CERN-TH.6571/92

THE POLARIZED PHOTON STRUCTURE FUNCTION g_1^γ

AS A PROBE OF CHIRAL SYMMETRY REALIZATIONS

G.M. Shore and G. Veneziano

CERN – *Geneva*

Abstract

The first moment of the polarized photon structure function $g_1^\gamma(y, Q^2; \kappa^2)$ is determined by the anomalous 3-current AVV correlation function. Its dependence on the off-shell momentum κ^2 of the target photon is a direct probe of the way chiral symmetry is realized in the AVV function. A careful account is given of the effect of different realizations of chiral symmetry on the AVV correlation function with special emphasis on the delicate chiral limit, where the occurrence of singularities in the invariant amplitudes makes the relation with the first moment of g_1^γ more subtle. The behaviour of g_1^γ as κ^2 is increased through the thresholds associated with the different quarks and leptons is described.

CERN-TH.6571/92
September 1992

1. Introduction

In ref.[1], we proposed a new sum rule relating the first moment of the polarized photon structure function $g_1^\gamma(y, Q^2; \kappa^2)$ to form factors characterizing the AVV (axial-vector-vector) 3-current correlation function in QCD. Experimentally, g_1^γ can be measured in a two-photon process (Fig.1) in deep-inelastic, polarized e^+e^- scattering. Theoretically, the AVV 3-point function is interesting since it involves both the strong and electromagnetic $U(1)$ anomalies and exhibits qualitatively different behaviour according to the realization of chiral symmetry. The sum rule therefore opens up the possibility of measuring directly a correlation function sensitive to some of the most subtle aspects of QCD dynamics.

In this paper, we extend the analysis of ref.[1] to give a detailed description of the AVV correlation function, contrasting the perturbative predictions, which would be relevant for leptons, heavy quarks or with the Wigner realization of unbroken chiral symmetry, with the expectations based on the Nambu-Goldstone realization, relevant for the light quark sector of QCD. We focus especially on the very delicate chiral limit and show that the singularity structure is quite different in the two realizations. In particular, the massless poles in the axial channel due to the Nambu-Goldstone bosons, which lead to discontinuities in the six independent amplitudes specifying the Green function, are absent in the perturbative realization except for the special case of on-shell photons.

We then explain carefully the precise relation between the first moment of g_1^γ and the AVV amplitudes in the chiral limit and show how, despite the discontinuities in these amplitudes, all physical quantities are smooth and continuous as the quark masses are taken to zero. Furthermore, we show that the first moment of g_1^γ does indeed distinguish between different realizations of chiral symmetry even in the chiral limit, contrary to the naive expectation that in this limit it is determined entirely by the anomaly and hence realization independent.

Finally, we summarise our results, describing the dependence of the first moment of $g_1^\gamma(y, Q^2; \kappa^2)$ on the off-shell target photon momentum κ^2 ($\equiv -k^2$) as the thresholds associated with the different quarks and leptons are encountered.

2. Chiral symmetry realizations and the AVV correlation function

It is well-known that the AVV correlation function

$$\Gamma^r_{\mu\lambda\rho} = \int dx_1\ dx_2\ e^{i(k_1.x_1+k_2.x_2)}\ \langle 0|T^*\ J^r_{\mu 5}(0)\ J_\lambda(x_1)\ J_\rho(x_2)|0\rangle, \tag{2.1}$$

where r is a flavour index, can be decomposed into six independent invariant amplitudes as follows:

$$\Gamma^r_{\mu\lambda\rho} = \sum_i A_i^{(r)}(p^2, k_1^2, k_2^2)\ I^i_{\mu\lambda\rho}, \tag{2.2}$$

where the amplitudes $A_i^{(r)}$ are analytic functions of $p^2 = (k_1 + k_2)^2$, k_1^2 and k_2^2, and are free of kinematical singularities. A convenient basis[2] for the tensors $I^i_{\mu\lambda\rho}$ $(i = 1, \ldots, 6)$ is

$$\begin{aligned} I^1_{\mu\lambda\rho} &= \epsilon_{\mu\lambda\rho\alpha} k_1^\alpha & I^2_{\mu\lambda\rho} &= \epsilon_{\mu\lambda\rho\alpha} k_2^\alpha \\ I^3_{\mu\lambda\rho} &= \epsilon_{\mu\lambda\alpha\beta} k_1^\alpha k_2^\beta k_{2\rho} & I^4_{\mu\lambda\rho} &= \epsilon_{\mu\rho\alpha\beta} k_1^\alpha k_2^\beta k_{1\lambda} \\ I^5_{\mu\lambda\rho} &= \epsilon_{\mu\lambda\alpha\beta} k_1^\alpha k_2^\beta k_{1\rho} & I^6_{\mu\lambda\rho} &= \epsilon_{\mu\rho\alpha\beta} k_1^\alpha k_2^\beta k_{2\lambda} \, . \end{aligned} \tag{2.3}$$

In all cases of interest to us, the two vector currents are identical, so that by crossing symmetry,

$$\begin{aligned} A_1^{(r)}(p^2, k_1^2, k_2^2) &= -A_2^{(r)}(p^2, k_2^2, k_1^2), & A_3^{(r)}(p^2, k_1^2, k_2^2) &= -A_4^{(r)}(p^2, k_2^2, k_1^2), \\ A_5^{(r)}(p^2, k_1^2, \kappa_2^2) &= -A_6^{(r)}(p^2, k_2^2, k_1^2). \end{aligned} \tag{2.4}$$

The Ward identities for vector current conservation immediately imply the two CVC constraints

$$\begin{aligned} A_1^{(r)} &= A_3^{(r)} k_2^2 \; - \; A_5^{(r)} \frac{1}{2}(k_1^2 + k_2^2 - p^2) \\ A_2^{(r)} &= A_4^{(r)} k_1^2 \; - \; A_6^{(r)} \frac{1}{2}(k_1^2 + k_2^2 - p^2). \end{aligned} \tag{2.5}$$

The anomalous chiral Ward identities give

$$A_1^{(r)} - A_2^{(r)} = -D^{(r)} \; - \; B\delta^{r0} \; + \; \frac{N_C}{4\pi^2} a^{(r)}, \tag{2.6}$$

where the amplitudes $D^{(r)}$ and B are defined from

$$2m \int dx_1 dx_2 \; e^{i(k_1 . x_1 + k_2 . x_2)} \; \langle 0 | T^* \; \phi_5^r(0) \; J_\lambda(x_1) \; J_\rho(x_2) | 0 \rangle \; = \; D^{(r)}(p^2, k_1^2, k_2^2) \; \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta, \tag{2.7}$$

with $\phi_5 = i\bar{\psi}\gamma_5 \lambda^r \psi$, and

$$2N_F \int dx_1 dx_2 \; e^{i(k_1 . x_1 + k_2 . x_2)} \; \langle 0 | T^* \; Q(0) \; J_\lambda(x_1) \; J_\rho(x_2) | 0 \rangle \; = \; B(p^2, k_1^2, k_2^2) \; \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta, \tag{2.8}$$

with $Q = \frac{\alpha_s}{8\pi} \mathrm{tr} \tilde{G}^{\mu\nu} G_{\mu\nu}$. The coefficients $a^{(r)}$ determined by the electromagnetic $U(1)$ anomaly are given by $a^{(r)} = \mathrm{tr} \; \hat{e}^2 \lambda^r$, where $\hat{e}$ is the numerical quark charge matrix. (For further notation, see ref.[1].)

