IUMPST: The Journal, Vol 1 (Content Knowledge), October 2007. [www.k-12prep.math.ttu.edu]

THE PEDAGOGICAL CONTENT KNOWLEDGE IN MATHEMATICS: PRE-SERVICE PRIMARY MATHEMATICS TEACHERS' PERSPECTIVES IN TURKEY

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Abstract

The purpose of this research was to determine the pre-service primary mathematics teachers' competency of pedagogical content knowledge in mathematics. The data were collected by means of four open ended problems from the participation of 45 primary mathematics teacher candidates. Teacher candidates' responses were analyzed based on pre-determined criteria. According to findings it was found that having a deep understanding of mathematical knowledge was necessary but not sufficient to teach mathematics. This finding pointed out the connection between knowledge of mathematics and knowledge of mathematics teaching. It is suggested that primary mathematics teacher candidates should be educated both from "mathematics knowledge" and "pedagogical content knowledge" aspects.

Key words: Pedagogical content knowledge, mathematics teacher education

Introduction

A number of factors may influence the teaching of mathematics but teachers play an important role in the teaching process. The common belief in society is if a mathematics teacher knows mathematics very well, he or she is the best person to teach mathematics. But what about "knowing to teach mathematics"? Fennema and Franke (1992) determined the components of mathematics teachers' knowledge as;

1) Knowledge of mathematics
    - Content knowledge
        - The nature of mathematics
        - The mental organization of teacher knowledge
2) Knowledge of mathematical representations
3) Knowledge of students
    - Knowledge of students' cognitions
4) Knowledge of teaching and decision making

The first item is about having conceptual understanding of mathematics. Fennema and Franke (1992) argue that if a teacher has a conceptual understanding of mathematics, this influences classroom instruction in a positive way; therefore, it is important to have mathematics knowledge for teachers. Teachers' interrelated knowledge is very important as well as procedural rules. They also emphasize the importance of knowledge of mathematical representations, because mathematics is seen as a composition of a large set of highly related abstractions. Fennema and Franke (1992: 153) state that 'if teachers do not know how to translate those abstractions into a form that enables learners to relate the mathematics to what they already know, they will not learn with understanding'.

Knowledge of students' cognitions is seen as one of the important components of teacher knowledge, because, according to Fennema and Franke (1992), learning is based on what happens in the classroom, and thus, not only what students do, but also the learning environment is important for learning. The last component of teacher knowledge is "knowledge of teaching and decision making". Teachers' beliefs, knowledge, judgments, and thoughts have an effect on the decisions they make which influence their plans and actions in the classroom (Fennema and Franke, 1992).

Knowledge of mathematics and knowledge of mathematical representations are related to content knowledge, while knowledge of students and knowledge of teaching are related to pedagogical content knowledge. Shulman (1995) defines *content knowledge* as the knowledge about the subject, for example mathematics and its structure. According to Shulman (1995: 130), pedagogical content knowledge includes,

> 'the ways of representing and formulating the subject that make it comprehensible to others'… 'an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons'.

Based on Shulman's (1987) notion of pedagogical content knowledge, effective teachers can possess an in-depth knowledge of how to represent the subject matter to learners (Parker & Heywood, 2000). Shulman (1987) also stated that pedagogical content knowledge must include the knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes and values, and their philosophical and historical bases. Additionally, pedagogical content knowledge refers to the ability of the teacher to transform content into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students (Shulman, 1987, cited in An, Kulm and Wu, 2004).

According to An, Kulm, and Wu (2004) pedagogical content knowledge has three components:

- Knowledge of content
- Knowledge of curriculum
- Knowledge of teaching

The interrelations of these components are shown in Figure 1. An, Kulm and Wu (2004) point out the importance of knowledge of teaching and they accept it as the core component of pedagogical content knowledge. To sum up, as Grouws and Schultz (1996) stated 'pedagogical content knowledge includes, but is not limited to, useful representations, unifying ideas, clarifying examples and counter examples, helpful analogies, important relationships, and connections among ideas (Grouws and Schultz, 1996: 443)'.

