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THE PERSISTENCE OF VOLATILITY AND STOCK MARKET FLUCTUATIONS

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ABSTRACT

This paper examines the potential influence of changing volatility in stock market prices on the level of stock market prices. It demonstrates that volatility is only weakly serially correlated, implying that shocks to volatility do not persist. These shocks can therefore have only a small impact on stock market prices, since changes in volatility affect expected required rates of return for relatively short intervals. These findings lead us to be skeptical of recent claims that the stock market's poor performance during the 1970's can be explained by volatility-induced increases in risk premia.

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This paper examines the potential influence of changing stock market volatility on the level of stock market prices. It demonstrates that volatility is only weakly serially correlated, implying that shocks to volatility do not persist. These shocks can therefore have only a small impact on stock market prices, since changes in volatility affect expected required rates of return for only short intervals. These findings lead us to be skeptical of recent claims that the stock market's poor performance during the 1970's can be explained by volatility-induced increases in risk premia, as suggested by Malkiel (1979) and Pindyck (1984). They also lead us to doubt that fluctuations in risk premia associated with changing return volatility can account for much of the observed variation in stock prices. The finding that volatility is not highly serially correlated is puzzling in light of Black's (1976) observation that stock market returns and changes in volatility are negatively correlated.

The paper is divided into five sections. The first clarifies the theoretical relationship between return volatility, the level of share prices, and required rates of return. The second section examines the time series properties of stock market volatility as measured using both monthly and daily data. The results suggest that although volatility is serially correlated, changes in current volatility should have only a negligible impact on volatility forecasts over intervals as short as one or two years. The third and fourth sections use data on the implied volatilities in option premia to re-examine the persistence question, and again find evidence of only weak serial correlation. The conclusion discusses the implications of our results for alternative explanations of recent stock market movements and for our understanding of the sources of asset price fluctuations more generally.

1. Volatility, Required Returns, and Stock Price Fluctuations

This section discusses the relationship between changes in volatility and changes in the level of stock market prices. For simplicity we assume that firms are not levered and that expected dividends grow at a constant rate. The former assumption allows us to ignore Black's (1976) important observation that the level of share prices, by affecting the degree of leverage, should have a direct impact on volatility. The latter assumption is maintained for convenience and could be relaxed easily. Because of the nature of the volatility estimates used in our empirical work, we use a discrete time formulation.

We assume that share prices satisfy the standard requirement that

$$\frac{E_t(P_{t+1}) - P_t}{P_t} + \frac{D_t}{P_t} = r_t + \alpha_t \tag{1}$$

where r_t is the risk-free interest rate, α_t is the risk premium, and D_t is the dividend paid in period t. Equivalently, equation (1) can be written as :

$$P_{t+1} = (1 + r_t + \alpha_t)P_t - D_t + \varepsilon_t P_t \tag{2}$$

where

$$\varepsilon_t = (P_{t+1} - E_t[P_{t+1}])/P_t \tag{3}$$

is a random disturbance assumed to be uncorrelated with any information available at time t. It reflects the impact of revisions in expectations about future values of D, α, and r which take place between periods t and t+1.

Equation (2) is a difference equation for P_t, and it can be solved forward subject to an appropriate transversality condition to yield

$$P_t = E_t[\sum_{j=0}^{\infty} \{\prod_{i=0}^{j} (1 + \alpha_{t+i} + r_{t+i})^{-1}\} D_{t+j}] . \tag{4}$$

Assuming that the risk free rate is constant over time, this expression may be linearized around the mean value of α, $\overline{\alpha}$, to obtain:

$$P_t = \sum_{j=0}^{\infty} \frac{E_t[D_{t+j}]}{(1+r+\overline{\alpha})^j} + \sum_{j=0}^{\infty} \frac{dP_t}{d\alpha_{t+j}}(E_t[\alpha_{t+j}] - \overline{\alpha}) \tag{5}$$

where

$$\frac{dP_t}{d\alpha_{t+j}} = -(1+r+\overline{\alpha})^{-j-1}. \sum_{k=0}^{\infty} \frac{E_t(D_{t+j+k})}{(1+r+\overline{\alpha})^k} \quad . \tag{6}$$

Equation (5) expresses current stock prices as a linear function of expected future risk premia. Assuming that expected dividends grow at a constant rate g, so that $E_t[D_{t+j}] = (1+g)^j D_t$, the derivative in (6) can be simplified as:

$$\frac{dP_t}{d\alpha_{t+j}} = \frac{-D_t(1+g)^j}{(1+r+\overline{\alpha})^{j+1}} \sum_{k=0}^{\infty} \frac{(1+g)^k}{(1+r+\overline{\alpha})^k} = \frac{-D_t(1+g)^j}{(1+r+\overline{\alpha})^j(r+\overline{\alpha}-g)} \, . \tag{7}$$

It is natural to postulate that α_t depends on σ_t^2, the variance of ε_t. We assume for simplicity that

$$\alpha_t = \gamma\sigma_t^2 \tag{8}$$

where γ is a constant of proportionality that depends on investors' levels of risk aversion. Merton (1973,1980) derives a similar relationship between α_t and the variance of returns in a continuous-time model.

To study the effect of changes in volatility on P_t it is necessary to adopt some assumption about the evolution of σ_t^2. We assume that σ_t^2 follows an AR(1) process:

$$\sigma_t^2 = \rho_0 + \rho_1\sigma_{t-1}^2 + \mu_t. \tag{9}$$

Evidence to support the AR(1) assumption is presented in subsequent sections.

From (8) and (9), it immediately follows that α_t also follows an AR(1) process:

$$\alpha_t = \gamma\rho_0 + \rho_1\alpha_{t-1} + \nu_t \tag{10}$$

where $\nu_t = \gamma\mu_t$. The mean value of α_t is therefore $\gamma\rho_0/(1-\rho_1)$, and the deviation between α_t and $\bar{\alpha}$ obeys

$$\alpha_t - \bar{\alpha} = \rho_1(\alpha_{t-1} - \bar{\alpha}) + \nu_t . \tag{11}$$

Equation (11) enables us to simplify (5) substantially, since $E_t(\alpha_{t+j} - \bar{\alpha}) = \rho_1^j(\alpha_t - \bar{\alpha})$. Substituting this relationship into (5) and using (7) yields

$$P_t = \sum_{j=0}^{\infty} \frac{E_t(D_{t+j})}{(1+r+\bar{\alpha})^j} - \sum_{j=0}^{\infty} \frac{D_t(1+g)^j\rho_1^j(\alpha_t-\bar{\alpha})}{(1+r+\bar{\alpha})^j(r+\bar{\alpha}-g)} \tag{12}$$

$$= \frac{D_t(1+r+\bar{\alpha})}{r+\bar{\alpha}-g} - \left[\frac{1+r+\bar{\alpha}}{1+r+\bar{\alpha}-\rho_1(1+g)}\right]\cdot\left[\frac{D_t}{r+\bar{\alpha}-g}\right](\alpha_t - \bar{\alpha})$$

The last expression shows the effect of risk premia shocks on share prices; the second term is $(dP_t/d\alpha_t)\cdot(\alpha_t - \bar{\alpha})$. This may be rewritten in terms of volatility shocks, using (8), as

$$\frac{dP_t}{d\sigma_t^2} = \frac{-\gamma(1+r+\bar{\alpha})}{[1+r+\bar{\alpha}-\rho_1(1+g)]}\cdot\left[\frac{D_t}{r+\bar{\alpha}-g}\right] \tag{13}$$

or

$$\frac{d\log P_t}{d\log\sigma_t^2} = \frac{-\gamma\sigma_t^2\lambda_t}{[1+r+\bar{\alpha}-\rho_1(1+g)]}\cdot\frac{[1+r+\bar{\alpha}]}{[r+\bar{\alpha}-g]} \tag{14}$$

where λ_t is the dividend yield, D_t/P_t. The numerator simplifies since $\gamma\sigma_t^2 = \alpha_t$ and $\lambda_t = r_t+\alpha_t-g$. Evaluating both expressions at $\bar{\alpha}$ yields

$$\frac{d\log P_t}{d\log \sigma_t^2} = \frac{-\bar{\alpha}[1+r+\bar{\alpha}]}{[1+r+\bar{\alpha}-\rho_1(1+g)]} . \qquad (15)$$

Notice that the absolute value of the derivative of share prices with respect to current volatility rises with ρ_1. This result is intuitively natural. If increases in volatility are expected to persist, they will have a greater impact on the discount factors applied to future cash flows, and therefore on share prices.