We now consider some aspects of the singularity structure of the amplitudes $A_i^{(r)}$, first in the chiral symmetry breaking phase of QCD, then in perturbation theory (which, as we justify later, is equivalent to the Wigner realization, i.e. unbroken chiral symmetry).

(i) NAMBU-GOLDSTONE REALIZATION

Consider first the flavour non-singlet case, where the strong $U(1)$ anomaly term B is absent from the chiral Ward identity (2.6).

Since there is a confinement mass gap in the vector channels, we do not expect any singularities at $k^2 = 0$. However, poles at $p^2 = 0$ in the form factors $A_i^{(r)}$ can arise in the chiral limit due to the massless Nambu-Goldstone bosons (hereafter referred to as "pions"). In the chiral limit, the pion contribution to $\Gamma^r_{\mu\lambda\rho}$ is clearly of the form

$$\Gamma^{\pi}_{\mu\lambda\rho} \;=\; \frac{1}{8\pi\alpha} F_\pi g_{\pi\gamma^*\gamma^*} \frac{p_\mu}{p^2} \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta. \tag{2.9}$$

This Lorentz structure does not appear in the Adler basis (2.3) of $I^i_{\mu\lambda\rho}$. Nonetheless, the remarkable identity

$$\delta_{\nu(\mu}\epsilon_{\lambda\rho\alpha\beta)} \;+\; \text{cyclic perms. of } (\ldots) \;=\; 0, \tag{2.10}$$

contracted once with $k_1^\nu k_1^\alpha k_2^\beta$ and once with $k_2^\nu k_1^\alpha k_2^\beta$ shows* that the expression (2.9) can in fact be expressed in this basis with the result

$$\begin{aligned}
A_1^\pi \;&=\; \frac{1}{16\pi\alpha}\, F_\pi g_{\pi^*\gamma^*\gamma^*}\, \frac{1}{p^2 - m_\pi^2}\, \left(p^2 - k_1^2 + k_2^2\right) \\
A_2^\pi \;&=\; \frac{1}{16\pi\alpha}\, F_\pi g_{\pi^*\gamma^*\gamma^*}\, \frac{1}{p^2 - m_\pi^2}\, \left(-p^2 - k_1^2 + k_2^2\right) \\
A_3^\pi \;&=\; -A_4^\pi \;=\; A_5^\pi \;=\; -A_6^\pi \;=\; \frac{1}{8\pi\alpha}\, F_\pi g_{\pi^*\gamma^*\gamma^*}\, \frac{1}{p^2 - m_\pi^2}\,.
\end{aligned} \tag{2.11}$$

Away from the chiral limit, the pion contribution is of exactly the same form except for the substitution $1/p^2$ for the poles $1/(p^2 - m_\pi^2)$.

We can immediately check that these expressions by themselves are consistent with the CVC Ward identities (2.5). Separating the form factors into a "regular" plus pion piece, we can use eqs.(2.11) to write the chiral Ward identity in the form

$$\left(A_1^{(r)} - A_2^{(r)}\right)_{\text{reg}} \;+\; \frac{1}{8\pi\alpha} F_\pi g_{\pi\gamma^*\gamma^*} \;=\; \frac{N_C}{4\pi^2} a^{(r)}, \tag{2.12}$$

---

* Specifically, if we define I_7 and I_8 by

$$I^7_{\mu\lambda\rho} \;=\; \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta k_{1\mu} \qquad\qquad I^8_{\mu\lambda\rho} \;=\; \epsilon_{\lambda\rho\alpha\beta} k_1^\alpha k_2^\beta k_{2\mu},$$

then we derive the identities

$$I_7 \;-\; I_4 \;+\; I_5 \;+\; k_1.k_2 I_1 \;-\; k_1^2 I_2 \;=\; 0$$

and

$$I_8 \;+\; I_3 \;-\; I_6 \;+\; k_2^2 I_1 \;-\; k_1.k_2 I_2 \;=\; 0.$$

in the chiral limit. Using the standard current algebra formula for the on-shell $\pi \to \gamma\gamma$ decay amplitude, viz.

$$F_\pi g_{\pi\gamma\gamma} \; = \; 2N_C \; a^{(r)} \; \frac{\alpha}{\pi}, \tag{2.13}$$

we can re-express eq.(2.12) for $p^2 = 0$ in the form

$$\left(A_1^{(r)} - A_2^{(r)}\right)_{\rm reg}(0, k^2, k^2) \; = \; \frac{N_C}{4\pi^2} a^{(r)} \left(1 \; - \; \frac{g_{\pi\gamma^*\gamma^*}(k^2)}{g_{\pi\gamma\gamma}(0)}\right). \tag{2.14}$$

This should be contrasted with the result for $\left(A_1^{(r)} - A_2^{(r)}\right)$ itself, viz.

$$\left(A_1^{(r)} - A_2^{(r)}\right)(0, k^2, k^2) \; = \; \frac{N_c}{4\pi^2} a^{(r)}, \tag{2.15}$$

which holds exactly in the chiral limit for all k^2.

Away from the chiral limit, the divergence term $D^{(r)}$ in eq.(2.6) also contributes, and the chiral Ward identity is saturated as follows:

$$\left(A_1^{(r)} - A_2^{(r)}\right)_{\rm reg} \; + \; \frac{1}{8\pi\alpha} F_\pi g_{\pi\gamma^*\gamma^*} \frac{p^2}{p^2 - m_\pi^2} \; = \; \frac{1}{8\pi\alpha} F_\pi g_{\pi\gamma^*\gamma^*} \frac{m_\pi^2}{p^2 - m_\pi^2} \; + \; \frac{N_C}{4\pi^2} a^{(r)}. \tag{2.16}$$

Rearranging terms, we see that eq.(2.12) actually holds also away from the chiral limit (up to corrections of $O(m/\Lambda)$). Also notice that for $m_\pi^2 \neq 0$, the pion does not contribute to $\left(A_1^{(r)} - A_2^{(r)}\right)$ at $p^2 = 0$, so that eq.(2.14) holds for either $\left(A_1^{(r)} - A_2^{(r)}\right)_{\rm reg}(0, k^2, k^2)$ or $\left(A_1^{(r)} - A_2^{(r)}\right)(0, k^2, k^2)$ itself.

