

The Plain-Strain Essential Work Of Fracture As A Measure Of The Fracture Toughness Of Ductile Polymers

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We have extended the essential work of fracture technique to allow for the determination of the plane-strain essential work of fracture. The new technique is to measure the specific work of fracture as a function of ligament length in deeply double edge notched samples. This type of data is then experimentally corrected to remove the plastic work of fracture and leave only the essential work of fracture as a function of ligament length. By extrapolating the essential work of fracture to zero-ligament length, we claim to be measuring the plane-strain essential work of fracture. This new technique was applied to two rubber toughened nylons and to a series of polyethylenes. The plane-strain essential work of fracture was found to be independent of thickness. Where comparison can be made to J -integral testing, the plane strain essential work of fracture was similar to the critical J -integral, J_{Ic} .

INTRODUCTION

Brittle polymers can often be characterized using techniques of linear elastic fracture mechanics (LEFM) [1]. LEFM is based on the assumption that when the magnitude of the stress intensity around a crack tip exceeds some critical value, K_{Ic} , fracture will occur. For this assumption to be valid, the plastic yielding must be limited to a restricted area immediately in the vicinity of the crack tip. In ductile polymers, the plastic zones are not small with respect to sample dimensions and the crack tip stress field can no longer be characterized by a stress intensity factor.

In ductile materials, it was suggested by Broberg [2], that the crack tip region can be divided into two parts — an end region and an outer region. The end region is specific to the crack tip and is the kernel of instability. Perhaps the fracture of ductile polymers could be studied by partitioning the total work of fracture into the work that goes into the end region and the work that goes into the outer region. The end region work initiates instability and as such has been termed the essential work of fracture [3]. The outer region work is associated with plastic work and is the nonessential or plastic work [3]. The essential work of fracture may be a useful material property for characterizing the fracture toughness of ductile materials.

Following Broberg's idea, Cotterell and Reddel [3,4] and later Mai and coworkers [5-14] developed a procedure for measuring the plane-stress essential work of fracture. By their procedure, the total work of fracture in deeply double edge notched specimens (DDENT) tested in tension is measured as a function of ligament length. Under restrictions to be explained later, the total work of fracture can be written as

$$w_{total} = w_{elt} + \beta w_p l^2 t \quad (1)$$

where w_e is the essential work of fracture per unit ligament area, w_p is the plastic work of fracture per unit volume, l is the ligament length, t is the sample thickness, and β is a shape factor that describes the size of the plastic zone. The geometric quantities and the plastic zone are shown in *Fig. 1*. The specific work of fracture or the work of fracture per unit ligament area is

$$w_{specific} = w_e + \beta w_p l \quad (2)$$

Plotting the specific work of fracture as a function of ligament length should yield a straight line. The intercept of the line will give the essential work of fracture and the slope of the line will give βw_p .

For *Eq (2)* to yield a straight line, we must restrict the experiment to samples for which w_e , w_p , and β are independent of the ligament length. First consider the shape of the plastic zone. Inherent in *Eqs (1) and (2)* is the assumption that the ligament length controls the size of the plastic zone and that the plastic zone volume is proportional to $l^2 t$ with the shape factor β as the proportionality constant. Two things can disrupt this proportionality. First, if the ligament length is not small compared to the total sample width (e.g., $l < W/3$ [3]), the size of the plastic zone can be disturbed by edge effects. Second, if the ligament length is larger than twice the plastic zone radius around a single crack tip than the ligament area will not be fully yielded at failure and it will not control the plastic zone size. To prevent either of these effects from occurring, it was proposed to keep the ligament smaller than the minimum of these two criteria [3]:

$$l \leq \min(W/3, 2r_p) \quad (3)$$

where W is the sample width and r_p is the radius of the plastic zone around a single crack tip at failure.

Another factor influencing the applicability of *Eqs (1) and (2)* is the stress state in the ligament length. For ligament lengths much longer than the sample thickness, ductile polymers will always be in a state of pure plane stress. Because we expect the materials properties w_e and w_p to be constant under constant stress state conditions, *Eqs (1) and (2)* will be valid for plane stress samples provided they also meet the restriction of *Eq (3)*. As the ligament length is reduced to values comparable to the sample thickness, the stress state will become mixed mode having both plane-stress and plane-strain character. In the mixed-mode region, w_e or w_p or both w_e and w_p may

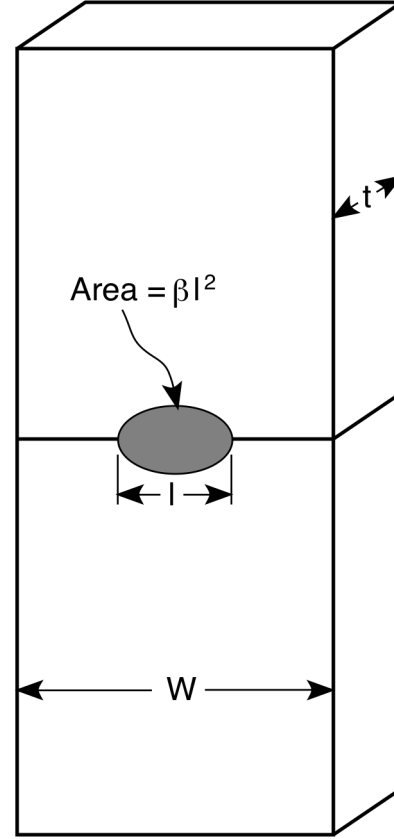


Fig. 1: The deeply double edge notched specimen geometry. l is the ligament length, W is the total sample width, t is the sample thickness, and the shaded area represents a yielded ligament.