According to Shulman (1986), mathematical content knowledge and pedagogical content knowledge are integrated parts of effective mathematics instruction. In order to construct mathematical concepts in students' mind, pedagogical knowledge as well as mathematical content knowledge is needed. The manner in which teachers relate their subject matter (what they know about what they teach) to their pedagogical knowledge (what they know about teaching) and how subject matter knowledge is a part of the process of pedagogical reasoning are seen as integrants of pedagogical content knowledge (Cochran, DeRuiter & King, 1993). Most researchers point out the importance of mathematical content knowledge as well as pedagogical knowledge. According to Kahan, Cooper and Bethea's (2003) review, the researchers frequently conclude that 'students would learn more mathematics if their teachers knew more mathematics but content knowledge in the subject area does not suffice for good teaching (p.223)'. However, they also outlined that the content of pedagogical content knowledge is 'content-specific and at the same time goes beyond simple knowledge of mathematics therefore a mathematician may not posses it (Kahan, Cooper and Bethea, 2003: 223)'.

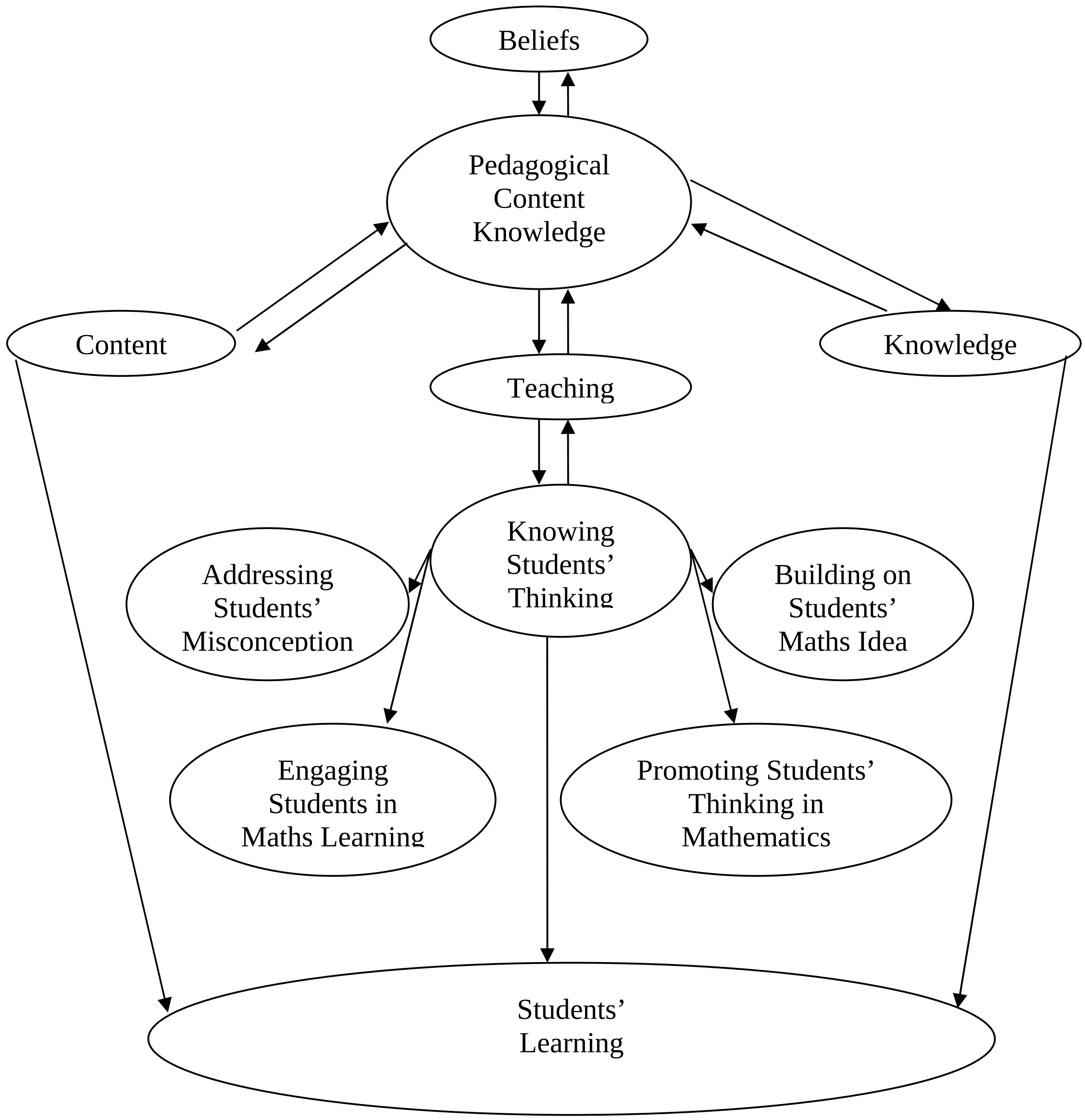


Figure 1.
The network of pedagogical content knowledge
(adapted from An, Kulm and Wu, 2004: 147)

A number of studies investigated different aspects of content knowledge and pedagogical content knowledge. Jones and Moreland (2004) described the frameworks and cognitive tools that have been developed to enhance practicing teachers' pedagogical content knowledge in primary school technology education. Daehler and Shinohara (2001) explored the potential of science teaching cases to deepen teachers' content knowledge and pedagogical content knowledge. An, Kulm and Wu (2004) compared the teachers' pedagogical content knowledge of mathematics in U.S. and Chinese middle schools. McDuffy (2004) examined the reflective practices of two elementary pre-service