For large $\kappa^2 = -k^2$, it is shown in refs.[3,1] that $g_{\pi\gamma^*\gamma^*}(\kappa^2)$ falls off as $1/\kappa^2$. Precisely,

$$\frac{1}{8\pi\alpha} F_\pi g_{\pi\gamma^*\gamma^*}(\kappa^2) \;\; \underset{\kappa^2 \to \infty}{\sim} \;\; 4a^{(r)} \; F_\pi^2 \; \frac{1}{\kappa^2}. \tag{2.17}$$

This implies the large κ^2 behaviour*

$$\left(A_1^{(r)} - A_2^{(r)}\right)(0, \kappa^2, \kappa^2) \;\; \underset{\kappa^2 \to \infty}{\sim} \;\; \frac{N_C}{4\pi^2} a^{(r)} \left(1 \; - \; \frac{16\pi^2}{N_C} \frac{F_\pi^2}{\kappa^2}\right). \tag{2.18}$$

The mass scale $16\pi^2 F_\pi^2 / N_C$ controlling the approach to the asymptotic value set by the anomaly is a typical hadronic mass of $O(m_\rho^2)$.

The CVC Ward identities (2.5) give (for $k_1^2 = k_2^2 = -\kappa^2$)

$$A_1^{(r)} - A_2^{(r)} \; = \; -\kappa^2 \left(A_3^{(r)} - A_4^{(r)} - A_5^{(r)} + A_6^{(r)}\right) \; + \; \frac{1}{2} p^2 \left(A_5^{(r)} - A_6^{(r)}\right). \tag{2.19}$$

---

* Notice that the same large κ^2 behaviour is shown in the chiral limit by the regular (pion-subtracted) part of the form factors, $(A_1^{(r)} - A_2^{(r)})_{\rm reg}(0, \kappa^2, \kappa^2)$ (see eq.(2.14)).

Since for $m_\pi^2 \neq 0$ there are no poles in the $A_i^{(r)}$ at either $\kappa^2 = 0$ or $p^2 = 0$, we can conclude immediately that for small κ^2

$$(A_1^{(r)} - A_2^{(r)})(0,\kappa^2,\kappa^2) \ = \ O(\kappa^2) \qquad\qquad (m_\pi^2 \neq 0). \tag{2.20}$$

We emphasise that this is only true for $m_\pi^2 \neq 0$. In the chiral limit, the demonstration fails because of the $1/p^2$ poles in $A_5^{(r)}$ and $A_6^{(r)}$ due to the pion.

It follows from this discussion that the form factor $\left(A_1^{(r)} - A_2^{(r)}\right)$ is actually discontinuous as we go to the chiral limit. For $m_\pi^2 = 0$, $\left(A_1^{(r)} - A_2^{(r)}\right)(0,\kappa^2,\kappa^2)$ is equal to the anomaly value $\frac{N_C}{4\pi^2}a^{(r)}$ for all κ^2. On the other hand, away from the chiral limit, it rises smoothly from zero at $\kappa^2 = 0$ to the anomaly value as $\kappa^2 \to \infty$ with a "turnover" scale which we estimate from eq.(2.19) to be of $O(m_\rho^2)$. The discrepancy is due to the contribution in the chiral limit of the massless pion. The regular piece, $\left(A_1^{(r)} - A_2^{(r)}\right)_{\rm reg}(0,\kappa^2,\kappa^2)$, in the chiral limit shows the same gradual rise from zero to the anomaly value as κ^2 is increased from zero to infinity.

Finally, in the singlet channel there is again no massless pseudoscalar even in the chiral limit. The form factors $\left(A_1^{(0)} - A_2^{(0)}\right)(0,\kappa^2,\kappa^2)$ therefore show the same κ^2 dependence as the non-singlet form factors for $m_\pi^2 \neq 0$. A more extensive discussion can be found in section 4 of ref.[1]. The approach to the large κ^2 limit is modified, the leading corrections now being logarithmic in κ^2 as a result of the anomalous dimension of the singlet axial current due to the strong $U(1)$ anomaly (see ref.[1]).

(ii) PERTURBATIVE REALIZATION

In perturbation theory, the invariant amplitudes are given in closed form as integrals over two Feynman parameters[2]. $A_1^{(r)}$ and $A_2^{(r)}$ are given in terms of $A_3^{(r)}, \ldots, A_6^{(r)}$ by the CVC relations (2.5), while $A_4^{(r)}$ and $A_6^{(r)}$ can be expressed in terms of $A_3^{(r)}$ and $A_5^{(r)}$ resp. by the crossing relations (2.4). This leaves the independent amplitudes, for the non-singlet $(r \neq 0)$ sector,

$$A_3^{(r)} \ = \ -2\,\frac{N_C}{4\pi^2}\,a^{(r)}\,\left(I_{0,0} - I_{1,0}\right) \tag{2.21}$$

$$A_5^{(r)} \ = \ 2\,\frac{N_C}{4\pi^2}\,a^{(r)}\,I_{1,1} \tag{2.22}$$

where*

$$I_{p,q} \ = \ \int d\alpha\, d\beta\, \theta(1-\alpha-\beta)\,\alpha^p\beta^q\left[\alpha\beta p^2 + \alpha(1-\alpha-\beta)k_2^2 + \beta(1-\alpha-\beta)k_1^2 - m^2\right]^{-1}. \tag{2.23}$$

Also,

$$D^{(r)} \ = \ -2\,\frac{N_C}{4\pi^2}\,a^{(r)}\,m^2 I_{0,0}, \tag{2.24}$$

* For simplicity, we take the light quarks to be degenerate with common mass m.

and the chiral Ward identity (2.16) is readily checked. In the singlet channel, there are further contributions of higher order in α_s, which are compensated by the strong $U(1)$ anomaly term B to ensure the consistency of the Ward identity.

The integrals (2.23) have been evaluated in several special cases in a recent paper by Achisov[4]. Here, we shall consider some particular limits relevant to the discussion in section 3. Various limits are of interest*.