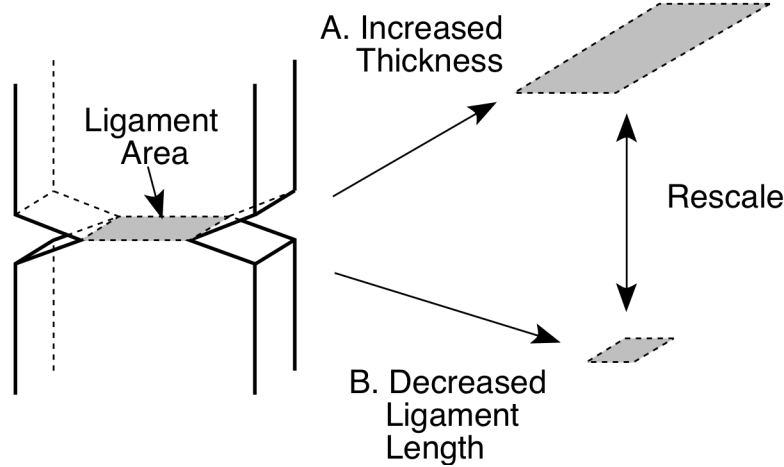


Figure 2: If the ligament on the left is not in the state of plane strain, plane strain conditions can be approached by two paths. A. Increase the thickness. B. Decrease the ligament length. The two paths lead to ligament areas that are equivalent except for a scaling factor.

depend on stress state and therefore on ligament length. If they do, Eqs (1) and (2) will no longer apply. To prevent the onset of mixed-mode stress states in some of the samples, most essential work of fracture experiments have been limited to samples with ligament lengths greater than 3 to 5 times the sample thickness [3,14]. Samples with $l < 3-5t$ will be called mixed-mode samples and samples with $l > 3-5t$ will be called plane-stress samples.

A physical picture of this effect is given in Fig. 2. Standard practice for achieving plane-strain stress states in fracture mechanics studies, is to increase the thickness at constant ligament length (Fig. 2A). The increased thickness promotes plane-strain character and reduces l/t — the ratio of the ligament length to sample thickness. Reducing the ligament length without changing the sample thickness also reduces l/t and therefore could be expected to also promote plane-strain character (Fig. 2B). We will give some experimental results in this paper that show that the value of $l=3-5t$ for the onset of mixed-mode stress states is probably appropriate.

Combining the plastic zone shape restriction with the stress state restriction gives the final sample restrictions for plane-stress essential work of fracture experiments:

$$3-5t \leq l \leq \min(W/3, 2r_p) \quad (4)$$

When plotting the specific work of fracture as a function of ligament length for DDENT samples that meet the restrictions in Eq (4), a linear relation as described in Eq (2) can be expected. The intercept of this linear plot will give the plane-stress essential work of fracture. The procedure has been shown to work for both ductile polymers and for ductile metals [3-14].

Although the plane-stress essential work of fracture may be a useful property, the major interest of fracture characterization is the plane-strain fracture properties. In this paper, we describe a method of extending the plane-stress essential work of fracture technique to include the determination of the plane-strain essential work of fracture. The basic approach is to extend the range of samples tested into the mixed-mode region ($l \leq 3-5t$). As the ligament gets shorter, the plane-strain character of the stress state increases. In the limit of zero ligament length, the stress state should approach pure plane-strain character. To measure the plane-strain essential work of fracture, we only need to extrapolate specific work of fracture data in the mixed-mode region to

zero ligament length. Unfortunately, the simple linear relation in *Eq* (2) does not apply to mixed-mode samples. We have adopted an experimental approach to making the extrapolation. The extrapolation procedure will be described and some results for Zytel® ST801 toughened nylon 66 (RTN66) will be given. Because the extrapolation procedure requires a lot of material and testing of many samples, we also present a quicker, albeit less reliable, procedure. This simpler procedure was used to test Zytel® ST901 toughened amorphous nylon (RTAN), moisture conditioned RTN66, and a series of polyethylenes (Alathon 7140, Alathon 7030, and Alathon 7170).

EXPERIMENTAL

Zytel® ST801 toughened nylon (RTN66) and Zytel® ST901 toughened amorphous nylon (RTAN) were provided by the DuPont Company in the form of 75 mm X 250 mm plaques. The samples were provided both in 3.175 mm thick and 1.587 mm thick sheets. The polyethylene samples (Alathon 7140, Alathon 7030, and Alathon 7170) were also provided by the DuPont Company. These materials were provided as 12.7 mm X 125 mm X 3.175 mm bars.

All specimens were tested in the deeply double edge notched (DDENT) configuration (See *Fig. 1*). The notches were cut using a razor blade mounted in a hand press. By using sample holding jigs we could reproducibly align the two notches on the opposite sides of each sample. We found notch alignment to be critical; samples with poorly aligned notches gave specific works of fracture above those of samples with well aligned notches. After notching the samples, it was difficult to see to location of the notch tip and therefore difficult to measure the ligament length. Because of this problem, the ligament lengths were measured after fracturing the sample; the notch tips were clearly visible on the fracture surfaces. The ligament length measurement was done using a traveling microscope. After cutting the notches, all samples, except moisture conditioned samples, were kept dry until testing.