teachers during their student teaching internship and found limits in pedagogical content knowledge and lack of confidence impeding the pre-service teachers' reflection while in the act of teaching. Stacey et al. (2001) investigated pre-service elementary school teachers' content knowledge and pedagogical content knowledge of decimal numeration. They asked the teacher candidates to complete a decimal comparison test, mark items they thought would be difficult for students, and explain why. Results pointed to the need for teacher education to emphasize content knowledge that integrates different aspects of number knowledge, and pedagogical content knowledge that includes a thorough understanding of common difficulties. Sánchez and Llinares (2003) attempted to identify the influence of teacher candidates' subject matter knowledge for teaching on the process of pedagogical reasoning. Their findings showed that the four teacher candidates in the study differed in their subject-matter knowledge for teaching both in the different aspects of concepts they emphasized and in the use of a representation repertoire to structure learning activities.

This paper aimed to discuss pedagogical content knowledge from the pre-service primary mathematics teachers' perspective in Turkey. It was also purposed to investigate the interrelationship between mathematical knowledge and pedagogical content knowledge. To this end, how student teachers use their mathematical knowledge and pedagogical content knowledge in dealing with some teaching problems that involve assessing pupils' ways of thinking and ways of constructing mathematical knowledge was also examined.

Methods

Participants

Participants were 45 primary mathematics teacher candidates studying in the last year of their education in a university, in Turkey, and taking teaching methods course. They have taken several courses throughout their university education. These are related to mathematical knowledge and pedagogical knowledge. These are given briefly in Table 1.