(a) $k^2 = 0,\ p^2 \to 0$:

In this case, $I_{1,1}\big|_{k^2=0} = 1/2p^2$, so the amplitudes $A_5^{(r)}$ and $A_6^{(r)}$ have a pole at $p^2 = 0$. In fact, in contrast to the Nambu-Goldstone realization just discussed, such a pole arises *only* in this limit with $k^2 = 0$ [4]. We then have

$$\lim_{p^2\to 0}(A_1^{(r)} - A_2^{(r)})(p^2,0,0) \;=\; \lim_{p^2\to 0}\frac{1}{2}p^2(A_5^{(r)} - A_6^{(r)})(p^2,0,0) \;=\; \frac{N_C}{4\pi^2}a^{(r)}. \qquad (2.25)$$

$A_3^{(r)}$ and $A_4^{(r)}$ are infra-red singular in this limit. However, they do not occur in physical quantities, since for on-shell photons the corresponding basis tensors vanish when contracted with transverse polarization tensors, i.e. $\epsilon^\rho(k_2)I^3_{\mu\lambda\rho} = 0$, etc.

---

* Although not so relevant for the subsequent discussion, it is interesting to consider also the following limits:

(c) $p^2 = 0,\ k^2 \to 0$:

Clearly, there is no pole in $1/p^2$ in $A_5^{(r)}$ or $A_6^{(r)}$. The CVC identity reads simply

$$\lim_{k^2\to 0}(A_1^{(r)} - A_2^{(r)})(0,k^2,k^2) \;=\; \lim_{k^2\to 0}k^2(A_3^{(r)} - A_4^{(r)} - A_5^{(r)} + A_6^{(r)}) \;=\; \frac{N_C}{4\pi^2}a^{(r)}$$

using the same calculation giving eq.(2.27).

(d) $p^2 \neq 0,\ k^2 \to 0$:

Here, by inspection of the integral $I_{2,0} - I_{1,0} + I_{1,1}$ we find a logarithmic singularity in $(A_3^{(r)} - A_5^{(r)})$ as $k^2 \to 0$, i.e.

$$(A_3^{(r)} - A_5^{(r)})(p^2,k^2,k^2) \underset{k^2\to 0}{\sim} O\Big(\frac{1}{p^2}\log\frac{k^2}{p^2}\Big)$$

so that $k^2(A_3^{(r)} - A_4^{(r)} - A_5^{(r)} + A_6^{(r)}) \to 0$ for $p^2 \neq 0$. The CVC identity is realized as

$$(A_1^{(r)} - A_2^{(r)})(p^2,0,0) \;=\; \frac{1}{2}p^2(A_5^{(r)} - A_6^{(r)})(p^2,0,0) \;=\; \frac{N_C}{4\pi^2}a^{(r)}$$

again using $I_{1,1}|_{k^2=0} = 1/2p^2$.

This makes clear the symmetry in the singularity structures in the vector and axial channels in perturbation theory.

(b) $k^2 \neq 0,\ p^2 \to 0$:

Here, the $1/p^2$ pole in $A_5^{(r)}$ is absent. Although there is no simple expression for the integral $I_{1,1}$, we can see by inspection that the $p^2 \to 0$ singularity is only logarithmic, i.e.

$$A_5^{(r)}(p^2, k^2, k^2) \underset{p^2 \to 0}{\sim} O\Big(\frac{1}{k^2} \log \frac{p^2}{k^2}\Big), \tag{2.26}$$

so in particular $p^2 A_5^{(r)} \to 0$ for $k^2 \neq 0$. The CVC identity (2.20) is realized this time by

$$\big(A_1^{(r)} - A_2^{(r)}\big)(0, k^2, k^2) \ = \ k^2\big(A_3^{(r)} - A_4^{(r)} - A_5^{(r)} + A_6^{(r)}\big) \ = \ \frac{N_C}{4\pi^2} a^{(r)}, \tag{2.27}$$

as can be checked directly from eqs.(2.23), which give $\big(I_{2,0} - I_{1,0} + I_{1,1}\big)\big|_{p^2=0} = -1/4k^2$.

Away from the chiral limit, we can still perform the integrals analytically to obtain a simple expression for $\big(A_1^{(r)} - A_2^{(r)}\big)$ for $p^2 = 0$. We find, for spacelike photon momenta $(k^2 \equiv \kappa^2 \leq 0)$,

$$\big(A_1^{(r)} - A_2^{(r)}\big)(0, \kappa^2, \kappa^2) \ = \ \frac{N_C}{4\pi^2} a^{(r)} \bigg(\ 1 \ - \ 2\frac{m^2}{\kappa^2}\ \frac{1}{\rho}\ \log \frac{\rho + 1}{\rho - 1}\bigg)\ , \tag{2.28}$$

where $\rho = \sqrt{1 + 4m^2/\kappa^2}$. This implies the large κ^2 behaviour,

$$\big(A_1^{(r)} - A_2^{(r)}\big)(0, \kappa^2, \kappa^2) \underset{\kappa^2 \to \infty}{\sim} \frac{N_C}{4\pi^2} a^{(r)} \Big(1 \ - \ 2\frac{m^2}{\kappa^2}\ \log \frac{\kappa^2}{m^2} \ + \ O\Big(\frac{m^4}{\kappa^4}\Big)\ \Big), \tag{2.29}$$

which should be compared with the corresponding result (2.18) in the Nambu-Goldstone realization. The turnover mass scale characterizing the evolution of $\big(A_1^{(r)} - A_2^{(r)}\big)$ is now $O(m^2)$, i.e. a light quark mass scale rather than the hadronic mass $O(m_\rho^2)$.

As for the small κ^2 behaviour, since there are no poles at $k^2 = 0$ or $p^2 = 0$ for non-zero m in the form factors, the CVC Ward identity again implies

$$\big(A_1^{(r)} - A_2^{(r)}\big)(0, \kappa^2, \kappa^2) \ = \ O(\kappa^2) \qquad\qquad (m^2 \neq 0). \tag{2.30}$$

For non-zero m, therefore, we recover the same type of κ^2 dependence of the amplitude $\big(A_1^{(r)} - A_2^{(r)}\big)(0, \kappa^2, \kappa^2)$ as in the Nambu-Goldstone realization for $m_\pi^2 \neq 0$, viz. a gradual rise from zero at $\kappa^2 = 0$ to the anomaly value $\frac{N_C}{4\pi^2} a^{(r)}$ as $\kappa^2 \to \infty$. However, here the characteristic mass scale is $O(m^2)$ rather than $O(m_\rho^2)$, so as we go to the chiral limit $\big(A_1^{(r)} - A_2^{(r)}\big)(0, \kappa^2, \kappa^2)$ smoothly approaches its limiting value for $m = 0$. There is no discontinuous jump as in the Nambu-Goldstone realization.

(iii) WIGNER REALIZATION

In confining theories with unbroken chiral symmetry (i.e. realized in the Wigner mode), anomaly matching[5] requires the existence of massless spin 1/2 composite fermions[5,6] coupling minimally to the external currents for momenta small compared to the confining scale.