All fracture tests were done on a 25 kN MTS System 810 servohydraulic testing frame. Load was measured using a 25 kN load cell and displacement was measured using the LVDT located in the System 810 actuator. Load and displacement data were digitized by an MTS 464 data display device which was interfaced to an IBM-PC by custom developed software. After collecting the data, the software analyzed the load-displacement data and reported the area under the curve or the total work of fracture.

RESULTS

As described in the introduction, as the ligament length becomes short relative to the sample thickness, the ligament stress state becomes mixed-mode having both plane-stress and plane-strain character. As the ligament length approaches zero, we expect the mixed-mode stress state to become pure plane strain. By extrapolating the specific work fracture of mixed-mode samples to zero ligament length, we should be measuring the plane-strain essential work of fracture.

We began with an experimental determination of the onset of the mixed-mode region. Most essential work of fracture studies have assumed that the onset of mixed-mode character occurs when the ligament length is 3 to 5 times the sample thickness ($l = 3-5t$) [3,14]. This assumption can be experimentally checked by measuring the net stress in the ligament at the point of full yielding. According to Hill [15], if σ_0 is the yield stress of an unnotched sample, then a fully yielded double

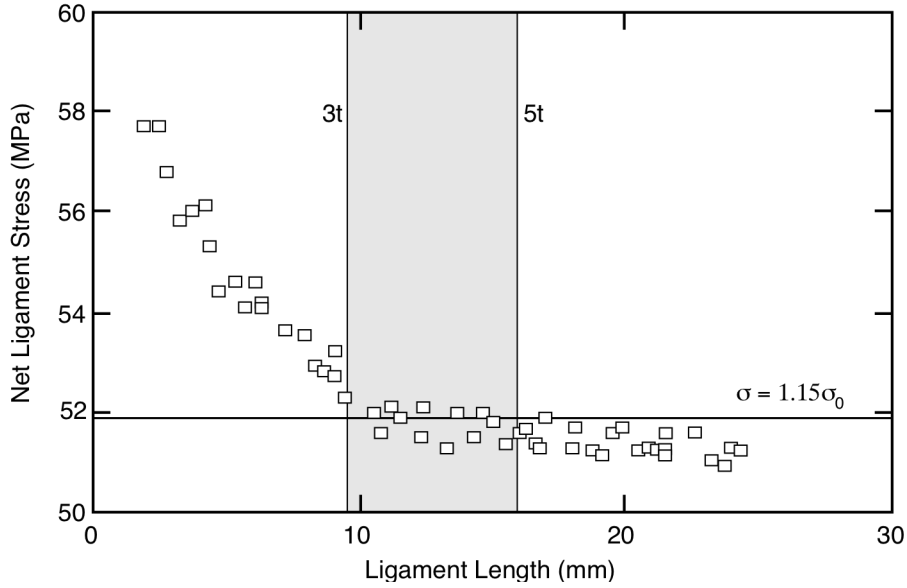


Figure 3: The net stress in the ligament of 3.175 mm thick ST801 at failure as a function of the ligament length. The horizontal line is at $\sigma = 1.15\sigma_0 = 1.15(45.1 \text{ MPa}) = 51.87 \text{ MPa}$. The shaded area marks the ligament length zone $3t < l < 5t$.

edge notched sample in plane stress will yield at $1.15\sigma_0$ while a sample in plane strain will yield at $2.97\sigma_0$. A plot of net stress on the ligament after yielding as a function of ligament length for RTN66 is given in Fig. 3. Also indicated in Fig. 3 is $1.15\sigma_0$ where σ_0 at rates comparable to our experiments is 45.1 MPa [16] (σ_0 is defined as the maximum in the stress-strain curve of an unnotched sample). For long ligament lengths, the yield stress was constant and is close to the $1.15\sigma_0$ predicted by Hill [15]. As the ligament length got short, the yield stress increased indicating the onset of a mixed-mode stress state. The shaded portion on the figure shows the range where $l=3-5t$. It can be seen that the lower end of this range is an appropriate marker for the onset of mixed-mode samples.

In Fig. 4, we show data for the specific work of fracture of 3.175 mm thick DDENT RTN66 specimens. The results span a range of ligament lengths from well within the mixed-mode region to the pure plane-stress region. The line in Fig. 4 is linear fit to the plane-stress data ($l > 3t$ or about 9 mm); the fit is excellent. The intercept gives a plane-stress essential work of fracture of $52 \pm 3 \text{ kJ/m}^2$.

As the ligament length was reduced into the mixed-mode region, there were clear deviations from the linear fit to the plane-stress region. These deviations are a consequence of the mixed-mode character of the short ligament length data. When we attempted to extrapolate this data to zero ligament length by fitting the mixed-mode data to a linear relation, the results were unsatisfactory. The data in this region had some curvature and as a result there were systematic deviations from the best linear fit. An additional problem with the attempted linear extrapolation of the mixed-mode data was that data from samples with different total widths extrapolated to different intercepts. As there should be no width effects on the plane-strain essential work of fracture, we concluded that linear extrapolation of mixed-mode sample data does not work.