In order to examine possible relationships between volatility and the level of share prices, it is useful to insert some plausible parameter values into (15). The mean annual return on common stocks for the period 1948-1983 was 11.6 percent.[1] The mean nominal return on Treasury bills was 4.6 percent per year over the same period, implying an average value of 7.0 percent for α. The average real return on Treasury bills, which we use to estimate r, was .4 percent. The estimated variance of the market return, expressed at annual rates, ranged from 26.83 in 1964 to 638.57 in 1974, averaging 238.3. The last statistic in conjunction with the mean estimate for α implies a value of .029 for γ. Merton (1980) estimated this parameter to be .032 for the period 1952-1978. The growth rate of nominal dividends on the S&P 500 during the 1948-1983 period was 5.2 percent annually. Combining this with our inflation rate of 4.2 percent yields an average growth rate for real dividends, g, of .01.

The effect of changes in volatility on the level of share prices is very sensitive to the level of ρ_1. The derivative in (15) equals -.070 when $\rho_1=0$, .131 when $\rho_1=.5$, and -.409 when $\rho_1=.9$. We have defined ρ_1 as the serial correlation in annual volatility; an annual value of ρ_1 = .90 implies a monthly autocorrelation of more than .99. Stated another way, a 50 percent increase in market volatility from its average level would reduce the value of the market by 3.5 percent if $\rho_1 = 0$, by 6.5 percent of $\rho_1 = .5$, and by 20 percent if $\rho_1 = .9$. It is clear that if fluctuations in volatility are to play a significant role in explaining market fluctuations, then ρ_1 must be quite large. The next two sections examine the serial correlation properties of several measures of volatility.

2. Serial Correlation in Market Volatility

As emphasized in Merton (1980), a great deal of work remains to be done on variance estimation. In this section, we use crude estimators for the variance of market returns to study the serial correlation properties of volatility.[2] We use two estimators of market variance, $\hat{\sigma}_t^2$ and $\hat{\hat{\sigma}}_t^2$, computed respectively from monthly and daily returns data. While daily data are preferable for variance estimation, they were available to us only for the 1968-84 period. Monthly returns data were available from 1926 to 1983.

2.1 Volatility Estimation Using Monthly Data

Our first variance estimator, based on monthly data, was calculated as:

$$\hat{\sigma}_t^2 = \sum_{i=1}^{12} (s_{it} - r_{it})^2/12 \qquad (16)$$

where s_{it} denotes the annualized return on common stocks in month i of year t, and r_{it} is the Treasury bill rate.[3] Our monthly returns data were obtained from Ibbotson (1984).

In Table 1, we report summary statistics on $\hat{\sigma}_t^2$ for two periods, 1926-1983 and 1948-83. It is immediately clear from the autocorrelogram and partial autocorrelogram that there is no substantial positive serial correlation in market volatility. For the 1948-1983 period, the first order autocorrelation coefficient is only .114; for the longer 1926-83 period, it is .675. The high autocorrelation coefficient for the whole period is sensitive to the inclusion of the Depression years in the data sample. The first order autocorrelation for the sample period 1935-1983 is .50, and for the 1940-1983 period, the estimate declines to .05. Results similar to those for the postwar period were obtained

Table 1: Autocorrelation in Annual Stock Market Volatilities

Lag Length (Years)	1926-1983 Sample: Autocorrelation	1926-1983 Sample: Partial Autocorrelation	1948-1983 Sample: Autocorrelation	1948-1983 Sample: Partial Autocorrelation
1	0.675 (.131)	0.675 (.131)	0.114 (.167)	0.114 (.167)
2	0.269 (.131)	-0.345 (.131)	-0.115 (.167)	-0.129 (.166)
3	0.110 (.131)	0.206 (.131)	-0.224 (.167)	-0.200 (.167)
4	0.077 (.131)	-0.058 (.131)	0.233 (.166)	0.287 (.167)
5	0.145 (.131)	0.217 (.131)	0.008 (.160)	-0.122 (.167)
6	0.215 (.131)	0.002 (.125)	0.197 (.167)	0.255 (.167)

Source: Annual volatility estimates were calculated as the average of twelve squared monthly values of the return on common stocks minus the return Treasury bills. These data were drawn from Ibbotson (1984) for the period 1926-1983, a total of 696 observations. The second sample, for the 1948-1983 period, contains 432 observations. Standard errors are shown in parentheses. See text for further details.

using data for only the 1960-1983 period, when the estimated first order serial correlation coefficient was .131. The hypothesis that annual volatility was a white noise process could be rejected at standard levels in the 1948-1983 or 1960-1984 periods.[4]

As a further check on the autocorrelation properties of our volatility estimates, we estimated some simple autoregressive models for $\hat{\sigma}_t^2$. The results, which are presented in Table 2, corroborate the conclusions reached above. They suggest no great persistence in volatility, and indicate that the simple first order autoregressive model used in the preceding section's theoretical development fits the data quite well.[5] Higher order models did not yield appreciably smaller sums of squared residuals for either sample period under consideration.

The point estimates of ρ_1 for each sample period are substantially less than unity. They imply that volatility shocks do not persist for long periods. For the 1926-1983 sample perod, where we find the greatest amount of persistence, ninety percent of a volatility shock will have dissipated by six years after the shock. In the AR(2) case for this period, only four years are required. The half life of the shock, the time required to move half way back to the long-run equilibrium, is two years for both processes. In the postwar period, the implied half life is much shorter -- less than one year.[6] These results confirm Schmalensee and Trippi's (1978) findings of relatively little persistence in volatility for individual firms.

The estimated autocorrelations all suggest little persistence in volatility, and conventional t-tests reject the hypothesis that volatility is a random walk ($\rho_1 = 1$). However, recent work on the estimation of time series models with unit roots, such as Dickey and Fuller (1981), has shown that the actual

Table 2

Time Series Models for Volatility Estimates Based on Monthly Data

Equation	Sample	Constant	ρ_1	ρ_2	Trend	SSR	Q(5)	Unit Root Test
1	1926-83	0.17 (.10)	0.68 (.10)	-	-	23.40	8.77 (9.48)	-3.26 (2.56)
2	1926-83	0.23 (.10)	0.91 (.13)	-0.34 (.13)	-	20.71	4.53 (7.81)	-4.19 (2.56)
3	1926-83	0.49 (.21)	0.60 (.11)	-	-0.98 (.56)	21.66	11.29 (9.48)	-3.75 (3.14)
4	1926-83	0.70 (.21)	0.84 (.13)	-0.40 (.13)	-1.37 (.55)	18.00	3.93 (7.81)	-5.06 (3.14)
5	1948-83	0.21 (.05)	0.11 (.17)	-	-	1.03	5.62 (9.48)	-5.24 (2.59)
6	1948-83	0.23 (.06)	0.12 (.17)	-0.11 (.17)	-	1.03	5.82 (7.81)	-4.26 (2.59)
7	1948-83	0.07 (.10)	.04 (.17)	-	0.39 (.25)	0.67	7.13 (9.48)	-5.58 (3.18)
8	1948-83	0.08 (.10)	0.05 (.17)	-0.17 (.17)	0.45 (.26)	0.65	6.57 (7.81)	-4.72 (3.18)

Notes: All equations are estimated by ordinary least squares, using estimates of annual market volatility based on averages of twelve squared monthly returns, measured at annual rates. The equation estimated is

$$\sigma_t = \rho_0 + \rho_1\sigma^2_{t-1} + \rho_2\sigma^2_{t-2} + \beta*TIME + \epsilon_t.$$

The reported Q-statistics are based on five lagged values of autocorrelations; critical values are shown in parentheses. Similarly, the critical values of the unit root tests are shown in parentheses. All other parenthetic quantities are standard errors.

size of these tests may be substantially different from their nominal size. Dickey and Fuller (1981) present tables of adjusted critical t-values for various sample sizes to test the hypothesis of a unit root. The last column in Table 2 shows that t-statistic against the hypothesis $\rho_1 = 1$ along with the appropriate critical t-value for the 95% confidence interval.[7] In each case we are able to reject the null hypothesis of a unit root.

