# The Pooling and Tranching of Securities: A Model of Informed Intermediation

Peter M. DeMarzo\*

Graduate School of Business Stanford University

July 1997

This Revision: October, 2003

**ABSTRACT.** This paper considers the problem faced by a financial intermediary with nassets to sell in the presence of asymmetric information. I show that when the intermediary has superior information about the value of each asset, the intermediary is better off selling shares in the assets individually rather than as a pool. In particular, pooling has an *information destruction* effect that operates to the disadvantage of the intermediary by preventing the intermediary from fully exploiting its information regarding each individual asset. If, however, the intermediary can create a derivative security that is collateralized by the assets, pooling and "tranching" may be optimal. Tranching allows the intermediary to take advantage of the risk diversification effect of pooling to create a low risk and highly liquid security. I show that if the residual risk of each asset is not too highly correlated, then for large enough n, the risk diversification effect dominates and pooling and tranching is optimal for the informed intermediary. I then contrast this with the case of an uninformed originator, selling to both informed intermediaries and uninformed investors. I show that for an uninformed seller, pooling is is preferred to separate asset sales. Finally, I combine these results in a dynamic model of financial intermediation: uninformed originators sell pools of assets, some of which are purchased by informed intermediaries. These intermediaries then further pool the assets and sell senior tranches to investors in order to raise cash to buy new securities in the origination market. By doing so, the intermediaries leverage their capital more efficiently, enhancing the returns to their private information.

<sup>&#</sup>x27;I thank Darrell Duffie for many enlightening conversations, Denis Gromb, Ming Huang, Kose John, and Jose Marin for very useful discussions, and seminar participants at the 1997 European Conference on Financial Markets, UC Berkeley, University of Chicago, Michigan, MIT, NYU, Stanford, Wharton, the WFA and the Maryland Conference on Financial Innovation. Financial support from the Q Group is gratefully acknowledged.

<sup>\*</sup> Stanford, CA 94305-5015. Phone: (650) 736-1082, Email: <a href="mailto:pdemarzo@stanford.edu">pdemarzo@stanford.edu</a>, Web: <a href="http://faculty-gsb.stanford.edu/demarzo">http://faculty-gsb.stanford.edu/demarzo</a>.

#### 1. Introduction

The repackaging of assets is ubiquitous in financial markets. Probably the most often discussed example of this phenomenon are mortgage-backed securities. These securities are created by first taking the cash flows from a large number of individual home mortgages and pooling them into a single financial trust. This trust is then sold to investors by selling separate classes, or tranches, of securities whose claims in aggregate represent a 100% interest in the trust, but which are individually highly heterogeneous.

Since the development of this process of securitization for residential mortgages, it has been generalized to include many other types of assets, including car loans, credit card receivables, Western Union deposits, commercial mortgages, etc. A recent example is the case of collateralized bond obligations (CBO's) which are created by pooling together dozens of different junk-bond issues. The pool is then tranched into an investment grade "debt" security that ranks first in interest and principal repayments, and a residual "equity" sliver in which the default risk is concentrated. Another innovation applies the concept recursively: the so-called "kitchen-sink bond" is formed by tranching a low risk debt security from a pool of residual pieces from other asset-backed securities.<sup>1</sup>

These examples share the common feature of a financial intermediary purchasing and pooling a number of distinct assets and reselling the pool as a collection of new securities. Note, however, that in a world of perfect capital markets, such repackaging would be irrelevant. This is at odds with the reality of the accelerated growth of the asset-backed securities market, and the substantial profits of the intermediaries involved. The goal of this paper is to develop a rational equilibrium model of this process that is consistent with these and other stylized facts.

In order to explain the gains from the repackaging of asset-backed securities, three important market imperfections seem relevant: transactions costs, market incompleteness, and asymmetric information. While all of these imperfections are likely to be important in reality, this paper will focus on the impact of asymmetric information. This focus is not intended to deny the presence and importance of transactions costs or market incompleteness. On the other hand, there are features of the market that seem best explained by an asymmetric information model. For example, market incompleteness does not explain the construction of simple "pass-through" pools, which do not augment the span of tradeable claims. It is also unlikely to explain the CBO market, since good substitutes already exist for the "debt" and "equity" tranches that are created. Transactions costs could be an advantage to pooling, but that explanation offers little help in rationalizing the pieces the pools are carved into. As we shall see, the asymmetric information model offers useful and consistent insights into many of these examples.

\_

<sup>&</sup>lt;sup>1</sup> At a more abstract level, even corporate securities can be viewed as a repackaging of the underlying human and physical assets of the firm into debt, equity, and other securities. Thus, while the model developed in this paper will focus on the application to asset-backed securities, many of the results may also be relevant for corporate finance.

Of course, one might question the importance of asymmetric information in these markets. For instance, for many asset-backed securities, such as mortgage-backed securities, the attributes of the underlying assets might be reasonably regarded as public information. However, information asymmetries may still exist because the models used to price these securities are largely proprietary. If these models produce heterogeneous results, then the value estimates produced by one's own model is an important piece of private information. Empirical evidence in favor of this is provided in a recent paper by Bernardo and Cornell (1997), who analyzed data from an auction of mortgage-backed securities (MBS's). Though all the bidders were sophisticated investors or investment banks, they found extreme variability in the bids, with the winning bid exceeding the median bid by over 17% on average. Further analysis of the data leads them to conclude that the most probable explanation for this variability is asymmetric information regarding valuation. Additional evidence on this is provided by Wallace (2001), who documents the degree of heterogeneity across similar MBS's.

The basic story of the paper is as follows. Consider a market with a sophisticated financial intermediary that has a superior ability in valuing some type of asset. Based on this ability, the financial intermediary can earn a profit by buying under-priced assets and holding them to maturity. On the other hand, in order to fully leverage its available capital, the intermediary would prefer to resell the securities at a fair price and reinvest the proceeds in newly identified under-priced assets.

The difficulty for the financial intermediary is that when it attempts to resell the assets, it has superior information about their value and so faces a "lemons" problem, as described by Akerlof (1970). This lemons problem results in illiquidity: the price the intermediary receives for the assets is decreasing in the quantity sold. The first part of this paper examines the possibility of pooling and tranching the assets prior to resale in order to mitigate this lemons problem.