Table 1.
Courses taken by primary mathematics teacher candidates

Mathematical Knowledge		Pedagogical Knowledge	
Course	**Credit**	**Course**	**Credit**
Calculus I,II,	5	Introduction to Teaching Profession	3
Calculus III, IV	4	School Experience in Elementary Education I,II	3
Geometry	3	Development and Learning	3
Abstract Mathematics	3	Instructional Planning and Evaluation	4
Linear Algebra I, II	3	Instructional Development and Media in Math. Education	3
Probability and Statistics I, II	3	Methods of Mathematics Teaching I,II	3
Analytical Geometry	3	Textbook Analysis in Math. Education	3
Algebra	3	Practice Teaching in Elementary Education	5
Number Theory	3	Guidance	3
		Classroom Management	3

Instrument

Data were collected through mathematical in-class problems. Four problems were asked in order to reveal the student teachers' approaches to teaching mathematics in topics of fractions, decimal numbers and integers (see Figure 2). Each of the problems fundamentally focused on teacher candidates' interpretations of students' misconceptions or misunderstandings of mathematical knowledge. In general, expectations from student teachers were; understanding students' current conceptions/reasoning, understanding the reason(s) of students' current reasoning, creating solutions to eliminate students' false reasoning, being able to ask appropriate questions to understand students' thought, forming appropriate criteria for assessment and assessing students' answers according to these criteria.

Problem 1

The following conversation took place among Aslı, Seda, Ali and Murat who are 8th grade students.

Aslı: Is -1/2 a proper fraction or an improper fraction?

Seda: In my opinion, since -1 is smaller than 2, it is a proper fraction.

Ali: -1/2 and - (1/2) are the same, aren't they? If ½ is a proper fraction - ½ is a proper fraction as well.

Murat: - ½ and 1/-2 are the same. Numerator is bigger than denominator, so it is an improper fraction.

-What might each of the student's be thinking?

-Why, do you think, Aslı has this kind of question?

-What can be done to overcome the pupils' misconceptions about fractions?

Problem 2

Orcun is a 7th grade student. The dialog between Orcun and his teacher is presented below.

Orcun: 5 minus 3 equal 2.

Teacher: Why do you think like this?

Orcun: I had five apples. I ate three of them. So I have two apples left.

Teacher: What is the result of -3 + 5?

Orcun: -3 + 5 is - 8.

Teacher: How did you do it?

Orcun: 3 plus 5 is 8. The sum has the sign of the first integer.

-What prerequisite knowledge might Orcun not have?

-What kind of questions can be asked to Orcun to understand his misconception?

-What kind of real world activity can be done to help him?

Problem 3

Hale is a 6th grade student. She does the multiplication of fractions as below.

Question: $\frac{1}{4} . \frac{1}{5} = ?$

I. Step $\underset{(5)}{\frac{1}{4}} . \underset{(4)}{\frac{1}{5}} = \frac{5}{20} . \frac{4}{20}$ II. Step $\frac{5}{20} . \frac{4}{20} = \frac{20}{400}$ III. Step $\frac{20}{400} = \frac{1}{20}$

-Discuss Hale's thought process.

-Is Hale's answer wrong? Why/why not?

-Determine assessment criteria for her answer.

-What is Hale's mark according to your criteria?

Problem 4

Serdar's teacher wanted Serdar to round 0.976 two times. Serdar does this task as below:

0.97$\underline{6}$	(6 is bigger than 5, so I add 1 to 7)	0.98
0.9$\underline{8}$	(8 is bigger than 5, so I add 1 to 9)	0.10

-Discuss Serdar's thought process.

-What kind of approach can be used to correct his understanding?

Figure2.
Mathematical in-class problems

Data Analyses

Data which were collected from teacher candidates were examined both quantitatively and qualitatively. In quantitative analyses, some criteria were determined according to the components of pedagogical content knowledge and student teachers' answers were assessed in terms of them. There were 14 criteria totally. Criteria for problem 1 and problem 4 are:

- Understanding students' misconception
- Understanding the reason(s) of students' misconception
- Creating solutions to remove students' misconception

Criteria for problem 2 are:

- Understanding students' misconception
- Understanding the reason(s) of students' misconception
- Creating solutions to remove students' misconception
- Asking appropriate questions to reveal misconception

Criteria for problem 3 are:

- Understanding students' reasoning
- Understanding the basis of student's reasoning
- Forming appropriate criterion for assessment
- Assessing students' answer in correct manner