In this realization, therefore, $\left(A_1^{(r)} - A_2^{(r)}\right)$ will behave for small κ^2 and p^2 in essentially the same way as in the perturbative realization just described. That is, in the chiral limit it takes the value set by the anomaly for all κ^2, while away from the chiral limit it approaches this asymptotic value with a turnover scale set by the mass of the light composite fermions.

3. Polarized photon structure function sum rule in the chiral limit

The first moment of the polarized photon structure function g_1^γ, which can be measured in deep-inelastic, polarized e^+e^- scattering, is intimately related to the AVV correlation function. The following sum rule was presented in ref.[1]:

$$\int_0^1 dy\ g_1^\gamma(y, Q^2; \kappa^2) \ = \ L\big(\alpha_s(Q^2)\big)\ 4\pi\alpha\ \sum_r c^{(r)}\Sigma^{(r)}(\kappa^2, Q^2), \qquad (3.1)$$

where $c^{(r)} = 2\text{tr}\ \hat{e}^2\lambda^r \ \ (r \neq 0), \quad c^{(0)} = N_F^{-1}\text{tr}\ \hat{e}^2$, and $L\big(\alpha_s(Q^2)\big)$ are correction factors arising from the Wilson coefficients in the OPE leading to eq.(3.1). The scattering kinematics is such that the target photon momentum is spacelike, $\kappa^2 = -k^2$. In ref.[1], it was shown that the form factors* $\Sigma^{(r)}$ are given in terms of the amplitudes $A_i^{(r)}$ by

$$\Sigma^{(r)}(\kappa^2, Q^2) \ = \ (A_1^{(r)} - A_2^{(r)})(0, \kappa^2, \kappa^2; Q^2) \qquad (m \neq 0), \qquad (3.2)$$

provided we are away from the chiral limit. (Here, Q^2 is the renormalisation scale, on which only the singlet form factors have a non-trivial dependence.)

Since the κ^2 dependence of $\left(A_1^{(r)} - A_2^{(r)}\right)$ is quite different in the Nambu-Goldstone and perturbative (Wigner) realizations of chiral symmetry, it follows that these can be clearly distinguished in a measurement of $\Sigma^{(r)}$ by following the κ^2 dependence of the first moment of g_1^γ in the range $0 \leq \kappa^2 \leq 1\text{GeV}^2$.

In the chiral limit, however, $\left(A_1^{(r)} - A_2^{(r)}\right)(0, \kappa^2, \kappa^2)$ is fixed at its anomaly value of $\frac{N_C}{4\pi^2}a^{(r)}$, independent of the realization. At first sight, therefore, it appears that the Nambu-Goldstone and perturbative realizations are indistinguishable in the chiral limit and that, in the Nambu-Goldstone realization, there is a discontinuous jump in $\Sigma^{(r)}$ as the quark mass is taken to zero.

* To compare with the notation of ref.[1], $\Sigma^{(r)} = \frac{N_C}{4\pi^2}a^{(r)}f^{(r)}$.

In fact, this is not true and in the remainder of this section we will demonstrate that indeed:

(a) the Nambu-Goldstone and perturbative realizations are clearly distinguishable in the chiral limit, and

(b) physical quantities such as $\Sigma^{(r)}$, and hence the first moment of g_1^γ, *are* smooth and continuous as we approach the chiral limit, despite the discontinuities in the amplitudes $A_i^{(r)}$.

To study the chiral limit, we keep a small regularizing momentum p for the axial current and consider the limit $p^2 \to 0$. From ref.[1], away from the chiral limit $\Sigma^{(r)}$ is then defined by

$$\lim_{p\to 0} \Gamma^r_{\mu\lambda\rho}(p,k_1,k_2) \;=\; \Sigma^{(r)}(\kappa^2)\epsilon_{\mu\lambda\rho\alpha}k^\alpha, \tag{3.3}$$

where $k_1 = k - p/2,\;\; k_2 = -k - p/2$, and we consider just the non-singlet channel ($r \neq 0$). (The chiral limit in the singlet channel is discussed already in ref.[1].) Provided that $m \neq 0$, so that there are no $1/p^2$ poles in the $A_i^{(r)}$, the identification of $\Sigma^{(r)}$ with $(A_1^{(r)} - A_2^{(r)})(0,\kappa^2,\kappa^2)$ is clear.

However, from eqs.(2.3),

$$\begin{aligned}\lim_{p\to 0} \Gamma^r_{\mu\lambda\rho}(p,k_1,k_2) \;=\;& \big(A_1^{(r)} - A_2^{(r)}\big)\epsilon_{\mu\lambda\rho\alpha}k^\alpha \;-\; \frac{1}{2}\big(A_1^{(r)} + A_2^{(r)}\big)\epsilon_{\mu\lambda\rho\alpha}p^\alpha \\ &+\; \big(A_3^{(r)} - A_5^{(r)}\big)\epsilon_{\mu\lambda\beta\alpha}k_\rho k^\beta p^\alpha \;-\; \big(A_4^{(r)} - A_6^{(r)}\big)\epsilon_{\mu\rho\beta\alpha}k_\lambda k^\beta p_\alpha \\ &+ \frac{1}{2}\big(A_3^{(r)} + A_5^{(r)}\big)\epsilon_{\mu\lambda\beta\alpha}p_\rho k^\beta p^\alpha \;+\; \frac{1}{2}\big(A_4^{(r)} + A_6^{(r)}\big)\epsilon_{\mu\rho\beta\alpha}p_\lambda k^\beta p^\alpha\end{aligned} \tag{3.4}$$

The question is how to identify the physically measurable $\Sigma^{(r)}$ when the $A_i^{(r)}$ may be singular. We can argue in different ways:

(i) If we assume that the target photons are essentially transversely polarized despite being off-shell with $\kappa^2 \ll Q^2$, then the physical $\Sigma^{(r)}$ can be extracted from

$$\lim_{p\to 0} \Gamma^r_{\mu\lambda\rho} \sum_{a,b=1,2} \epsilon^{ab}\epsilon_a^\lambda(k_1)\epsilon_b^\rho(k_2) \;=\; \Sigma^{(r)}(\kappa^2)\epsilon_{\mu\lambda\rho\alpha}k^\alpha \sum_{a,b=1,2} \epsilon^{ab}\epsilon_a^\lambda(k)\epsilon_b^\rho(-k). \tag{3.5}$$

We find*

$$\Sigma^{(r)}(\kappa^2) \;=\; \big(A_1^{(r)} - A_2^{(r)}\big)(0,\kappa^2,\kappa^2) \;-\; \lim_{p^2\to 0}\frac{1}{2}p^2\big(A_5^{(r)} - A_6^{(r)}\big). \tag{3.6}$$