To do this, the paper builds on an earlier asymmetric model of security design introduced by DeMarzo and Duffie (1999) and the signaling model of Leland and Pyle (1977). These papers develop signaling models of the sale of a security, in which the issuer signals a high value security by the issuer's willingness to retaining a portion of the issue. In the context of such a model, this paper shows that in fact, pooling of the individual assets prior to sale is not advantageous to the informed intermediary. The reason for this is that pooling the assets destroys the asset-specific information held by the intermediary, eliminating the intermediary's "option" regarding how aggressively to sell each asset. This *information destruction effect* reduces the payoff to the intermediary.

Next, I consider the effect of both pooling *and* tranching by the informed intermediary. Here again I rely on earlier results by DeMarzo and Duffie (1999) and DeMarzo (1997) that demonstrate that in such an environment, the optimal security to issue is a debt security that is backed by the asset pool. Here I show that there is a beneficial *risk diversification effect* of pooling, which allows the intermediary to construct a low-risk debt security from a large pool. This low-risk debt is less sensitive to the intermediary's private information, and hence is more liquid. Finally, I show that as the size of the pool

grows large, the risk diversification effect dominates the information destruction effect, so that pooling and tranching is optimal for the intermediary.

Having analyzed the intermediary's problem once it chooses to sell the assets, the paper then considers the asset acquisition process for the intermediary. The assets are generally created by "originators" specializing in marketing and other customer services, who then sell the assets to the market. Here I look at the special case in which the originator is less sophisticated and therefore less informed about the value of the asset than the financial intermediary. The originator puts the assets up for sale to similarly uninformed investors, but cannot prevent the financial intermediary from participating. Thus, the intermediary will buy those assets it knows to be of high quality. This skews the allocation to the uninformed investors so that they are more likely to purchase a low quality asset. The net result is that to attract the uninformed investors, the originator must under-price the asset initially, for the same reason that IPO's are under-priced in the model of Rock (1986).

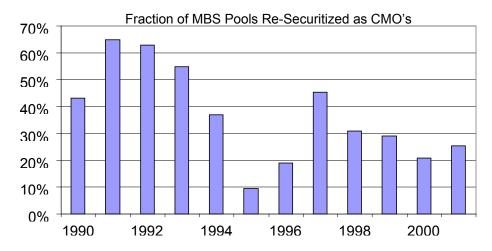
In the context of this model of origination, I then consider whether the originator has an incentive to pool the assets prior to issue. Here I find that in contrast to the financial intermediary, the originator has an incentive to pool the assets even if they are not tranched prior to sale. Pooling prevents the intermediary from selectively purchasing just the highest quality components of the pool, reducing the adverse selection problem of the uninformed investors. Finally, I show that this model of origination allows for the endogenous determination of many of the parameters of the asset-sale model.

Combining these results leads to a model of informed intermediation. Uninformed originators pool assets so as to reduce under-pricing when issued. Informed intermediaries use their information to selectively purchase the highest quality pools in the origination market. In order to raise capital for future purchases, the intermediaries further pool the asset pools and issue low-information-sensitive security tranches backed by these large pools. The model demonstrates how the ability to repackage securities enhances the returns to information.

This stylized model fits well the market for asset-backed securities. For example, in the case of mortgages, mortgage originators generally pool the mortgages they originate into pass-through mortgage backed securities (MBS's) consisting of 20-30 mortgages. Indeed, data since 1995 reveal that over 50% of all mortgages originated in the U.S. are ultimately pooled into MBS's. These pools are then sold to intermediaries who generally pool the mortgages further, typically combining 100-300 of these MBS pools into a real estate mortgage investment conduit, or REMIC. The intermediaries then issue securities backed by the REMIC known as collateralized mortgage obligations (CMO's). The most liquid of these CMO's are generally designed to be relatively insensitive to the rate of mortgage prepayment, consistent with the notion that the intermediaries themselves are likely to be best able to evaluate and price prepayment risk.<sup>2</sup> The figure below illustrates

<sup>&</sup>lt;sup>2</sup> Prepayment risk (the risk that the borrower repays the mortgage prior to maturity) is the most important risk for MBS's. Credit risk is not an issue due to guarantees typically provided by the various agencies (Fannie Mae, Freddie Mac, Ginnie Mae). In contrast, for other types of asset backed securities (ABS's),

the fraction of MBS's resecuritized as CMO tranches over the last decade.<sup>3</sup> Also consistent with the model of this paper is that while intermediaries generally sell off many of these CMO's, they also retain significant fractions of them in their own portfolios. See Wallace (2001) for a detailed analysis of this market and its participants.



#### 1.1. Related Literature

There are a number of papers related to the topics addressed here. Leland and Pyle (1977) develop the idea of a signaling-based model of liquidity in which a risk averse owner-entrepreneur wishes to diversify and sell his or her equity stake in the firm to investors. They demonstrate that the entrepreneur can signal his or her private valuation for the security by retaining some fraction of it, and bearing the costs of being under-diversified. Owners of good firms are more willing to retain shares at a given price, so the fraction sold becomes a signal of quality. In equilibrium, the greater the fraction of the equity the entrepreneur sells, the lower the market price of the equity; hence demand is not perfectly elastic. Leland and Pyle also conjecture that signaling costs might be reduced by combining many projects, leading to specialization in information production.

This paper considers both the Leland and Pyle signaling model and a variation in which the asset holder is risk neutral but has an above market discount rate (due to the availability of other positive NPV investments). By a similar reasoning, an equilibrium results in which owners signal an asset's high quality by their willingness to retain some of the cash flows. I show, however, that in both settings the conjecture of Leland and Pyle is false in the sense that a simple pooling of assets will not reduce asymmetric information costs and enhance liquidity.

Diamond (1984) develops a model of financial intermediation based on monitoring costs. In his model, there is an ex-post asymmetry of information because the cash flows of firms are not observed by investors. To ensure repayment, investors must either pay a

4

(

credit risk is generally quite important. Consistent with the model, for these ABS's, pools are generally tranched into prioritized securities, with the most senior securities being relatively insensitive to information regarding credit quality.

<sup>&</sup>lt;sup>3</sup> Data from The Bond Market Association, www.bondmarkets.com.

cost to monitor the firm's payoff, or impose a non-pecuniary penalty to make disclosure of the payoff incentive compatible for the firm's manager. If a firm has more than one investor, monitoring requires investors to duplicate their efforts. If monitoring is delegated to a financial intermediary, however, this duplication of effort can be avoided, and monitoring borrowers is more efficient than using penalties. Moreover, as the size of the financial intermediary grows large (through the pooling of independent securities), the intermediary can offer a debt contract to investors with a probability of default approaching zero. Since the intermediary rarely defaults, the expected penalties necessary to maintain the intermediary's incentives become negligible.

