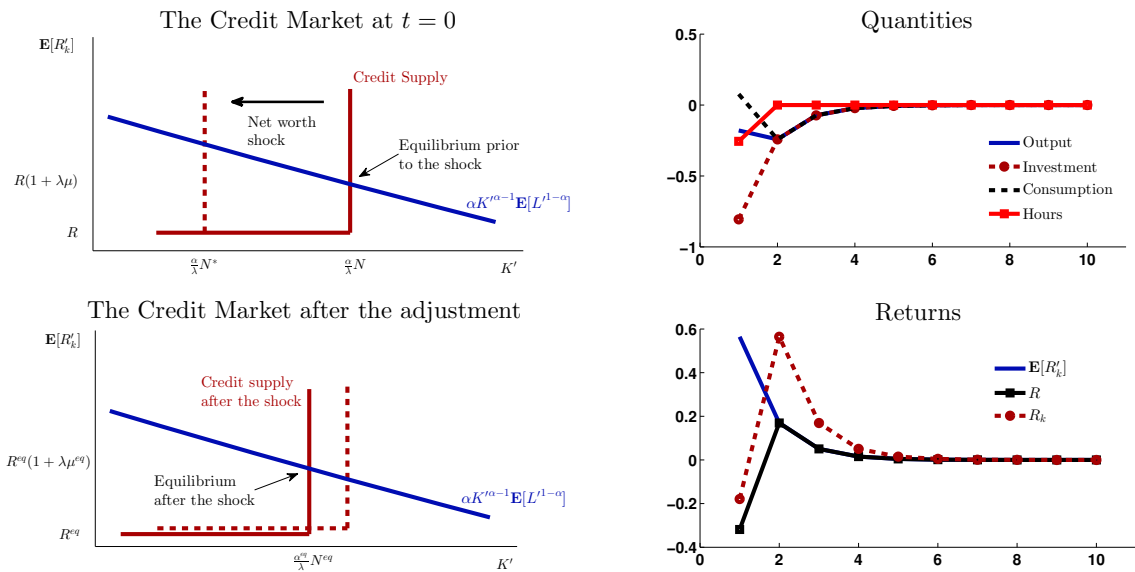
Notes: The figure reports the transitional dynamics induced by a transitory and unexpected 5% decline in net worth. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10]$. The right panels report variables expressed as percentage deviations from their steady state.

The top left panel of Figure 1 plots the equilibrium in the credit market prior to the decline in ω. The supply of funds is derived from the bankers' optimization problem: if the leverage constraints were not binding, bankers would be willing to lend at the risk free rate R since this economy is non-stochastic. The supply of funds to firms is inelastic at $K' = \frac{\alpha}{\lambda} N$, the point at which the leverage constraint binds. The demand for credit is downward sloping and equal to the expected marginal product of capital, $\alpha K'^{\alpha-1} \mathbb{E}[L'^{\alpha}]$. Since the leverage constraint binds, expected returns to capital equal $R(1 + \lambda\mu)$.

The unanticipated decline in ω tightens the leverage constraint ($\frac{\alpha}{\lambda} N^* < \frac{\alpha}{\lambda} N$), and the inelastic part of the supply schedule shifts leftward. The right panels of the figure describe adjustments for quantities and prices. The tightening of credit has adverse effects on capital accumulation. Consumption increases because agency costs makes savings in banks less attractive. The risk free rate declines in order to accommodate this rise in consumption. Aggregate hours falls because the low returns to savings make working unattractive. The decline in hours leads to a drop in output.

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[22]The qualitative behavior of this transition is robust across a wide range of parameter values.

There are three important things to note about this example. First, consumption and output move in opposite directions conditional on a tightening of the leverage constraint of banks. This "comovement problem" arises frequently in neoclassical settings, see Barro and King (1984) for a general formulation and Hall (2011) and Bigio (2012) for specific analysis in models with financial frictions.[23] One way to restore comovement would be to allow the demand for labor to be directly affected by the tightenss of bank leverage constraint. This could be done, for example, by introducing working capital constraint as in Mendoza (2010) and using preferences that mute the wealth effect on labor. While this extention is straightforward to pursue in the current set up, I focus on a benchmark real model for comparability with previous research. Second, variation in bank net worth is amplified in the full model because of endogenous response in Tobin's q. This occurs if there are frictions in the production of capital goods, $\xi > 0$. Brunnermeier et al. (2013) provide a detailed discussion of these amplification effects in models with financial frictions. Third, the tightening of the leverage constraint induces negative comovement between bankers' marginal value of wealth $\alpha = \frac{1}{1-\mu}$, and realized return on holding capital. When the constraint tightens, the former increases while the latter declines. This is intuitive: an additional unit of wealth for bankers is more valuable when the constraints are tight because it allows them to arbitrage away part of the difference between $\mathbb{E}[R'_K]$ and R. Moreover, the decline in credit to firms leads to a reduction in output per unit of capital, which translates into lower firms' profits. As we will see in the subsequent analysis, this negative comovement between R_K and α is the key mechanism that generates endogenous risk in the model.

An increase in the probability of a future sovereign default in the model of Section 2 triggers a decline in bank net worth and this may induce their leverage constraints to bind. An increase in s, in fact, leads to a decline in the market value of government bonds because investors anticipate a future haircut. Thus, current realized returns on government bond holdings decline. From equation (17) we can see that this effect is stronger the longer the maturity of bonds.[24] Low realized returns on bonds have a negative impact on bank net worth as we can see from equation (10). The parameters governing the exposure of banks to government bonds determine the quantitative importance of the elasticity of net worth to R_B. Thus, an increase in s can activate the process of Figure 1 through its adverse effects on bank net worth. I will refer to this mechanism as the *leverage-constraint* channel.

3.2 Bad *News* about Future Net Worth

Besides affecting the current net worth of financial intermediaries, sovereign credit risk acts as a bad news regarding their *future* wealth. As I will show in this section, this carries important consequences on the

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[23]See also Jaimovich and Rebelo (2009), **?** and **?** for a discussion of related comovement problems in different environments.

[24]The parameter π also has an indirect effect on the elasticity of R_B to the s-shock: when the maturity is longer, bond prices are more elastic to the sovereign risk shock.

way banks discount risky assets. It is helpful at this stage to derive an equilibrium relation describing the pricing of assets in the economy of Section 2. From equation (8) and (5) we find that expected returns to asset j equal

$$\mathbb{E}_{\mathbf{S}}[R_j(\mathbf{S}', \mathbf{S})] = R(\mathbf{S}) \left[1 + \frac{\lambda \mu(\mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})]} \right] - \frac{R(\mathbf{S}) \text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}), R_j(\mathbf{S}', \mathbf{S})]}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})]}. \tag{25}$$

Equation (25) defines the cross-section of assets' returns. Expected returns to capital typically carry a risk premium represented by $\text{cov}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}), R_K(\mathbf{S}', \mathbf{S})]$. In the model, this component is sensitive to news about how tight bank leverage constraints will be in the future.

This can be illustrated with a simple modification of the previous set-up. I now allow ψ to be greater than 0.[25] Moreover, I assume that there are two regimes in the economy.

1. "Normal times": transfers are fixed at their steady state, and bank leverage constraints are not binding.

2. "Financial crises": bankers are hit by the transitory decline in transfers described in the previous section.

I assume that the economy is currently in the normal time regime, and I denote by p the probability that in the next period it switches to a financial crisis regime. Once in a financial crisis, the economy experiences the temporary decline in ω described in the previous section. I assume that $p = 0$ at $t = 0$. In period $t = 1$, the economy experiences an unexpected increase in p to 0.1. In period $t = 2$, p returns to 0 and no further changes are anticipated. The agents are surprised by the initial increase in p, but they are aware of its future path from $t = 1$ onward, and they make rational choices based on this path. Figure 2 describes how the credit market and equilibrium quantities are affected by this increase in p.

The increase in p shifts the elastic component of the credit supply schedule upward because of a decline in $\text{cov}(\hat{\Lambda}', R_k')$. The top right panels of the figure explain where this change in the covariance originates. The first panel plots the joint distribution for $(\hat{\Lambda}', R_k')$ conditional on being in normal times when $p = 0$. This is a point distribution: the pricing kernel equals β while realized returns to capital are equal to β^{-1}. When p increases to 0.1, banks assign a higher probability of switching to the financial crises regime. As shown in the previous section, realized returns to capital are low in this state while bankers' marginal valuation of wealth is high. Capital is therefore a "bad" asset to hold during a financial crisis because it pays little precisely when bankers are most in need of wealth. For this reason, it commands a risk premium in normal times, and these premia are typically increasing in p.[26]

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[25]The decision problem of bankers is static when $\psi = 0$.

[26]In this example this is true only when $p < 0.5$.

Figure 2: **Bad *News* about Future Net Worth**
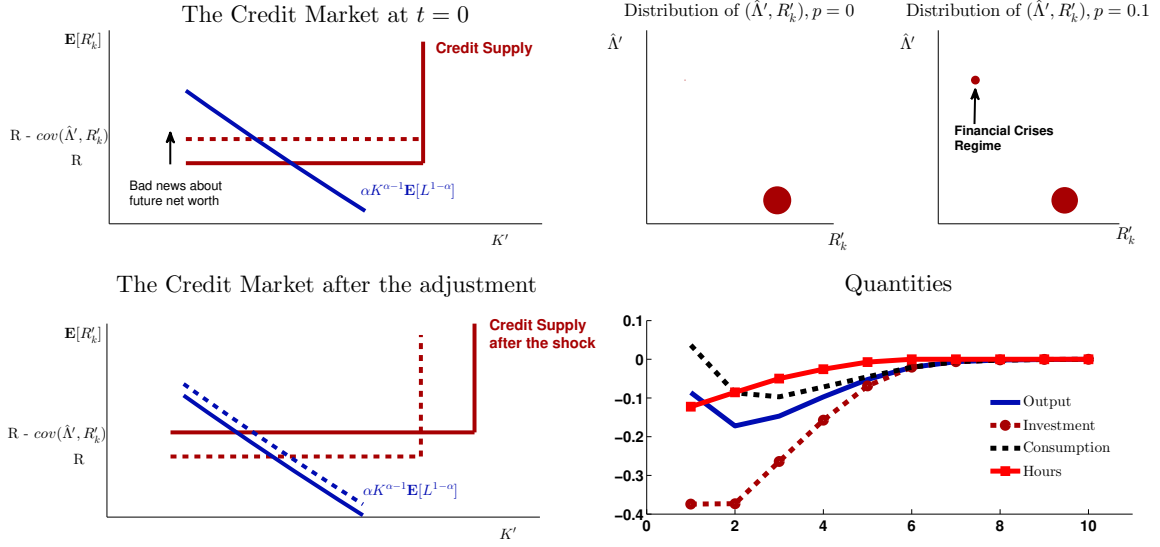
Notes: The figure reports the transitional dynamics induced by a transitory and unexpected increase in p from 0 to 0.10. The parametrization adopted is $[\alpha = 0.33, \nu = \infty, \beta = 0.995, \lambda = 0.44, \omega = 0.10, \psi = 0.95]$. The bottom right panel reports variables expressed as percentage deviations from their steady state.

A sovereign default in the model of Section 2 resembles the financial crisis regime discussed here: banks suffer large balance sheet losses because of the haircut imposed by the government. Claims on firms pay off badly in this state because of low firm profits and the decline in their market value. These low payouts are highly discounted by banks because they are already facing large balance sheet losses, and their marginal valuation of wealth is high. When the likelihood of this event increases, banks have a precautionary incentive to deleverage because the economy is approaching a state where firms' claims are not particularly valuable, and this deleveraging results in a decline in capital accumulation. I will refer to this second mechanism through which sovereign credit risk propagates to the real economy as the *risk channel*.

3.3 Policy Relevance

While these two propagation mechanisms have similar implications for quantities and prices, they carry substantially different information. This can be seen by comparing the credit markets in Figure 1 and Figure 2. In Figure 1, excess returns over firms' claims arise because the constraints on bank leverage prevent profitable investment opportunities: if banks had an additional unit of wealth, they would invest it in firms' claims. In Figure 2, instead, excess returns reflect fair compensation for increased risk: bank leverage constraint are not binding, and there are no unexploited profitable opportunities.

This distinction has important implications for the evaluation of credit policies in the model. For example, it is reasonable to expect that an injection of equity to the banking sector may be more effective in stimu-

lating banks' lending when these latter are facing tight constraints on their leverage, while their aggregate implications may be muted when risk premia are high. We will see in Section 5 that this intuition holds in the model. First though, I move to the empirical analysis.

4   Empirical Analysis

The model is estimated using Italian quarterly data (1999:Q1-2011Q4). This section proceeds in three steps. Section 4.1 describes the data used in estimation and discusses how they help identifying the mechanisms of interest. Section 4.2 illustrates the estimation strategy. I place a prior on parameters and conduct Bayesian inference. Because of the high computational costs involved in solving the model repeatedly, I adopt a two-step procedure. In the first step, I estimate a version of the model without sovereign default risk on the 1999:Q1-2009:Q4 subsample. In the second step, I estimate the parameters for the $\{s_t\}$ shock using a time series for sovereign default probabilities for the Italian economy. Section 4.3 presents an assessment of model fit based on posterior predictive checks for a set of sample moments computed from the data.

4.1   Data

As discussed in the previous section, the transmission of sovereign risk to the real economy is the result of two key mechanisms. Their strength in the model is governed by three "parameters": i) the elasticity of government bond returns to sovereign risk; ii) the elasticity of bank net worth to variation in realized returns on government bonds; iii) the macroeconomic implications of tighter leverage constraints for banks. The selection of the data aims to making the model consistent with three sets of facts that can empirically inform these aspects of the model.

First, I ensure that the time-varying nature of sovereign risk in the model is realistic. Indeed, the behavior of government bonds' prices in response to sovereign risk is partly determined by how persistent agents perceive these changes to be. For this purpose, I use credit default swaps (CDS) spreads on Italian government securities with a five-year maturity. This time series is available at daily frequencies starting in January 2003 from Markit. See Appendix C.1 for further details.

Second, I measure the exposure of banks to this risk. I collect data on the exposure of the five largest Italian banks to domestic government debt obtained from the 2011 European Banking Authority stress test.[27] As detailed in Appendix C.2, these data include holdings of domestic government securities, loans to central government and local authorities and other provisions, and these items are classified in terms of

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[27]The five banks are: Unicredit, Intesa-San Paolo, MPS, BPI and UBI. Their total assets at the end of 2010 accounted for 82% of the total assets of domestic banking groups in Italy.

their maturity. I match this information with the end of 2010 consolidated balance sheet data obtained from Bankscope. This allows me to measure the size of the exposure of these five banks to the Italian government in terms of their total assets.[28]

Third, I measure the cyclical behavior of the leverage constraint. The agency frictions studied in this paper are fairly abstract at this level of aggregation and they have poorly measured empirical counterparts. For this reason, I use the model's restrictions to relate the tightness of banks' leverage constraint to a set of observable variables. Result 2 in Appendix C.3 shows that the Lagrange multiplier on the leverage constraint of banks can be expressed as a function of financial leverage (lev_t) and of the spread between a risk free security (R_t^f) that is traded only by bankers and the risk free rate (R_t)

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t}\right] \text{lev}_t}{1 + \left[\frac{R_t^f - R_t}{R_t}\right] \text{lev}_t}. \tag{26}$$

I use equation (26) to generate a time series for the multiplier μ_t. I measure R_t^f with the prime rate on interbank loans (EURIBOR). This is the natural rate to consider because we can interpret the model from Section 2 as having a frictionless interbank market of the type considered in Gertler and Kiyotaki (2010). The risk free rate R_t is matched with the yields on German government securities. The leverage of financial intermediaries is measured using the Italian flow of funds. Appendix C.3 describes in detail the steps involved in measuring μ_t. Figure 3 reports this time series along with GDP growth. Two main facts stand out from a visual inspection of the figure. First, the Lagrange multiplier is countercyclical, rising substantially in periods in which GDP growth is markedly below average. Second, it is very close to 0 until 2007:Q2. Thus, the constraints seem to bind only occasionally in our sample.

While these three sets of facts are important to identify the effects of interest, they are not informative for all model parameters. Thus, I complement this information with time series for the labor income share, the investment-output ratio, the government spending-output ratio and hours worked. Appendix C.4 provides detailed definitions and data sources.