---

* The proof of eq.(3.6) follows that explained in (iii) below. The rôle of the leptonic tensor is played by the quantity

$$\sum_{a,b=1,2} \epsilon^{ab}\epsilon_a^\lambda(k)\epsilon_b^\rho(-k) \;=\; \frac{1}{k.\bar{k}}\epsilon^{\lambda\rho\gamma\delta}k_\gamma \bar{k}_\delta$$

where $\bar{k}_\mu$ ($\equiv k^\mu$) is the photon momentum with the sign of the spacelike components reversed, and the antisymmetric tensor ϵ^{ab} ($a,b = 1,2$) ensures the appropriate combination of transverse polarizations for the contribution to g_1^γ. The amplitudes $A_3^{(r)}$ and $A_4^{(r)}$ do not contribute because of the transversality conditions $k_{1\lambda}\epsilon_a^\lambda(k_1) = 0$, etc.

The additional term is non-trivial in the Nambu-Goldstone realization, where $A_5^{(r)}$ and $A_6^{(r)}$ have poles in $1/p^2$ due to the massless pion.

(ii) The polarized photon structure function g_1^γ is directly related to the corresponding "electron structure function" g_1^e defined[1] in deep-inelastic e^+e^- scattering.* Just as for the familiar case of the proton, g_1^e is determined by the pseudovector form factor $G_A^{(r)}(0)$ in the forward matrix element $\langle e|J^r_{\mu 5}(0)|e\rangle$, i.e.

$$\langle e|J^r_{\mu 5}(p)|e\rangle \;=\; G_A^{(r)}(p^2)\bar{u}\lambda^r\gamma_\mu\gamma_5 u \;+\; G_P^{(r)}(p^2)p_\mu\bar{u}\lambda^r\gamma_5 u, \tag{3.7}$$

and is independent of the pseudoscalar form factor $G_P^{(r)}(0)$. On the other hand, the pion (in the Nambu-Goldstone realization), clearly contributes only to the pseudoscalar $G_P^{(r)}$, and therefore decouples from the first moment of g_1^e (and hence g_1^γ). We may thus conclude that $\Sigma^{(r)}$ is given by $\left(A_1^{(r)} - A_2^{(r)}\right)$ with the pion contribution subtracted, i.e.

$$\Sigma^{(r)}(\kappa^2) \;=\; \left(A_1^{(r)} - A_2^{(r)}\right)_{\rm reg}(0,\kappa^2,\kappa^2). \tag{3.8}$$

(iii) Finally, we can explicitly take the regularised expression (3.4) for $\Gamma^r_{\mu\lambda\rho}$ and multiply it into the leptonic tensor[1] arising in the e^+e^- scattering amplitude, to see which piece contributes to the physical $\Sigma^{(r)}$. The leptonic tensor is $\Delta L_{\lambda\rho} = 2i\epsilon_{\lambda\rho\gamma\delta}p_2^\gamma k^\delta$, where p_2 is the momentum of the target electron. Contracting with $\Gamma^r_{\mu\lambda\rho}$, using the fact that $p.p_2$ and $p.k$ are of $O(p^2)$, and with the assumption that $A_1^{(r)}$ and $A_2^{(r)}$ are non-singular, we find

$$\lim_{p\to 0}\Gamma^r_{\mu\lambda\rho}\,\Delta L^{\lambda\rho} \;=\; \Sigma^{(r)}(\kappa^2)\,\epsilon_{\mu\lambda\rho\alpha}k^\alpha\,\Delta L^{\lambda\rho}, \tag{3.9}$$

with

$$\Sigma^{(r)}(\kappa^2) \;=\; \left(A_1^{(r)} - A_2^{(r)}\right)(0,\kappa^2,\kappa^2) \;-\; \lim_{p^2\to 0}\frac{1}{4}p^2\left(A_3^{(r)} - A_4^{(r)} + A_5^{(r)} - A_6^{(r)}\right). \tag{3.10}$$

In perturbation theory, since away from the exceptional point $\kappa^2 = 0$ the amplitudes $A_i^{(r)}$ never have a singularity as strong as $1/p^2$ in the $p^2 \to 0$ limit, the extra terms in eqs.(3.6) or (3.10) do not contribute. For $\kappa^2 = 0$, where eq.(3.6) is applicable, we have

* In fact, in the notation of ref.[1],

$$\int_0^1 dx\, g_1^e(x,Q^2) \;=\; \frac{3}{2}\frac{\alpha}{2\pi}\int_0^\infty \frac{d\kappa^2}{\kappa^2}\int_0^1 dy\, g_1^\gamma(y,Q^2;\kappa^2)$$

$\lim_{p^2\to 0} \frac{1}{2}p^2(A_5^{(r)} - A_6^{(r)})(p^2,0,0) = \lim_{p^2\to 0}(A_1^{(r)} - A_2^{(r)})(p^2,0,0)$ from eq.(2.25). So we conclude,

$$\Sigma^{(r)}(\kappa^2)\big|_{\text{P.T.}} = (A_1^{(r)} - A_2^{(r)})(0,\kappa^2,\kappa^2) \qquad (\kappa^2 \neq 0) \qquad (3.11)$$

$$= 0 \qquad (\kappa^2 = 0) \qquad (3.12)$$

However, in the Nambu-Goldstone realization, the amplitudes $A_3^{(r)},\ldots,A_6^{(r)}$ do have $1/p^2$ poles in the chiral limit due to the massless pion. The consistency of the expressions (3.6), (3.8) and (3.10) follows from the result (2.11) for the pion contribution to the $A_i^{(r)}$. Eqs.(3.6) or (3.10) become

$$\Sigma^{(r)}(\kappa^2) = (A_1^{(r)} - A_2^{(r)})(0,\kappa^2,\kappa^2) - \frac{1}{8\pi\alpha}F_\pi g_{\pi\gamma^*\gamma^*}, \qquad (3.13)$$

$$= (A_1^{(r)} - A_2^{(r)})_{\text{reg}}(0,\kappa^2,\kappa^2). \qquad (3.14)$$

The conclusion is that as we approach the chiral limit the physical form factor $\Sigma^{(r)}(\kappa^2)$ remains smooth and continuous. The naive identification of $\Sigma^{(r)}$ with $(A_1^{(r)} - A_2^{(r)})$ fails, and the discontinuity in $(A_1^{(r)} - A_2^{(r)})$ is precisely compensated by the extra contributions to $\Sigma^{(r)}$ from the singular amplitudes $A_3^{(r)},\ldots,A_6^{(r)}$ in the chiral limit. The form factor $\Sigma^{(r)}$, unlike $(A_1^{(r)} - A_2^{(r)})$, is therefore not simply given by the anomaly and its κ^2 dependence does indeed distinguish the perturbative and Nambu-Goldstone realizations of the AVV correlation function.