Diamond (1993) and Winton (1995) also motivate different classes of investors through seniority of claims. These models suggest that multiple tranches are important when investors need to take actions to prevent borrower misbehavior. For example, Winton (1995) shows how by issuing prioritized claims, the borrower can reduce investors' monitoring costs.

This paper has some similarities to these models. In particular, debt is also derived as an optimal contract for a large intermediary to issue. On the other hand, I focus on an exante information asymmetry in which sophisticated investors become intermediaries because they have superior information about the asset values. This type of information is probably much more relevant for many asset-backed securities. For example, monitoring the cash flows of the underlying mortgages of CMO's is not typically discussed as a problem with these securities, whereas there is general agreement that the major investment banks have a superior ability to value them.<sup>4</sup>

Gorton and Pennachi (1990) develop a model of financial intermediation in which there are two "clienteles" of investors, informed and uninformed. Informed investors exploit the uninformed investors when the uninformed are in need of liquidity. In their model, an optimal response of the uninformed is to form an intermediary that splits cash flows into a riskless debt and equity claim. The uninformed can then use debt claims to satisfy their liquidity needs and avoid losses from trading with the informed. Note that, in contrast with our model, the intermediary in the Gorton and Pennachi model is uninformed.

Winton (2001) considers a model in which the intermediary (a bank) has an incentive to monitor and acquire information about the underlying assets (a firm) in order to reduce agency costs. The intermediary may suffer a liquidity shock, however, and be forced to sell its holdings. In a model similar to DeMarzo and Duffie (1999), he shows that as an informed seller, the intermediary's liquidity costs are reduced if it holds debt rather than equity securities. He does not consider the possibility of pooling and tranching the securities of multiple firms, however.<sup>5</sup>

<sup>&</sup>lt;sup>4</sup> On the other hand, the ex-post monitoring may be a more important issue for other types of asset-backed securities, such as collateralized loan obligations (CLO's).

In fact, the pooling and tranching of bank loans into collateralized loan obligations (CLO's) is an important and rapidly growing class of asset-backed securities.

Glaeser and Kallal (1997) have a model of asset-backed securities in which they look at the incentives for the issuer to gather information. They note that pooling many underlying assets together has ambiguous effects on the incentives for the issuer to become informed and therefore on the liquidity of the pool. Our model extends their analysis by allowing the issuer to issue derivative tranches rather than simple pass-through securities, which I demonstrate to be critical. Riddiough (1997) also has a model of security design for asset-backed securities. He notes that splitting off a completely riskless security is beneficial since the issuer will suffer no asymmetric information losses on that security. The model of this paper is more general and does not require that the security tranche be riskless. His paper focuses on the agency and governance issues that arise in this setting, which I do not address here.

A number of papers have explored the benefits of pooling in reducing adverse selection when uninformed traders compete with informed traders. Subrahmanyam (1991) shows that security index baskets, such as stock index futures, can be more liquid than the underlying stocks in a Kyle (1985)-type model in which the uninformed liquidity demand is exogenously distributed across assets. Also in a Kyle-type model, Gorton and Pennachi (1993) look at the endogenous portfolio choice of the uninformed, and consider the construction of an optimal composite security. Axelson (1999) explores this issue in an auction context in which buyers have differential information. He shows that as the number of assets grows large, auction revenues can be improved by pooling assets prior to sale. His analysis corresponds most closely to that of Section 5, where I also demonstrate that pooling helps reduce the adverse selection problem when there is one informed buyer and many uninformed buyers.

## 2. The Underlying Assets

Consider the problem faced by an intermediary who holds *n* assets, but who would prefer to hold cash. This intermediary must choose whether to sell the assets separately, as a pool, or as an asset-backed security based on the pool. To evaluate the acquisition and sale decisions of this intermediary, I introduce the following assumptions regarding the underlying assets.

Each asset *i* has a final non-negative payoff of  $Y_i = X_i + Z_i$ . The component  $X_i$  represents the private information of the intermediary, and  $Z_i$  is the remaining risk the intermediary faces. Let  $Y = (Y_1, ..., Y_n)$  denote the vector of payoffs, and  $Y^n = \sum_{i=1}^n Y_i$  denote the cumulative payoff of the assets, and similarly for X,  $X^n$  and X,  $X^n$  and X,  $X^n$  introduce the notation  $X_{-i} = (X_1, ..., X_{i-1}, X_{i+1}, ..., X_n)$ .

The first assumption is without loss of generality:

1.  $E[Z_i | X] = 0$ ; or equivalently,  $X_i = E[Y_i | X]$ .

This assumption simply states that  $X_i$  embodies all of the information known to the intermediary regarding the expectation of the cash flow  $Y_i$ . Note that this assumption could have been derived from the primitive assumption that I represents the intermediary's information, and the definition  $X_i \equiv E[Y_i | I]$ .

Next, introduce the technical assumption:

2. Given any  $X_{-i}$ , the conditional support of  $X_i$  is a closed interval.

This assumption is useful since it implies that whatever information the intermediary has revealed about the assets other than i, there still remains a continuum of possible information states regarding asset i.

The last assumption is substantive, but only mildly restrictive:

3. Given any  $X_{-i}$ , the conditional support of  $X_i$  has greatest lower bound  $x_{i0} > 0$ .

That is, the "worst case" outcome of  $X_i$  is independent of  $X_{-i}$ . Given this assumption, note that the support of  $X^n$  is an interval with greatest lower bound  $x_0^n = \sum_i x_{i0}$ .

Note that while this last assumption implies that any two assets  $X_i$  and  $X_j$  are not perfectly correlated, it does *not* rule out any other correlation. For example, if the assets represent loans, knowing the quality of loan i might affect the probability that loan j is "bad." The assumption simply states that this probability does not go to zero – whatever the quality of other loans, it remains *possible* that loan j is bad.<sup>6</sup>

Assume that ownership of each asset is perfectly divisible. In particular, owning a fraction  $q_i \in [0,1]$  of asset i entitles the owner to the cash flows  $q_i$   $Y_i$ . In the various applications of the rest of the paper, I investigate the relationship between the market price for the assets and the fraction sold by the intermediary when there is illiquidity resulting from asymmetric information.

Finally, the remaining market assumption is that there exists a large number of risk neutral investors. For convenience, assume also that the market interest rate is zero. Hence, in the absence of asymmetric information (i.e., if X were public), each asset could be sold by the intermediary for a market price of  $X_i$ .

