THE PERFECT BOUNDARY APPROXIMATION TECHNIQUE FACING THE BIG CHALLENGE OF HIGH PRECISION FIELD COMPUTATION*

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Abstract

Computational tools for the design of accelerating structures are in use since decades. While highly accurate methods exist for quasi two dimensional cavities, fully three dimensional modeling with high precision is still a ‘big challenge’.

The most widely used computer code in this area is MAFIA, basing on the Finite Integration Theory (FIT, [1,2]). While being well known for its robustness and reliability, MAFIA nevertheless suffers somewhat from a deficiency in being able to model very complicated 3D-cavities including curved boundaries with high precision.

In this paper we present two recently developed algorithms, facing this challenge within FIT: the usage of generalized non-orthogonal computational grids (NO-FIT), and the so-called Perfect Boundary Approximation (PBA) technique. Both methods represent consistent extensions of FIT, preserving all important properties as second order accuracy and stability of the transient solver. Especially the PBA technique reveals to be a highly efficient method, as it combines easy-to-use Cartesian grids with a perfect approximation of boundaries.

We compare MAFIA with the PBA technique for typical accelerator components, and it turns out, that the PBA technique is more than one order of magnitude faster than the conventional method if many non Cartesian metallic boundaries appear inside the modeled structure.

1 INTRODUCTION

The most common disadvantage of the Finite Integration Technique in three dimensions is the usage of Yee-type [3] cartesian meshes (cf. Fig. 1a), being quite inflexible, if complex, non-orthogonal structures have to be discretized. Even with the concept of triangular fillings (cf. Fig. 1b), not only the local electric and magnetic field, but sometimes also global quantities like resonance frequencies and Q-values suffer from the modeling errors along curved boundaries.

In the field of the FDTD-method, which is equivalent to FIT for transient calculations, several approaches have been published in the last years, trying to overcome this problem. Most of these algorithms, however, either suffer from stability problems, or do not show the same high efficiency properties as FDTD referred to both memory and CPU-time requirements. In this paper we present two algorithms, which can handle curved boundaries and are not only provably stable, but also highly accurate and efficient. Included in the matrix-formalism of the FIT, these algorithms are not only applicable to RF-problems, but also to the calculation of static fields, time-harmonic fields, the interaction with charged particles, and other related problems.

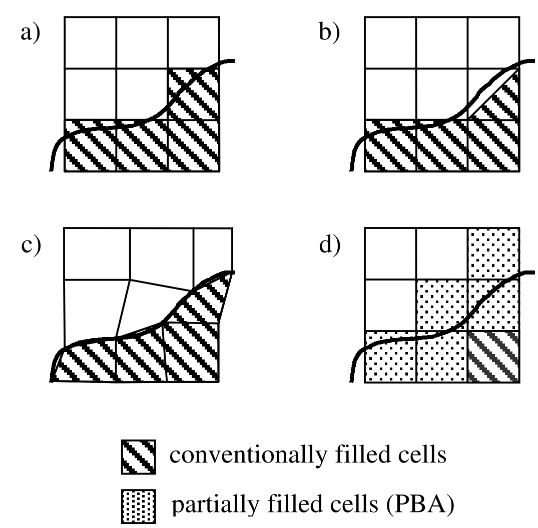

Figure 1: Grid approximations of rounded boundaries: Standard (a), triangular (b), non-orthogonal (c), PBA

2 ALGORITHMS

2.1 FIT on Non-Orthogonal Grids

The most general approach to handle curved boundaries is to allow generalized non-orthogonal grids (Fig. 1c). The basic idea to extend the FDTD-method on such grids has already been formulated in 1983 [4], including a local interpolation scheme for field components. Fulfilling the symmetry-condition in this interpolation process, we get an explicit time-stepping method with proven stability properties [5].

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The non-orthogonal algorithm has been implemented in MAFIA (experimental status), and successfully applied to several transient RF-calculations, as well as to 2D and 3D eigenvalue computations. As the method reduces to standard FIT for orthogonal meshes, interfacing orthogonal with non-orthogonal grids is trivial. The numerical cost is increased by the interpolation scheme by a factor between 2 and 3 for 2D- or 3D-problems, respectively.

2.2 Perfect Boundary Approximation Technique

The application of the non-orthogonal algorithm is sometimes limited by the increase of the numerical cost, and, last but not least, by the requirement to supply a body-fitted, structured, non-orthogonal grid.