4. Thresholds and κ^2 dependence of the g_1^γ sum rule

We are now ready to trace the behaviour of the first moment of $g_1^\gamma(y,Q^2;\kappa^2)$ as κ^2 is increased through the thresholds associated with the different quarks and leptons. First, we discuss which realization of the AVV correlation function is relevant for each type of fermion.

(i) *Leptons* :

Clearly the perturbative realization is appropriate. The contribution to $\Sigma^{(r)}(\kappa^2)$ from each lepton is a constant fixed by the electromagnetic $U(1)$ anomaly as κ^2 is increased past the perturbative turnover scales at m_e^2, m_μ^2 and m_τ^2.

(ii) *Light quarks* :

Since chiral symmetry is spontaneously broken in QCD, the Nambu-Goldstone realization is relevant for the light (u, d, s) quarks. The total magnitude of the contribution to $\Sigma^{(r)}(\kappa^2)$ does not distinguish between the Nambu-Goldstone and Wigner realizations, but these are clearly distinguished by the scale at which the rise in $\Sigma^{(r)}(\kappa^2)$ occurs, viz. $O(m_\rho^2)$ for the Nambu-Goldstone realization in contrast to $O(m_u^2, m_d^2, m_s^2)$ for the Wigner realization.

(iii) *Heavy quarks* :

Since chiral symmetry is badly broken at these scales (m_c^2, m_b^2), the explicit breaking term dominates in the chiral Ward identity (2.6) and the characteristic effects of spontaneous chiral symmetry breaking are negligeable. The contribution to $\Sigma^{(r)}(\kappa^2)$ is reliably given by perturbation theory, since the running coupling $\alpha_s(m^2)$ is small at the heavy quark mass. The turnover scale is therefore simply the quark mass.

The κ^2 dependence of the first moment of g_1^γ is therefore as follows. At $\kappa^2 = 0$ (just outside the physical region for polarized e^+e^- scattering), $\int_0^1 dy\ g_1^\gamma(y, Q^2; \kappa^2) = 0$. The first rise occurs around $\kappa^2 = m_e^2$ with magnitude α/π, followed by another at $\kappa^2 = m_\mu^2$. These contributions are due of course to the electron and muon pair production processes and are described by perturbative Feynman diagrams. As discussed above, the next rise does not occur until $\kappa^2 = O(m_\rho^2)$, *not* at the light quark masses m_u^2, m_d^2, m_s^2, as a consequence of the Nambu-Goldstone realization of the AVV correlation function in QCD. The total contribution from the u, d and s quarks to the first moment of g_1^γ is approximately $2/3(\alpha/\pi)$, with small corrections (for details, see ref.[1]) depending on $\log \kappa^2$ and $\log Q^2$ arising from the effect of the strong $U(1)$ anomaly on the singlet form factor and the usual leading QCD corrections of $O\big(\alpha_s(Q^2)\big)$. Finally, there are further rises at κ^2 around m_τ^2 (with magnitude α/π) and $\kappa^2 \simeq m_c^2, m_b^2$ (with magnitudes $16/27(\alpha/\pi)$ and $1/27(\alpha/\pi)$ resp.) due to the production of heavy lepton and heavy quark pairs.

These contributions are sketched individually in Figs. 2 and 3, using the exact expression (from eq.(2.28)) for the leptons and heavy quarks and approximating the light quark contribution very roughly by the expression $2/3\big(\kappa^2/(\kappa^2 + m_\rho^2)\big)$ to give an idea of the final non-perturbative result. The sum, giving the total result for the first moment of g_1^γ as a function of κ^2, is plotted in Fig. 4. Unfortunately, the width of the various perturbative contributions means that they overlap extensively, and the individual rises associated with the different thresholds are not separately visible. Experimentally, however, the interesting light quark contribution should be easily extracted by separating out the leptonic final states and those associated with a heavy flavour.

Acknowledgements

We are grateful to Richard Ball and Elliot Leader for interesting discussions and Han-qing Zheng and Steven Thomas for their help at different stages of this work.

References

[1] S. Narison, G.M. Shore and G. Veneziano, "A Sum Rule for the Polarised Photon Structure Function g_1^γ", CERN-TH.6467/92 Rev.

[2] S.L. Adler, Phys. Rev. 177 (1969) 2426

[3] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, Nucl. Phys. B237 (1984) 525

[4] N.N. Achisov, ZhETF 101 (1992) 1713, Phys. Lett. B287 (1992) 213

[5] G. 't Hooft, in Recent developments in gauge theories, Cargese 1979, eds. G. 't Hooft et al (Plenum, New York and London)

[6] Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, Nucl. Phys. B177 (1981) 157

S. Coleman and B. Grossman, Nucl. Phys. B203 (1982) 205

Figure Captions

FIGURE 1 :

The two-photon reaction $e^+e^- \to e^+e^- X$.

FIGURE 2 :

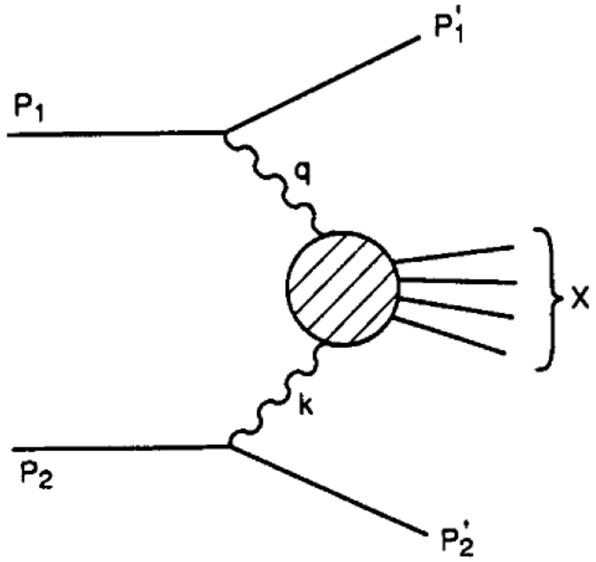
The contributions to the first moment of $g_1^\gamma(y, Q^2; \kappa^2)$ from the leptons e, μ and τ as κ^2 is increased through the thresholds with turnover scales of $O(m_e)$, $O(m_\mu)$ and $O(m_\tau)$ resp. The magnitude of the contributions, in units of α/π, is 1 in each case.