## 3. Pooling and Information Destruction

In this section I consider the case in which the intermediary has private information X and attempts to sell the assets either individually or as a pool. For a given demand function, an intermediary with a low valuation optimally sells a greater quantity of the asset than an intermediary with a high valuation does. Hence, the greater the quantity

<sup>&</sup>lt;sup>6</sup> Note also that the positivity of  $x_{i0}$  does *not* imply strictly positive cash flows  $Y_i$ . It merely requires that the security retains some positive "option" value even given the worst case information.

sold the lower the implied valuation for the asset. Since uninformed investors interpret the sales decision of the informed intermediary as a signal of the asset's value, the equilibrium demand schedule for the assets is downward sloping.

I show that the intermediary's payoff exhibits a natural convexity resulting from the intermediary's option to choose the quantity of each asset to sell based on the information regarding that asset. Pooling destroys the option to use the asset specific information to determine the quantity of each asset to sell.<sup>7</sup> This *information destruction effect* implies that it is not optimal for the intermediary to sell the assets as a single pool. Rather, the intermediary's payoff is highest if each asset is sold individually to the market.

The central model of the paper developed in this section involves a risk-neutral intermediary who discounts future cash flows at higher rate than other investors. Thus, the intermediary would prefer to sell the assets for cash. This corresponds to the model of DeMarzo and Duffie (1999) – henceforth D&D – and can be motivated by supposing the intermediary has access to other investment opportunities with an above market return. In particular, if the intermediary earns a profit buying and selling assets, the intermediary may wish to raise cash to fund new asset purchases. Later in the paper, I model this process and discuss the possibility of endogenously determining this preference for cash.

Thus suppose the intermediary is risk neutral and has a discount factor  $\delta < 1$ . Suppose the intermediary sells a fraction q of the entire pool of assets to investors, at a market price for the pool of p. Then the payoff to the intermediary is given by

$$E \left\lceil \delta(1-q)Y^n + qp \left| X \right| \right] = \delta X^n + q \left( p - \delta X^n \right).$$

If the intermediary anticipates a market demand schedule given by P(q) for the pool, then the intermediary with conditional value  $X^n = x$  will choose a quantity  $Q^P(x)$  to issue that solves

$$\max_{q} \delta x + q(P(q) - \delta x) = \delta x + \max_{q} q(P(q) - \delta x). \tag{1}$$

Equation (1) reveals that the total payoff to the intermediary is the intermediary's discounted value of the asset plus any "profit" from the sale of the security. Thus, the intermediary's asset sale decision is to choose a quantity to sell to maximize this profit. Let  $\Pi^P(x)$  denote the associated profits:

<sup>&</sup>lt;sup>7</sup> Of course, in a strategic setting options are not always valuable. In this case, the asset-by-asset quantity option makes it easier for good types to separate from bad types.

<sup>&</sup>lt;sup>8</sup> Note that some motive for trade is necessary to avoid the No Trade Theorem (see Milgrom and Stokey (1982)). While a higher discount rate is one possible motive for trade, an alternative is risk-sharing, as in Leland and Pyle (1977). The intuitions of this model are robust to that setting as well, as I show in the appendix.

<sup>&</sup>lt;sup>9</sup> Alternatively, the intermediary may be a bank facing minimum regulatory capital requirements.

$$\Pi^{P}(x) = \max_{q \in [0,1]} q(P(q) - \delta x). \tag{2}$$

The following important properties follow immediately from the definition of  $\Pi^P$ :

**LEMMA 1.** For any demand function P, the intermediary's profit  $\Pi^P$  is decreasing and convex in x. Also, the fraction  $Q^P(x)$  that the intermediary sells is decreasing in x.

**PROOF:** For fixed q, the intermediary's objective is decreasing and linear in x with slope  $-q\delta$ . Hence  $\Pi^P$  is the upper-envelope of linear functions and is therefore convex. The fact that q is decreasing follows from the convexity of  $\Pi^P$  and the fact that it has  $-q\delta$  as a subgradient.  $\bullet$ 

The properties described in **Lemma 1** are clearly robust, and will drive much of the subsequent analysis. However, it is useful to describe an equilibrium in this setting. Specifically, the demand schedule of the investors is not arbitrary, but is based on their perceived value of the pool given the intermediary's decision. In a standard rational expectations or Bayes-Nash equilibrium, the demand function should satisfy

$$P(Q^{P}(X^{n})) = E[X^{n}|Q^{P}(X^{n})].$$
(3)

Note that from **LEMMA 1**, for any such equilibrium the demand schedule P is (weakly) downward sloping in the range of Q. We say that the equilibrium is *separating* if  $P(Q(X^n)) = X^n$ . The following characterization of the equilibrium is from D&D:

**LEMMA 2.** Given the worst case asset value  $x_0 > 0$ , there is a unique separating equilibrium, given by  $Q^*(x) = (x/x_0)^{\frac{-1}{1-\delta}}$  and  $P^*(q) = x_0 q^{\delta-1}$ . The equilibrium payoff function  $\Pi^*(x) = \pi(x/x_0) x_0$ , where

$$\pi(x/x_0) = (1-\delta)(x/x_0)^{\frac{-\delta}{1-\delta}}.$$

**PROOF:** See D&D. The solution follows from differentiation of (2) and (3), and the boundary condition that  $O^*(x_0) = 1$ .

As with all signaling models, multiple equilibria are possible in the absence of restrictions on out of equilibrium beliefs. For the simple model considered here, the separating equilibrium is the natural one to consider given that the intermediary's payoff satisfies the single-crossing property, and can be shown to be the unique equilibrium satisfying standard refinements (see D&D).

Note that the intermediary's quantity choice depends upon the ratio of the asset value to its worst case value,  $x/x_0$ . Thus the intermediary's equilibrium payoff is homogeneous of degree 1 in x and  $x_0$ . While having an explicit functional form for  $\pi$  is convenient, recall that **LEMMA 1** implies that  $\pi$  is decreasing and convex without further calculation. This describes the equilibrium payoff for the intermediary if the intermediary chooses to sell the entire pool as a single asset.