Our analysis so far has assumed that market volatility can be modelled as a stationary time series. Some research, however, has suggested that volatility may trend over time. If so, this would imply that our estimated autocorrelations are biased upwards. The equations in rows 3,4,7, and 8 of Table 2 present simple tests for the existence of trends over various sample periods. They suggest a positive trend for the time period 1948-1983, and a negative trend for 1926-1983. However, the findings also show that the null hypothesis of no trend cannot be rejected in the 1948-1983 period cannot be rejected.

These results contradict Pindyck's (1984) claim that there has been a clear upward trend in market volatility over the last thirty years. He pre-smooths monthly volatility estimates by computing twelve-month moving averages before looking for trends, and this makes his conclusions difficult to evaluate. Even if there were trends, however, it is important to recognize that only unexpected deviations from trend should affect asset returns. If the trend rate of volatility growth rises abruptly, that should lead to a one-time adjustment in share prices, but it cannot account for a low rate of return over an extended period.

2.2 Volatility Estimates Based on Daily Data

Our second volatility estimator was based on daily data; it follows closely on Merton's (1980) estimator.[8] Using daily returns on the Standard and Poor's

500 Stock Index, we computed

$$\hat{\hat{\sigma}}_t^2 = \sum_{i=1}^{21} \bar{s}_{it}^2/21 \tag{17}$$

where $\bar{s}_{it}$ is the daily return adjusted for non-trading days. In daily data, measuring the returns around the risk-free rate would have virtually no effect on the estimated volatilities. The twenty-one trading day intervals which we use correspond roughly to months of calendar time.

The autocorrelogram and partial autocorrelogram for this volatility estimator are shown in Table 3. In this case, the data clearly exhibit positive serial correlation. Again, the persistence of volatility is relatively unimportant from an economic perspective. The first order autocorrelation coefficient obtained using monthly data is .596, which is equivalent to an autocorrelation coefficient of only (.596)12, or .002, in annual data. This is not inconsistent with the estimates obtained using the post-war monthly volatility data in the last section, since we could not reject the null hypothesis that there was no serial correlation in volatility.

Table 4 shows estimated autoregressive models for the volatility series calculated from daily data. Once again, the first or second order autoregressive process provides an adequate description of the data. The hypothesis that the coefficients on all variables lagged more than two periods equalled zero could never be rejected. Higher lagged terms have small, as well as statistically insignificant, coefficients. None of the estimates of ρ_3 or ρ_4 ever exceeded .10 in absolute value. The tests against the null hypothesis of $\rho_1=1$ again clearly reject the hypothesis that volatility follows a random walk. The half life for a shock in the AR(1) model is less than three months.

Table 3: Autocorrelation in Monthly Stock Market Volatility

Lag Length (Months)	Autocorrelation	Partial Autocorrelation
1	0.596 (.071)	0.596 (.071)
2	0.446 (.071)	0.142 (.071)
3	0.330 (.071)	0.027 (.072)
4	0.225 (.071)	-0.025 (.072)
5	0.198 (.071)	0.059 (.071)
6	0.186 (.071)	0.056 (.072)
7	0.169 (.071)	0.024 (.071)
8	0.154 (.071)	0.016 (.072)
9	0.172 (.071)	0.067 (.071)
10	0.192 (.071)	0.069 (.071)
11	0.168 (.071)	-0.009 (.074)
12	0.050 (.071)	-0.161 (.071)
13	0.021 (.072)	-0.009 (.069)
14	0.009 (.071)	0.023 (.070)
15	0.021 (.071)	0.034 (.072)
16	-0.003 (.075)	-0.063 (.071)
17	0.013 (.069)	0.020 (.073)
18	-0.036 (.070)	-0.070 (.071)
19	-0.097 (.071)	-0.097 (.072)
20	-0.113 (.071)	-0.047 (.071)
21	-0.107 (.071)	0.017 (.072)
22	-0.115 (.071)	-0.003 (.070)
23	-0.085 (.071)	0.045 (.072)
24	-0.062 (.071)	0.002 (.091)

Source: The table shows the estimated autocorrelogram for monthly estimates of market volatility. Each month's estimate is based on the average of squared daily returns on the S&P 500 Index for the period 1968:001 to 1984:180. A total of 197 monthly observations, based on 4137 daily observations, are used. Standard errors are shown in parentheses.

Table 4

Time Series Models for Volatility Estimates Based on Daily Data

Equation	Constant(x10-5)	ρ_1	ρ_2	ρ_3	ρ_4	Trend (x10-4)	SSR(x10-5)	Q(12)	Unit Root Test
1	2.53 (.479)	0.60 (.06)	-	-	-	-	39.04	10.67 (19.67)	-7.04 (2.53)
2	2.17 (.511)	0.50 (.07)	0.12 (.07)	-	-	-	38.35	7.60 (18.30)	-5.42 (2.53)
3	2.10 (.537)	0.49 (.08)	0.11 (.07)	-0.01 (.07)	-	-	38.19	7.52 (16.91)	-4.88 (2.53)
4	2.16 (.562)	0.49 (.08)	0.12 (.07)	0.02 (.08)	-0.06 (.07)	-	38.14	7.46 (15.51)	-4.69 (2.53)
5	2.11 (.704)	0.59 (.06)	-	-	-	4.65 (5.72)	38.91	10.33 (19.67)	-7.08 (3.10)
6	1.83 (.722)	0.51 (.07)	0.14 (.07)	-	-	3.89 (5.75)	38.16	7.46 (18.30)	-5.45 (3.10)

Notes: All equations are estimated by ordinary least squares, using estimates of monthly market volatility based on averages of twenty-one squared daily returns. The equation estimated is

$$\sigma_t = \rho_0 + \rho_1\sigma^2_{t-1} + \rho_2\sigma^2_{t-2} + \rho_t\sigma^2_{t-3} + \rho_t\sigma^2_{t-4} + \beta{*}TIME + \varepsilon_t.$$

The reported Q-statistics are based on twelve autocorrelations; critical values are shown in parentheses. Similarly, the critical values of the unit root tests are shown in parentheses. All other parenthetic quantities are standard errors.

Substantive economic conclusions about the importance of volatility shocks are unaffected by the choices between the different autoregressive models. All of our results suggest very little persistence.[9] Moreover, the results again question the importance of trends in volatility. None of the estimated time trend coefficients is statistically significant. In both the AR(1) and AR(2) cases, the estimate trend coefficients have t-statistics of less than unity and the addition of trend variables has little effect on the other coefficients in these equations.

3. Market Volatilities Implied by Option Premia

The estimates presented in the last section suggest that volatility shocks are short-lived. However, the estimated serial correlation parameters might be biased downward by measurement error, and they may also fail to reflect market participants' beliefs about volatility persistence. To address these problems, we analyzed the persistence in volatilities inferred from option premia. Unfortunately, options on stock market indices such as the S&P 500 have been traded for too short a period to make analyzing them informative. However, the Chicago Board Options Exchange (CBOE) has computed an index of the price of a standardized stock option on every Thursday since January 8, 1976.[10]

> ...the CBOE Call Option Index is an average of percent option premiums; for each CBOE underlying stock, a market premium is estimated for a hypothetical six-month, at the money option using the market premiums of existing option series. This estimated market premium is expressed as a percentage of the stock price. The CBOE Call Option Index for a given day is the arithmetic average of all such percent premiums on CBOE underlying stocks on that day.[CBOE(1979), p. 1]

These data are now available for a period of eight and one half years and it is possible to analyze the persistence of volatility expectations using them.