In quantitative analyses 3 points were given for completely correct answers of problems, 2 points were given for correct answers which lacked enough explanation and 1 point was given to wrong answers. Because there were 14 criteria in total, the highest score was 42 and the lowest score was 14. Twenty-one points was determined as the beginning score for level 2, by calculating half of the highest score to be taken. Student teachers who had 3 points from 8 criteria and 2 points from 6 criteria were determined as the beginning score for level 3. Teacher candidates' total points were calculated. Scores between 42-36 were determined as Level 3 (Excellent), scores between 35-21 were determined as Level 2 (Mediocre) and scores between 20-14 were determined as Level 1 (Insufficient). Teacher candidates' total scores were calculated for assessing their performance on all 4 problems combined and they were interpreted according to following levels:

Level 3 (Excellent)

- Understanding students' difficulties and understanding the reasons for students difficulties
- Being able to ask proper and meaningful questions in order to understand their thought process.
- Having the ability to create solutions to overcome students' learning difficulties.
- Forming appropriate criteria for assessment and assessing students' answers according to these criteria.

Level 2 (Mediocre)

- Understanding students' difficulties and understanding the reasons for students difficulties
- Failing to ask proper and meaningful questions to understand their thought process
- Having difficulty to create solutions to misconceptions
- Having difficulty to form appropriate criteria for assessment and assessing students' answers according to criteria.

Level 1 (Insufficient)

- Having difficulty to understand both students' difficulties and the reasons for students' difficulties
- Neither being able to understand students' thought process with questions nor having the ability to create solutions to students' learning difficulties
- Having difficulty to form appropriate criteria for assessment and not assessing students' answers according to these criteria.

Teacher candidates' responses to each problem were also analyzed qualitatively to express their mistakes on content and pedagogical content knowledge. Examples of their own misconceptions were given and discussed separately.

Results

Percentages of teacher candidates' responses according to the categories are presented in table 2.

Table2.
Percentages of teacher candidates' responses according to categories

	Categories	Given Points					
		1 point		2 points		3 points	
		f	(%)	f	(%)	f	(%)
Problem 1	Understanding students' misconception	32	71	3	7	10	22
	Understanding the reason(s) of students' misconception	34	76	2	4	9	20
	Creating solutions to remove students' misconception	35	78	9	20	1	2
Problem 2	Understanding students' misconception	12	48	20	44	13	28
	Understanding the reason(s) of students' misconception	20	44	18	40	7	16
	Asking appropriate questions to reveal misconception	11	24	10	22	24	54
	Creating solutions to remove students' misconception	6	13	21	47	18	40
Problem 3	Understanding students' reasoning	10	22	19	42	16	36
	Understanding the basis of student's reasoning	17	38	9	20	19	42
	Forming appropriate criterion for assessment	34	76	9	20	2	4
	Assessing student's answer in correct manner	27	60	17	38	1	2
Problem 4	Understanding students' misconception	21	47	17	38	7	15
	Understanding the reason(s) of students' misconception	28	63	7	15	10	22
	Creating solutions to remove students' misconception	24	54	16	36	5	10

According to these results, teacher candidates have difficulty in determining students' misconceptions about fractions and decimal fractions. Also they do not have sufficient assessment knowledge and cannot form appropriate criteria either.

Teacher candidates were asked to create solutions to remove students' misconceptions in problem 1, problem 2 and problem 4. They produced different kinds of solutions to these problems. Percentages of teacher candidates' preferred solution types in problems 1, 2 and 4 are presented in table 3. (There are no data about problem 3, in table 3, because it wasn't requested to produce any solution in this problem.)