Next suppose that the intermediary sells the assets separately. Consider the sale of asset i. Essentially, the intermediary faces the same problem as in (2) above with  $X_i$  in place of  $X^n$ . The investors are also in an analogous position, with the possible exception that they may have learned information about  $X_j$  from the prior sale of asset  $j \neq i$ . This might alter their conditional distribution for  $X_i$ . However, note that the equilibrium described above only depends on the worst-case outcome of the expected asset payoff, and not on the distribution itself. By our initial assumptions, the worst case is not affected by  $X_{-i}$ . Hence the equilibrium is unchanged. This leads to the following:

**LEMMA 3.** If the intermediary sells each asset *i* separately, there is a unique separating equilibrium in which the intermediary's total payoff is given by

$$\sum_{i=1}^{n} \pi(X_{i}/x_{i0}) x_{i0}$$

Thus, the intermediary's payoff from a separate or a pooled sale of the assets can be compared, yielding the main result of this section:

**THEOREM I.** The intermediary prefers a separate sale of the assets to a pooled sale; that is,

$$\sum_{i=1}^n \pi \left( \frac{X_i}{x_{i0}} \right) x_{i0} \ge \pi \left( \frac{X^n}{x_0^n} \right) x_0^n,$$

where the inequality is strict if  $X_i/x_{i0}$  is not equal for all *i*.

**PROOF:** By the convexity of  $\pi$ , and the fact that  $x_0^n = \sum_i x_{i0}$ , we have

$$\sum_{i=1}^{n} \frac{x_{i0}}{x_{0}^{n}} \pi \left( \frac{X_{i}}{x_{i0}} \right) \ge \pi \left( \sum_{i=1}^{n} \frac{x_{i0}}{x_{0}^{n}} \frac{X_{i}}{x_{i0}} \right) = \pi \left( \frac{X^{n}}{x_{0}^{n}} \right)$$

<sup>&</sup>lt;sup>10</sup> As in all separating equilibria, the equilibrium is highly sensitive to the support assumption,  $x_0$ . This extreme sensitivity to the support and insensitivity to the distribution is a concerning feature of signaling equilibria. In Section 6 of the paper, I embed this static model in a dynamic setting and show how  $x_0$  arises endogenously based on the full distribution of X.

<sup>&</sup>lt;sup>11</sup> See appendix for proofs not in the text.

The strict inequality follows from that fact that  $Q^*$  is strictly decreasing so that  $\pi$  is strictly convex.  $\bullet$ 

Thus, **THEOREM I** demonstrates that an informed seller will not prefer to a sell a single "pass through" pool of securities, but will instead prefer to sell the securities individually. The intuition for this result is that because the intermediary holds an option" regarding the quantity of the asset to sell, the intermediary's payoff is convex in the privately-observed quality of the asset. Therefore, the intermediary would prefer not to combine high and low quality assets to create a medium quality pool. I refer to this as the *information destruction effect* associated with pooling.

Note that the proof of this result relies purely on the convexity of  $\pi$  and not its explicit functional form. Thus, the result is more general than the explicit setting considered here. As an example of its robustness, I show in the appendix that this result extends to the Leland and Pyle (1977) model, in which the issuer is risk averse.

Before concluding this section, it is worthwhile to remark that **THEOREM I** relies upon the assumption that  $x_0^n = \sum_i x_{i0}$ ; that is, the worst possible pool is equal to a pool of the worst possible assets. There may be cases for which this does not hold. For example, investors may have data regarding characteristics of the pool that improves the worst-case scenario, so that  $x_0^n > \sum_i x_{i0}$ . In this case there may be benefits associated with pooling even for an informed issuer. <sup>12</sup>

## 4. Tranching and Risk Diversification

In the previous section, the intermediary could either issue the assets separately or as a pool, and pure pooling was shown to be sub-optimal. In this section, I allow the intermediary to issue a derivative security based on the cash flows of the underlying asset or pool of assets. I then show that pooling the assets and selling a derivative tranche is superior to both pure pooling and separate asset sales.

Consider the case of an intermediary with assets with payoff Y = X + Z. Rather than sell shares in asset i directly, the intermediary may create a security or tranche that pays  $F(Y_i)$  for some measurable function F. Restricting attention to limited-liability securities that are backed solely by the underlying assets implies  $F(y) \in [0,y]$ . Such a security F is referred to as an "asset-backed security." As a final restriction, I will consider only non-decreasing functions F.<sup>13</sup> The goal of this section is to compare the intermediary's payoff from selling optimal tranches  $F_i(Y_i)$  of each asset separately, to the payoff from pooling the assets and selling an optimal tranche  $F^n(Y^n)$  backed by the pool.

<sup>13</sup> This restriction is not without loss of generality, but is made for the purpose of tractability. Since the goal is to show that pooling and tranching is superior to both individual sales and pure pooling, restricting attention to monotone tranches only strengthens the result.

<sup>&</sup>lt;sup>12</sup> An example may be credit card issuers securitizing their accounts. Investors may know average default rates, whereas the private information of the issuer is the *identity* of the bad accounts, rather than the number of them.

Recall that in the previous section, the remaining risk Z played no role whatsoever, since both intermediary and the investors are risk neutral. In this section, however, the creation of non-linear securities F implies that Z plays a critical role. Indeed, the risk inherent in Z will determine the degree to which a security can be designed that minimizes the degree of asymmetric information between the intermediary and investors. I show that the *risk diversification effect* of pooling is beneficial in this regard. In fact, it can overcome the information destruction effect, so that pooling and tranching becomes optimal for the issuer.

At the time of issue, X is private information of the intermediary. There remains, however, the question as to whether or not X is known at the time that the security design F is chosen. Because there are usually significant delays between the design of a security and its sale, it is possible that significant new information may be acquired by the intermediary during this time. Thus, both cases are reasonable. I therefore explore both cases in the subsections below, and show that they lead to similar conclusions.

#### 4.1. Ex-Ante Security Design

Suppose that the security design F is chosen prior to the realization of the information X. In this case, the intermediary's choice of F does not reveal any information. This timing is relevant in several applications. First, asset-backed security designs may be standardized, and thus not reflective of private information relating to a particular issue. Second, there are often significant delays between the design of the security and its sale. If private information is acquired continuously, significant information may learned during this delay. Third, the informed intermediary may be an underwriter who did not directly control the design.

To simplify notation, suppose for the moment that there is a single asset (n = 1) with payoff Y. Given a security design F and private information X, let the expected payoff of the security be given by  $f = E[F(Y) \mid X]$ . Because the intermediary discounts future cash flows by the discount factor  $\delta < 1$ , the security has a private value to the intermediary of  $\delta f$ . Suppose the intermediary sells a fraction f0 of the security f1 for price f2. Then the intermediary's portfolio consists of f3 in cash, together with assets worth f4 of f5. The intermediary's expected payoff is then given by

$$E\left[\delta(Y-qF(Y))+qp|X\right]=\delta X+q(p-\delta f).$$

Suppose the intermediary anticipates a market demand schedule given by  $P^F(q)$  for the security F. Then given the conditional value f of the security, the intermediary's problem is to choose the optimal quantity to solve

$$\Pi^{P^F}(f) = \max_{a} q(P^F(q) - \delta f).$$

\_

<sup>&</sup>lt;sup>14</sup> For example, if the private information of the intermediary corresponds to the output of a proprietary valuation model, it is the output of the model on the day of sale (using information such as the current yield curve) that is relevant. This information is only known well after the initial security design is chosen.