The CBOE Index does not correspond to the option premium of any traded security. It is a measure of this option premium on the "representative share" for which options are traded on the CBOE. As such, the implied volatility should be substantially higher than the volatility of the market, since the market is a weighted average of many imperfectly correlated shares. While our estimates of the implied volatility on a representative share are not directly comparable to the volatilities estimated in the last section, our assumption is that their serial correlation properties should be reasonably similar.[11]

To estimate the volatility of the "representative stock" implied by the CBOE index, we assumed that the dividend yield on this share equalled that on the S&P 500.[12] We followed Black's (1976) suggestion for dividend adjustment and subtracted the present value of dividend payments over the life of the option from the price of the stock. We assumed that the option on the representative stock was priced according to the Black-Scholes (1973) formula, and applied a numerical search algorithm to determine the variance of returns which was consistent with the observed option price, risk free rate, and market dividend yield. The CBOE Index is standardized to apply to an option on a stock with a current price of $40.00, and since the index applies to at the money options, the strike price is $40.00 as well. The weekly data on the CBOE Index, as well as our estimates of the implied volatilities, are shown in the Data Appendix.

The movements our implied volatilities were compared with those of six-month ex post volatilities estimated from daily returns on the S&P 500; the two series cohere reasonably well. Figure 1 shows the movments in these two series, each divided by its mean, for the 1976-1984 period. A positive association between the series is readily apparent. Both series rise throughout the late 1970s, and decline during the 1982-1984 period. The CBOE implied volatility does not rise as dramatically as the ex poste volatility series during the 1981 stock market rally, although it does increase.

Table 5 shows the estimated autocorrelogram and partial autocorrelogram for the implied volatility series. These are <u>weekly</u> data, and so the estimated autocorrelations are higher than those in the earlier sections. The first order autocorrelation, for example, is .971. However, these results confirm the

Figure 1: Time Pattern of Actual and Implied Volatilities

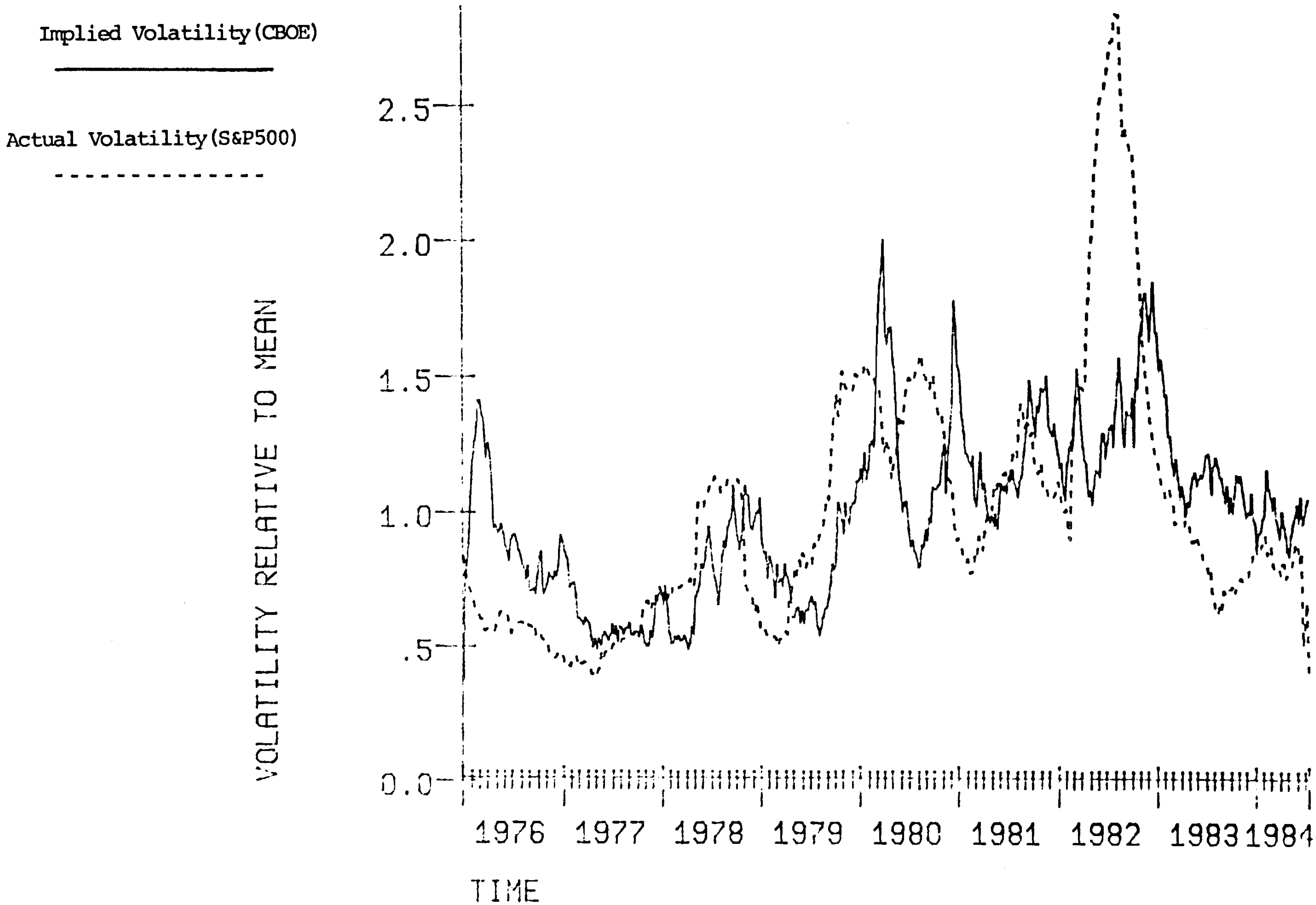


Table 5: Autocorrelation in Volatility Forecasts Implied by Option Premia

Lag Length (weeks)	Autocorrelation	Partial Autocorrelation
1	0.971 (.047)	0.971 (.047)
2	0.939 (.082)	-0.056 (.047)
3	0.903 (.106)	-0.087 (.047)
4	0.869 (.125)	0.029 (.047)
5	0.835 (.142)	-0.028 (.047)
6	0.802 (.157)	-0.003 (.048)
7	0.767 (.171)	-0.043 (.047)
8	0.734 (.184)	-0.003 (.043)
9	0.703 (.195)	0.032 (.047)
10	0.673 (.195)	-0.003 (.056)
11	0.650 (.195)	0.092 (.047)
12	0.629 (.195)	-0.002 (.044)
13	0.612 (.195)	0.062 (.047)
14	0.589 (.195)	-0.130 (.047)
15	0.566 (.195)	-0.026 (.047)
16	0.545 (.195)	0.064 (.047)
17	0.525 (.195)	-0.014 (.046)
18	0.507 (.195)	0.012 (.046)
19	0.493 (.195)	0.059 (.047)
20	0.482 (.195)	0.053 (.047)
21	0.473 (.195)	0.025 (.047)
22	0.466 (.195)	0.028 (.048)
23	0.458 (.195)	-0.032 (.047)
24	0.450 (.195)	-0.008 (.048)
25	0.443 (.195)	0.004 (.046)
26	0.439 (.195)	0.027 (.048)

Source: Estimates of volatility forecasts were determined by inverting the Black-Scholes option valuation formula to obtain the volatility implied by CBOE option premia indices. These data were available for the period 1976:1 to 1984:26, for a total of 447 weekly observations. See appendix for further details and data description. Standard errors are shown in parentheses.

earlier conclusions that volatility changes are not persistent. One year after a shock to volatility, expected volatility will exceed its mean by only $(.971)^{52} = .22$, or twenty two percent, of the initial shock. The partial autocorrelogram again suggests that a first order autoregressive representation is appropriate for this series. The statistical insignificance of partial autocorrelations at lags of more than one week is indicative of an AR(1) structure in these data.

Table 6 reports estimates of several time series models for these data. Equations for our entire data period, comprising 447 weeks, are reported in the first four rows of the table. The hypothesis that the residuals from the AR(1) model are white noise is nearly rejected at standard levels, as shown by the reported Q-statistics. The AR(2) results do not suffer from this difficulty. The higher order (third and fourth order) autocorrelation parameters are never statistically significant. The implied responses to a volatility shock are similar in all of the estimated models. They suggest that the half life for a volatility shock is about six months.[13] Although the estimated weekly autocorrelation is near unity, the last column of the table shows that the hypothesis of a unit root is still rejected in each case.