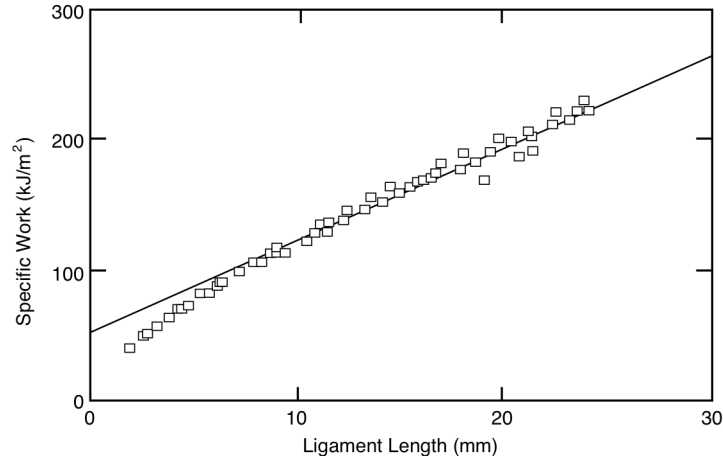


Figure 4: The total specific work of fracture for 3.175 mm thick ST801 as a function of ligament length. The line is a fit through the plane stress data or the data for $l > 3t$.

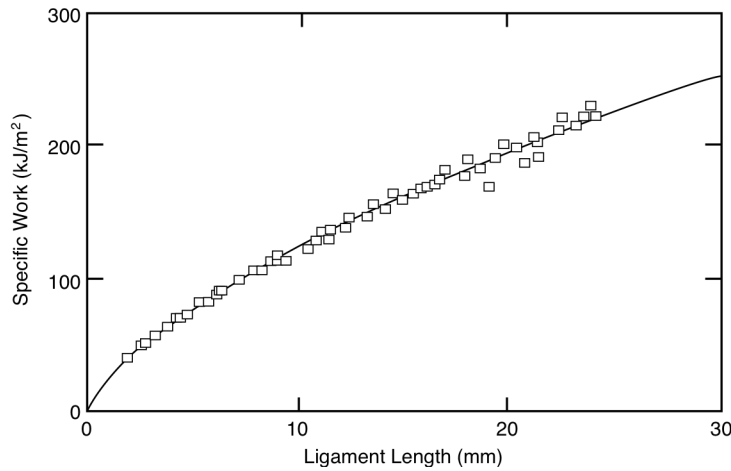


Figure 5: The total specific work of fracture for 3.175 mm thick ST801 as a function of ligament length. The smooth line through the data is the best fit to a power-law relation.

Plane-Strain Essential Work of Fracture Technique

Recent work on experimental J integral fracture techniques proposed fitting J integral vs. crack growth data using a power law relation [17]. In essential work of fracture specimens, the crack growth is the total ligament length. An analogy with J -integral techniques suggests plotting the specific work of fracture using the power-law relation

$$w_{specific} = A l^n \quad (5)$$

A fit to the RTN66 data using Eq (5) is shown in Fig. 5. Eq (5) fits the data through both the mixed-mode regions and through the plane-stress region.

The intercept of the power-law relation is always zero at zero ligament length. Although the power-law relation can correlate the data, it has no theoretical basis and it can not be used to determine the plane-strain essential work of fracture by extrapolation to zero ligament length. We could propose a failure criterion as the specific work of fracture to fracture some specified ligament

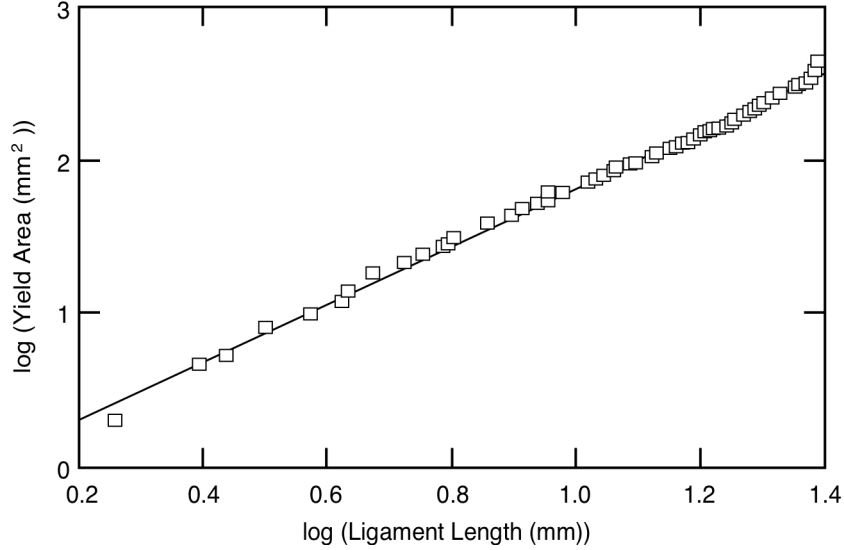


Figure 6: The area of the plastic zone in 3.175 mm thick ST801 as a function of the ligament length. The line is a linear fit to the log-log plot.

length. This proposal would correspond to the J -integral standard technique which proposes the critical J integral as the J integral required to cause crack growth of 0.2 mm. There are two problems with this approach. First, we cannot justify the choice of any particular ligament length for use in determining a failure criterion. Second, any non-zero ligament length will be a sample in a mixed-mode stress state and the resulting specific work of fracture will not give a plane-strain fracture property.