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[28]These data do not correct for the possibility that banks insured part of this debt via CDS. Acharya and Steffen (2013) impute the exposure of major banks to distressed sovereigns in the euro-area, finding that insurance via CDS is likely to be small.

Figure 3: **Lagrange Multiplier on Leverage Constraint and GDP Growth: 1999:Q1-2012:Q4**
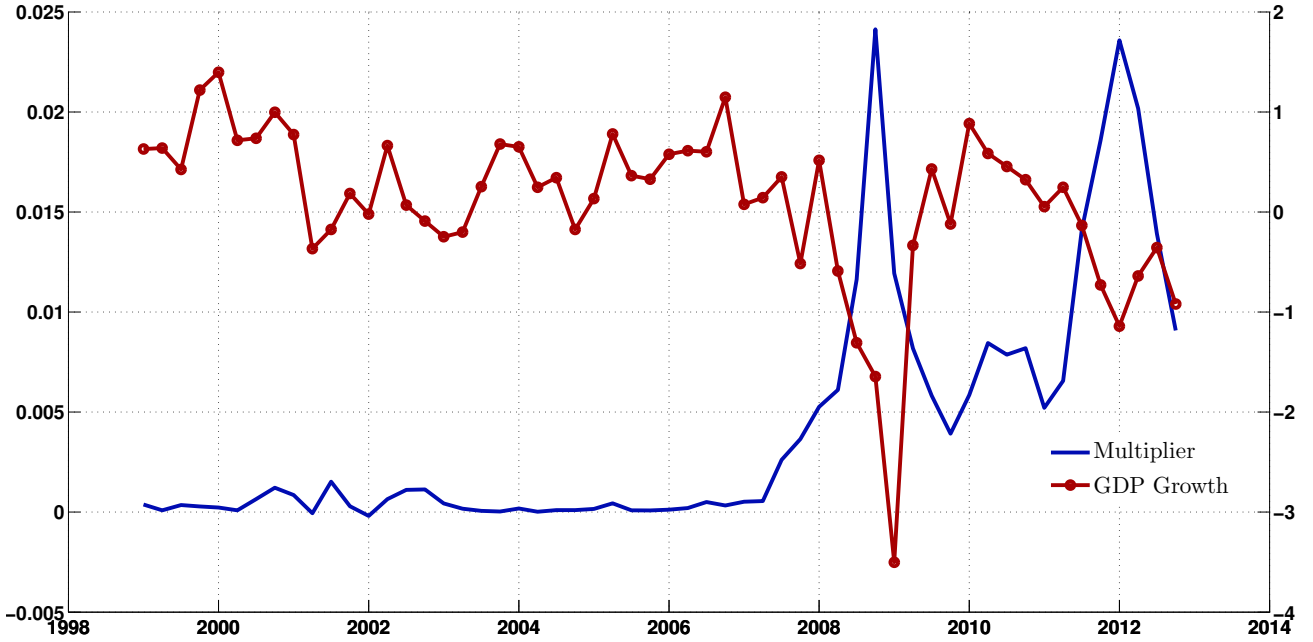
Notes: The Lagrange multiplier on banks' leverage constraint is the solid line (left axis). The circled line is GDP growth (right axis). Appendix C provides detailed information on data sources.

4.2 Estimation Strategy

I denote by $\theta \in \Theta$ the vector of model parameters. It is convenient to organize the discussion around the following partition, $\theta = [\theta_1, \theta_2]$

$$\theta_1 = \left[\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma, \pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}, l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg}\right], \qquad \theta_2 = [D, s^*, \rho_s, \sigma_s].$$

Conceptually, we can think of θ_1 as indexing a restricted version of the model without sovereign risk, while θ_2 collects the parameters determining the sovereign default process. I have reparametrized $[\lambda, \omega, \delta, \chi, \iota, \tau^*, a_1, a_2]$ with balanced growth values for, respectively, the Lagrange multiplier on leverage constraints (μ^{bg}), the leverage ratio (lev^{bg}), the investment-output ratio $\left(\frac{i^{bg}}{y^{bg}}\right)$, worked hours ($l^{bg}$), the price of government securities (q_b^{bg}), the ratio of government securities held by bankers to their total assets (exp^{bg}) and the size of capital adjustment costs (adj^{bg}).

While a nonlinear analysis of the model is necessary to capture time variation in risk premia and the fact that leverage constraints bind only occasionally, it complicates inference substantially since repeated numerical solutions of the model are computationally costly. I therefore estimate θ using a two-step procedure. In the first step, I infer θ_1 by estimating the model without sovereign risk on the 1999:Q1-2009:Q4 subsample using Bayesian methods. This restricted version of the model has fewer state variables and is

easier to analyze numerically. Moreover, focusing on this restricted model should not substantially alter the inference over θ_1 because i) the 1999:Q1-2009:Q4 period was characterized by low sovereign risk for the Italian economy; and ii) the decision rules of the restricted model closely approximate those of the full model in this area of the state space. In the second step, I estimate θ_2 using a retrieved time series of sovereign default probabilities.

4.2.1 Estimating the Model without Sovereign Risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$. The parameters are

$$\theta_1 = \left[\underbrace{\mu^{bg}, \psi, \xi, \sigma_z, \rho_z, \gamma}_{\tilde{\theta}_1}, \underbrace{\pi, g^*, \rho_g, \sigma_g, \gamma_t, \nu, \alpha, \frac{i^{bg}}{y^{bg}}, l^{bg}, lev^{bg}, R^{bg}, exp^{bg}, q_b^{bg}, adj^{bg}}_{\theta_1^*} \right].$$

I construct the likelihood function of the model using time series for GDP growth and the Lagrange multiplier on banks' leverage constraint described earlier. As explained in the earlier section, the cyclical behavior of the model's financial friction is key to assess the impact of sovereign risk on the real economy: a likelihood-based approach guarantees a high degree of consistency between the model implied behavior for these variables and their data counterparts.

This choice has limitations. First, I am discarding potentially important information as one could incorporate the components of the multiplier into the likelihood function: the risk free rate, the interbank rate and the leverage of banks. I verified though that the model is too restrictive to track the time series behavior of financial leverage in the earlier part of the sample, because structural shocks do not generate enough variation in asset prices when leverage constraints are far from binding.[29] Second, certain model parameters are only weakly affected by the information in the likelihood and their identification is problematic. For this reason, and prior to conduct full information inference, I determine a subset of θ_1, θ_1^*, prior to the estimation using external information. Table 1 reports the numerical values for these parameters. I set $[\frac{i^{bg}}{y^{bg}}, lev^{bg}, l^{bg}, R^{bg}]$ to the sample average of their empirical counterparts while α is determined using the sample average of the labor income share. I use the information in Table A-1 in Appendix C.2 to determine $[exp^{bg}, \pi]$: holdings of government securities account for 8% of banks' total assets in the model, and the average maturity of those bonds is set to 23 months. I select $[g^*, \rho_g, \sigma_g]$ from the estimation of an AR(1) on the spending-output ratio over the 1999:Q1-2011:Q4 period. The remaining parameters in θ^* are determined through normalizations or previous research. I set the Frisch elasticity of labor supply to 2 and γ_τ to 0.5. The former is in the high range of the estimates obtained using U.S. data (Rios-Rull et al., 2012), but it is

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[29]This aspect is related to one shortcoming of pure neoclassical models, namely their inability to generate volatility in asset prices. See for example Bocola and Gornemann (2013) for a discussion.

not an uncommon value in the profession for the analysis of Real Business Cycle models. Since taxes are non-distortionary in the model, γ_τ has little implications for the model's endogenous variables other than debt. I set adjustment costs to zero in a balanced growth path while I normalize q_b^{bg} to 1 (bonds trade at par in a balanced growth path).

Table 1: **Parameters Determined with External Information**

	Parameters				**Source**
$\frac{i^{bg}}{y^{bg}}$	lev^{bg}	l^{bg}	R^{bg}	α	OECD, EU-KLEMS,
0.25	4.34	0.30	1.0034	0.30	ECB, BoI
exp^{bg}	π				EBA,
0.079	0.044				Bankscope
e^{g^*}	ρ_g	σ_g			OECD
0.22	0.92	0.010			
q_b^{bg}	adj^{bg}	ν	γ_τ		Normalizations,
1	0	2	0.5		Previous Research

Notes: See Appendix C for information on data sources.

I next turn to the estimation of $\tilde{\theta}_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z]$. Let $\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t]'$, and let $\mathbf{Y}^t = [\mathbf{Y}_1, \ldots, \mathbf{Y}_t]'$. The model defines the nonlinear state space system

$$\mathbf{Y}_t = f_{\tilde{\theta}_1}(\mathbf{S}_t) + \eta_t \qquad \eta_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$$

$$\mathbf{S}_t = g_{\tilde{\theta}_1}(\mathbf{S}_{t-1}, \varepsilon_t) \qquad \varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}),$$

where η_t is a vector of measurement errors and ε_t are the structural shocks.[30] Measurement errors, absent from the structural model, are included to help the evaluation of the likelihood function. I approximate the likelihood function of this nonlinear state space model using sequential importance sampling (Fernández-Villaverde and Rubio-Ramírez, 2007).[31] The posterior distribution of model parameters is

$$p(\tilde{\theta}_1|\mathbf{Y}^T) = \frac{\mathcal{L}(\tilde{\theta}_1|\mathbf{Y}^T)p(\tilde{\theta}_1)}{p(\mathbf{Y}^T)},$$

where $p(\tilde{\theta}_1)$ is the prior, $\mathcal{L}(\tilde{\theta}_1|\mathbf{Y}^T)$ the likelihood function and $p(\mathbf{Y}^T)$ the marginal data density. I characterize the posterior density of $\tilde{\theta}_1$ using the Random Walk Metropolis Hastings for DSGE models developed in Schorfheide (2000) with an adaptive variance-covariance matrix for the proposal density. Appendix D

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[30]The functions $g_{\tilde{\theta}_1}(.)$ and $f_{\tilde{\theta}_1}(.)$ are approximated following the steps described in Appendix B for a version of the model that does not feature sovereign credit risk. Since θ_1^* is fixed, I omit from the notation the dependence of decision rules on these parameters.

[31]I use the auxiliary particle filter of Pitt and Shephard (1999) which, in this application, substantially improves the efficiency of the likelihood evaluation. See Aruoba and Schorfheide (2013) for a recent application to economics. I consider a diagonal matrix $\mathbf{\Sigma}$ where the nonzero elements are equal to 25% of the sample variance of $\{\mathbf{Y}_t\}$. Appendix D provides a description of the evaluation of the model's likelihood function.

provides a description of the estimation algorithm. Table 2 reports the prior along with posterior statistics for $\tilde{\theta}_1$.

Table 2: **Prior and Posterior Distribution of $\tilde{\theta}_1$**

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
$\mu^{bg} \times 100$	Uniform	0	∞	0.18	[0.13,0.21]
ψ	Uniform	0	1	0.97	[0.95,0.98]
ξ	Beta	0.5	0.25	0.42	[0.35,0.50]
$\gamma \times 400$	Normal	1.25	0.5	0.36	[0.06,0.75]
ρ_z	Beta	0.3	0.25	0.08	[0.04,0.14]
$\sigma_z \times 100$	Inverse Gamma	0.75	2	0.94	[0.84,1.06]

Notes: Para 1 and Para 2 list the mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. The prior on γ is truncated at 0. Posterior statistics are computed using 10000 draws from the posterior distribution of model's parameters. The table reports equal tail probability 90% credible sets.

The prior on the TFP process is centered using presample evidence while I center ξ to 0.5, a conventional value in the literature. Priors on these three parameters are fairly diffuse. I choose uniform priors over μ^{bg} and ψ, implying that the shape of the posterior is determined by the shape of the likelihood. Regarding posterior estimates, the multiplier is estimated to be close to 0 in a deterministic balanced growth path while ψ is close to unity. This suggests that agency costs are fairly small on average in the model. This is not surprising given the time series behavior of μ_t in Figure 3.[32] Capital adjustment costs and the TFP process are in the range of what is typically obtained in the literature when using U.S. data.

4.2.2 Estimating Sovereign Risk

I next turn to the estimation of $\theta_2 = [D, s^*, \rho_s, \sigma_s]$. The empirical strategy consists of i) constructing a time series for the probabilities of a sovereign default and ii) using this time series to estimate θ_2.

I accomplish the first task by exploiting the model's pricing equation. In fact, using equation (8) and equation (5), we can define the the *risk neutral* measure as:[33]

$$\hat{p}(\mathbf{S}'|\mathbf{S}) = \frac{R^f(\mathbf{S}) p(\mathbf{S}'|\mathbf{S}) \hat{\Lambda}(\mathbf{S}', \mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda \mu(\mathbf{S})}.$$

After integrating the above expression over states $\mathbf{S}'$ associated with a sovereign default next period, I

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[32]One way of assessing the size of financial friction in the model is to ask how large the distortion is that they generate on returns to capital. The posterior mean of μ^{bg} tells us that this distortion is approximately equal to 18 basis points in a balanced growth path of the model.

[33]Note that $\hat{p}(\mathbf{S}'|\mathbf{S})$ is nonnegative and it integrates to 1. To see the last property, note that the return on a risk free security traded by bankers can be written as $R^f(\mathbf{S}) = \frac{\alpha(\mathbf{S})[1-\mu(\mathbf{S})]+\lambda\mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}',\mathbf{S})]}$ using equation (8) and equation (5).

obtain an expression for the actual probability of a sovereign default, p_t^d. This time series is related to its *risk neutral* counterpart, $\hat{p}_t^d$, as follows

$$p_t^d = \hat{p}_t^d \frac{\alpha_t(1 - \mu_t) + \lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]}. \tag{27}$$

Because of bankers' risk aversion, there is a wedge between these two probabilities, represented by the risk correction $\frac{\alpha_t(1-\mu_t)+\lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1}=1]}$. Equation (27) is important because it allows us to measure actual probabilities of sovereign default using empirical counterparts to risk neutral probabilities and the risk correction.

First, I obtain a time series for $\{\hat{p}_t^d\}$ using CDS spread on Italian government securities, up to a normalization of the haircut parameter D. I fix D to 0.45, consistent with the historical experience on recent sovereign defaults in emerging economies (Cruces and Trebesh, 2013).[34] While Pan and Singleton (2008) show that D could be estimated using information from the term structure of sovereign CDS spreads, their Monte Carlo analysis suggests that this parameter is typically poorly identified in small samples.

Second, I construct a time series for $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$, the conditional expectation of the pricing kernel in the event of a sovereign default. This is a difficult task because of the absence of a sovereign default in the sample. I indirectly use the model's restrictions to conduct this extrapolation. In particular, I approximate the object of interest as follows

$$\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1] \approx \mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}, \tag{28}$$

where $\kappa > 0$ is a hyperparameter. The idea underlying equation (28) is that the pricing kernel in the model is above its unconditional average in the event of a sovereign default because of banks' implicit risk aversion: κ parametrizes the number of standard deviations by which $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$ is above $\mathbb{E}_t[\hat{\Lambda}_{t+1}]$.

The terms $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$ are generated using an empirical counterpart to the model's pricing kernel defined in equation (9). The pricing kernel, in turn, is a function of observables and model parameters estimated in the first step

$$\hat{\Lambda}_t = \beta e^{-\Delta \log(c_t)}[(1 - \psi) + \psi\lambda\mathrm{lev}_t], \tag{29}$$

where Δc_t is consumption growth and lev_t is financial leverage. $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}], \mathrm{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}$ are constructed using equation (29), the posterior mean for $[\beta, \psi, \lambda]$ and a time series for the conditional forecasts of $[\Delta c_{t+1}, \mathrm{lev}_{t+1}]$ generated by a first order Bayesian Vector Autoregressive model. I then select the hyperparameter κ with the help of the structural model. I consider a set of values $\kappa^i \in \{1, 3, 5\}$ and select the value that minimizes,

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[34]Zettelmeyer et al. (2013) document a larger haircut (on average between 59% and 65%) in the Greek debt restructuring event of 2012. A value of D equal to 0.45 is conservative, and corrects for potential transfers that the government may give to its domestic bondholders.

in model simulated data, average root mean square errors for the approximation of $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. This gives a value of $\kappa = 3$.