Because each weekly observation on the CBOE Index depends on forecasts of volatility for each of the next twenty six weeks, two consecutive observations on the implied volatility will have twenty five weeks of forecast volatilities in common. This may bias our estimated autocorrelations. We therefore estimated autoregressive models using non-overlapping data periods, corresponding to every twenty-sixth observation in our data set. The estimated AR(1) and AR(2) models are reported in the last two rows of the table. The estimated six-month

Table 6

Time Series Models for Volatility: Estimates Based on CBOE Data

Equation	Constant ($x10^{-4}$)	ρ_1	ρ_2	ρ_3	ρ_4	SSR($x10^{-4}$)	Q(26)	Unit Root Test
1	3.284 (1.093)	0.964 (.011)	-	-	-	2.034	36.41 (37.65)	-3.03 (2.52)
2	2.847 (1.075)	1.024 (.046)	-0.056 (.046)	-	-	1.918	31.53 (36.41)	-2.74 (2.52)
3	3.071 (1.081)	1.017 (.047)	0.043 (.067)	-0.096 (.046)	-	1.899	28.22 (35.17)	-2.95 (2.52)
4	3.067 (1.093)	1.016 (.047)	0.039 (.067)	-0.079 (.067)	-0.011 (.046)	1.898	28.93 (33.92)	-2.92 (2.52)
5	66.025 (19.761)	0.294 (.212)	-	-	-	.886	---	-4.93 (2.66)
6	46.012 (26.292)	0.207 (.256)	0.310 (.220)	-	-	.769	---	-4.92 (2.68)

Notes: All equations are estimated by ordinary least squares, using estimates of implied market volatility derived by inverting the Black-Scholes option formula as applied to the CBOE Option Index. The equation estimated is

$$\sigma_t = \rho_0 + \rho_1\sigma^2_{t-1} + \rho_2\sigma^2_{t-2} + \rho_t\sigma^2_{t-3} + \rho_t\sigma^2_{t-4} + \varepsilon_t.$$

The reported Q-statistics are based on twenty-six autocorrelations; critical values are shown in parentheses. Similarly, the critical values of the unit root tests are shown in parentheses. All other parenthetic quantities are standard errors. The data period comprises 1976:1-1984:26, a total of 444 observations. The last two equations are estimated using only the non-overlapping data periods, or observations separated by twenty-six weeks.

autocorrelation coefficient from these data is .43, which is only slightly lower than the six-month autocorrelation implied by our weekly estimates. The results again suggest that over half of a volatility shock vanishes within six months.

4. The Term Structure of Implied Volatilities

The CBOE Call Option Index data provide strong support for the transient character of volatility shocks. However, they do not permit us to directly investigate how long-term expectations of volatility respond to changing short-term volatility expectations. A second source of option data can illuminate this issue. Since 1979, Valueline has computed indices of option premia at three and six month maturities. We inverted these option premia indices using the same procedure which we applied to the CBOE data.[14]

The availability of two different maturity option indices provides an opportunity for additional tests of the persistence hypothesis. The implied volatility for the six month options was assumed to equal the average of the expected three month volatilities for the next three months as well as the three months following them:

$$ {}_{t,6}\sigma_t^2 = ({}_{t,3}\sigma_t^2 + {}_{t,3}\sigma_{t+13}^2)/2 \;. \tag{18}$$

In this notation, ${}_{s,k}\sigma_t^2$ is the volatility expected to prevail, as of time t, over the k months beginning in week s. The assumption in (18) allows us to solve for an estimate of the implied forward volatility which is expected to prevail for the three month period beginning three months from the current week:

$$ {}_{t+13,3}\hat{\sigma}_t^2 = 2{}_{t,6}\hat{\sigma}_t^2 - {}_{t,3}\hat{\sigma}_t^2 \tag{19}$$

We can use the estimated forward volatilities to study the change in the implied forward volatility which occurs when the current three-month "spot" implied volatility changes. The results of this estimation are shown below:

$$ {}_{t+13,3}\hat{\sigma}^2_t - {}_{t+12,3}\hat{\sigma}^2_{t-1} = \underset{(.0769)}{-.0233} + \underset{(.050)}{0.511} \left[{}_{t,3}\hat{\sigma}^2_t - {}_{t-1,3}\hat{\sigma}^2_{t-1} \right] \quad (20) $$

These results indicate that when current volatility expectations change, expected volatility in future periods also changes. However, they also constitute further evidence for our contention that volatility shocks are not persistent. One year after a volatility shock, these estimates imply that only seven percent of a shock will still persist. The half life of a volatility shock according to these data is just over three months. One year after a ten percent shock to volatility, the three-month forecast of volatility would only be 1.3 percent greater than in the initial period.

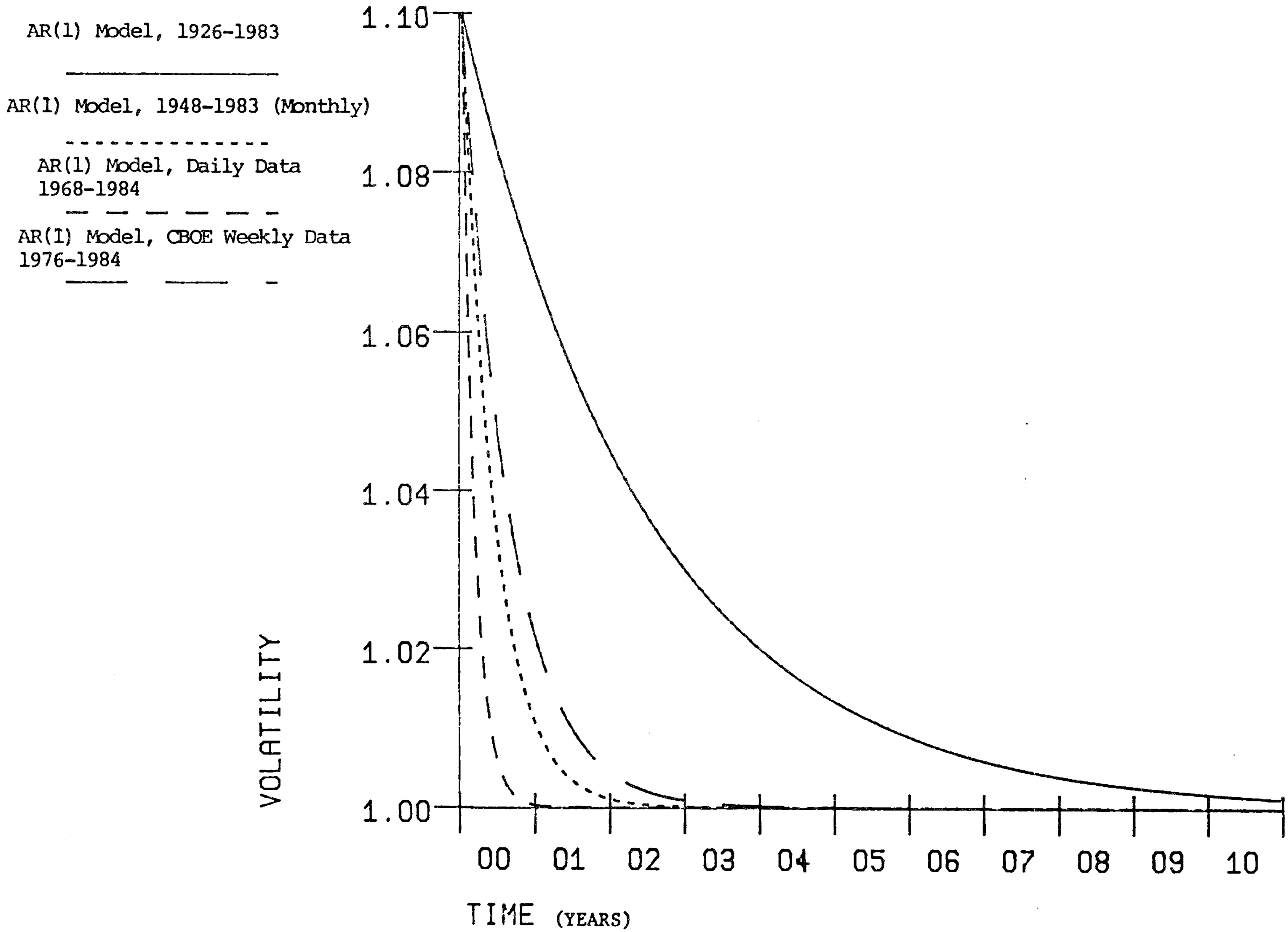
5. Conclusions and Implications

Our findings suggest that shocks to stock market volatility are not persistent. This is illustrated in Figure 2, which shows the impulse response functions corresponding to several of the estimated time series models for volatility. These functions, which show the moving average representation of each process, depict the evolution of volatility following a "shock" equal to ten percent of the steady-state value of volatility. While the speed with which the shock dissipates varies across models, the half life of the shock never exceeds two years. For the equations corresponding to the CBOE and daily volatility estimators, the half-life of a volatility shock is less than half a year.