Instead we choose an alternative approach [18,19]. We began by generalizing Eq (2) to account for ligament length dependence and thickness dependence of the quantities that are assumed to be constant in the plane-stress region. The generalized equation applies to all ligament lengths:

$$w_{specific} = w_e(l,t) + w_p(l,t)V_p(l,t)/lt \quad (6)$$

where $V_p(l,t)$ is the volume of the plastic zone. Solving Eq (6) for the essential work of fracture yields:

$$w_e(l,t) = w_{specific} - w_p(l,t)V_p(l,t)/lt \quad (7)$$

If we can get some independent information about the plastic work, we can use Eq (7) to correct the specific work of fracture data and yield a direct measure of the essential work of fracture as a function of ligament length. Extrapolating the essential work of fracture data to zero ligament length turned out to be simpler than extrapolating the uncorrected specific work of fracture data.

To get independent information about the plastic work term, we observed the plastic zone in RTN66 samples as a function of ligament length. In RTN66 samples, the plastic zone could be seen as a whitened zone confined to the ligament section. For all samples, the plastic zone was observed to be elliptical. The ligament length defines the major axis of the ellipse and the height of the whitened area at the center of the sample defines the minor axis of the ellipse. A log-log plot of the area of the plastic zone as a function of the ligament length is given in Fig. 6. The resulting plot is close to linear and the same linear relation applies to both the mixed-mode region and the

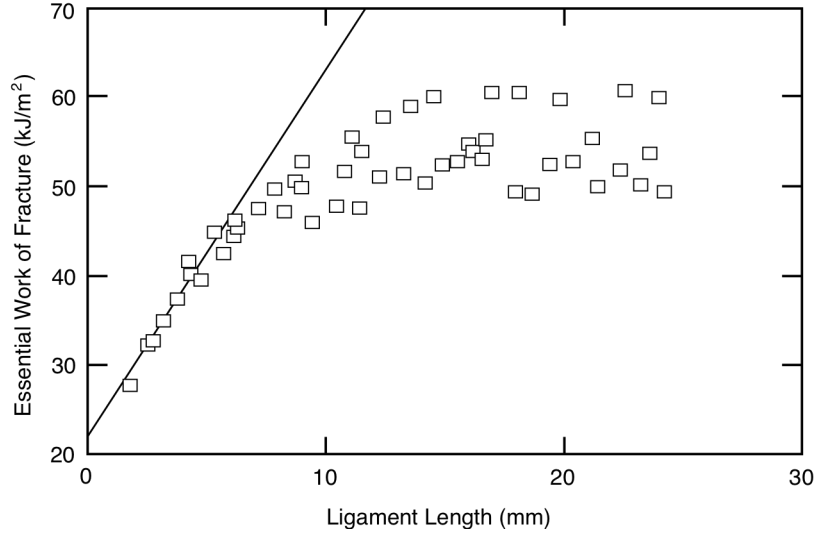


Figure 7: The essential work of fracture for 3.175 mm thick ST801 as a function of ligament length. The plotted line is a linear fit through the short ligament length data.

plane-stress region. The slope of the line in Fig. 6 is 1.88. Similar experiments on other sample types gave similar fits with slopes always around 2 ± 0.15 .

From our observations on the plastic zone size, we can say that the volume of the plastic zone is proportional to l^2t with a proportionality constant $\beta(t)$ that is independent of the ligament length

$$V_p(l,t) = \beta(t)l^2t \quad (8)$$

The overall character of the yielding process appears to be unaffected by the onset of mixed-mode character. From this observation, we suggest that the plastic work per unit volume, $w_p(l,t)$, is also be independent or nearly independent of the ligament length. This assumption, based purely on experimental observation, should hold for samples with ligament lengths in the range of those tested in Fig. 6. In brief, we are assuming that the drop in specific work of fracture at short ligament lengths is due to a drop in the essential work of fracture. With these new assumptions, Eq (7) can be rewritten as

$$w_e(l,t) = w_{\text{specific}} - w_p(t)\beta(t)l \quad (9)$$

In Eq (9) we retain the thickness dependence of $w_p(t)$ and $\beta(t)$ because these quantities may depend on sample thickness.

If we know $w_p(t)\beta(t)$ for a particular material, we can correct the specific work of fracture and directly measure the essential work of fracture as a function of ligament length. Extrapolation of $w_e(l,t)$ to zero ligament length would be the plane-strain essential work of fracture. Because, as expressed, $w_p(t)\beta(t)$, is independent of ligament length, we can find it by experiments in the plane stress region where the specific work of fracture is known to be linear with slope equal to $w_p(t)\beta(t)$.