Third, I combine the retrieved time series for $\{\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]\}$ with observations on banks' financial leverage, the multiplier and the prime interbank rate to generate the risk correction $\frac{\alpha_t(1-\mu_t)+\lambda\mu_t}{R_t^f \mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1}=1]}$. I make use of the fact that the marginal value of wealth for bankers is proportional to financial leverage when the constraint binds, and measure the risk correction as follows:

$$\frac{\lambda \text{lev}_t(1 - \mu_t) + \lambda\mu_t}{R_t^f\{\mathbb{E}_t[\hat{\Lambda}_{t+1}] + \kappa\text{Var}_t[\hat{\Lambda}_{t+1}]^{\frac{1}{2}}\}}. \tag{30}$$

Figure 4: **Sovereign Default Probabilities**
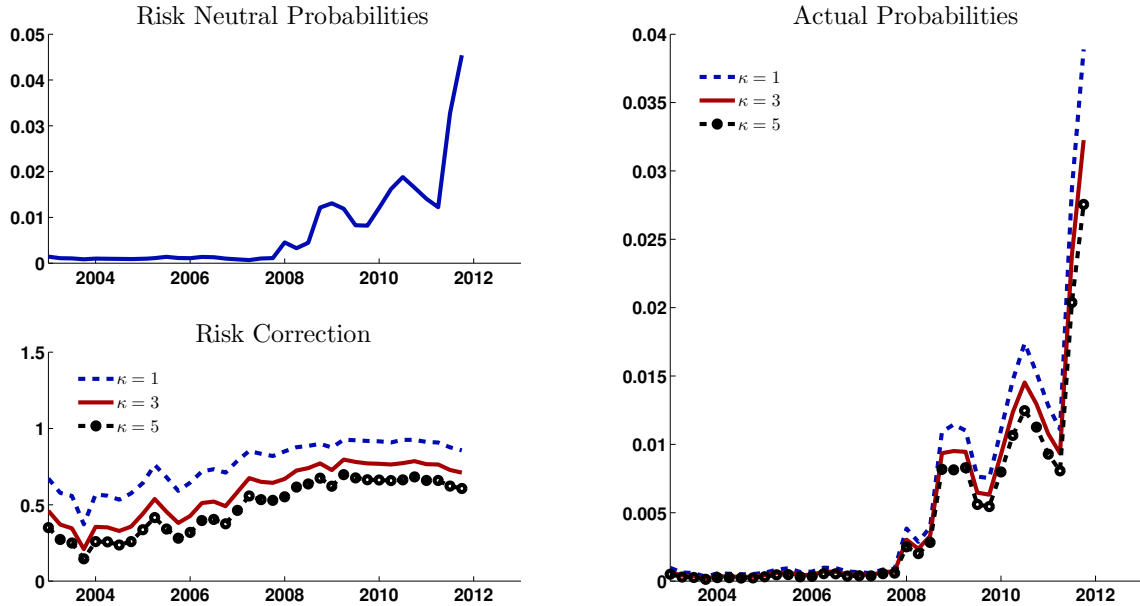
Notes: The top left panel reports risk neutral probabilities of a sovereign default. The bottom right panel reports the risk correction, defined in equation (30). The right panel reports actual probabilities of a sovereign default, defined in equation (27).

Figure 4 plots $\{p_t^d\}$ along with its decomposition of equation (27) for the different values of κ. The top left panel reports the risk neutral probabilities, the bottom-left panel plots the risk correction and the right panel reports the time series for actual sovereign default probabilities. The estimates imply that roughly 30% of actual sovereign default probabilities in the sample is due to risk premia, consistent with the empirical evidence reported in Longstaff et al. (2011) for a group of developing countries.

I then use $\{p_t^d\}$ to estimate the parameters of the sovereign risk shock s_t. Indeed, the two are related in the model as follows

$$\log\left(\frac{p_t^d}{1 - p_t^d}\right) = s_t, \qquad s_t = (1 - \rho_s)s^* + \rho_s s_{t-1} + \sigma_s\varepsilon_{s,t}, \tag{31}$$

where $\varepsilon_{s,t}$ is a standard normal random variable. I use the Kalman filter to evaluate the likelihood function of this linear state space model. Table 3 reports prior and posterior statistics for $[s^*, \rho_s, \sigma_s]$. As I do not have presample information, I consider fairly uninformative priors. Posterior statistics are computed from a canonical Random Walk Metropolis Hastings algorithm.

Table 3: **Prior and Posterior Distribution of** $[s^*, \rho_s, \sigma_s]$

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
s^*	Normal	-7	5	-6.17	[-8.88,-3.35]
ρ_s	Beta	0.5	0.3	0.95	[0.87,0.98]
σ_s	Inverse Gamma	0.75	4	0.55	[0.44,0.70]

Notes: Para 1 and Para 2 liststhe mean and standard deviation for Beta and Normal distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. Posterior statistics are computed using 10000 draws from the posterior distribution of model's parameters. The table reports equal tail probability 90% credible sets.

4.3 Model Fit

In order to determine if the estimated model fits the time series described in the previous section, I verify whether model simulated trajectories for the multiplier, GDP growth and sovereign default probabilities resemble those observed in the data. This is accomplished through posterior predictive checks.[35] I generate model implied densities for sample statistics and check how they compare with the same statistics computed from actual data.

First, I examine the performance of the model regarding GDP growth and the multiplier. I summarize their joint behavior using the following sample statistics: mean, standard deviation, first order autocorrelation, skewness, kurtosis and their correlation. These are collected in $\mathcal{S}$. The model implied densities for $\mathcal{S}$ are generated using the following algorithm

Posterior Predictive Densities: Let θ^i denote the i'th draw from the posterior density of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i simulate a realization for GDP growth and the multiplier of length T=100.[36] Let $\{\mathbf{Y}_t^i\}$ denote this realization.

2. Based on the simulated trajectories $\{\mathbf{Y}_t^i\}$, compute a set of sample statistics $\mathcal{S}^i$. □

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[35]See Geweke (2005) for a general discussion of predictive checks in Bayesian analysis and Aruoba et al. (2013) for a recent application to the evaluation of estimated nonlinear Dynamic Stochastic General Equilibrium models.

[36]These simulations are generated from the restricted model (no sovereign risk). Simulations are initialized at the ergodic mean of the state vector.

Given the draws $\{\mathcal{S}^i\}$, I use percentiles to describe the predictive density $p(\mathcal{S}(.)|\mathbf{Y}^T)$. Figure 5 shows the 5th and 95th percentile of the model implied density (the box) along with its median (the bar) and their sample counterpart (the dot).

Figure 5: **Posterior Predictive Checks: Multiplier and GDP Growth**
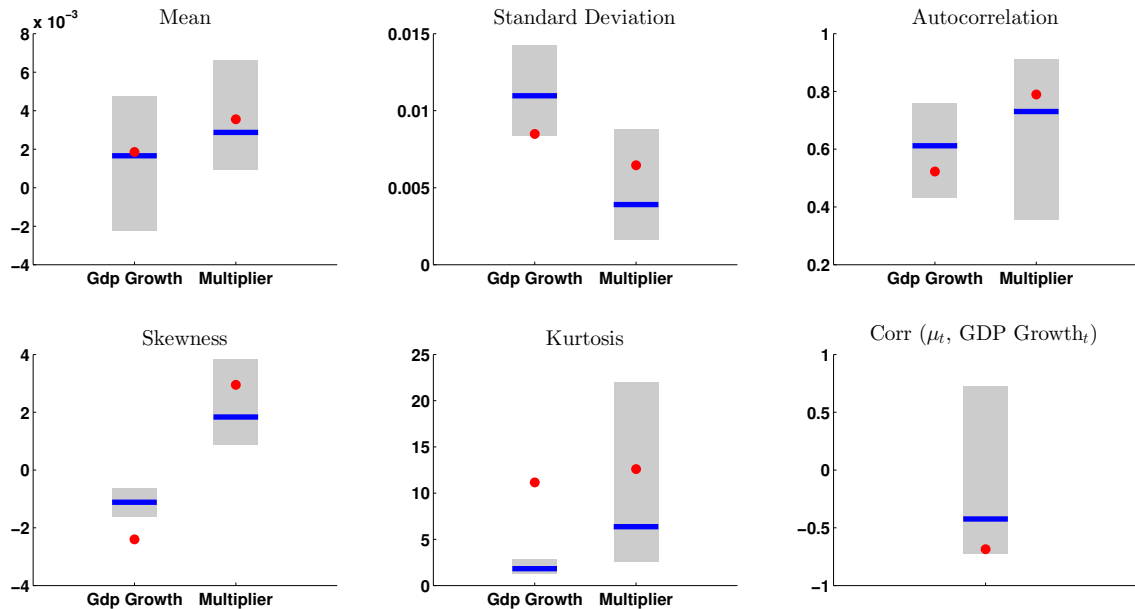
Notes: Dots correspond to the value of the statistic computed from actual data. Solid horizontal lines indicate medians of posterior predictive distribution for the sample statistic and, the boxes indicate the equal tail 90% credible set associated with the posterior predictive distribution.

The model generates trajectories for the multiplier and GDP growth whose moments are in line with those observed in the data. The main discrepancy with the data is in the excess kurtosis for the GDP growth trajectory: the model is too restrictive to replicate this feature of the data. In addition, the model captures part of the left skewness of GDP growth. This derives from two properties: the amplification of the leverage constraint and the fact that it binds in recessions. In fact, GDP growth is more sensitive to structural shocks when the leverage constraint binds. Since these constraints are only occasionally binding, this amplification generates asymmetry in the unconditional distribution for GDP growth. Left skewness is then the result of GDP growth and the multiplier being negatively correlated. Guerrieri and Iacoviello (2013) discuss the asymmetry generated by occasionally binding credit constraints in a model of housing.

Second, I ask whether the behavior of sovereign default probabilities in the model is in line with what was observed in the data. The posterior predictive checks are reported in Table 4. We can verify that the specification adopted to model time-variation in sovereign risk captures key features of the empirical distribution of sovereign default probabilities.

Overall, the results in this section suggest that: i) the cyclical behavior of the leverage constraint in

Table 4: **Posterior Predictive Checks: Sovereign Default Probabilities**

Statistic	Data	Posterior Median	90% Credible Set
Median	0.07	0.25	[0.01,6.81]
Mean	0.53	0.53	[0.03,11.7]
Standard Deviation	0.76	0.63	[0.03,13.5]
Autocorrelation	0.91	0.83	[0.69,0.94]
Skewness	2.03	2.04	[0.96,3.78]
Kurtosis	7.37	7.23	[3.04,20.1]

Notes: Based on 1000 draws from the posterior distribution of $[s^*, \rho_s, \sigma_s]$. For each draw, I simulate the $\{s_t\}$ process for 100 periods. Statistics are computed on each of these 1000 samples. The table reports the posterior median and equal tail probability 90% credible set for the posterior predictive distributions.

the estimated model is empirically reasonable; and that ii) agents in the model have beliefs about the time-varying nature of sovereign credit risk that closely track what was observed in the data.

5 Model Analysis

This section analyzes some properties of the estimated model that are important for the interpretation of the main experiments of this paper, which will be presented in Section 6. There are three key points that emerge from this analysis:

1. A sovereign default leads to a deep decline in real economic activity. This occurs because the haircut on government bonds tightens the leverage constraints of banks and triggers a decline in aggregate investment (Section 5.1).

2. An increase in the probability of a sovereign default when the economy is in the non-default state leads to an increase in expected excess returns. This occurs through two mechanisms. First, sovereign credit risk tightens the leverage constraint of banks (*leverage-constraint channel*). Second, it increases the required premia that banks demand for holding firms' assets (*risk channel*). This increase in the financing premia of firms is associated with a decline in capital accumulation and output (Section 5.2).

3. The aggregate effects of equity injections into the banking sector are highly state dependent, even if implemented at times of high financial stress.[37] These interventions are more successful in stimulating real economic activity in regions of the state space where leverage constraints are tight. Conversely, these policies have substantially weaker effects when risk premia on firms' assets are high (Section

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[37] Periods of high financial stress will be defined as periods during which expected excess returns are above a threshold and output growth is below a threshold.

Since the aim of this section is purely illustrative, the model's parameters are fixed at their posterior mean.

5.1 A Sovereign Default

Figure 6 shows the behavior of key model's variables around a typical sovereign default. I apply event study techniques to the simulated time series and report their average path around the default. The window covers 10 quarters before and after the event.

Figure 6: **A Sovereign Default**
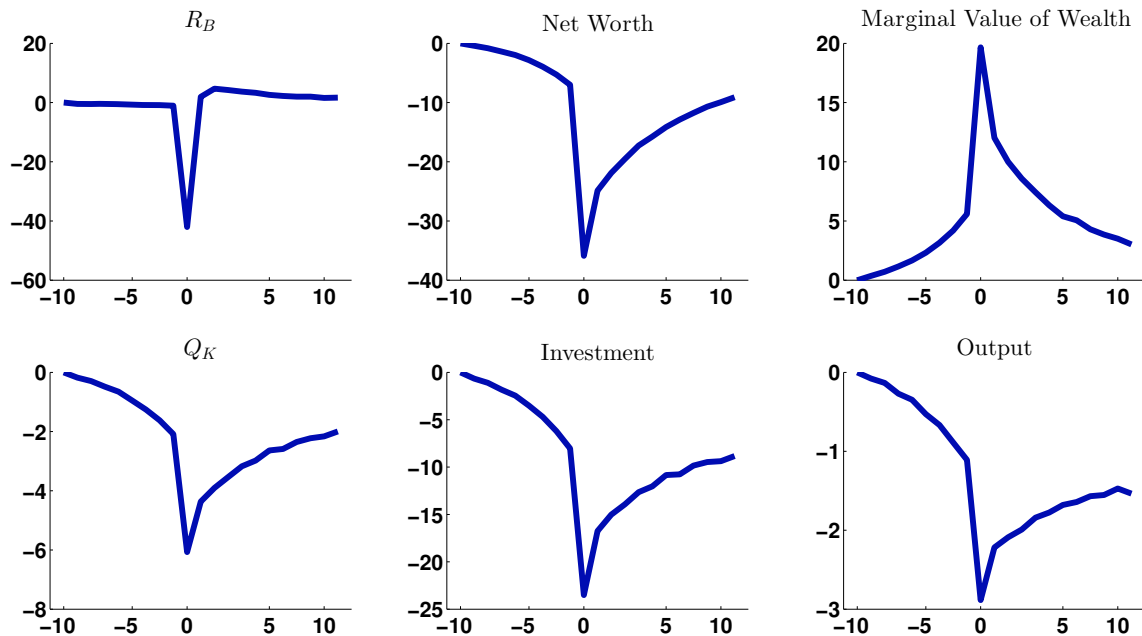
Notes: The panels are constructed as follows. Simulate $M = 15000$ realizations of length $T = 300$. Each simulation is initialized at the ergodic mean of the state vector. For each realization, select time series around a sovereign default event. Net worth, output and investment are linearly detrended. The figure reports medians across the simulations. Returns on government bonds are expressed as deviations from their $t = -10$ value in annualized basis points. The other variables are expressed as percentage deviations from their $t = -10$ value.

At $t = 0$ the government imposes a haircut on bondholders. As a consequence, bank net worth declines and the leverage constraint tightens, thus forcing them to reduce their holdings of firms' assets. This has adverse effects on aggregate investment and output: at $t = 0$, they are respectively 25% and 2.9% below their trend. From $t = 1$ onward, bank net worth recovers because excess returns are above average. This loosens the leverage constraint, and the economy slowly returns to its balanced growth path.

It is important to stress two important facts about a sovereign default in the model. First, the behavior of asset prices substantially amplifies this event (Kiyotaki and Moore, 1997; Mendoza, 2010). The tightening of the leverage constraint forces banks to restrict lending to firms. The associated decline in capital demand

puts downward pressure on asset prices because of Tobin's Q and further depresses the net worth of banks. As we can see from the figure, the market value of firms is 6% below trend at $t = 0$, while bank net worth is roughly twice the size of the haircut imposed by the government. Second, as the bottom-right panel of the figure shows, the marginal value of wealth for bankers is high during a sovereign default.

It is also interesting to note that a sovereign default is preceded by a deep slowdown in real economic activity, which conforms with historical evidence on these episodes, see Yeyati and Panizza (2011). This observation is typically rationalized in the literature via a selection argument: equilibrium models of sovereign defaults predict that incentives for the government to renege on debt are high in bad economic times, see Arellano (2008) and Mendoza and Yue (2012) for example. In the model analyzed here, the "V" shape behavior of output around a default event occurs purely because of *anticipation* effects: increases in the probability of a future sovereign default are, in fact, recessionary. The next section explains why.