As an even more efficient approach we implemented the Perfect Boundary Approximation (PBA) Technique. In this method, the (orthogonal) computational grid does not have to be conformal to the rounded boundaries (Fig. 1d). Instead, also sub-cellular information is taken into account, leading to an algorithm with second order accuracy for arbitrary shaped boundaries. Except for a somewhat more complicated preprocessing, there is only slightly additional numerical cost during the iteration (the factor being near to one). Moreover, the grid generation becomes very easy, as there is no need for a highly resolved mesh near by non-orthogonal shapes. In most cases, even equidistant meshes produce highly accurate results. However, adaptive mesh generation has been implemented to achieve the highest possible accuracy for a given number of mesh cells, including user defined accuracy requirements.

The application of the PBA-technique to several problems from microwave- and accelerator-components is presented in the next chapter.

3 NUMERICAL EXAMPLES

3.1 Twisted Waveguide

In the first example, the reflection parameter S11 at the input port of a twisted waveguide has been calculated. Fig. 2 shows the discrete model using a non-orthogonal mesh, and in Fig. 3 the broadband results for three grid resolutions are shown. All three curves are very close

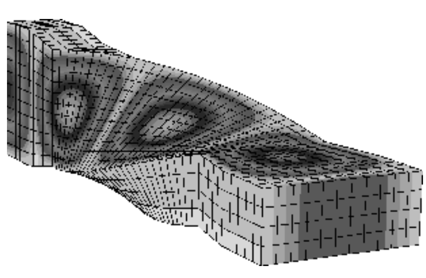

Fig. 2: Twisted Waveguide: non-orthogonal grid-model and electric field (steady state).

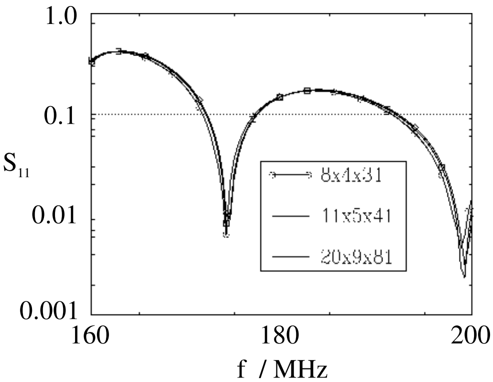

Fig. 3: Twisted Waveguide: reflection coefficient S_{11} for different grid resolutions

together. The result is already well converged for the coarsest grid.

3.2 TESLA Input Coupler

This example (cf. Fig. 4) shows a planned new input coupler for the TESLA superconducting cavities[1]. A rectangular coupler in-between two cavity cells is connected to an elliptical waveguide. A ceramic window is located

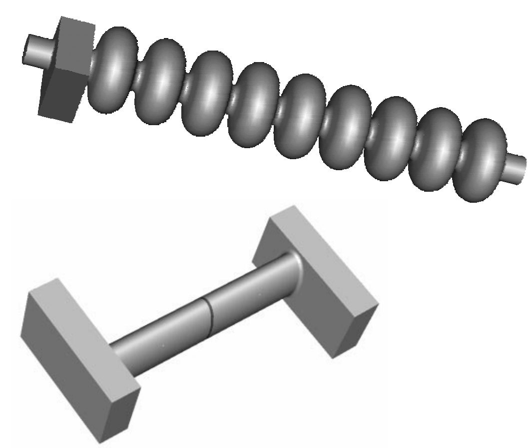

Fig. 4: TESLA 9-cell cavity with planned input coupler structure.

in the middle of the elliptical guide, followed by a second rectangular waveguide.

Fig. 5 shows the geometry of the transition of the rectangular guide to the elliptical one. The connection be-

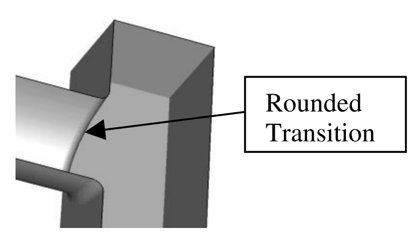

Fig. 5: Transition rectangular to elliptical waveguide.

[1] Design by M. Dohlus and A. Jöstingmeier (DESY)

tween the two waveguides was rounded with a rounding radius of 1cm.

Some more geometric details, the dielectric window and two matching rods (radius 2mm) inside the elliptic waveguide, are shown in Fig. 6.

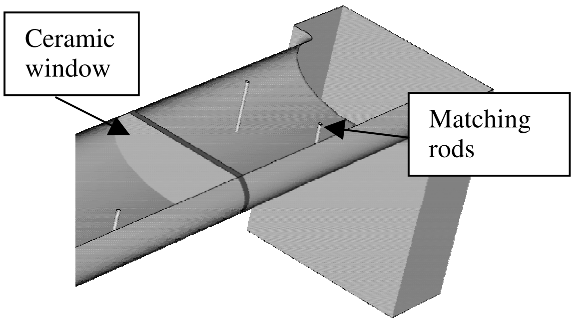

Fig. 6: Details inside the elliptical waveguide.