Our empirical results suggest that changes in volatility should affect expected required returns for periods not substantially greater than two years. This means that they can only have a very limited impact on the level of share prices. A doubling of volatility would reduce the level of the market by only about nine percent.[15] Since actual volatility fluctuations are usually smaller than this, we doubt that changing volatility accounts for any large fraction of market fluctuations. This observation applies both to the problem of explaining recent events and to the deeper problem of explaining the sources of stock price fluctuations.

Our work deepens the puzzle of explaining the strong negative correlation, observed by Black (1976) and Schmalensee and Trippi (1978), between stock market returns and volatility. Black (1976) showed that the inverse correlation between volatility and returns was so strong that a positive one percent return on a stock implied more than a one percent reduction in volatility; this

Figure 2: Impulse Response Functions for Volatility Shocks



implies that raising the share price actually reduces the dollar volatility of the stock. The finding that volatility is not highly persistent suggests that autonomous changes in volatility should have only a relatively small effect on share prices. If Black's (1976) "leverage effect" explanation of the relationship between returns and volatility were correct, then one would expect to observe that volatility, like prices, would follow a random walk.

One possible explanation for the observed data is that the same events which make the returns to capital more uncertain also reduce their expected value. This coincidence in the arrival of stochastic shocks would lead to an apparent relationship between volatility changes and share prices, although in fact no such causal link exists. Another possibility is that when adversity strikes, firms are expected to respond with new strategies. Between the time the market anticipates that some new policy will be chosen and the time this policy is actually announced, uncertainty and therefore volatility may increase.

Endnotes

1. These data are based on Ibbotson (1984).

2. To the extent that there is measurement error in our estimates of volatility, the estimated serial correlation coefficients may be biased downward. In the next section we use estimates of volatility expectations which are less susceptible to these difficulties.

3. Merton (1980) observes that although estimators of the variance which center the estimated returns around a point which is not their sample mean will be biased, these biases are trivial. He estimates variances using uncentered returns; Pindyck (1984) estimates variances for twelve month periods by computing second moments centered around the overall mean return in his sample.

4. We used Box-Jenkins (1970) Q-statistics to test the null hypothesis of no serial correlation of up to fifth order. The values of the test statistics were 32.9 for the 1926-1983 period, 4.70 for 1948-1983, and 4.47 for 1960-1983. The critical value for rejecting the null hypothesis of at the .05 level is 12.59.

5. The finding that low-order autoregressive processes provide an adequate description of the variance of market returns suggest that it may be possible to apply the econometric techniques for time-varying heteroscedasticity, suggested by Engle(1982), when studying the behavior of security returns.

6. The same conclusion emerged when we examined changes in volatility. This is shown by the following equation, estimated for the 1948-1983 period:

$$\hat{\sigma}_t^2 - \hat{\sigma}_{t-1}^2 = \underset{(3079.1)}{1083.3} - \underset{(.155)}{.438}\,[\hat{\sigma}_{t-1}^2 - \hat{\sigma}_{t-2}^2]$$

The coefficient on the lagged volatility change is negative and has a t-value of 2.83, clearly different from zero.

7. Critical values of the unit root tests are drawn from Dickey and Fuller (1981), Tables I and II.

8. Merton (1980, Appendix A) discusses two adjustments to estimated volatility series. The first, for nontrading days, is implemented in our study. The second, which corrects for nontrading shares in the stock index, multiplies the estimated volatility by a constant. Since our study is concerned with the autocovariance and not the level of the volatility series, and the former is unaffected by multiplication by a constant, we did not make the correction.

9. The results of estimating a model in changes were:

$$\hat{\hat{\sigma}}_t^2 - \hat{\hat{\sigma}}_{t-1}^2 = \underset{(3.42\text{x}10^{-6})}{1.44\text{x}10^{-7}} - \underset{(.068)}{.315} \cdot [\hat{\hat{\sigma}}_{t-1}^2 - \hat{\hat{\sigma}}_{t-2}^2] .$$

Again, this suggests negative serial correlation.

10. A further description of this series may be found in CBOE(1979).

11. Latané and Rendelman (1976) computed implied standard deviations for a series of options over a thirty nine week period and discovered that these implied standard deviations tended to move together. Schmalensee and Trippi (1978) found similar results. These coincident movements in volatility are the market-wide volatility shifts we hope to capture.

12. We assumed that dividends were paid as a continuing flow at rate λ per year, where λ is the current yield on the S&P 500.

13. We also considered the serial correlation of changes for the non-overlapping differences:

$$\hat{\sigma}^2_{I,t} - \hat{\sigma}^2_{I,t-26} = \underset{(0.00013)}{0.00012} - \underset{(.044)}{.425}\,[\hat{\sigma}^2_{I,t-26} - \hat{\sigma}^2_{I,t-52}]$$

where the subscript I denotes an implied volatility.

14. The Value Line data were available for the period 1980:16 to 1984:26; this constitutes a total of 220 weeks. However, there were 17 weeks of missing data. This precluded calculating the long autocorrelograms which are reported for the other volatility series. However, the first order autocorrelations for these series, .88 for the three month implied volatility and .87 for the six month, were roughly consistent with earlier findings.

15. This calculation is based on $\rho_1 = .21$, the estimate from the CBOE implied volatilities, and the other parameter values described in Section 1.

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Appendix Table A-1: Values of CBOE Index and Implied Volatilities

	CBOEINDEX	SIGMASQ
1976:1	8.7750	0.00338161
1976:2	8.2100	0.00688129
1976:3	8.3600	0.00718768
1976:4	8.7300	0.00786170
1976:5	9.5200	0.00941007
1976:6	10.1100	0.01056881
1976:7	10.4200	0.01123941
1976:8	10.6400	0.01168084
1976:9	11.0500	0.01259923
1976:10	11.1300	0.01260401
1976:11	10.9000	0.01215222
1976:12	10.7600	0.01184704
1976:13	10.1900	0.01065771
1976:14	10.4600	0.01121048
1976:15	10.3000	0.01091529
1976:16	9.9100	0.01015934
1976:17	9.0500	0.00840304
1976:18	9.1300	0.00851522
1976:19	8.9900	0.00824749
1976:20	9.0700	0.00832446
1976:21	9.2400	0.00853808
1976:22	9.1200	0.00826295
1976:23	8.9100	0.00786020
1976:24	8.7800	0.00767974
1976:25	8.6000	0.00735197
1976:26	8.9600	0.00801350
1976:27	9.0700	0.00816900
1976:28	9.0700	0.00821424
1976:29	8.9300	0.00800708
1976:30	8.7900	0.00771343
1976:31	8.6800	0.00756029
1976:32	8.4900	0.00724738
1976:33	8.4000	0.00711491
1976:34	8.1800	0.00674421
1976:35	8.4100	0.00716195
1976:36	7.9500	0.00634817
1976:37	7.9500	0.00635975
1976:38	7.9800	0.00640790
1976:39	7.8700	0.00625543
1976:40	8.1700	0.00676683
1976:41	8.5100	0.00744239
1976:42	8.5600	0.00762658
1976:43	7.7400	0.00622341
1976:44	7.8200	0.00634018
1976:45	7.9000	0.00654311
1976:46	8.1200	0.00692030
1976:47	8.0300	0.00682771
1976:48	7.9100	0.00669973
1976:49	8.0500	0.00694907
1976:50	7.9900	0.00684404
1976:51	8.1900	0.00717758
1976:52	8.7300	0.00819981
1976:53	8.6400	0.00802094
1977:1	8.4900	0.00769729
1977:2	8.4100	0.00749805
1977:3	8.3000	0.00728983
1977:4	7.8300	0.00645759
1977:5	7.8900	0.00659028
1977:6	7.8600	0.00661912
1977:7	7.6100	0.00622771
1977:8	7.1400	0.00546088
1977:9	7.1100	0.00543530
1977:10	7.0400	0.00534762