5.2 An Increase in the Probability of a Future Sovereign Default

From equation (25) we obtain a decomposition of expected excess returns to capital into two pieces: the multiplier component and the covariance component.

$$\underbrace{\frac{\mathbb{E}_t[R_{K,t+1} - R_t]}{R_t}}_{\text{EER}_t} = \underbrace{\frac{\lambda \mu_t}{\alpha_t[1 - \mu_t]}}_{\text{Multiplier component}_t} - \underbrace{\frac{\text{cov}_t[\hat{\Lambda}_{t+1}, R_{K,t+1}]}{\alpha_t[1 - \mu_t]}}_{\text{Covariance component}_t}. \tag{32}$$

According to equation (32), expected excess returns can be high because of two distinct sources. First, banks face tight leverage constraints and this restricts the flow of funds to firms (Multiplier component). Second, banks require a premium for lending to firm because this intermediation is risky (Covariance component). Sovereign credit risk influences both of these components.

Figure 7 plots Impulse Response Functions (IRFs) to an *s*-shock when the economy is at the ergodic mean. The initial impulse in *s* is such that the probability of a future sovereign default goes from 0.17% to 5%. This represents roughly a 6 standard deviations shock. The figure shows that this shock tightens the leverage constraint of banks. The price of government bonds declines by 18%, leading to a reduction in their realized returns of roughly the same magnitude. The net worth of banks declines by 15%. Because of this decline in net worth, the leverage constraints of banks start binding, as the behavior of the multiplier shows. Expected excess returns increase by 200 basis points in annualized terms on impact, 140 of which are attributable to the multiplier component. Also the covariance component respond to the *s*-shock: this risk channel explains 30% of the impact increase in expected excess returns.

Figure 7: **IRFs to an s-shock: Expected Excess Returns**
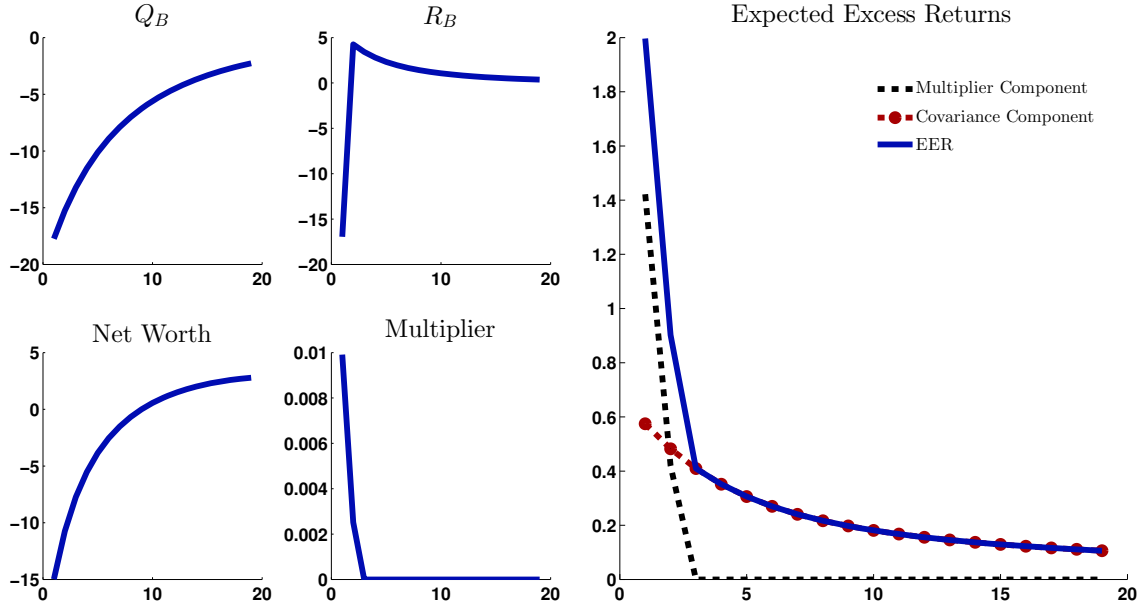
Notes: IRFs are computed via simulations initialized at the ergodic mean of the state vector. Q_B and Net Worth are expressed as percentage deviations from their ergodic mean value. Returns are reported in annualized basis points.

Figure 8 explains why firms' are perceived to be riskier when a sovereign default approaches. The figure reports the joint probability density function (contour lines) for the next period pricing kernel and realized returns to capital. The left panel reports it when the state vector is at its ergodic mean. The right panel reports the same object with the only exception that the probability of a sovereign default next period equals 5%. We can see from the left panel of the figure a clear negative association between realized returns to capital and the pricing kernel, suggesting that the model generates a non-trivial compensation for risk at the ergodic mean. As the economy approaches a sovereign default (right panel), these variables become more negatively associated. This motivates an increase in the compensation for holding claims on firms in their balance sheet. Intuitively, capital is a "bad" asset to hold during a sovereign default because the decline in its market value has adverse effects on bank net worth, and these balance sheet losses are very costly since banks' marginal value of wealth is high. This makes the s-shock a priced risk factor for firms' claims.

The rise in expected excess returns after an s-shock is associated with a decline in capital accumulation. Figure 9 reports the response of aggregate investment and output to the s-shock. The increase in the probability of a sovereign default leads to a decline in output and aggregate investment of, respectively, 1.5% and 12%. The mechanisms through which this happens are those described in Section 3.

Figure 8: **The s-shock and Risk Premia**

Ergodic Mean ($p^d = 0.0017$)   High Sovereign Risk ($p^d = 0.05$)
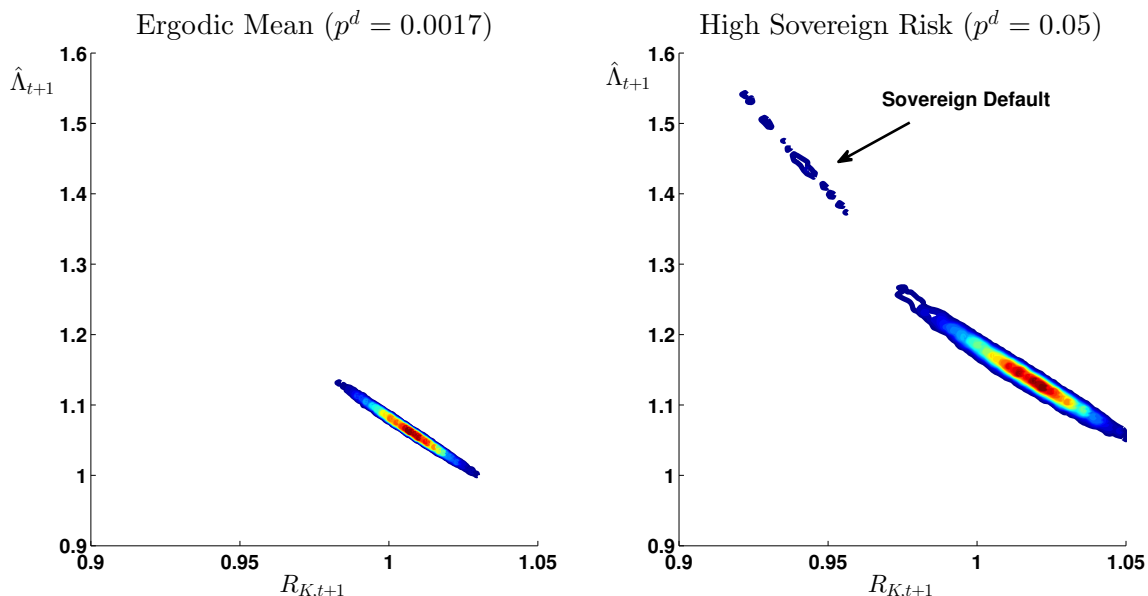
Notes: The left panel reports the joint density of $\{\hat{\Lambda}_{t+1}, R_{K,t+1}\}$ at the ergodic mean. This is constructed as follows. Simulate $M = 15000$ realizations for $\{\hat{\Lambda}, R_K\}$. Each simulation is initialized at the ergodic mean of the state vector and it has length $T = 2$. The contour lines are generated from a nonparametric density smoother applied to $\{\hat{\Lambda}_2^m, R_{K,2}^m\}_{m=1}^M$. The right panel reports the same information, at a different point in the state space. The procedure to construct the figure is the same as above, but the simulations are initialized as follows: i) s-shock is set so that $p_t^d = 0.05$; ii) the other state variable are set at their ergodic mean.

5.3 An Injection of Equity into the Banking Sector

The distinction between the two propagation mechanisms studied in this paper is key for the assesment of credit policies. I illustrate this point by studying the effects of an equity injection into the banking sector. This type of interventions has been already studied in the literature, see for example Gertler and Kiyotaki (2010) and He and Krishnamurthy (2012a). I assume that at $t = 1$ the government transfers resources from households to banks using lump sum taxes. This policy is not anticipated by agents, and no further policy interventions are expected in the future. The policy has the effect of changing the liability structure of banks, raising their net worth relative to their debt.

To make the experiment realistic, I implement the policy when the economy is in a "financial recession". I define this as a state in which output growth is 1.5 standard deviations below average while expected excess returns are 1.5 standard deviations above average. Qualitatively, the results do not depend on these cut-offs. I denote by $\{\mathbf{S}_i^*\}_i$ a set of states variables that is consistent with this definition of financial recession.[38] For each element of $\{\mathbf{S}_i^*\}_i$, I compute the expected path for selected endogenous variables under the policy and without the intervention. The policy effects are reported as percentage differences between these two paths.

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[38]Operationally, this set is constructed by simulating time series of length $T = 20000$ from the model and selecting $\{\mathbf{S}_i^*\}_i$ so that output growth and expected excess returns satisfy the threshold restrictions.

Figure 9: **IRFs to an s-shock: Quantities**
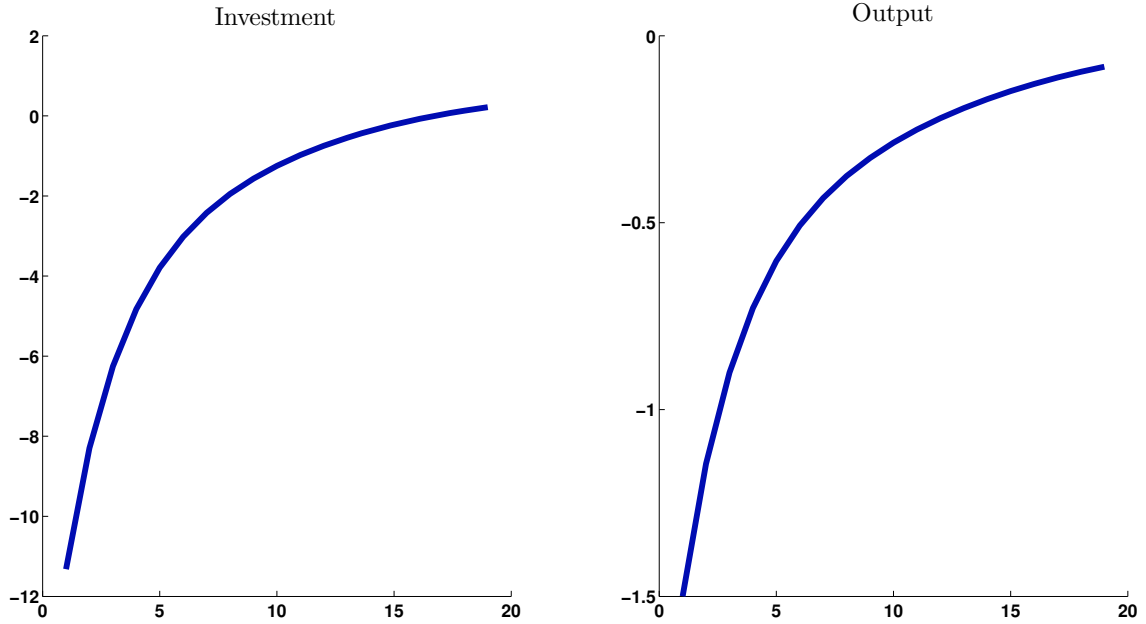
In order to interpret the results, I define

$$\delta_i = \frac{-\text{Covariance component}_i}{\text{EER}_i}, \tag{33}$$

where the Covariance component and EER are defined in equation (32). The variable $\delta_i \in [0, 1]$ gives us an indication of how risky are firms in state $\mathbf{S}_i^*$. In fact, when $\delta_i = 0$, expected excess returns exclusively reflects agency costs while $\delta_i = 1$ means that they reflect fair compensation for risk. Figure 10 plots two sets of results. The solid line reports policy responses conditioning on $\delta_i \leq 0.25$, while the dotted line conditions on $\delta_i \geq 0.75$.

The figure shows that equity injections are particularly effective in stimulating real economic activity when agency costs are large ($\delta \leq 0.25$). The policy relaxes bank leverage constraints and leads to an increase in capital accumulation. This effect is reinforced by general equilibrium forces since the increase in capital demand pushes up the market value of firms, strenghtening the balance sheet of banks and relaxing further their leverage constraint.

The same policy has substantially weaker effects in regions of the state space where firms' risk is high ($\delta \geq 0.75$). The red dotted line reports this case. We can observe that the response of investment and output to the equity injection is 2.5 times smaller with respect to the previous case. Moreover, the general equilibrium effects are substantially muted.

Figure 10: **An Injection of Equity into the Banking Sector**
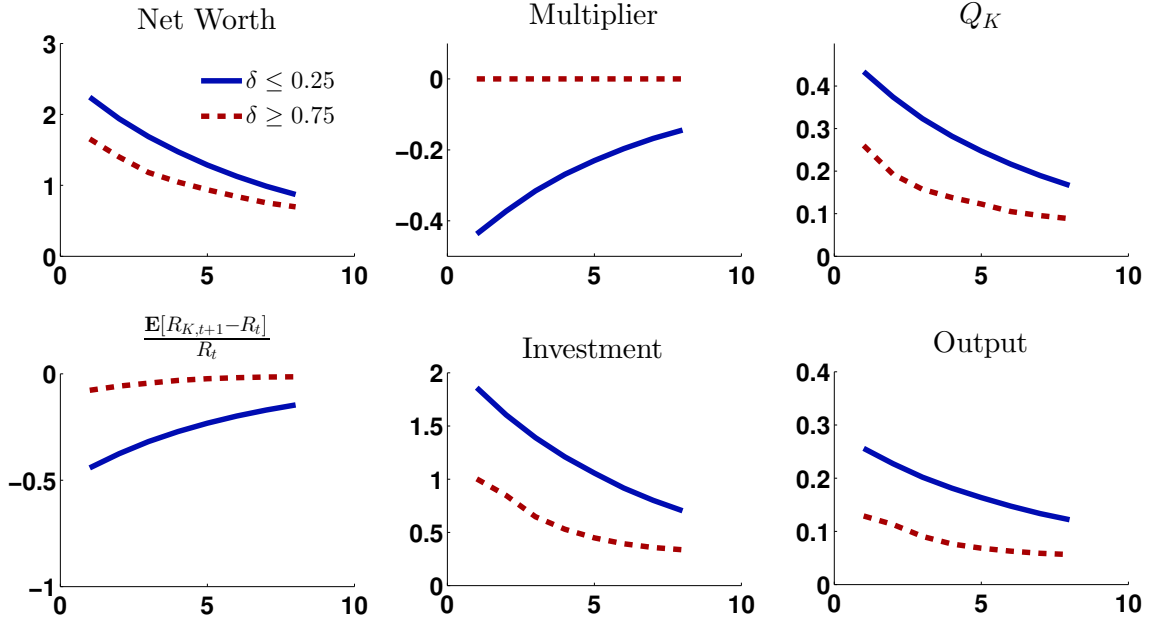
Notes: The figure is constructed as follows: i) simulte the model for $T = 20000$ periods; ii) select state variables such that output growth is 1.5 standard deviations below average and expected excess returns 1.5 standard deviations above average; iii) for each of these states as initial condition, compute the expected path under the equity injection and in absence of the policy; iv) take the difference between these expected paths. The figure reports the effects of the policy on outcome variables when conditioning on different values of δ, see equation (33). Net Worth, Output, and Investment are linearly detrended in simulations. Returns are reported in basis points. The other variables as percentage changes.