For the RTN66 data in Fig. 4, the slope of the plane-stress fit gave $w_p(t)\beta(t) = 7.1 \pm 0.4$ mJ/mm³. Using this result and Eq (9), we calculated the essential work of fracture as a function of ligament length. The result is plotted in Fig. 7. For long ligament lengths, the essential work of

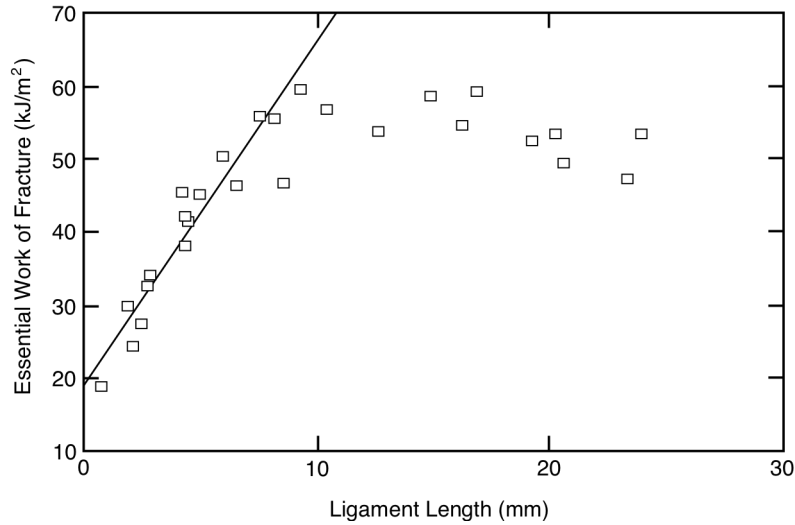


Figure 8: The essential work of fracture for 1.587 mm thick ST801 as a function of ligament length. The plotted line is a linear fit through the short ligament length data.

fracture reached a plateau at a value equal to the plane-stress essential work of fracture (52 ± 3 kJ/m²). For ligament lengths less than 10 mm there was a clear transition to the mixed-mode region and the essential work of fracture decreased. The curvature of the short ligament data was reduced compared to the uncorrected data and a simple linear extrapolation appeared adequate. The result of an extrapolation of this data to zero ligament length (using a linear fit to $l < 6$ mm) gave a plane-strain essential work of fracture of 21.0 kJ/m².

The plain-strain J_{Ic} of RTN66 was measured by Huang to be 29.2 kJ/m² [20]. While there is a qualitative similarity between the J_{Ic} result and the plane-strain essential work of fracture, there is still a significant difference. The differences could be due to the test measuring different material properties or to differences in testing conditions. The J_{Ic} experiments were done on single edge notched specimens tested in bending at high rates and included data from thick samples. The essential work of fracture experiments were done on double edge notch specimens tested in tension at low rates and used only thin samples.

Thickness Independence

If the plane-strain essential work of fracture is a true plane-strain property, then it should be independent of the sample thickness. This thickness independence should hold even though we might expect a thickness dependence of the specific work of fracture data. We repeated the above procedure on 1.587 mm thick RTN66 samples. The essential work of fracture as a function of ligament length is plotted in Fig. 8. Extrapolating the short ligament length data to zero ligament length resulted in a plane-strain essential work of fracture of 19.0 kJ/m². This result is close to the result obtained for the thicker samples.

A Simpler Procedure

The plane-strain essential work of fracture procedure described above is tedious and involves much material. In the RTN66 experiments, we tested 55 specimens and in order to restrict the ligament length to less than $W/3$ and to still obtain results for long ligament length samples, each specimen had to be 75 mm wide. In an attempt to simplify the procedure, we decided to make use

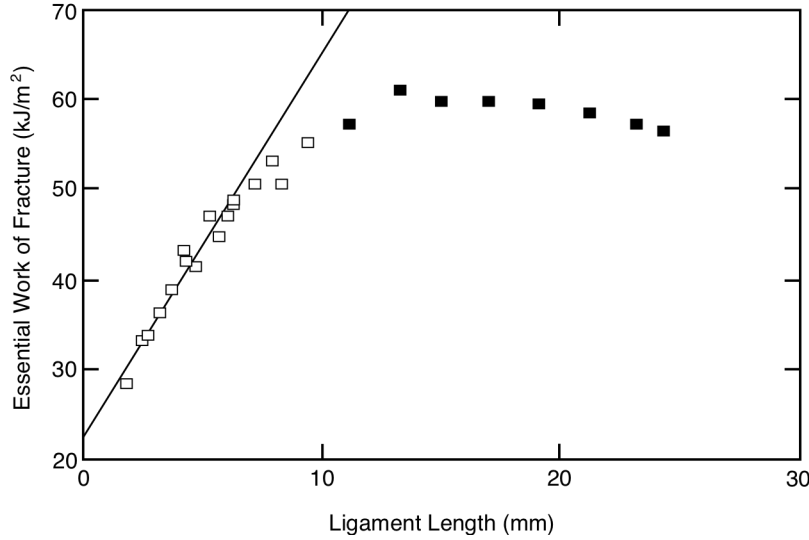


Figure 9: The essential work of fracture for 3.175 mm thick ST801 as a function of ligament length. The short ligament length data (open symbols) are measured data. The long ligament length data (filled symbols) are calculated data found by extrapolating short ligament length specific work of fracture data to long ligament lengths using a power-law relation. The plotted line is a linear fit through the short ligament length data.

of the power-law relation in Eq (5) to correct specific work of fracture of mixed-mode region data without doing any experiments in the plane-stress region. The procedure is as follows:

1. Fit the specific work of fracture data from mixed-mode region samples to the power law in Eq (5).
2. Use the power-law fit to extrapolate the data into the plane stress region and calculate a number of plane-stress data points
3. Analyze this full data set by the procedure described earlier.