The structure of this problem is identical to (2) of Section 3. This leads immediately the following characterization of the equilibrium payoff (see D&D):

**LEMMA 4.** Let  $[f_0,f_1]$  be the support of  $f=E[F(Y)\mid X]$ . There is a unique separating equilibrium with equilibrium payoff function  $\Pi^*(f)=\pi(f|f_0)\,f_0$ , where  $\pi$  is defined in **LEMMA 2**.

This result gives the intermediary's profit given a security design F and a conditional value of f = E[F(Y) | X]. Recall, however, that at the time F is chosen, the information X is not yet known to the intermediary. Thus, security design F yields the intermediary an ex-ante expected profit of  $E[\pi(f|f_0) f_0]$ . Hence, the ex-ante security design problem to be solved by the intermediary is the following:

$$G[Y] = \max_{F(\cdot)} E\left[\pi(f/f_0) f_0\right]. \tag{4}$$

Before proceeding, note the following properties of the ex-ante payoff function  $G^{15}$ :

**LEMMA 5.** G is homogeneous of degree 1; that is, G[aY] = aG[Y]. Also,  $G[Y] \le (1-\delta) x_0$ , and the inequality is strict if X and Z are independent and continuously distributed.

The upper bound in **Lemma 5** states that the issuer can at best recover the "retention cost"  $(1 - \delta)$  on the worst-case value  $x_0$ . Intuitively, if a security design provided a higher payoff, it would be imitated by the worst type.

In general, the set of possible security designs is an infinite dimensional space, and solving (4) may be intractable. Under an additional assumption regarding the nature of the residual risk, however, D&D show that the security design problem can be reduced to a one-dimensional optimization. Intuitively, (4) suggests that the issuer should make the worst case payoff  $f_0$  of the security as high as possible, while at the same time minimizing the information "sensitivity"  $f/f_0$  (since  $\pi$  is decreasing). For standard distributions, this is accomplished by using a standard debt contract, which pays the lowest (and most information insensitive) cash flows first.

**LEMMA 6.** Suppose Z is independent of X and has a log-concave<sup>16</sup> density function. Then the optimal monotone security design is a standard debt contract. That is,  $F^*(Y) = \min(d, Y)$  for some constant d.

**PROOF:** Given the additive separable construction of Y, the assumption on Z implies that the conditional distribution of Y given X satisfies the MLRP. This is stronger than the

<sup>16</sup> The density function g is log-concave if log(g(s)) is concave in s. This property is satisfied by many standard distributions, such as uniform, normal (possibly truncated), and exponential (possibly truncated).

<sup>&</sup>lt;sup>15</sup> Note also that G operates on the random variable Y, not its outcome, analogous to an expectation operator. I use square brackets,  $[\cdot]$ , to denote such operations.

"uniform worst case" condition of D&D. They demonstrate that this condition implies standard debt is the optimal monotone security design. •

Thus it is sufficient to consider standard debt contracts, allowing (4) to be replaced with

$$G[Y] = \max_{d} E\left[\pi(f^{d}/f_{0}^{d})f_{0}^{d}\right],$$

where  $f^d = E[\min(d, Y) \mid X]$ . Let  $D^*[Y]$  represent the optimal face value of the debt.

Having characterized the security design problem, now consider whether an intermediary with multiple assets (n > 1) has an incentive to pool them prior to creating and issuing a security. This requires a comparison of the intermediary's payoff from issuing a single debt security backed by the pooled assets, versus the payoff from issuing separate debt securities each backed by a single asset.<sup>17</sup> In order to take advantage of the prior results, I assume the following structure for the residual risk:

**ASSUMPTION A.**  $Z_i = \varepsilon_i + \eta$ , where the idiosyncratic risk  $\varepsilon_i$  is independent of  $(\varepsilon_{-i}, \eta, X)$  and the common risk  $\eta$  is independent of  $(\varepsilon_{-i}, X)$ . Also,  $\varepsilon_i$  and  $\eta$  have log-concave density functions.

Under this assumption, for a pool of size n,  $Z^n = (\sum_i \varepsilon_i) + n \eta$ . Since log-concavity is preserved by convolution (Prékopa (1973)), the conditions of **LEMMA 6** are satisfied. Hence, if the assets are pooled, the intermediary's ex-ante expected payoff is given by

$$G\left[\sum_{i=1}^{n} Y_{i}\right]. \tag{5}$$

If instead the intermediary does not pool the assets, they can be sold individually. Rather than selling each asset outright, however, the intermediary can construct a new security for each asset *i* which is backed by that asset. By an argument identical to that in the proof of **Lemma 3**, this problem is separable across assets, and the aggregate ex-ante profits to the intermediary from separate sales is given by

$$\sum_{i=1}^{n} G[Y_i] .$$
(6)

Therefore, the decision to pool or not amounts to a comparison of (5) and (6). Using the homogeneity of G, the intermediary prefers to pool the assets prior to tranching if it leads to a higher per-asset payoff:

14

<sup>&</sup>lt;sup>17</sup> Of course, we could consider the payoff from pooling and tranching versus the payoff from simply selling the assets individually. Since issuing debt against an individual asset is superior to selling the asset outright, the comparison we undertake is a stricter test of the superiority of pooling.

$$G\left[\frac{1}{n}\sum_{i=1}^{n}Y_{i}\right] \geq \frac{1}{n}\sum_{i=1}^{n}G[Y_{i}].$$

Next I show the key result of this section: if the residual risk of the assets is diversifiable, then for large enough n, it is optimal for the intermediary to pool the assets prior to tranching them. To state this result, we first suppose that the per-asset worst case payoff is well-defined in the limit,  $\lim_{n\to\infty}\frac{1}{n}\sum_i x_{i0}=x_0$ , and that the asset payoffs are non-degenerate in the sense of **Lemma 5** so that there is some loss relative to the theoretical maximum payoff of  $(1-\delta)x_{i0}$  for each asset:

$$\overline{\lim_{n \to \infty}} \frac{G[Y_i]}{(1 - \delta)x_{i0}} < 1. \tag{7}$$

Now we show that with pooling, when the residual risk is diversifiable the payoff per asset approaches the theoretical maximum of  $(1-\delta) x_0$ .