	CBOEINDEX	SIGMASQ
1977:11	6.9300	0.00521295
1977:12	7.0600	0.00542736
1977:13	7.0000	0.00535914
1977:14	6.9300	0.00524969
1977:15	6.6200	0.00482955
1977:16	6.3700	0.00448103
1977:17	6.5400	0.00470608
1977:18	6.3600	0.00441166
1977:19	6.6300	0.00473286
1977:20	6.5000	0.00453360
1977:21	6.7100	0.00485059
1977:22	6.7700	0.00494615
1977:23	6.7200	0.00487269
1977:24	6.5800	0.00468729
1977:25	6.6900	0.00483722
1977:26	6.9300	0.00520197
1977:27	6.7800	0.00493429
1977:28	6.9400	0.00514416
1977:29	6.6100	0.00465198
1977:30	6.9500	0.00513408
1977:31	6.9400	0.00508132
1977:32	6.8500	0.00492127
1977:33	6.9600	0.00505946
1977:34	6.9700	0.00510101
1977:35	7.0600	0.00524610
1977:36	6.8500	0.00491914
1977:37	6.8500	0.00488985
1977:38	6.8900	0.00496018
1977:39	6.9200	0.00498673
1977:40	6.8200	0.00480824
1977:41	7.0800	0.00514363
1977:42	6.9700	0.00499883
1977:43	6.6800	0.00462291
1977:44	6.6300	0.00451075
1977:45	6.6000	0.00448933
1977:46	6.9300	0.00498952
1977:47	6.7900	0.00479773
1977:48	7.0900	0.00524759
1977:49	7.6600	0.00615420
1977:50	7.5800	0.00603659
1977:51	7.8400	0.00647884
1977:52	7.7600	0.00634210
1978:1	7.4400	0.00585736
1978:2	7.9000	0.00649788
1978:3	7.5300	0.00594276
1978:4	6.8900	0.00497713
1978:5	6.6100	0.00459639
1978:6	6.6100	0.00460496
1978:7	6.7500	0.00480364
1978:8	6.6700	0.00471525
1978:9	6.7200	0.00483195
1978:10	6.5900	0.00467760
1978:11	6.5900	0.00469976
1978:12	6.6800	0.00482168
1978:13	6.6100	0.00467355
1978:14	6.4600	0.00440390
1978:15	6.5900	0.00457666
1978:16	6.9700	0.00510703
1978:17	6.8700	0.00490297
1978:18	7.6900	0.00610570
1978:19	7.8200	0.00626383
1978:20	7.9700	0.00648260
1978:21	8.4100	0.00720867

	CBOEINDEX	SIGMASQ
1978:22	8.3600	0.00712354
1978:23	8.4700	0.00732725
1978:24	8.8300	0.00797387
1978:25	9.1100	0.00845399
1978:26	8.8400	0.00791705
1978:27	8.4300	0.00718380
1978:28	8.2200	0.00679031
1978:29	8.0900	0.00656333
1978:30	7.6900	0.00589536
1978:31	8.0600	0.00647796
1978:32	8.6200	0.00744731
1978:33	8.9700	0.00797325
1978:34	8.8700	0.00776597
1978:35	9.2500	0.00841402
1978:36	9.3600	0.00863600
1978:37	9.4500	0.00874145
1978:38	9.9700	0.00971461
1978:39	9.5100	0.00871857
1978:40	9.2500	0.00817690
1978:41	8.9900	0.00770376
1978:42	9.1500	0.00794984
1978:43	9.4300	0.00854194
1978:44	10.1600	0.00981296
1978:45	10.0300	0.00951896
1978:46	9.9200	0.00948997
1978:47	9.3900	0.00842432
1978:48	9.4000	0.00835449
1978:49	9.5300	0.00859450
1978:50	9.6800	0.00888856
1978:51	9.7700	0.00891035
1978:52	9.9800	0.00937878
1979:1	9.2600	0.00792409
1979:2	9.1100	0.00761064
1979:3	9.1300	0.00762039
1979:4	8.6200	0.00676814
1979:5	8.9600	0.00746687
1979:6	8.8400	0.00725698
1979:7	8.5200	0.00671193
1979:8	8.2100	0.00612251
1979:9	8.8600	0.00722444
1979:10	8.5200	0.00663488
1979:11	8.5900	0.00672468
1979:12	8.4400	0.00647413
1979:13	8.8500	0.00720183
1979:14	8.6800	0.00686085
1979:15	8.6000	0.00663969
1979:16	8.3300	0.00626611
1979:17	7.8100	0.00546674
1979:18	7.8100	0.00545594
1979:19	7.9600	0.00571500
1979:20	7.9600	0.00577634
1979:21	7.6200	0.00524077
1979:22	7.8800	0.00565844
1979:23	7.6100	0.00526619
1979:24	7.8100	0.00567132
1979:25	7.8000	0.00565864
1979:26	7.9700	0.00596934
1979:27	8.0700	0.00610925
1979:28	8.0100	0.00592628
1979:29	8.0600	0.00595224
1979:30	7.6000	0.00517521
1979:31	7.4000	0.00485923
1979:32	7.5600	0.00504987

	CBOEINDEX	SIGMASQ
1979:33	7.8300	0.00541512
1979:34	8.0200	0.00569435
1979:35	8.1000	0.00576217
1979:36	8.3800	0.00605275
1979:37	8.6800	0.00651536
1979:38	9.0500	0.00720488
1979:39	8.9300	0.00704910
1979:40	9.1300	0.00735832
1979:41	10.3300	0.00925825
1979:42	10.2900	0.00907736
1979:43	10.2400	0.00867629
1979:44	9.9100	0.00823610
1979:45	10.4300	0.00924267
1979:46	10.0800	0.00871213
1979:47	9.9900	0.00854469
1979:48	10.1400	0.00908108
1979:49	10.3200	0.00921439
1979:50	10.4400	0.00923739
1979:51	10.7200	0.00993933
1979:52	10.7300	0.00991950
1980:1	10.9400	0.01032783
1980:2	10.7000	0.00988432
1980:3	11.1700	0.01087619
1980:4	10.8000	0.00998422
1980:5	10.9700	0.01028936
1980:6	11.4000	0.01111371
1980:7	11.5600	0.01123861
1980:8	11.6400	0.01107375
1980:9	12.4000	0.01274044
1980:10	13.3500	0.01483056
1980:11	13.8600	0.01627521
1980:12	14.0000	0.01685384
1980:13	14.4600	0.01787010
1980:14	13.1900	0.01474667
1980:15	12.9200	0.01442156
1980:16	12.8900	0.01494987
1980:17	12.6500	0.01498084
1980:18	12.0300	0.01401120
1980:19	11.4500	0.01319024
1980:20	10.6300	0.01141237
1980:21	10.0400	0.01025086
1980:22	9.7200	0.00969588
1980:23	9.3700	0.00894882
1980:24	9.3600	0.00924997
1980:25	9.4000	0.00928266
1980:26	9.0800	0.00838532
1980:27	8.8700	0.00780473
1980:28	9.0200	0.00800098
1980:29	8.8500	0.00767564
1980:30	8.7700	0.00751073
1980:31	8.6800	0.00713038
1980:32	8.7400	0.00714588
1980:33	9.1700	0.00782808
1980:34	9.3600	0.00778220
1980:35	9.7900	0.00836030
1980:36	9.5200	0.00799956
1980:37	9.9800	0.00872841
1980:38	10.0100	0.00861878
1980:39	10.6500	0.00973721
1980:40	10.7400	0.00970511
1980:41	10.6800	0.00970261
1980:42	10.7400	0.00981516
1980:43	10.8600	0.00982599