After a transient field simulation including an on-line DFT transformation we obtain the entire spectrum for the S-parameters, as shown in Fig. 7:

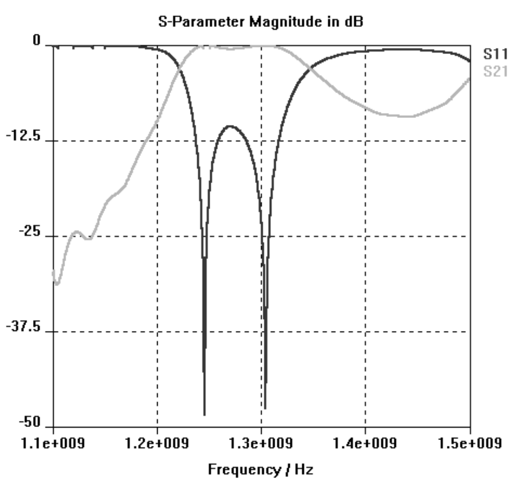

Fig. 7: S-Parameter of the Waveguide Transition.

The design shows a very good reflection property at the design frequency of 1.3 Ghz, although it was not yet optimized and was meant to serve here only as a demonstration example for the new PBA-algorithm.

3.3 Coax to Waveguide Coupler

The last example is the (mismatched) coax to waveguide coupler structure shown in Fig. 8. It has been simulated with both the new PBA-technique and standard MAFIA.

The results for the transmission coefficient demonstrate the high accuracy of the PBA-method: even the calculation with the coarsest grid resolution (10 mesh steps per wavelength) yields a S_{12}-curve close to the final result, whereas the standard method needs more than 60 steps/λ to achieve a comparable accuracy. The numerical cost of the simulation thus can be reduced by about two orders of magnitudes, which is a typical result for structures containing many non Cartesian geometrical details.

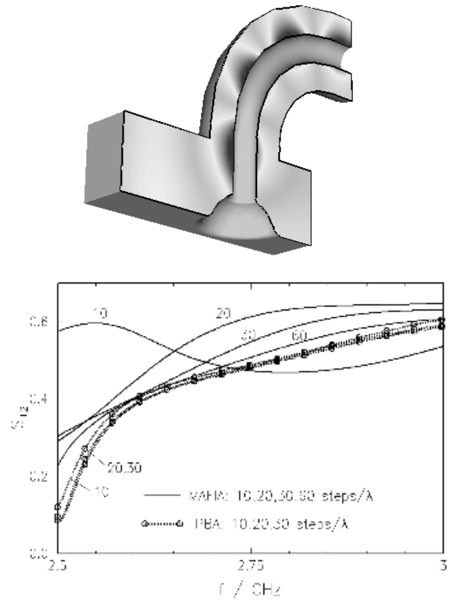

Fig. 8: Coax to waveguide coupler (with electric field in steady state), transmission coefficient for different grid resolutions (MAFIA and PBA results).

4 CONCLUSION

The Finite Integration Theory, combined with either non-orthogonal computational grids or with the newly developed Perfect Boundary Approximation Technique, is able to model structures with very fine geometric details with high accuracy. The application especially of the PBA-technique to typical accelerator devices demonstrates the high efficiency of the method compared to conventional FD- or FE-approaches.

5 REFERENCES

[1] T.Weiland, "A Discretization Method for the Solution of Maxwell's Equations for Six-Component Fields," Electronics and Communication 31, p.116 (1977).

[2] T.Weiland, "Time Domain Electromagnetic Field Computation with Finite Difference Methods," Int. Journal of Numerical Modelling 9, 295-319 (1996).

[3] K.S.Yee, "Numerical Solution of Initial Boundary Value Problems in Isotropic Media," IEEE Trans. on Antennas and Propagation AP-14, 302-307 (1966).

[4] R.Holland, "Finite-Difference Solution of Maxwell's Equations in Generalized Nonorthogonal Coordinates," IEEE Trans. on Nucl. Sciences NS-30,6, 4589-4591 (1983).

[5] R.Schuhmann, T.Weiland, "Stability of the FDTD Algorithm on Nonorthogonal Grids Related to the Spatial Interpolation Scheme," Proc. of the 11th Conf. on the Computation of Electromagnetic Fields (COMPUMAG), 211-212 (1997).