FIGURE 3 :

As Fig. 2, for the heavy quarks c and b and the sum of the light quark contributions. The c and b quark contributions have magnitudes 16/27 and 1/27 with turnover scales of $O(m_c)$ and $O(m_b)$ resp. The light quarks give a total contribution of magnitude 2/3 with a typical hadronic scale of $O(m_\rho)$.

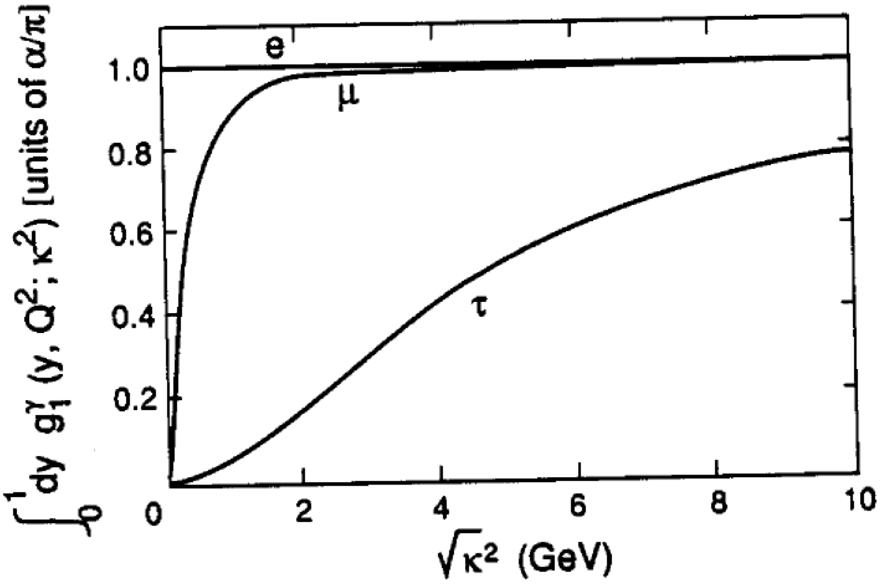
FIGURE 4 :

A sketch of the first moment of $g_1^\gamma(y, Q^2; \kappa^2)$ in the range $0 \leq \kappa^2 \leq 10\mathrm{GeV}^2$.



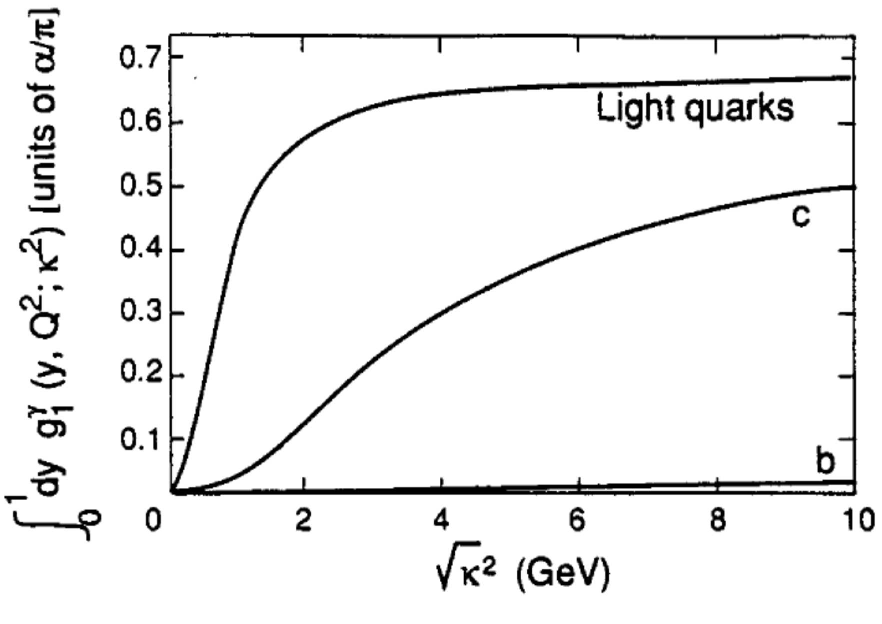
P_1
P_1'
q
X
k
P_2
P_2'

Fig. 1



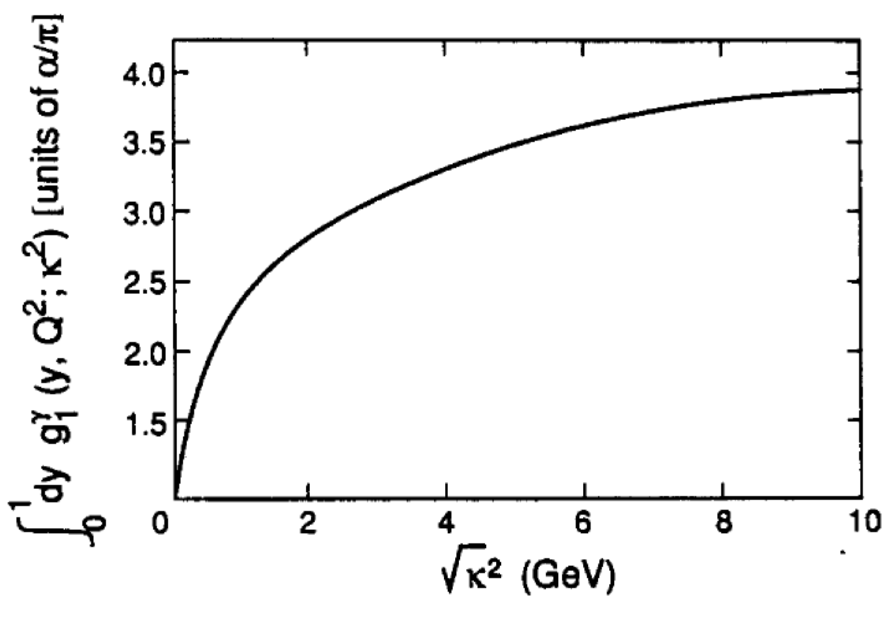
$\int_0^1 dy\ g_1^\gamma (y, Q^2; \kappa^2)$ [units of α/π]
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
e
μ
τ
$\sqrt{\kappa^2}$ (GeV)

Fig. 2



$\int_0^1 dy\ g_1^\gamma\ (y, Q^2; \kappa^2)$ [units of α/π]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2
4
6
8
10
Light quarks
c
b
$\sqrt{\kappa^2}$ (GeV)

Fig. 3



$\int_0^1 dy\ g_1^\gamma\ (y, Q^2; \kappa^2)$ [units of α/π]
4.0
3.5
3.0
2.5
2.0
1.5
0
2
4
6
8
10
$\sqrt{\kappa^2}$ (GeV)

Fig. 4