Appendix Table A-1 Continued

	CBOEINDEX	SIGMASQ
1980:44	11.5500	0.01085849
1980:45	11.8300	0.01114440
1980:46	11.0900	0.00955733
1980:47	11.8400	0.01087747
1980:48	12.2400	0.01164973
1980:49	12.7000	0.01253447
1980:50	14.1500	0.01588009
1980:51	13.7900	0.01503705
1980:52	12.9900	0.01371105
1981:1	12.8700	0.01344185
1981:2	12.2700	0.01200133
1981:3	12.1800	0.01160932
1981:4	11.9700	0.01088636
1981:5	11.8000	0.01081941
1981:6	11.6500	0.01060041
1981:7	11.7200	0.01044572
1981:8	11.7200	0.01075329
1981:9	11.0400	0.00936380
1981:10	11.0100	0.00913633
1981:11	11.1100	0.00974962
1981:12	11.3800	0.01089692
1981:13	10.9600	0.00969187
1981:14	10.9600	0.00984027
1981:15	11.0000	0.00937052
1981:16	10.7900	0.00886873
1981:17	10.6300	0.00846321
1981:18	10.8700	0.00879952
1981:19	11.0000	0.00845689
1981:20	11.2300	0.00886318
1981:21	10.9500	0.00837479
1981:22	10.9800	0.00888526
1981:23	11.4700	0.00988313
1981:24	11.1800	0.00956571
1981:25	11.2400	0.00976889
1981:26	11.1400	0.00952214
1981:27	11.3900	0.01007909
1981:28	11.4000	0.00992833
1981:29	11.5500	0.01030993
1981:30	11.5600	0.00989064
1981:31	11.4200	0.00972691
1981:32	11.3900	0.00940933
1981:33	11.6000	0.00991624
1981:34	11.7200	0.01008077
1981:35	12.0100	0.01072651
1981:36	12.2500	0.01149344
1981:37	12.3700	0.01204629
1981:38	12.6400	0.01323766
1981:39	12.3800	0.01262776
1981:40	12.2300	0.01205959
1981:41	11.7900	0.01138902
1981:42	12.1500	0.01240760
1981:43	12.1200	0.01222583
1981:44	12.3500	0.01296956
1981:45	12.1200	0.01282830
1981:46	11.9500	0.01296335
1981:47	11.9900	0.01337835
1981:48	11.2800	0.01176248
1981:49	11.2400	0.01156737
1981:50	11.2200	0.01141285
1981:51	11.4800	0.01180492
1981:52	11.3400	0.01128414
1981:53	11.1700	0.01085111
1982:1	10.9500	0.01041743
1982:2	11.0700	0.01051316
1982:3	10.6800	0.00956625
1982:4	10.5300	0.00934528
1982:5	11.2500	0.01052816
1982:6	11.5000	0.01096971
1982:7	11.6100	0.01123639
1982:8	11.3000	0.01115374
1982:9	11.8200	0.01260211
1982:10	12.2000	0.01359571
1982:11	12.0300	0.01300703
1982:12	11.5000	0.01173950
1982:13	11.3500	0.01108346
1982:14	11.1500	0.01063849
1982:15	11.0900	0.01052543
1982:16	10.5300	0.00945601
1982:17	10.6000	0.00961872
1982:18	10.3900	0.00916293
1982:19	10.6000	0.00969986
1982:20	10.7700	0.01027714
1982:21	10.6800	0.01024523
1982:22	10.6400	0.01007040
1982:23	11.2800	0.01151841
1982:24	11.3900	0.01150065
1982:25	11.1000	0.01064148
1982:26	11.2900	0.01113419
1982:27	11.4100	0.01166637
1982:28	11.3500	0.01178384
1982:29	10.8500	0.01105213
1982:30	11.3600	0.01214476
1982:31	11.6000	0.01309818
1982:32	11.9300	0.01396577
1982:33	11.2100	0.01292117
1982:34	10.6000	0.01146265
1982:35	10.6300	0.01106050
1982:36	11.1900	0.01221877
1982:37	11.1800	0.01211309
1982:38	11.0900	0.01199737
1982:39	11.3300	0.01266198
1982:40	10.6700	0.01103392
1982:41	11.4400	0.01327706
1982:42	11.3600	0.01297836
1982:43	12.2200	0.01486174
1982:44	12.1600	0.01476073
1982:45	12.6400	0.01590957
1982:46	12.7200	0.01609052
1982:47	12.4200	0.01541761
1982:48	12.1300	0.01453853
1982:49	12.4700	0.01552308
1982:50	12.7700	0.01648078
1982:51	12.1500	0.01480244
1982:52	12.1700	0.01481338
1983:1	11.6500	0.01351308
1983:2	11.7700	0.01390449
1983:3	11.6500	0.01352743
1983:4	11.3900	0.01274035
1983:5	11.4300	0.01276328
1983:6	10.8700	0.01141967
1983:7	10.8600	0.01141162
1983:8	10.4300	0.01056348
1983:9	10.2500	0.01015734
1983:10	10.5500	0.01065370
1983:11	10.3200	0.01008719
1983:12	10.0300	0.00935783
1983:13	10.2600	0.00976925
1983:14	10.1100	0.00950418
1983:15	9.7300	0.00882642
1983:16	9.9500	0.00924470
1983:17	9.8900	0.00915605
1983:18	10.1400	0.00971890
1983:19	10.3100	0.01004804
1983:20	10.4300	0.01021658
1983:21	10.3000	0.00979555
1983:22	10.4300	0.00997857
1983:23	10.5600	0.01015231
1983:24	10.5100	0.01007238
1983:25	10.6600	0.01028382
1983:26	10.8800	0.01078110
1983:27	10.9700	0.01082858
1983:28	10.8500	0.01052765
1983:29	10.3600	0.00952208
1983:30	10.7800	0.01039404
1983:31	10.9500	0.01070475
1983:32	10.8500	0.01046306
1983:33	10.6400	0.01012247
1983:34	10.5800	0.01003862
1983:35	10.4600	0.00964602
1983:36	10.2200	0.00921041
1983:37	10.4700	0.00977768
1983:38	10.0100	0.00891679
1983:39	10.2200	0.00940462
1983:40	9.9400	0.00887760
1983:41	10.2200	0.00937654
1983:42	10.5200	0.01012786
1983:43	10.4600	0.00993398
1983:44	10.5100	0.01009931
1983:45	10.3300	0.00966249
1983:46	10.1600	0.00929908
1983:47	9.8800	0.00874782
1983:48	9.9800	0.00890122
1983:49	9.9900	0.00887383
1983:50	10.3300	0.00953422
1983:51	9.9200	0.00871650
1983:52	9.8000	0.00849457
1984:1	9.4200	0.00773678
1984:2	9.6600	0.00822500
1984:3	9.6800	0.00831236
1984:4	9.8700	0.00870594
1984:5	9.8900	0.00885205
1984:6	10.6000	0.01032103
1984:7	10.3200	0.00971661
1984:8	10.2100	0.00944311
1984:9	10.0100	0.00901549
1984:10	10.2300	0.00942646
1984:11	9.9000	0.00869088
1984:12	9.8100	0.00838877
1984:13	9.6300	0.00805474
1984:14	10.0900	0.00892699
1984:15	9.8500	0.00850303
1984:16	9.7100	0.00820591
1984:17	9.4200	0.00769460
1984:18	9.3500	0.00749246
1984:19	9.7600	0.00815544
1984:20	9.9500	0.00853854
1984:21	10.0800	0.00875475
1984:22	10.3000	0.00915754
1984:23	9.9900	0.00859372
1984:24	10.3800	0.00940144
1984:25	9.9600	0.00853563
1984:26	10.0600	0.00877988
1984:27	10.2300	0.00914805
1984:28	10.3300	0.00932848
1984:29	10.4800	0.00962470

Notes:

CBOEINDEX is the value of the CBOE's index of six month option premia as a percent of share price

SIGMASQ is the implied monthly volatility computed by the authors