This procedure involves two extrapolations. First we extrapolate mixed-mode data to calculate plane-stress data. Second, we extrapolate essential work of fracture data to zero-ligament length. We call this procedure the double extrapolation technique.

To test the double extrapolation procedure, we used it to determine the plane-strain essential work of fracture of RTN66 using only the data from the mixed-mode region samples. The plane-strain essential work of fracture of 3.175 mm thick RTN66 analyzed by this simpler procedure is given in Fig. 9. Extrapolating the short ligament length data to zero ligament length resulted in a plane-strain essential work of fracture of 22.4 kJ/m² — a result that was close to the result derived using all the data (21.0 kJ/m²). Analyzing only the mixed-mode region of the 1.587 mm thick RTN66 sample gave a plane-strain essential work of fracture of 21.4 kJ/m² versus the result from using all the experimental data of 19.0 kJ/m².

Although obtaining plane-stress data by extrapolation of mixed-mode region data is certainly not the most accurate approach, it appears to give results close to those obtained using all the data and does achieve two useful goals. First, the plane-strain essential work of fracture can be measured with tests on fewer specimens. Second, and more significantly, because the longest ligament length sample tested can be limited to about 8 mm, samples with total widths of about 25 mm instead of the 75 mm used in our experiments would be sufficient. In summary, by using the

power-law to calculate the plane-stress data, the essential work of fracture experiments can be done in about 1/3 the time and they require 1/6 to 1/10 the amount of material.

Examples

RTAN is another form of toughened nylon. By J -integral testing [16,20] and by impact toughness measurements, its toughness is slightly lower than RTN66. We measured the essential work of fracture on 3.175 mm and 1.587 mm thick RTAN samples. We used 25 mm wide samples and tested only mixed-mode region samples. The data was analyzed by the double extrapolation technique. The plane-strain essential work of fracture for the 3.175 mm and the 1.587 mm thick samples were 16.0 and 16.2 kJ/m², respectively. As expected the plane-strain essential work of fracture of RTAN is slightly lower than RTN66. As with RTN66, the plane-strain essential work of fracture is independent of the sample thickness.

The plain-strain J_{Ic} of RTAN was measured by Huang to be 16.3 kJ/m² [20]. The plane-strain J_{Ic} and the plane-strain essential work of fracture of RTAN are identical. In view of the differences noted between these two quantities for RTN66, their similarity for RTAN may be fortuitous.

In extrapolating uncorrected specific work of fracture data, we noted that the essential work of fracture obtained by extrapolating to zero ligament length depended on the sample width. To test for similar effects on extrapolation of corrected specific work of fracture, we repeated the RTN66 experiments for 25 mm wide samples. The results (analyzed by the double extrapolation technique) for 3.175 mm and 1.587 mm thick were 19.0 and 21.4 kJ/m². These results were again independent of thickness and also were close to the results obtained for the 75 mm wide samples.

RTN66 will absorb a significant amount of water. The water absorption plasticizes the polymer and significantly lowers its yield stress. Although plane-strain J -integral type testing can be done on dry RTN66 the lower yield stress of moisture conditioned RTN66 increases the sample thickness requirements for plane-strain conditions [17]. The thickness requirement calls for moisture conditioned samples too thick to be practical. There are no thickness requirements in the plane-strain essential work of fracture method and the method can therefore be applied to moisture conditioned RTN66. We soaked 1.587 mm thick and 25 mm wide RTN66 samples in water for 37 days. During this period, these samples absorbed 10.47% water by weight. The specific work of fracture was measured for mixed-mode region samples and the plane strain essential work of fracture was determined by the double extrapolation procedure. The moisture conditioned samples had a plane-strain essential work of fracture of 24.3 kJ/m². This result should be compared to the dry RTN66 result (for the 25 mm wide samples) of 21.4 kJ/m². We observed little or no change in the plane-strain essential work of fracture due to moisture. There is little change despite the fact that the yield stress, as observed by net section stress, drops by a factor of three on going from dry to wet conditions.

The plane-strain essential work of fracture was measured for three different polyethylenes (Alathon 7140, Alathon 7030, and Alathon 7170). Again, experimental data was taken in the mixed-mode region and the analysis was done by the double extrapolation procedure. The plane-strain essential work of fractures of Alathon 7140, Alathon 7030, and Alathon 7170 were determined to be 23.8 kJ/m², 7.3 kJ/m², and 0.7 kJ/m², respectively. By melt index data [21], it is known that Alathon 7140 and Alathon 7030 are fairly high molecular weight and that Alathon 7170 is low molecular weight. The plane-strain essential work of fracture is therefore significantly higher for the higher molecular weight polyethylenes.