Table 3.
Percentages of preferred solution types

	Solution types	f	Percent (%)
Problem 1	Use of questions or tasks	5	11
	Use of rule or procedure	42	93
	Use of hands-on activity	1	2
	Use of worksheets	0	0
	Draw picture or table	11	49
	Connect to concrete model	0	0
	Give example	7	16
Problem 2	Use of questions or tasks	12	27
	Use of rule or procedure	1	2
	Use of hands-on activity	18	40
	Use of worksheets	0	0
	Draw picture or table	15	33
	Connect to concrete model	4	9
	Give example	20	44
Problem 4	Use of questions or tasks	11	49
	Use of rule or procedure	16	36
	Use of hands-on activity	2	4
	Use of worksheets	0	0
	Draw picture or table	7	16
	Connect to concrete model	2	4
	Give example	6	13

Teacher candidates tend to explain procedures or rules to pupils instead of inspiring them to discover the mathematical relations by themselves. They sometimes use questions but these questions do not give a sufficient insight to students to tackle the problem.

Student teachers' total scores were calculated. Their levels were determined according to their answers to all four problems. Teacher candidates' levels are presented in table 4.

Table4.
Teacher candidates' levels

Levels	f	Percent (%)
Level 1 (Insufficient)	8	18
Level 2 (Mediocre)	37	82
Level 3 (Excellent)	0	0
Total	45	100

No teacher candidates are at level 3, 82% are at level 2, and 18% at level 1.

Teacher candidates' responses to each problem were also analyzed qualitatively. Their responses to each problem are discussed below.

Teacher candidates' responses to 'problem 1'

Teacher candidates have difficulty in identifying what the problem is with the students' answers because they themselves do not have a deep understanding of fractions. Some of their explanations are stated below:

- *Negative fractions cannot be determined as "proper fraction" or "improper fraction".*

- No connections are constructed between integers and fractions. Only positive fractions are being emphasized in teaching fractions, not negative ones.

- Absolute values of numerator and denominator must be compared in order to decide if the fraction is proper fraction or improper fraction.

- In order to decide if $-\frac{1}{2}$ *is a proper fraction or not, we must think without taking into consideration its sign, and later the sign can be added. .*

- If the absolute value of numerator is smaller than the absolute value of denominator, the fraction is called as 'proper fraction'.

It's not possible to teach mathematics without having enough knowledge about the subject. If teachers have misconception about fractions, it's hard for them to understand students' misconceptions about fractions, and it's hard for them to create solutions to eliminate students' misconception either.

Teacher candidates' responses to 'problem 2'

Forty-four percent of the teacher candidates understand the students' misconception, while 56% of them do not understand because they have misconception about addition and subtraction of integers themselfves. They do not have the exact connection between addition and subtraction. Some of their sentences which reveal their misconceptions are stated below:

- 5-3 is an addition operation. It means "5 plus -3".

- Students must understand 5-3 and -3+5 are the same operations.

- There is no difference between 5-3 and 3-5.

- The set of integers is commutative under subtraction.

Teacher candidates who have appropriate mathematical knowledge about integers try to create solutions without understanding the reason of their misconception. Additionally they are asking questions to explain the task, not to understand their thought process. It is like giving a medicine to a patient without understanding what her/his illness is. Because of this approach, their solutions do not satisfy students' expectations.

Teacher candidates' responses to 'problem 3'

Teacher candidates' assessment abilities are evaluated by the problem 3. Their answers revealed that they have difficulty in assessing students' knowledge because of two reasons. The first reason is; they don't understand Hale's solution stages. Most of the teacher candidates mentioned that Hale's solution is not correct or Hale's result is correct but her way of solving it is not correct. Some of their comments on Hale's solution are stated below:

- Hale mixes addition and multiplication.

- Hale performs multiplication of fraction based on addition of fractions. So, she doesn't know the properties of multiplication of fractions.

- She equals the denominator although it's not necessary so she does not know the multiplication of the fractions

- Hale solved the question but the result is the same with the correct result accidentally.