This state-dependence in the effects of equity injections has an intuitive explanation. Expected excess returns in the model can be high because of two reasons: tight leverage constraints (low δ-regions) and high risk premia (high δ-regions). Leverage constraints prevents banks from undertaking otherwise profitable investment opportunities. Therefore, policies that relax their constraints stimulate investment because they facilitate the flow of funds from households to firms. A large value of δ, instead, indicates that the high excess returns we observe are fair compensation for risk: aggregate investment respond little to equity injections since these latter have only indirect effects on this risk.[39]

6   Measurement and Policy Evaluation

I now turn to the two main quantitative experiments of this paper. In Section 6.1, I measure the effect of sovereign credit risk on the financing premia of firms and output, and I assess the contribution of the leverage-constraint channel and the risk channel. More specifically, I use the estimated model along with the particle filter to generate trajectories for variables of interest under the assumption that sovereign

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[39]By strengthening bank net worth, the policy provides a buffer when a sovereign default hits the economy. This dampens the effects of a sovereign default on realized returns to capital and lowers risk premia ex-ante. The size of the equity injection is thus an important determinant of the policy effects.

default probabilities in Italy were constrant over the sample. I then study the difference between these counterfactual trajectories and the actual trajectories in order to assess the impact of sovereign credit risk on the variables of interest. Section 6.2 proposes a quantitative assessment of the Longer Term Refinancing Operations (LTROs) implemented by the European Central Bank (ECB) in the first quarter of 2012. As we saw in the earlier section, the effects of policy interventions are state and size dependent due to the highly nonlinear nature of the model. Therefore, an integral part of the policy evaluation is to specify the "initial conditions". I do so by estimating the state of the Italian economy in 2011:Q4 using the particle filter. The evaluation of LTROs is conducted from an *ex-ante* perspective.

## 6.1 Sovereign Risk, Firms' Borrowing Costs and Output

What were the effects of sovereign credit risk on the financing premia of firms and on real economic activity in Italy? What was the relative strength of the leverage-constraint channel and the risk channel in driving this propagation? In order to answer these questions, I conduct a counterfactual experiment. First, I use the particle filter to extract the historical sequence of shocks for the Italian economy. Second, I feed the model with counterfactual trajectories for these shocks: these are equivalent to the estimated ones, with the exception that the innovations to s_t are set to 0 for the entire sample. I then compare the actual and counterfactual path for a set of the model's endogenous variables. Their difference reflects the effects of sovereign risk on the variables of interest. More specifically, I use the following algorithm

Counterfactual Experiment: Let θ^i denote the i'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i, apply the particle filter to $\{\mathbf{Y}_t = [\text{GDP Growth}_t, \mu_t, \pi_t^d]\}_{t=2003:Q1}^{2011:Q4}$ and construct the densities $\{p(\mathbf{S}_t|\mathbf{Y}_t, \theta^i)\}_{t=2003:Q1}^{2011:Q4}$.

2. Sample N realizations of the state vector from $\{p(\mathbf{S}_t|\mathbf{Y}_t, \theta^i)\}_{t=2003:Q1}^{2011:Q4}$.

3. Feed into the model each of these realizations, $n \in N$, and generate a path for a set of outcome variables, $\{\mathbf{x}_t(i,n)\}_t$.

4. For each realization n, replace the sovereign risk shock with its unconditional mean. Feed the model with this counterfactual realization of the state vector and collect in $\{\mathbf{x}_t^c(i,n)\}_t$ the implied outcome variables of interest.

5. The effect of sovereign credit risk for the outcome variable x is measured as $x_t^{\text{eff}}(i,n) = x_t(i,n) - x_t^c(i,n)$.
   □

Figure 11: **Sovereign Risk, Firms' Borrowing Costs and Output**
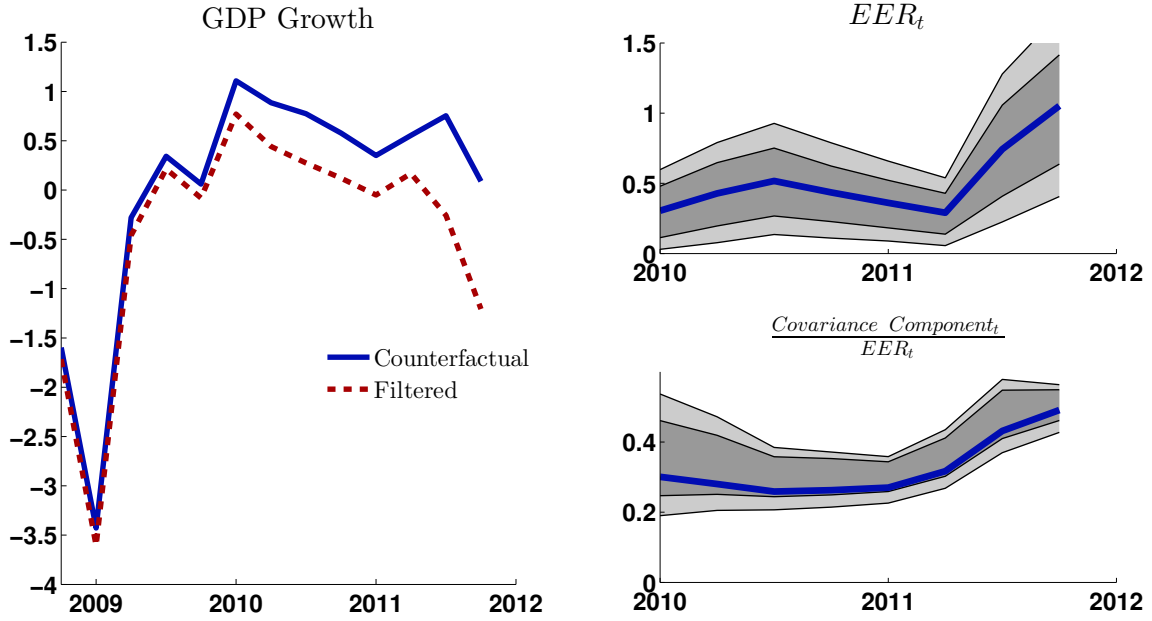
Notes: The solid line in the left panel reports the posterior mean for the filtered GDP growth series, and the dotted line reports the posterior mean of its counterfactual. The solid lines in the right panels represent the posterior mean. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets for the variables of interest.

Regarding the specifics of this experiment, I select the parameters' draws by subsampling, picking 1 of every 100. Thus, $M = 100$. In the filtering state, I set measurement errors to 0.5% of the sample variance of $\{\mathbf{Y}_t\}_t$ and I use 500,000 particles. This implies that the filtered time series $\{\mathbf{Y}_t\}_t$ are essentially equivalent to the actual data. I first analyze the effect of sovereign risk on the financing costs of firms and on output. I then decompose these effects into the two transmission mechanisms.

The left panel of Figure 11 reports the filtered and counterfactual trajectories for GDP growth while the top-right panel reports the effects of sovereign risk on expected excess returns. The rise in sovereign risk in Italy over the 2010:Q1-2011:Q4 period led to an increase in the financing costs of firms and a decline in output growth. The model predicts that expected excess returns increased by 50 basis points on average over this period, with a peak of 100 basis points in the last quarter of 2011. GDP growth would have been on average 0.5422% higher throughout the 2010-2011 period if sovereign default probabilities were fixed at their unconditional mean.

The bottom-right panel of the figure reports the covariance component defined in equation (32) as a fraction of expected excess returns. The model shows that the risk channel played quantitatively a first order role in the propagation of sovereign credit risk in Italy, and its relevance grew over time: at the end of 2011:Q4, the covariance component explains on average 47% of the effects of sovereign risk on the financing premia of firms. Table 5 reports posterior statistics for variables of interest.

Table 5: **Sovereign Risk, Firms' Borrowing Costs and Output: 2010:Q1-2011:Q4**

Statistic	Posterior Mean	90% Credible Set
Cumulative Output Losses	4.7576	[2.0890,8.0290]
Average Expected Excess Returns	0.4768	[0.1633,1.0741]
Covariance Component	0.2619	[0.1940,0.5129]

Notes: Cumulative output losses: sum of GDP growth losses (difference between counterfactual and filtered GDP growth) over the 2010:Q1-2011:Q4 period. Average expected excess returns: average difference between filtered and counterfactual expected excess return, expressed in annualized basis points. Covariance component: fraction of expected excess returns explained by the covariance component.

6.2 Longer Term Refinancing Operations

The ECB undertook several interventions in response to the euro-area sovereign debt crisis. Some of these policies were explicitly targeted toward easing the tensions in the market for bonds of distressed governments. The Security Markets Program (SMP) and the Outright Monetary Transactions (OMTs) fall within this category.[40] Other interventions, instead, had the objective of loosening the funding constraints of banks exposed to distressed government debt. The unconventional LTROs launched by the ECB in December 2011 and February 2012 were the most important in this class. Relative to canonical open market operations in Europe,[41] these interventions featured a long maturity (36 months), a fixed-interest rate (1%) and special rules for the collateral that could be used by banks. Moreover, the two LTROs were the largest refinancing operations in the history of the ECB, as more than 1 trillion euros were lent to banks through these interventions.

A full assessment of the policy is beyond the scope of this paper. LTROs, in fact, are not sterilized and the real model considered here misses this aspect. Moreover, the policy may have resulted in a reduction of sovereign credit risk, and the analysis in this paper does not capture this effect either. However, we can use the model to ask whether the provision of liquidity to banks, by itself, stimulated lending. I model LTROs as a nonstationary version of the discount window lending considered in Gertler and Kiyotaki (2010). The government gives banks the option at $t = 1$ of borrowing resources up to a threshold $\overline{m}$. These resources are financed through lump sum taxes. The loans have a fixed interest rate R_m. Banks repay the loan (principal plus interest) at a future date T and no interests are payed between t and T. Finally, the government has

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[40]In May 2010, the ECB started the SMP. Under the SMP, the ECB could intervene by buying, on secondary markets, the securities that it normally accepts as collateral. This program was extensively used for sustaining the price of government securities of southern European countries. The program was replaced by OMTs in August 2012. This latter program had two main differences compared with SMP: i) OMTs are *ex-ante* unlimited; ii) their approval is subject to a conditionality program from the requiring country.

[41]Open market operations in the euro-area are conducted through refinancing operations. These are similar to repurchase agreements: banks put acceptable collateral with the ECB and receive cash loans. Prior to 2008, there were two major types of refinancing operations: main refinancing operations (loans of a weekly maturity) and LTROs, with a three month maturity.

perfect monitoring of banks, so that these liabilities do not count for their leverage constraint.[42] Within the logic of the model, this intervention has the effect of relaxing the leverage constraint of banks, and it has a positive effect on their net worth. These two points are explained in Appendix E, along with a description of the numerical algorithm used to implement the policy.

The evaluation of LTROs is conducted using the following algorithm

Evaluating LTROs: Let θ^i denote the i'th draw from the posterior distribution of the model's parameter. For $i = 1$ to M

1. Conditional on θ^i, sample N realizations of the state vector from $p(\mathbf{S}_{t=2011:Q4}|\mathbf{Y}^T, \theta^i)$.

2. For each $\{\mathbf{S}^n_{t=2011:Q4}\}_n$, simulate the model forward J times with and without the policy intervention.

3. For each outcome variable x, compute the difference between these two paths $x^{\text{eff}}_t(i, n, j) = x^{\text{ltro}}_t(i, n, j) - x^{\text{no ltro}}_t(i, n, j)$. Collect these paths in $\mathbf{x}^{\text{eff}}_t(i, n, j)$. □

The density $p(\mathbf{S}_{t=2011:Q4}|\mathbf{Y}^T, \theta^i)$ is computed using the particle filter. The vector of variables $\{\mathbf{x}^{\text{eff}}_t(i, n, j)\}_t$ denotes the effect of the policy on variable x. The results of this experiment can be interpreted as an *ex-ante* evaluation of the policy, since I am conditioning on retrospective estimates for the state vector in the 2011:Q4 period. In order to make the experiment more realistic, I calibrate the policy to the actual ECB intervention. I set $R_m = 1.00$, $T = 12$ and $\overline{m} = 0.1\hat{Y}^{ss}$.

As a benchmark, I first discuss the forecasted path for GDP growth in absence of the policy. The left panel of Figure 12 reports the posterior median of the model's forecast for GDP growth in absence of the policy along with its 60% and 90% credible set. The model predicts a "risky" recovery for GDP growth from the 2011:Q4 point of view. While on average GDP growth returns to its trend value by 2013, we can see a long left tail in these forecasts, especially in the early part of 2012. That is, the model indicates some probability that economic outcomes substantially worsen in the absence of the policy.[43]

The right panel of Figure 12 shows how LTROs influence these forecasts. I use box plots to describe the predictive densities $p(\text{GDP Growth}_{T+h}|\mathbf{Y}^T)$ with and without LTROs for $h = \{1, 6, 11\}$. The box stands for the interquartile range, the line within the box is the median while the circle represents the mean. The refinancing operations have a clear positive effect on GDP growth in the first quarter of 2012. Indeed, the median forecast for GDP growth under the policy is 0.5% while in its absence is 0.16%. More strikingly, the policy removes most of the downside risk: the left tail of the predictive density for GDP growth in 2012:Q1

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[42]If that were not the case, the loans would perfectly crowd out households' deposits by construction: see Gertler and Kiyotaki (2010).

[43]The long left tail is induced by two factors: i) the asymmetries induced by the leverage constraint; ii) the probability of a sovereign default.

Figure 12: **Ex-Ante Assessment of LTROs**

GDP Growth (Without LTROs) $p(\text{GDP Growth}_{T+h}|\mathbf{Y}^T)$
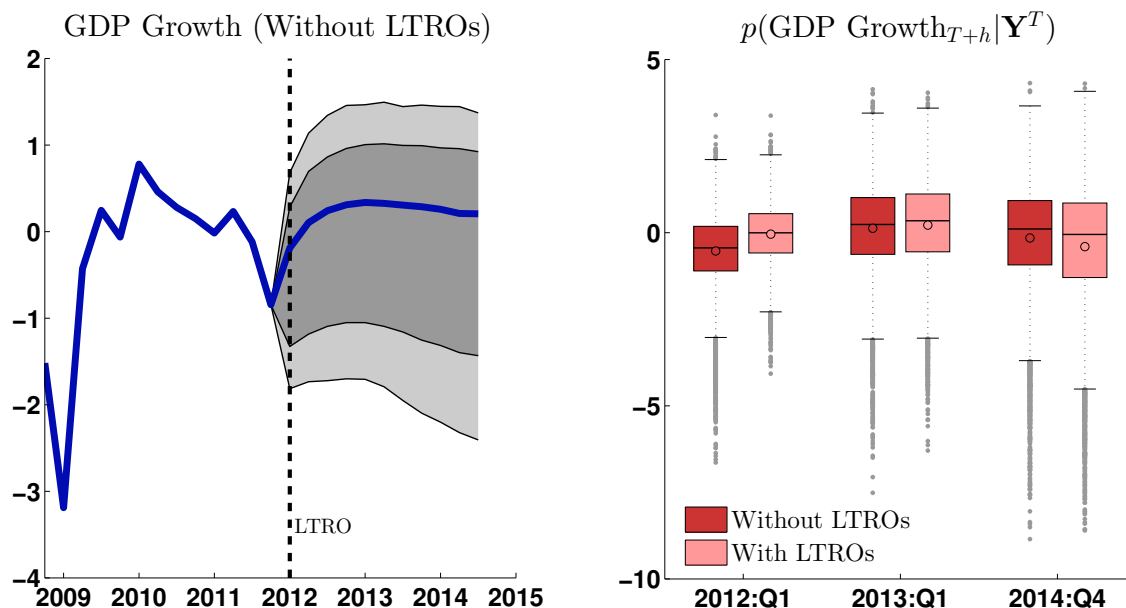
Notes: The solid line in the left panel is the conditional mean forecasts of the GDP growth time series from 2012:Q1 to 2014:Q4. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets. The right panels reports the predictive densities for GDP growth with and without LTROs (box plots).

almost disappears. This happens because the policy increases the maturity of banks' liabilities, which makes their balance sheet less sensitive to adverse shocks. As time goes on and the repayment date approaches, though, GDP growth forecasts under LTROs become fairly similar to those in absence of this policy. At the scheduled repayment date, the predictive density for GDP growth is actually more left-skewed relative to that obtained in absence of the policy.

Figure 13 reports posterior statistics on the policy effects for the level of output, expected excess returns and their decomposition into multiplier and covariance components. The policy lowers expected excess returns on impact and most of these effects are due to looser funding constraints of banks. The covariance component is barely affected by refinancing operations at early stages because of the reasons discussed in Section 5.3.