**THEOREM II.** Suppose  $\eta = 0$  and that  $Y_i$  have bounded second moments. Then as  $n \to \infty$ ,

$$G\left[\frac{1}{n}\sum_{i=1}^{n}Y_{i}\right] \rightarrow (1-\delta)x_{0} \text{ and } D^{*}\left[\frac{1}{n}\sum_{i=1}^{n}Y_{i}\right] \rightarrow x_{0}.$$

Thus, pooling and tranching is optimal for sufficiently large n.

It is useful to contrast this result with **THEOREM I** of Section 3, which showed that due to the information destruction effect, pooling the assets reduced the intermediary's profits. In contrast, **THEOREM II** shows that with securitization, the intermediary can benefit from pooling a large number of securities. The intuition behind this result is a second effect associated with pooling, the *risk diversification effect*. When the residual risk is idiosyncratic, this effect allows the intermediary to issue debt with a face value of  $x_0$  that is nearly risk-free. Since the debt is nearly risk-free, its value is insensitive to the intermediary's private information and so has no lemon's cost associated with it. Note, however, that the consequences of asymmetric information are still present in the limit – the intermediary's payoff is bounded by  $(1-\delta)x_0$ , whereas in the first-best the intermediary could recover  $\lim_{n\to\infty} (1-\delta)\frac{1}{n}X^n$ .

To verify the intuition that the gain from pooling results from risk diversification, consider an alternative scenario in which the residual risks of the assets are perfectly correlated. In this case, there is no risk diversification from pooling, and so only the information destruction effect should apply. This is confirmed by the following result:

15

<sup>&</sup>lt;sup>18</sup> It is interesting to note that diversification is valuable even though all agents are risk neutral. Diversification has the indirect benefit of allowing the issuer to construct a low-risk security with less of a "lemons" problem and thus greater liquidity.

**THEOREM III.** Suppose  $\varepsilon_i = 0$  and  $x_{i0} = x_0$  for all i. Then for any n, pooling is not optimal.

Thus far, I have demonstrated that the benefits from pooling depend on the degree of diversification that results. Since the cost of pooling is the information destruction effect, the gain from pooling should naturally also depend on the nature of the information. In particular, if the information  $X_i$  is specific to each asset and thus independent across assets, this information destruction should be relatively severe. On the other hand, if the information  $X_i$  is more general, and hence positively correlated across assets, less information destruction should result, enhancing the gains to pooling. This is supported by the following result, which states that the payoff from pooling is increasing in the riskiness of  $X^n$ :

**THEOREM IV.** Let 
$$Y_i = X_i + \varepsilon_i + \eta$$
 and  $\hat{Y}_i = \hat{X}_i + \varepsilon_i + \eta$ , and suppose  $x_{i0} = \hat{x}_{i0}$ . If  $X^n$  is a mean-preserving-spread of  $\hat{X}^n$ , then  $G\left[\frac{1}{n}\sum_i Y_i\right] \ge G\left[\frac{1}{n}\sum_i \hat{Y}_i\right]$ .

**THEOREM IV** implies that the issuer is better off if the private information is general, rather than specific to each asset. For example, the following corollary is immediate:

**COROLLARY.** Let  $Y_i = X_i + \varepsilon_i + \eta$  and suppose the private information  $X_i$  is composed of J independent factors  $X_i = \sum_{j=1}^J \xi_{ji}$ . Suppose each factor j is either common, such that  $\xi_{ji} \equiv \xi_j$  for all i, or unique, such that  $\xi_{ji}$  are independent draws with the same distribution as  $\xi_j$  for all i. Then the issuer's payoff from pooling and tranching is increasing in the number of common factors.

**PROOF:** Note that the distribution of each  $X_i$  is the same whether the factors are common or unique. Only the distribution of  $X^n$  changes. The result then follows from the observation that  $\xi_i$  is a mean-preserving-spread of  $n^{-1} \sum_i \xi_{ii}$ .

I conclude this section with a simple numerical example. In this example,  $E[Y_i] = 100$ ,  $\varepsilon_i = \hat{\varepsilon}_i - 95$ , where  $\hat{\varepsilon}_i$  is exponential with mean 95, and  $\eta = 0$ .  $X_i$  is uniform on [95,105] with probability 0.001 and symmetric binomial on {95,105} with probability 0.999. Also, the  $X_i$  are independent.

Given these distributional assumptions, Figure 1 shows the expected payoff per asset if the intermediary issues debt with face value d per asset that is backed by a pool of n assets. Note that for n = 1, the optimal face value of the debt for the issuer is  $d = \infty$ ; that is, the issuer chooses  $F(Y) = \min(d, Y) = Y$ , a pure pass-through security. As n increases, however, the payoff associated with the pure pass-through decreases, as implied by

<sup>&</sup>lt;sup>19</sup> Effectively,  $X_i$  is binomial, with the small chance of a uniform distribution used to make its support an interval.

<sup>&</sup>lt;sup>20</sup> Of course, the graph only illustrates up to d = 300. Given the parameter choices, the value at d = 300 is very close to the asymptote at  $d = \infty$ .

**THEOREM I** of Section 3. For  $n \ge 3$ , it is optimal for the intermediary to issue a debt security that is backed by the asset pool. As n increases, the issuer's payoff increases due to the increased risk diversification. Ultimately, for n sufficiently large, pooling and tranching dominates individual asset sales.

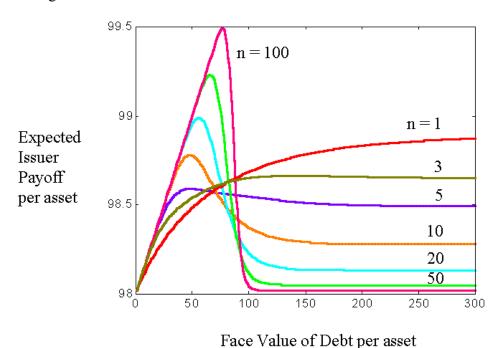


Figure 1: Per-Asset Payoff for Different Levels of Debt and Pool Size

Figure 2 illustrates the effect of correlation on the payoff to the intermediary. Shown is the payoff per asset as function of the size of the pool, assuming the issuer constructs the optimal debt tranche. Consider the case in which the  $Z_i$  are perfectly positively correlated. In this case, there is no diversification from pooling. Hence, by **THEOREM** III, pooling makes the issuer worse off due to the information destruction effect. On the other hand, suppose the  $X_i$  are perfectly positively correlated. Then there is no information destruction due to pooling. Thus pooling benefits the issuer. Finally, for intermediate cases, both effects are operative. If the  $Z_i$  are uncorrelated, then pooling is optimal for n exceeding some minimum threshold.