Table 1. The plane-strain essential work of fracture and the critical J integral for toughened nylons (RTN66 and RTAN) and for several polyethylenes (Alathon 7140, Alathon 7030, and Alathon 7170). (Note, the J_{Ic} data from Ref. [20] was done on samples of different thicknesses)

Material	W(mm)	t (mm)	w_e^* (kJ/m ²)	w_e^{**} (kJ/m ²)	J_{Ic} (kJ/m ²)
RTN66	75	3.175	21.0	22.4	29.5[20]
RTN66	75	1.587	19.0	21.4	29.5[20]
RTN66	25	3.175		19.0	29.5[20]
RTN66	25	1.587		21.4	29.5[20]
RTN66+H ₂ O	25	1.587		24.3	
RTAN	25	3.175		16.0	16.3[20]
RTAN	25	1.587		16.2	16.3[20]
Alathon 7140	12.7	3.175		23.8	
Alathon 7030	12.7	3.175		7.3	
Alathon 7170	12.7	3.175		0.7	

* Analysis done using full set of experimental data spanning both the mixed-mode and the plane-stress regions

** Analysis done by the double extrapolation procedure

Table 1 summarizes the plane-strain essential work of fracture experiments described in this paper. Where available, the critical plane-strain J -integral for polymer is also listed.

DISCUSSION

It is reasonable to assume that by extrapolating the specific work of fracture of mixed-mode regions samples to zero ligament length, one would be measuring the plane-strain essential work of fracture. The problem is how to accomplish the difficult extrapolation. After trying several different procedures, we concluded that by subtracting the plastic work of fracture from the specific work of fracture, the resulting essential work of fracture could be extrapolated to zero-ligament length more easily than could the uncorrected raw data.

Although we make no rigorous argument as to why the proposed procedure helps the extrapolation effort, we suggest two reasons. First, the extrapolation of the raw data is a long extrapolation. The data in *Fig. 4* shows that at long ligament lengths the specific work of fracture exceeds 200 kJ/m². Because the plane-strain essential work fracture is around 20 kJ/m², extrapolation of the raw data involves extrapolation over a ten-fold decrease in toughness. Such long and steep extrapolations are difficult and sensitive to small experimental errors. In contrast, after removing the plastic work terms, the resulting essential work of fracture is smaller (See *Fig. 7*) and the extrapolation to zero ligament length is a shorter extrapolation. It is reasonable to expect that the shorter extrapolation will be less sensitive to experimental errors.

The second reason is the ability to decide which data points to include in the fitting procedure. Implicit in our fit of the short ligament length essential work of fracture data to a linear relation is the assumption that $w_e(l,t) \approx w_e(0) + \alpha(t)l$ for l less than some critical value — l_{crit} . From *Eq (9)* the specific work of fracture for $l < l_{crit}$ is also linear

$$w_{specific} \approx w_e(0) + [\alpha(t) - w_p(t)\beta(t)]l \quad (10)$$

and a linear extrapolation of $w_{specific}$ would yield the same result as the linear extrapolation of $w(l,t)$. The problem with fitting $w_{specific}$ is the uncertainty in selecting l_{crit} . While it is easy to see

when $w(l,t)$ deviates from a linear relation, it is typically not possible to make the same observation from uncorrected $w_{specific}$ data. For example, the data in *Fig. 7* clearly deviates from linearity around 6 mm while the data in *Fig. 5* shows no such clear indication. If nothing else, the process of subtracting the plastic work of fracture from the specific work of fracture enables a direct measure of the $w(l,t)$ and more clear indications of how the data should be extrapolated to zero ligament length.

Where J_{Ic} and the plane-strain essential work of fracture are available for the same material, we found qualitative agreement. We suggest that J_{Ic} and the plane-strain essential work of fracture are similar material properties that characterize the plane-strain fracture toughness of ductile materials. An advantage of the plane-strain essential work of fracture technique is that it can be used on samples which could not be studied by standard J -integral procedures [17]. In particular, as the yield strength is lowered, the sample thickness requirements for valid J -integral testing increases. For very low yield strengths, it is impractical or impossible to obtain samples of sufficient thickness. The plane-strain essential work of fracture technique, however, does not require thick specimens and can therefore be used for materials where J -integral tests fail. The results on the moisture conditioned RTN66 samples reported in this paper represent an example of a material for which the J -integral tests are inadequate but for which the plane-strain essential work of fracture technique appears to work.

Our suggestion that J_{Ic} and the plane-strain essential work of fracture are similar material properties is based purely on experimental evidence. The theoretical basis for the initiation of failure in non-linear materials taking place a critical level of the J -Integral is based on the presence of an HRR (Hutchinson [22], Rice and Rosengren [23]) type stress field at the crack tip. The HRR stress field analysis is for samples in which all areas of the specimen experience proportional loading. In the essential work of fracture specimens, we can expect that as the crack propagates across the sample there will be significant amounts of non-proportional unloading behind the crack front and therefore significant deviations from an HRR stress field. In the limit of zero-ligament length, however, there will be no crack growth and no nonproportional unloading. We therefore speculate that while all experimental measurements are for samples that are neither in a state of pure plane strain, nor exhibit an HRR stress field, in the limit of zero ligament length the specific work of fracture characterizes the failure of a pure plain strain crack with an HRR stress field. These comments provide some theoretical justification for the equivalence of J_{Ic} and the plain-strain essential work of fracture.

Although the complete plane-strain essential work of fracture procedure is a multi-specimen test, the double extrapolation procedure described in this paper significantly reduces the amount of time and material required to get results. The double extrapolation procedure simplifies the procedure, but it probably also limits the accuracy. For quantitative work, it is better to measure the data in the plane-stress region rather than to calculate it by extrapolating a power-law fit.

ACKNOWLEDGMENTS

This work was supported by the American Chemical Society Petroleum Research Fund Grant #18824-G7 and by E. I. duPont deNemours & Company monitored by D. D. Huang.

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