These initial positive effects are reversed over time. Starting from 2013:Q1, the model places a probability of at least 20% on LTROs increasing the financing costs of firms and reducing the level of output. This result, which may appear paradoxical, is driven by the behavior of the model at the repayment stage. In 2014:Q4, banks need to repay the loans they took on and adverse net worth shocks at that date are very costly for them. The anticipation of the repayment stage makes banks more cautious *ex-ante* and leads them to demand higher compensation for risk. This counteracts the initial positive effects of the policy. Under certain circumstances, this second effect may dominate and lead to an increase in expected excess returns and to a decline in output relative to the no-policy benchmark. Overall, these results suggest

Figure 13: **Effects of LTROs on Output and Expected Excess Returns**
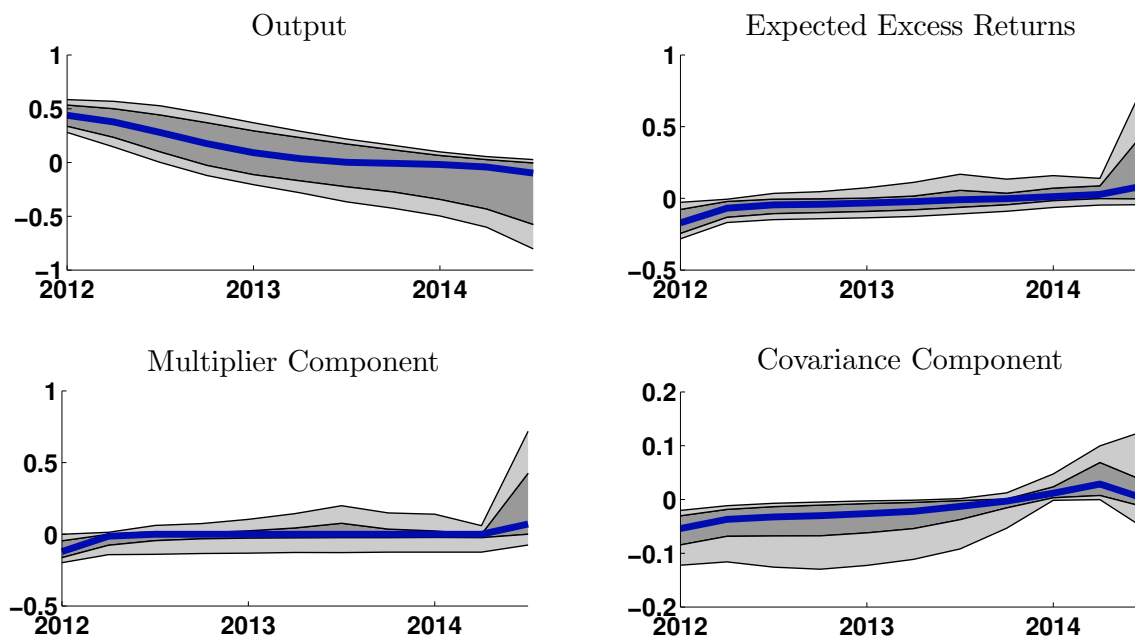
Notes: The solid line reports the posterior mean of the expected policy effects on the time series 2012:Q1 to 2014:Q4. The Dark and light shaded area represents, respectively, a 60% and 90% equal tail probability credible sets. Output is linearly detrended and expressed in percentages. The other variables are expressesed in annualized basis points.

that refinancing operations are forecasted to be quite ineffective in stimulating real economic activity if we condition to empirically reasonable regions of the state space in 2011:Q4.

This result does not imply that refinancing operations are a bad policy instrument. Rather, that their effects depend on the economic environment in which they are implemented. In order to see this last point, I implement LTROs in a different region of the state space, drawn from the density $p(\mathbf{S}_{t=2008:Q3}|\mathbf{Y}^T, \theta^i)$. In contrast to the 2011:Q4 period, the model interprets the financial distress of 2008:Q3 as driven mainly by banks liquidity problems. Table 6 reports the effects of the policy on output and expected excess returns on impact and at the repayment stage. Two main differences stand out compared to the previous analysis. First, the policy has a substantially stronger effect on the financing premia of firms and on output when implemented in 2008:Q3. Expected excess returns decline on impact by 79 basis points while the level of output increases by 0.52%. Second, the downside risk at the repayment stage is substantially reduced. This can be seen by comparing the credible sets for output at the repayment stage for the two cases.

The reasons underlying this state dependence are related to the discussion of equity injections in Section 5.3. In 2008:Q3, agency costs are estimated to be high. This indicates that there are profitable investment opportunities in the economy and banks use the funds from the LTROs to lend to firms. General equilibrium, then, generates a positive loop: the market value of firms' claims is positively influenced by higher demand for capital, and this strengthens bank net worth. As a result, banks arrive at the repayment stage with a

Table 6: **Effects of LTROs on Impact and at Repayment: 2008:Q3 vs. 2011:Q4**

	2008:Q3		2011:Q4	
Statistic	Impact	Repayment	Impact	Repayment
Output	0.52	0.01	0.34	-0.01
	[0.29,0.73]	[-0.24,0.15]	[0.11,0.56]	[-0.86,0.12]
Expected Excess Returns	-0.79	-0.01	-0.35	0.11
	[-1.30,-0.26]	[-0.39,0.25]	[-0.71,-0.01]	[-0.2054,2.14]

Notes: Posterior statistics on the effects of LTROs on output and expected excess returns on impact (period 1) and at the repayment stage (period 12). The first two columns initialize the state vector at $p(\mathbf{S}_{t=2008:Q3}|\mathbf{Y}^T,\theta^i)$. The last two columns initialize the vector at $p(\mathbf{S}_{t=2011:Q4}|\mathbf{Y}^T,\theta^i)$.

buffer that makes their balance sheet less sensitive to adverse shocks. These general equilibrium effects are, instead, muted when implementing the policy in 2011:Q4.

7   Conclusion

In this paper I have conducted a quantitative analysis of the transmission of sovereign credit risk to the borrowing costs of firms and real economic activity. I studied a model where banks are exposed to risky government debt and they are the main source of finance for firms. An increase in the probability of a sovereign default has negative effects on credit markets through two channels. First, by reducing the market value of government securities, higher sovereign risk reduces the net worth of banks and hampers their funding ability: their increased financing costs pass-through into the borrowing rates of firms (leverage-constraint channel). Second, an increase in the probability of a sovereign default raises the risks associated with lending to firms: if the default occurs in the future, in fact, claims on the productive sector will pay out little and banks will have to absorb these losses. I referred to this second mechanisms as the risk channel. The structural estimation of the model on Italian data suggests that the sovereign debt crisis significantly increased the financing premia of firms, with the risk channel explaining up to 47% of these effects. Moreover, the rise in the probability of a sovereign default had severe adverse consequences for the Italian economy: cumulative output losses were 4.75% at the end of 2011. In counterfactual experiments, I use the estimated model to evaluate the policy response adopted by the ECB, with particular emphasis on the LTROs of the first quarter of 2012. The model estimates that these interventions have minor effects on lending and output. This happens because risk premia, which were sizable when the policy was enacted, discourage banks' lending to firms. More generally, the analysis shows that the stabilization properties of these interventions are state dependent in the model, and their aggregate effects depend on the relative strength of the leverage-constraint channel and of the risk-channel.

There are a number of dimensions in which the model could be extended. The most important is to allow sovereign default risk to respond to macroeconomic conditions. This could be done in different ways, for example by introducing distortionary taxation in the model and considering the optimal default policy of a Ramsey government. Incorporating these aspects would allow for a more complete evaluation of policy responses adopted by the ECB. A second extension would be that of considering an open economy. I believe this dimension would help the empirical identification of the mechanisms discussed in this paper, since they are likely to generate differential implications for international capital flows. While both of these issues are challenging, and require a substantial departure from this framework, they represents exciting opportunities for future work.

Abstracting from the current application, recent research advocates the use of indicators of credit spreads as observables when estimating quantitative models with financial intermediation. This paper adds to that by underscoring the importance of measuring the *sources* driving the movements in these indicators of financial stress. Understanding whether firms' financing premia during crises are high because of "frictions" in financial markets or because of fair compensation for increased risk is a key information for policy makers. Incorporating the nonlinearities emphasized in this paper in larger scale models used for policy evaluation is technically challenging. Moreover, given the policy relevance of these nonlinearities, there is a need for developing tools for their empirical validation in the data. I plan to address these issues in future work.

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Online Appendix: The Pass-Through of Sovereign Risk

Luigi Bocola

A Derivation of Results 1

Combine equation (3) and (4) to eliminate the demand for deposits from the decision problem of the banker. The decision problem is then

$$
v_b(n; \mathbf{S}) = \max_{a_B, a_K} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v_b(n'; \mathbf{S}') \right] \right\},
$$

$$
n' = \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + R(\mathbf{S})n,
$$

$$
\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right] \leq v_b(n; \mathbf{S}),
$$

$$
\mathbf{S}' = \boldsymbol{\Gamma}(\mathbf{S}).
$$

Guess that the value function is $v(n, \mathbf{S}) = \alpha(\mathbf{S})n$. Necessary and sufficient conditions for an optimum are

$$
\mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi) + \psi \alpha(\mathbf{S}') \right] \left[R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S}) \right] \right\} = \lambda \mu(\mathbf{S}) \quad j = \{B, K\}, \tag{A.1}
$$

$$
\mu(\mathbf{S}) \left(\alpha(\mathbf{S})n - \lambda \sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j \right) = 0. \tag{A.2}
$$

Substituting the guess in the dynamic program, and using the law of motion for n', we obtain

$$
v_b(n, \mathbf{S}) = \max_{a_B, a_K} \left\{ \sum_{j=\{B,K\}} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')][R_j(\mathbf{S}') - R(\mathbf{S})] \right\} Q_j(\mathbf{S}) a_j \right\} +
$$

$$
\mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi \alpha(\mathbf{S}')] \right\} R(\mathbf{S})n.
$$

Note that the first term on the right hand side of the above equation equals $\mu(\mathbf{S})\alpha(\mathbf{S})n$. Indeed, when the leverage constraint does not bind ($\mu(\mathbf{S}) = 0$), expected discounted returns on assets held by bankers equal the discounted risk free rate by equation (A.1). This implies that the term equals 0. When the constraint

binds ($\mu(\mathbf{S}) > 0$), instead, this term can be written as

$$\lambda\mu(\mathbf{S}) \sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j.$$

Using the complementary slackness condition in (A.2), we can express $\lambda\mu(\mathbf{S}) \sum_{j=\{b,k\}} Q_j(\mathbf{S})a_j$ as $\mu(\mathbf{S})\alpha(\mathbf{S})n$. Thus, the value function under the guess takes the following form:

$$\alpha(\mathbf{S})n = \mu(\mathbf{S})\alpha(\mathbf{S})n + \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')] \right\} R(\mathbf{S})n.$$

Solving for $\alpha(\mathbf{S})$, we obtain

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]\}R(\mathbf{S})}{1 - \mu(\mathbf{S})}.$$

The guess is verified if $\mu(\mathbf{S}) < 1$. From equation (A.2) we obtain:

$$\mu(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S})]\}R(\mathbf{S})n}{\lambda \left(\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \right)}, 0 \right\} < 1.$$

Finally, notice that financial leverage equals across bankers whenever $\mu(\mathbf{S}) > 0$. This implies that $\frac{n}{\lambda \sum_{j=\{B,K\}} Q_j a_j}$ is equal to $\frac{N}{\lambda\left[\sum_{j=\{B,K\}} Q_j A_j\right]}$ when the constraint binds.

B Numerical Solution

B.1 Equilibrium Conditions

The state variables of the model are $\mathbf{S} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s, d]$. The control variables $\{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ solve the residual equations

$$E_{\mathbf{S}}\left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'} R(\mathbf{S})\right] - 1 = 0, \tag{A.3}$$

$$\mathbb{E}_{\mathbf{S}}\left\{\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'}[(1-\psi) + \psi\alpha(\mathbf{S}')]\left[\frac{(1-\delta)Q_K(\mathbf{S}') + \alpha\frac{\tilde{Y}(\mathbf{S}')}{\tilde{K}'(\mathbf{S})}e^{\Delta z'}}{Q_K(\mathbf{S})}\right]\right\} - \lambda\mu(\mathbf{S}) = 0, \tag{A.4}$$

$$\mathbb{E}_{\mathbf{S}}\left\{\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'}[(1-\psi) + \psi\alpha(\mathbf{S}')]\left[1 - d'D\right]\left[\frac{\pi + (1-\pi)\left[\iota + Q_b(\mathbf{S}')\right]}{Q_B(\mathbf{S})}\right]\right\} - \lambda\mu(\mathbf{S}) = 0, \tag{A.5}$$

$$\alpha(\mathbf{S}) - \frac{(1-\psi) + \psi R(\mathbf{S})\mathbb{E}_{\mathbf{S}}\left[\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'}\alpha(\mathbf{S}')\right]}{1 - \mu(\mathbf{S})} = 0, \tag{A.6}$$

where $Q_K(\mathbf{S})$ is the market value of the capital stock and the multiplier $\mu(\mathbf{S})$ is given by

$$\mu(\mathbf{S}) = \max\left\{1 - \left[\frac{\mathbb{E}_{\mathbf{S}}\left\{\beta \frac{\tilde{C}(\mathbf{S})}{\tilde{C}(\mathbf{S}')} e^{-\Delta z'}[(1-\psi) + \psi\alpha(\mathbf{S}')]R(\mathbf{S})\right\}\tilde{N}(\mathbf{S})}{\lambda[Q_K(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_B(\mathbf{S})\tilde{B}(\mathbf{S})]}\right], 0\right\}. \tag{A.7}$$

The endogenous state variables $[\tilde{K}, \tilde{B}, \tilde{P}]$ evolve as follows

$$\tilde{K}'(\mathbf{S}) = \left\{(1-\delta)\tilde{K} + \Phi\left[e^{\Delta z}\left(\frac{\tilde{Y}(\mathbf{S})(1 - e^g) - \tilde{C}(\mathbf{S})}{\tilde{K}}\right)\right]\tilde{K}\right\}e^{-\Delta z}, \tag{A.8}$$

$$\tilde{B}'(\mathbf{S}) = \frac{[1 - dD]\{\pi + (1-\pi)[\iota + Q_B(\mathbf{S})]\}\tilde{B}e^{-\Delta z} + \tilde{Y}(\mathbf{S})\left[g - \left(t^* + \gamma_\tau \frac{B}{\tilde{Y}(\mathbf{S})}\right)\right]}{Q_B(\mathbf{S})}, \tag{A.9}$$

$$\tilde{P}'(\mathbf{S}) = R(\mathbf{S})[Q_k(\mathbf{S})\tilde{K}'(\mathbf{S}) + Q_b(\mathbf{S})\tilde{B}'(\mathbf{S}) - \tilde{N}(\mathbf{S})]. \tag{A.10}$$

The state variable $\tilde{P}$ measures the detrended *cum* interest deposits that bankers pay to households at the beginning of the period and it is sufficient to keep track of the evolution of aggregate bankers' net worth.

Indeed, the aggregate net worth of banks can be expressed as

$$\tilde{N}(\mathbf{S}) = \psi \left\{ \left[Q_K(\mathbf{S}) + \alpha \frac{\tilde{Y}(\mathbf{S})}{\tilde{K}} e^{\Delta z} \right] \tilde{K} + [1 - dD] \left[\pi + (1 - \pi) \left[\iota + Q_B(\mathbf{S}) \right] \right] \tilde{B} - \tilde{P} \right\}$$

$$\text{(A.11)}$$

$$+ \omega [Q_K(\mathbf{S}) \tilde{K} + Q_B(\mathbf{S}) \tilde{B}].$$

Using the intratemporal Euler equation of the household, we can express detrended output as

$$\tilde{Y}(\mathbf{S}) = \left[\chi^{-1} \frac{(\tilde{K} e^{-\Delta z})^\alpha}{\tilde{C}(\mathbf{S})} \right]^{\frac{1-\alpha}{\alpha + \nu - 1}} (K e^{-\Delta z})^\alpha. \qquad \text{(A.12)}$$

The exogenous state variables $[\Delta z, \log(g), s]$ evolve as follows

$$\Delta z' = (1 - \rho_z)\gamma + \rho_z \Delta z + \sigma_z \varepsilon_z, \qquad \text{(A.13)}$$

$$g' = (1 - \rho_g)g^* + \rho_g g + \sigma_g \varepsilon_g, \qquad \text{(A.14)}$$

$$s' = (1 - \rho_s)s^* + \rho_s s + \sigma_s \varepsilon_s, \qquad \text{(A.15)}$$

while d follows

$$d' = \begin{cases} 1 & \text{with probability} \quad \frac{e^s}{1+e^s} \\ 0 & \text{with probability} \quad 1 - \frac{e^s}{1+e^s}. \end{cases} \qquad \text{(A.16)}$$

It will be convenient to express detrended state and control variables as log-deviations from their deterministic steady state. I denote this transformation for variable x as $\hat{x}$.