- Hale's way to reach the solution is wrong (long)

Actually there is no conceptual mistake in her solution. If Hale actually confused her knowledge about addition and multiplication, her answer probably would be as below:

$$\frac{5}{20} \cdot \frac{4}{20} = \frac{20}{20}$$

But she is able to do multiplication correctly. So she does not confuse addition and multiplication. Additionally if she can construct her knowledge based on her previous knowledge, it's better to form the mathematical concept in her mind. Some of the teacher candidates stated that she found the correct result accidentally if the fractions were different she couldn't have found the correct result this way. Most of them stated that her answer is wrong because her way is too long. According to them solving a problem in a long way means it is solved wrong.

The second reason for having difficulty to assess students' knowledge is teacher candidates' lack of assessment knowledge. They could not determine the exact criteria in order to assess Hale's answer. Teacher candidates mentioned that they absolutely needed to recognize the student to assess him/her a grade. Their comments on her solution and their grade for her solution aren't consistent with each other. For example, although they thought Hale's answer was wrong, they gave Hale a high grade. Likewise although they like her answer, they tend to give her a low grade. This happens because of not forming criteria for answer. Some of their comments on assessment of Hale's solution are listed below:

- *If I knew the student's characterizations, I could have given a higher grade.*
- *If I were Hale's teacher, I would never teach something like this.*
- *Because Hale does not solve the question in the way I taught in the classroom, I think she doesn't know the multiplication of fraction.*
- *Hale solves this question in an uncommon way. Even though the solution is correct, the solution way of the question is wrong.*

Teacher candidates' responses to 'problem 4'

Teacher candidates have difficulty in determining Serdar's misconception about decimal fractions. Some of them failed to understand Serdar's problem. Some of their comments are listed below;

- *What is wrong here is expecting Serdar to round decimal fraction twice.*
- *Her misconception is; writing 0.10 instead of 0.1*
- *Serdar knows the rule about rounding decimal fraction.*
- *I think Serdar's problem is being careless not lack of knowledge.*
- *Although the result is correct, Serdar's thought process is wrong.*

They maintain that Serdar understands rounding decimal fractions. But, in order to say "students understand the mathematical concept or rule" it is necessary to see if they can apply this knowledge in different cases. Maybe, if Serdar's decimal fraction was 0.325, he could have applied the rules. But when different style is requested, he is confused. So it is not possible to say he internalized the knowledge about rounding decimal fractions properly.
However, some of the teacher candidates have correct point of view. They stated the basic reason as; "Serdar has problem about "ordering decimal fractions" and "what it means to round a decimal fraction". Teacher candidates' solutions to eliminate Serdar's misconception are focused on explaining the rule or procedure to him.

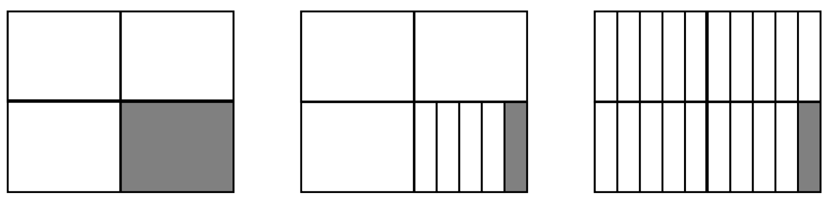
Discussion

Leinhardt and Greeno (1986: 75) considered skill in teaching to rest on two fundamental systems of knowledge, *subject matter* and *lesson structure*. The first is the knowledge of the content to be thought. In the present study, the results showed that teacher candidates do not have sufficient mathematical content knowledge. They have difficulty to understand the relationship between addition and subtraction. They mentioned 5-3 means "+5 plus -3". Although both of them have the same result, they need to know the former is +5 minus +3. If the teacher does not explain different and similar features of (+5) - (+3) and (+5) + (-3), it's not possible to teach these operations to pupils. The data also revealed that some of the student teachers don't know the properties of integers both under addition and subtraction.