B.2 Algorithm for Numerical Solution

I approximate the control variables of the model using piece-wise smooth functions, parametrized by $\gamma = \{\gamma^x_{d=0}, \gamma^x_{d=1}\}_{x=\{\hat{C}, \hat{\alpha}, \hat{Q}_b, \hat{R}\}}$. The law of motion for a control variable x is described by

$$x(d, \hat{\mathbf{S}}) = (1 - d)\gamma^x_{d=0}{}'\mathbf{T}(\hat{\mathbf{S}}) + d\gamma^x_{d=1}{}'\mathbf{T}(\hat{\mathbf{S}}), \qquad \text{(A.17)}$$

where $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta\hat{z}, \hat{g}, \hat{s}]$ and $\mathbf{T}(.)$ is a vector collecting Chebyshev's polynomials.[44] Define $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ to be a 4×1 vector collecting the left hand side of the residual equations (A.3)-(A.6) for the candidate solution γ^c evaluated at $\{d, \hat{\mathbf{S}}\}$. The numerical solution of the model consists in choosing γ^c so that $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\}) = 0$

---

[44]The shocks are expressed as deviation from their mean.

for a set of collocation points $\{d, \hat{\mathbf{S}}^i\} \in \{0,1\} \times \mathcal{S}$.

The choice of collocation points and of the associated Chebyshev's polynomials follows Krueger et al. (2010). The rule for computing conditional expectations when evaluating $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}\})$ follows Judd et al. (2011). To give an example of this latter, suppose we wish to compute $\mathbb{E}_{d,\mathbf{S}^i}[y(d', \hat{\mathbf{S}}')]$, where y is an integrand of interest.[45] Given a candidate solution γ^c, we can compute y at every collocation point using the model's equilibrium conditions. Next, we can construct an implied policy function for y, $\{\gamma^y_{d=0}, \gamma^y_{d=1}\}$, via a Chebyshev's regression. Using the law of total probability, the conditional expectation of interest can be expressed as

$$\mathbb{E}_{d,\mathbf{S}^i}[y(d', \hat{\mathbf{S}}')] = (1 - \text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\})\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma^y_{d=0}{}'\mathbf{T}(\hat{\mathbf{S}}')] + \text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\}\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma^y_{d=1}{}'\mathbf{T}(\hat{\mathbf{S}}')], \quad \text{(A.18)}$$

where $\text{Prob}\{d' = 1|\hat{\mathbf{S}}^i\} = \frac{e^{s^i}}{1+e^{s^i}}$. Judd et al. (2011) propose a simple procedure to evaluate integrals of the form $\mathbb{E}_{\hat{\mathbf{S}}^i}[\gamma^y_{d=1}{}'\mathbf{T}(\hat{\mathbf{S}}')]$. In proposition 1 of their paper, they show that, under weak conditions, the expectation of a polynomial can be calculated via a linear transformation $\mathcal{I}$ of the coefficient vector γ^y_d, where $\mathcal{I}$ depends exclusively on the deep parameters of the model. The authors provide general formulas for the transformation $\mathcal{I}$.

The algorithm for the numerical solution of the model goes as follows

Step 0.A: Defining the grid and the polynomials. Set upper and lower bounds on the state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}, \Delta z, g, s]$. Given these bounds, construct a μ-level Smolyak grid and the associated Chebyshev's polynomials $\mathbf{T}(.)$ following Krueger et al. (2010).

Step 0.B: Precomputing integrals. Compute $\mathcal{I}$ using Judd et al. (2011) formulas.

Step 1: Equilibrium conditions at the candidate solution. Start with a guess for the model's policy functions γ^c. For each $(d, \hat{\mathbf{S}}^i)$, use γ^c and equation (A.17) to compute the value of control variables $\{\hat{C}(d, \hat{\mathbf{S}}^i), \hat{\alpha}(d, \hat{\mathbf{S}}^i), \hat{Q}_b(d, \hat{\mathbf{S}}^i), \hat{R}(d, \hat{\mathbf{S}}^i)\}$. Given the control variables, solve for the endogenous state variables next period using the model's equilibrium conditions. Given the value of control and state variables, compute the value of every integrand in equations (A.3)-(A.6) at $(d, \hat{\mathbf{S}}^i)$. Collect these integrands in the matrix $\mathbf{y}$.

Step 2: Evaluate conditional expectations. For each $d = \{0, 1\}$, run a Chebyshev regression for the integrand in $\mathbf{y}$, and denote by γ^y_d the implied policy function for an element $y \in \mathbf{y}$. Conditional expectations are calculated using equation (A.18) and the matrix $\mathcal{I}$.

Step 3: Evaluate residual equations. Given conditional expectations, compute the multiplier

---

[45]For example, y could be $\frac{e^{-\Delta z'}}{C(\mathbf{S}')}$ in equation (A.3).

using equation (A.7). Evaluate the residual equations $\mathcal{R}(\gamma^c, \{d, \hat{\mathbf{S}}^i\})$ at every collocation point $(d, \hat{\mathbf{S}}^i)$. The dimension of the vector of residuals equals 4 times the cardinality of the state space. Denote by r the Euclidean norm for this vector.

Step 4: Iteration. If $r \leq 10^{-20}$, stop the algorithm. Else, update the guess and repeat Step 1-4. $\square$

The specific for the algorithm are as follows

Choice of bounds. The bounds on $[\Delta\hat{z}, \hat{g}]$ are +/- 3 standard deviations from their mean. The bounds on $\hat{s}$ are larger and set to $[-5, +5]$. The bounds on the endogenous state variables $\hat{\mathbf{S}} = [\hat{K}, \hat{P}, \hat{B}]$ are set to +/- 4.5 their standard deviations in the model without sovereign risk. The standard deviation is calculated by simulating a third order perturbation of the model without sovereign risk. A desirable extention of the algorithm is to select different bounds depending on whether the economy is in a default state or not.

Smolyak Grid. For tractability, I choose $\mu = 3$ for the Smolyak grid. This implies that I have 389 distinct points in $\mathcal{S}$.

Precomputation of Integrals. I use Gaussian numerical quadrature for computing the matrix $\mathcal{I}$.

Iterative Algorithm. I find the zeros of the residual equation using a variant of the Newton algorithm. To speed up computations, numerical derivatives are computed in parallel.

I denote the model solution by γ^*.

B.3 Accuracy of Numerical Solution

I check the accuracy of the numerical solution by computing the errors of the residual equations (Judd, 1992). More specifically, I proceed as follows. First, I simulate the model forward for 5000 periods. This gives a simulation for the state variable of the model $\{d_t, \hat{\mathbf{S}}_t\}_{t=1}^{5000}$. Second, for each pair $(d_t, \hat{\mathbf{S}}_t)$, I calculate the errors of the residual equations $\mathcal{R}(\gamma^*, \{d_t, \hat{\mathbf{S}}_t\})$. As an example, let's consider equation (A.4). Then, the residual error at $(d_t, \hat{\mathbf{S}}_t)$ for this equation is defined as

$$1 - \frac{\mathbb{E}_{\mathbf{d_t}, \hat{\mathbf{S}}_\mathbf{t}}\left\{\beta\frac{\tilde{C}(d_t, \hat{\mathbf{S}}_t)}{\tilde{C}(\mathbf{S}')}e^{-\Delta z'}[(1-\psi) + \psi\alpha(\mathbf{S}')]\left[\frac{(1-\delta)Q_K(\mathbf{S}')+\alpha\frac{\tilde{Y}(\mathbf{S}')}{\tilde{K}'(d_t, \hat{\mathbf{S}}_t)}e^{\Delta z'}}{Q_K(d_t, \hat{\mathbf{S}}_t)}\right]\right\}}{\lambda\mu(d_t, \hat{\mathbf{S}}_t)},$$

where the model's policy functions are used to generate the value for endogenous variables at $d_t, \hat{\mathbf{S}}_t$. Note that, by construction, the residual errors are zero at the collocation points. This residual equation errors provide a measure of how large are the discrepancy between the decision rule derived from the numerical

Figure A-1: **Residual Equations Errors**
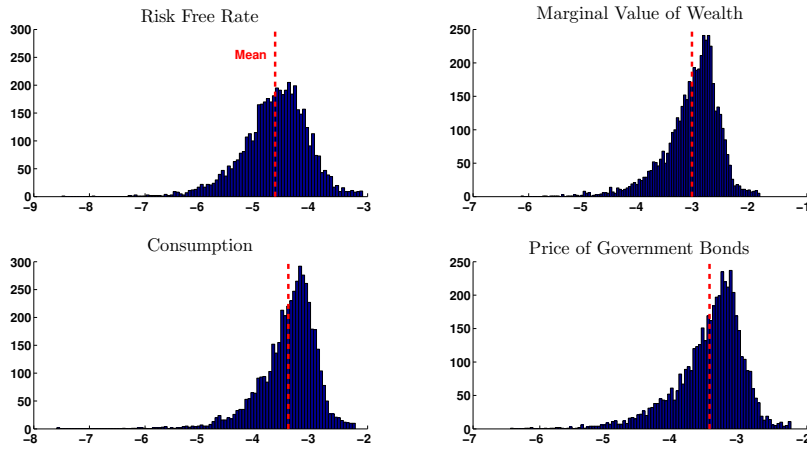
Notes: The histograms reports the residual equations errors in decimal log basis. The dotted line marks the mean residual equation error.

algorithm and those implied by the model's equilibrium condition in other points of the state space. Following standard practice, I report the decimal log of the absolute value of these residual errors. Figure A-1 below reports the density (histogram) of those errors.

On average, residual equation errors are in the order of -4.75 for the risk free rate, -3.5 for consumption and the price of government securities and -3 for the marginal value of wealth. These numbers are comparable to values reported in the literature for models of similar complexity, and they are still very reasonable. Figure A-2 reports residual equation errors for a sequence of states $\{\mathbf{S}_t\}_{t=2004:Q1}^{2011:Q4}$ extracted using the particle filter (See Section 4). The figure shows that residual equation errors are reasonable in empirically relevant region of the state space.

Figure A-2: **Residual Equations Errors: Empirically Relevant Region**
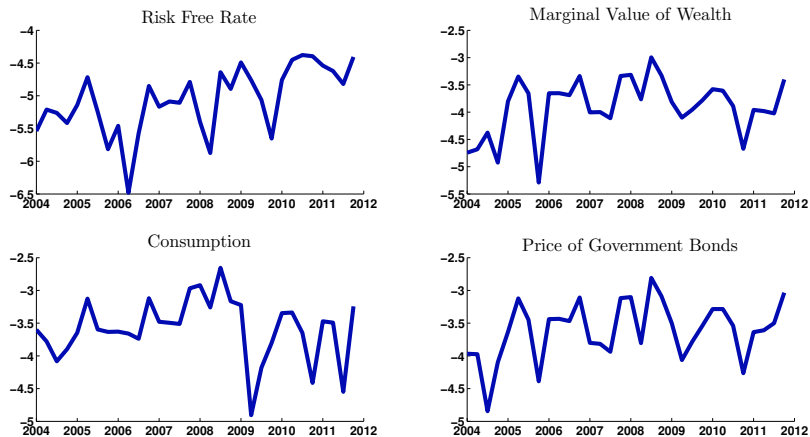
C Data Source

C.1 Credit Default Swap (CDS) Spread

Daily CDS spreads on 5 years Italian governmet securities (RED code: 4AB951). The restructuring clause of the contract is CR (complete restructuring). The spread is denominated in basis points and paid quarterly. The source is Markit, accessed from the Wharton Research Data Services.

C.2 Banks' exposure to the Italian government

The European Banking Authority (EBA) published information on holdings of government debt by European banks participating to the 2011 stress test. Five Italian banks were in this pool: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena (MPS), Banco Popolare (BPI) and Unione di Banche Italiane (UBI). Results of the stress test for each of these five banks are available at `http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results`. I measure exposure of each bank to Italian central and local government as gross direct long exposure (accounting value gross of specific provisions). This information is available by maturity of financial instrument, and it reflects positions as of 31st of December 2010. I match these data on exposure with end of 2010 total financial assets for each of the five institutions. This latter information is obtained using consolidated banking data from Bankscope, accessed from the Wharton Research Data Service. Table A-1 reports these information.

Table A-1: **Exposure to Domestic Sovereign by Major Italian Banks: End of 2010**

	3Mo	1Yr	2Yr	3Yr	5Yr	10Yr	15Yr	Tot.	Tot. Assets
Intesa	17.71	9.86	2.82	5.16	8.16	6.64	9.77	60.15	658.76
Unicredit	17.97	10.14	3.04	6.21	4.47	6.39	0.87	49.07	929.49
MPS	5.67	4.99	4.00	3.58	1.45	3.75	9.02	32.47	240.70
BPI	3.89	1.65	1.14	3.64	0.78	0.40	0.25	11.77	134.17
UBI	1.33	3.56	0.30	0.31	0.72	2.53	1.76	10.54	129.80
Total	46.57	30.02	11.30	18.9	13.76	19.71	21.67	164.00	2092.99

Notes: Data is reported in billions of euros.

C.3   Construction of the Multiplier

Result 2. *In equilibrium, the multiplier on the incentive constraint of bankers is a function of financial leverage and of the spread between a risk free security traded by bankers and the risk free rate*

$$\mu_t = \frac{\left[\frac{R_t^f - R_t}{R_t}\right] lev_t}{1 + \left[\frac{R_t^f - R_t}{R_t}\right] lev_t}. \tag{A.19}$$

Proof. Since the asset is risk free, one has that $\text{cov}_t(\hat{\Lambda}_{t+1}, R_{t+1}^f) = 0$. Therefore, using equation (25) in the main text, one has:

$$\left[\frac{R_t^f - R_t}{R_t}\right] = \frac{\lambda}{\alpha_t}\frac{\mu_t}{1 - \mu_t}.$$

Equation (A.19) follows from the fact that $\frac{\alpha_t}{\lambda}$ equals financial leverage when $\mu_t > 0$. □

Essentially, Result 2 tells us that the agency friction can be interpreted as a markup on financial interme- diation. To measure this markup, one needs to focus on returns on assets, traded only by bankers, that have the same risk properties of households' deposits. I use the prime interbank rate to measure R_t^f since the model of Section 2 can be interpreted as having a frictionless interbank market as in Gertler and Kiyotaki (2010). The time series used in the construction of the multiplier are the following

Financial Leverage: The definition of financial leverage in the model is banks' equity divided by the market value of total assets. I use quarterly data from the Italian flow of funds (*Conti Finanziari*) to construct these two time series. First, I match banks in the model with Monetary and Financial Institutions (MFIs). This category includes commercial banks, money market funds and the domestic central bank. I use balance sheet information for the Bank of Italy to exclude the latter from this pool. Second, I construct a time series for bank equity as the difference between "total assets" and total debt liabilities. This latter is defined as "total liabilities" minus "shares and other equities" (liabilities) and "mutual fund shares" (liabilities). Financial leverage is the ratio between equity and total assets. Data can be downloaded at `http://bip.bancaditalia.it/4972unix/`. See Bartiloro et al. (2003) for a description of the Italian flow of funds.

TED Spread: The prime interbank rate (EURIBOR 1yrs.) is obtained from the ECB Statistical Data Warehouse, under *Market Indexes* in the section *Monetary and Financial Statistics*. The frequency of obser- vation is monthly. Data can be downloaded at `http://sdw.ecb.europa.eu/`. I match the model's risk free

rate with the yields on German bonds with a 1 year maturity. A monthly time series for this latter is obtained from the time series database of the Deutsche Bundesbank. Data can be downloaded at http://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/time_series_databases.html. I construct a quarterly measure of the TED spreads by averaging the computed series over three months.