The second is the knowledge required to construct and conduct a lesson (Leinhardt and Greeno, 1986). It is possible to graduate teachers who have mastered the use of manipulatives, board buddies, cooperative group learning, technology, and "good" educational resources, but who still do not appreciate how children come to know mathematics and how guided flexible discourse invites mathematical thinking (Grouws and Schultz, 1996). In this study, the teacher candidates with their mathematical knowledge try to create solutions without understanding the reason for students' misconception. Additionally they tend to ask questions to explain the task not to understand students thought processes. However it's very important to understand students' ways of thinking for teaching mathematics.

These results show that teacher candidates are not thinking about 'what they are teaching'. For example in problem 3, Hale actually is able to do multiplication of fractions. Most of them said 'numerators of fraction are multiplied and written to numerator; denominators of fractions are multiplied and written to denominator.' But when somebody asks 'why don't we equal the denominators of fraction?' teacher candidates aren't able to answer this question. Furthermore, although they don't know the reason for not equaling the denominators, they maintained that it is not correct to equal them. Actually it might be better to explain "it is not necessary to equal them" to pupils with equaling denominators. We can multiply fractions with equaling the denominators. But is it really necessary?

$\frac{1}{4} . \frac{1}{5}$ means;



Students tend to equal denominators of fractions because of their prior knowledge about addition. However it's important to emphasize the relationship between addition and multiplication. Students could be convinced why we do not need to equal them although it is not wrong.

The findings of this study are consistent with previous research results. Stanford project shows that inexperienced teachers have incomplete and superficial levels of pedagogical content knowledge (Carpenter, Fennema, Petersen and Carey, 1998; Feiman - Nemser and Parker, 1990; Shulman, 1987 cited in Cochran, DeRuiter and King, 1993). A novice teacher often relies on unmodified subject matter knowledge most often directly extracted from the text or curriculum materials and may not have coherent framework from which to present information (Cochran, DeRuiter and King, 1993). The results of the studies suggested that novice teachers are sometimes not developmentally ready to assume the roles required of them as good mathematics teachers (Brown and Borko, 1992: 232). In a number of studies, novice teachers showed evidence of growth in pedagogical content knowledge as a result of teaching and preparing to teach. According to Brown and Borko (1992)'s study, teacher candidates struggled, and sometimes failed, to come up with powerful means of representing subject areas to students. Further, their efforts are often time-consuming and inefficient (Brown and Borko, 1992). The teacher must be informed of students' ways of thinking so that his or her teaching can be based on existing mathematical conceptions and misconceptions of the students (Grouws and Schultz, 1996).

Methods instructors can influence teacher candidates' beliefs in a positive way if they consistently encourage individuals and collaborative groups to reflect on a limited but powerful set of pedagogical principles (Langrall, Thornton, Jones and Malone, 1996). The importance of these two competences can be explained to lecturers to make them emphasize these in their courses.

Conclusion

Mathematics education is a ‘science’ like pure mathematics. Although mathematics and mathematics education have a dynamic interaction with each other, they have different aspects as well. One of the most common debates among pure mathematicians and mathematics educators is ‘whether having a deep understanding of mathematics is sufficient to teach mathematics?’ In order to find an answer to this question, the divides between pure mathematics and mathematics education need to be bridged. The results of this study revealed that having a deep understanding of mathematical knowledge is necessary but not sufficient to teach mathematics. Additionally, it’s not possible to teach mathematics without having mathematical knowledge as well. Mathematics teachers must be educated both from “mathematics knowledge” and “pedagogical content knowledge” aspects in universities.

In this research primary mathematics teacher candidates’ general competency of pedagogical content knowledge was investigated. In order to get detailed information, more studies which aimed to evaluate pedagogical content knowledge of teacher candidates on specific content areas like integers, decimals can be done. In this way mathematics teacher candidates’ knowledge level on specific areas could be revealed. A cross-cultural study can be designed for searching cultural aspects of pedagogical content knowledge in mathematics.

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