C.4 Other Time Series

GDP growth: Real GDP growth is the growth rate relative to previous quarter of real gross domestic product (*B1_EG*). Data are quarterly, 1980:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Consumption growth: Consumption growth is the growth rate relative to previous quarter of real private final consumption expenditure (*P31S14_S15*). Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Spending-Output Ratio: Spending-output ratio is general government final consumption expenditure divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Investment-Output Ratio: Investment-output ratio is gross capital formation divided by gross domestic product. Both series are seasonally adjusted and in volume estimates. Data are quarterly, 1991:Q1-2012:Q4. The source is *OECD Quarterly National Accounts*.

Labor income share: I obtain annual data (1970-2007) for and labor compensation (*LAB*) and value added (*VA*) from EU KLEMS database. Labor share is defined as $\frac{LAB}{VA}$. Data can be downloaded at http://www.euklems.net/.

Worked hours: Average numbers of hours worked per year by person engaged. I scale the series by $(24 - 8) \times 7 \times 52$. Data are annual (1970-2007), and obtained from EU KLEMS. Data can be downloaded at http://www.euklems.net/.

D Estimating the Model without Sovereign Risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{P}_t, \hat{B}_t, \Delta z_t, g_t]$. Let $\mathbf{Y}_t$ be a 2×1 vector of observables collecting output growth and the time series for the multiplier on the leverage constraint. The state-space representation is

$$\mathbf{Y}_t = f_{\tilde{\theta}}(\mathbf{S}_t) + \eta_t \qquad \eta_t \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}) \tag{A.20}$$

$$\mathbf{S}_t = g_{\tilde{\theta}}(\mathbf{S}_{t-1}, \varepsilon_t) \qquad \varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \tag{A.21}$$

The first equation is the measurement equation, with η_t being a vector of Gaussian measurement errors. The second equation is the transition equation, which represents the law of motion for the model's state variables. The vector ε_t represents the innovation to the structural shocks Δz_t and g_t. The function $f_{\tilde{\theta}}(.)$ and $g_{\tilde{\theta}}(.)$ are generated using the numerical procedure described in Appendix B applied to the model without sovereign risk. I characterize the posterior distribution of $\tilde{\theta}$ using full information Bayesian methods. I denote by $p(\tilde{\theta})$ the prior on $\tilde{\theta}$. In what follows, I provide details on the likelihood evaluation and on the posterior sampler adopted.

D.1 Likelihood Evaluation

Let $\mathbf{Y}^t = [\mathbf{Y}_1, \ldots, \mathbf{Y}_t]$, and denote by $p(\mathbf{S}_t|\mathbf{Y}^{t-1}; \theta)$ the conditional distribution of the state vector given observations up to period $t-1$. The likelihood function for the state-space model of interest can be expressed as

$$\mathcal{L}(\mathbf{Y}^T|\theta) = \prod_{t=1}^{T} p(\mathbf{Y}_t|\mathbf{Y}^{t-1}; \theta) = \prod_{t=1}^{T} \left[\int p(\mathbf{Y}_t|\mathbf{S}_t; \theta) p(\mathbf{S}_t|\mathbf{Y}^{t-1}; \theta) d\mathbf{S}_t \right]. \tag{A.22}$$

While the conditional density of $\mathbf{Y}_t$ given $\mathbf{S}_t$ is known and Gaussian, there is no analytical expression for the density $p(\mathbf{S}_t|\mathbf{Y}^{t-1}, \theta)$. I use the auxiliary particle filter of Pitt and Shephard (1999) to approximate this density via a set of pairs $\{S_t^i, \tilde{\pi}_t^i\}_{i=1}^{N}$. This approximation is then used to estimate the likelihood function.

Step 0: Initialization. Set $t = 1$. Initialize $\{S_0^i, \tilde{\pi}_0^i\}_{i=1}^{N}$ from the model's ergodic distribution and set $\tilde{\pi}_0^i = \frac{1}{N} \; \forall i$.

Step 1: Prediction. For each $i = 1, \ldots, N$, draw $\mathbf{S}_{t|t-1}^i$ values from the proposal density $g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)$.

Step 2: Filtering. Assign to each $\mathbf{S}_{t|t-1}^i$ the particle weight

$$\pi_t^i = \frac{p(\mathbf{Y}_t|\mathbf{S}_{t|t-1}^i; \theta) p(\mathbf{S}_t|\mathbf{S}_{t|t-1}^i; \theta)}{g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)}.$$

Step 3: Sampling. Rescale the particles $\{\pi_t^i\}$ so that they add up to unity, and denote these rescaled values by $\{\tilde{\pi}_t^i\}$. Sample N values for the state vector with replacement from $\{\mathbf{S}_{t|t-1}^i, \tilde{\pi}_t^i\}_{i=1}^N$. Call each draw $\{\mathbf{S}_t^i\}$. If $t < T$, set $t = t + 1$ and go to Step 1. Else, stop. $\square$

The likelihood function of the model is then approximated as

$$\mathcal{L}(\mathbf{Y}^T|\theta) \approx \frac{1}{N} \left(\prod_{t=1}^T \left[\frac{1}{N} \sum_{i=1}^N p(\mathbf{Y}_t|\mathbf{S}_{t|t-1}^i; \theta) \right] \right).$$

Regarding the tuning of the filter, I set $N = 20000$. The matrix $\boldsymbol{\Sigma}$ is diagonal, and the diagonal elements equal 25% of the variance of the observable variables. The choice for the proposal density $g(\mathbf{S}_t|\mathbf{Y}^t, \mathbf{S}_{t-1}^i)$ is more involved. I sample the structural innovations ε_t from $\mathcal{N}(m_t, \mathbf{I})$. Then, I use the model's transition equation (A.21) to obtain $\mathbf{S}_{t|t-1}^i$. The center for the proposal distribution for ε_t is generated as follows:

- Let $\overline{\mathbf{S}}_{t-1}$ be the mean for $\{\mathbf{S}_{t-1}^i\}$ over i.

- Set m_t to the solution of this optimization program

$$\text{argmin}_\varepsilon \left\{ [\mathbf{Y}_t - f_{\tilde{\theta}}(g_{\tilde{\theta}}(\overline{\mathbf{S}}_{t-1}, \varepsilon))]'[\mathbf{Y}_t - f_{\tilde{\theta}}(g_{\tilde{\theta}}(\overline{\mathbf{S}}_{t-1}, \varepsilon))] + \varepsilon'\boldsymbol{\Sigma}^{-1}\varepsilon \right\}.$$

The first part of the objective function pushes ε toward values such that the state vector can rationalize the observation $\mathbf{Y}_t$. The second part of the objective function imposes a penalty for ε that are far away from their high density regions. I verify that this proposal density results in substantial efficiency gains relative to the canonical particle filter, especially when the model tries to fit extreme observations for $\mathbf{Y}_t$.

D.2 Posterior Sampler

I characterize the posterior density of $\tilde{\theta}$ using a Random Walk Metropolis Hastings with proposal density given by

$$q(\tilde{\theta}^p|\tilde{\theta}^{m-1}) \sim \mathcal{N}(\tilde{\theta}^{m-1}, c\mathbf{H}).$$

The sequence of draws $\{\tilde{\theta}^m\}$ is generated as follows

1. Initialize the chain at $\tilde{\theta}^1$.

2. For $m = 2, \ldots, M$, draw $\tilde{\theta}^p$ from $q(\tilde{\theta}^p|\tilde{\theta}^{m-1})$. The jump from $\tilde{\theta}^{m-1}$ to $\tilde{\theta}^p$ is accepted ($\tilde{\theta}^m = \tilde{\theta}^p$) with probability $\min\{1, r(\tilde{\theta}^{m-1}, \tilde{\theta}^p|\mathbf{Y}^T)\}$, and rejected otherwise ($\tilde{\theta}^m = \tilde{\theta}^{m-1}$). The probability of accepting the draw is

$$r(\tilde{\theta}^{m-1}, \tilde{\theta}^p | \mathbf{Y}^T) = \frac{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^p) p(\tilde{\theta}^p)}{\mathcal{L}(\mathbf{Y}^T | \tilde{\theta}^{m-1}) p(\tilde{\theta}^{m-1})}.$$

First, I run the chain for $M = 10000$ with $\mathbf{H}$ being the identity matrix and $c = 0.001$. The chain is initialized from an estimation of the model using the Method of Simulated Moments.[46] I drop the first 5000 draws, and I use the remaining draws to initialize a second chain and to construct a new candidate density. This second chain is initialized at the mean of the 5000 draws. Moreover, the variance-covariance matrix $\mathbf{H}$ is set to the empirical variance-covariance matrix of these 5000 draws. The parameter c is fine tuned to obtain an acceptance rate of roughly 60%. I run the second chain for $M = 20000$. Posterior statistics are based on the latter 10000 draws.

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[46]The moments used in this step are: i) mean, standard deviation and autocorrelation for GDP growth and the multiplier; ii) correlation between GDP growth and the multiplier.

E Policy Experiments

E.1 Refinancing Operations

It is instructive to first consider the stationary problem. The government allows bankers to borrow up to $\overline{m}$ at the fixed interest rate R_m, and this intervention is financed through lump-sum taxation. Moreover, these loans are not subject to limited enforcement problems. The decision problem of the banker becomes

$$
\begin{aligned}
v_b(n; \mathbf{S}) &= \max_{a_B, a_K, b} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v_b(n'; \mathbf{S}') \right] \right\}, \\
n' &= \sum_{j=\{B,K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + [R_m - R(\mathbf{S})]m - R(\mathbf{S})b, \\
\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j &= n + b, \\
\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j - m \right] &\leq v_b(n; \mathbf{S}), \\
m &\in [0, \overline{m}], \\
\mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}).
\end{aligned}
$$

Assuming that $m \geq 0$ does not bind,[47] the first order condition with respect to m is

$$
\mathbb{E}_s \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi) + \psi \frac{\partial v_b(n'; \mathbf{S})}{\partial n'} \right] \right\} [R(\mathbf{S}) - R_m] + \lambda \mu(\mathbf{S}) = \chi(\mathbf{S})
$$

It can be showed, following a similar logic of Result 1, that $v_b(n; \mathbf{S}) = \alpha(\mathbf{S})n + x(\mathbf{S})$, with $x(\mathbf{S}) \geq 0$. The leverage constraint becomes

$$
\frac{\sum_j Q_j(\mathbf{S}) a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda} + \frac{x(\mathbf{S})}{\lambda} + \overline{m}
$$

Notice that refinancing operations have two main direct effects on banks. First, they represent an implicit transfer to banks. Indeed, to the extent that $(R_m < R(\mathbf{S}))$, banks benefit from the policy as their debt is subsidized. This has a positive effect on the net worth of banks relative to what would happen in the no-policy case. Second, the policy relaxes the leverage constraint of banks. This happens because of two distinct reasons: i) the loan from the government does not enter in the computation of the constrained level of leverage (the $\overline{m}$ component); and ii) the value function of bankers increase as a result of the subsidized

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[47] This is not a restriction, as the policy considered involves an R_m substantially below R, meaning that bankers are willing to accept the loan.

loan, this lowering the incentives of the banker to walk away.

E.2 Longer Term Refinancing Operations (LTROs)

The LTROs are a non-stationary version of the refinancing operations discussed above. The government allows banker to borrow up to $\overline{m}$ in period $t = 1$, and they receive the pricipal and the interest in a later period T. Figure A-3 describes the timing of transfers between government and banks under LTROs.

Figure A-3: **Timing of LTROs**
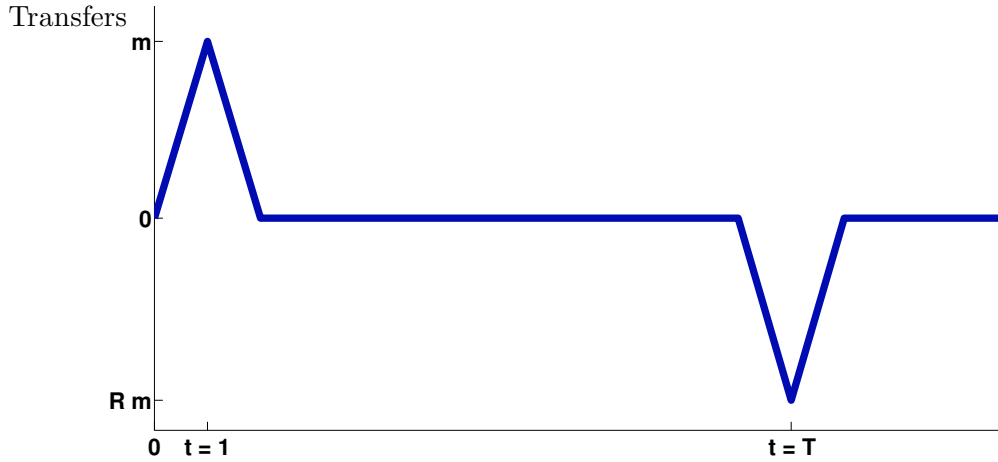
I assume that the policy was unexpected by agents. At time $t = 1$, agents are perfectly informed about the time path of the loans and they believe that the policy will not be implemented in the future. Note that the decision rules under LTROs are time dependent: the dynamics at $t = 1$ will be different from those at $t = T - 1$ as in the latter case we are getting closer to the repayement stage and banks will have a different behavior. In order to solve for the path of model's decision rules, I follow a backward induction procedure. From period $t = T + 1$ onward, the decision rules are those in absence of policy. Thus, at $t = T$, agents use those decision rules to form expectations. By solving the equilibrium conditions under this assumption and the repayment of the loan, we can obtain decision rules for $c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), Q_{B,T}(\mathbf{S})$. At $t = T - 1$ we proceed in the same way, this time using $c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), Q_{B,T}(\mathbf{S})$ to form expectations. More specifically, the policy functions in the transition $\{c_t(\mathbf{S}), R_t(\mathbf{S}), \alpha_t(\mathbf{S}), Q_{b,t}(\mathbf{S})\}_{t=1}^{T}$, are derived as follows:

1. **Period** T: Solve the model using $\{c(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), q(\mathbf{S})\}$ to form expectations. The multiplier is modified as follows

$$\mu_T(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}}\{\Lambda_{T+1}(\mathbf{S}')[(1 - \psi) + \psi\alpha_{T+1}(\mathbf{S})]\}R_T(\mathbf{S})(N' - m)}{\lambda \left(Q_{b,T}(\mathbf{S})B_T' + Q_{k,T}(\mathbf{S})K_T'\right)}, 0 \right\}$$

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Denote the solution by $\{c_T(\mathbf{S}), R_T(\mathbf{S}), \alpha_T(\mathbf{S}), q_T(\mathbf{S})\}$.

2. **Period** $t = T - 1, \ldots, 1$: Solve the model using $\{c_{t+1}(\mathbf{S}), R_{t+1}(\mathbf{S}), \alpha_{t+1}(\mathbf{S}), q_{t+1}(\mathbf{S})\}$ to form expectations. The multiplier is modified as follows

$$\mu_t(\mathbf{S}) = \max\left\{ \frac{\lambda(TA_t(\mathbf{S}) - m\mathbf{1}_{t=1}) - \mathbb{E}_{\mathbf{S}}\{\Lambda_{t+1}(\mathbf{S}')[(1-\psi) + \psi\alpha_{t+1}(\mathbf{S})]\}R_t(\mathbf{S})(N' + m\mathbf{1}_{t=1}) - \psi\mathbb{E}_{\mathbf{S}}[\Lambda_{t+1}(\mathbf{S}')\alpha_{t+1}(\mathbf{S}')x_{t+1}(\mathbf{S}')]}{\lambda\left(Q_{b,t}(\mathbf{S})B_t' + Q_{k,t}(\mathbf{S})K_t'\right)}, 0 \right\}$$

where x_t follows the recursion $x_t(\mathbf{S}) = \frac{\lambda m\mu_t(\mathbf{S}) + \psi\mathbb{E}_S[\Lambda_{t+1}(\mathbf{S}')\alpha_{t+1}(\mathbf{S}')x_{t+1}(\mathbf{S}')]}{1 - \mu_t(\mathbf{S})}$. The initial condition of this recursion is $x_T(\mathbf{S}) = -\alpha_T(\mathbf{S})m$. Store the solution $\{c_t(\mathbf{S}), R_t(\mathbf{S}), \alpha_t(\mathbf{S}), q_t(\mathbf{S}), x_t(\mathbf{S})\}$. $\square$