In conclusion, then, these results imply that the intermediary will benefit most from pooling and tranching when the issuer's private information is "general" (the  $X_i$ 's are positively correlated) and the risks are specific (the  $Z_i$ 's are uncorrelated). This may explain the desire for geographic diversification in mortgage pools, or industry diversification in collateralized bond obligations. It may also explain the tendency not to combine types of underlying assets (e.g., mortgages and corporate bonds), since for these different asset classes the private information is likely to be uncorrelated.<sup>21</sup>

<sup>&</sup>lt;sup>21</sup> Of course, it might also be true that issuers have different information about different industries or locales. The argument here is most relevant when the issuer's information is related to variables common

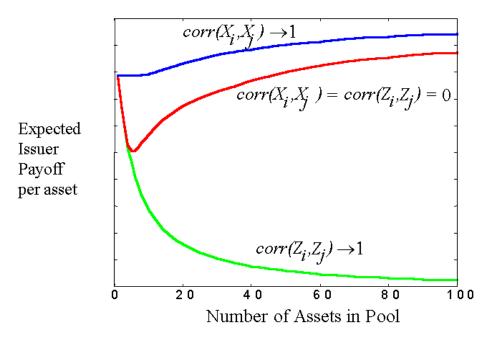


Figure 2: Per-Asset Payoff Based on Pool Size For Different Correlations Between Information and Residual Risk

#### 4.2. Multiple Tranches and Ex-post Security Design

Thus far, we have allowed the issuer to sell a single tranche for each asset pool. The tranche is designed prior to learning X, and the quantity to be sold is determined after X is known. However, the issuer may be able to do better by (i) using multiple tranches, and/or (ii) postponing the security design until after X is known. In this section I explore this possibility, and show that while the solution to the signaling equilibrium is very different, the qualitative results of Section 4.1 continue to hold. In particular, the risk diversification benefit of pooling is still present, and leads to pooling and tranching being optimal given sufficient diversification.

If the issuer creates multiple tranches for an asset pool, then once the information X is learned the issuer will choose a quantity of each tranche, or a tranche portfolio, to sell to investors. This portfolio itself can be interpreted as a security design, in that its payoff is equivalent to some function F of the payoff of the asset pool. In a companion paper (see DeMarzo 2003), I show that issuer's payoff is increasing in the number of tranches. Further, if the number of tranches is unlimited, and are restricted so that each tranche has a monotone payoff, the equilibrium is equivalent to a setting in which the security design

to all assets in a given class (such as risk premia), as opposed to cash flow data on specific to an industry or locale.

is chosen ex-post. Thus, we describe here the ex-post security design problem,<sup>22</sup> but note that it is equivalent to the case of unlimited tranching.

Consider an intermediary with a single asset with payoff Y = X + Z, and for simplicity maintain **ASSUMPTION A** so that X and Z are independent. Given the private information X, the asset has a private valuation of  $\delta X$  to the intermediary. If, rather than hold the asset, the intermediary designs and sells the asset-backed security F for price P, the intermediary's payoff is given by<sup>23</sup>

$$E \lceil \delta(Y - F(Y)) + p \mid X \rceil = \delta X + (p - \delta E \lceil F(Y) \mid X \rceil).$$

That is, the intermediary receives the private valuation  $\delta X$  plus the surplus generated by the sale of the security F.

The intermediary chooses the security design F taking as given the market demand function for securities, given by some function P such that P[F] is the price that investors will pay for security F. Thus, given the private information X = x, the intermediary will choose a security design F to solve the following:

$$\Gamma^{P}(x) = \max_{F(\cdot)} P[F] - \delta E[F(x+Z)]. \tag{8}$$

Denote by  $F_x$  the solution to (8) corresponding to X = x. Given this solution, the price investors will pay should correspond to the expected payoff of the security conditional on the information revealed by the issuer's security choice. That is, the security design chosen by the issuer serves as a signal of the assets value. In equilibrium,

$$P[F_X] = E \lceil F_X (X + Z) \mid F_X \rceil. \tag{9}$$

A signaling equilibrium corresponds to a simultaneous solution to (8) and (9). This is an infinite-dimensional signaling problem, generally intractable. In DeMarzo (2003), I show that if attention is restricted to securities such that both security payoff, F(y), and the residual retained by the issuer, y - F(y), are non-decreasing, there is a unique equilibrium satisfying the Intuitive Criterion of Cho and Kreps (1987). In this equilibrium, the security design chosen is a debt contract, with the face value of the debt depending on the private information X. That is, for each x, there is a face value d(x) such that<sup>24</sup>

$$F_{x}(Y) = \min(d(x), Y).$$

\_

<sup>&</sup>lt;sup>22</sup> The ex-post design problem is also considered by Nachman and Noe (1994). However, in their model the issuer raises a fixed amount of capital, and so a pooling equilibrium results. In this model, the amount of cash raised is variable, allowing for separation based on the security issued.

Note that in this case, there is no need to separate the quantity decision from the design decision, since selling fraction q of security F is equivalent to a selling all of the security qF.

With multiple tranches, this is equivalent to tranches being prioritized, and the issuer selling the most

<sup>&</sup>lt;sup>24</sup> With multiple tranches, this is equivalent to tranches being prioritized, and the issuer selling the most senior tranches first up to a "hurdle" class.

Given this result, rewrite the intermediary's problem (8) as

$$\Gamma^{P}(x) = \max_{d} P(d) - \delta E \left[ \min(d, x + Z) \right], \tag{10}$$

where P(d) is the market price of debt with face value d.

We have the following immediate result:

**LEMMA 7.** For any demand function P, the intermediary's profit  $\Gamma^P$  is continuous, decreasing and convex in x. Also, the face value of the debt can be assumed to be decreasing in x.

**PROOF:** For any fixed d, the objective in (10) is continuous, decreasing and convex in x, with a subgradient of  $-\delta \Pr(d-x > Z)$ . Since  $\Gamma^P$  is the upper envelope of such functions, it is also continuous, decreasing and convex. Finally, optimal d can be chosen to be non-increasing follows from the super-modularity of  $E\lceil \min(d, x+Z) \rceil$ .

This result demonstrates that the key property of convexity of the intermediary's payoff holds for this setting as well. Also, since the intermediary will optimally choose a face value of the debt that is decreasing in X, investors will naturally interpret large debt issues as a negative signal about the value of the assets. This leads to a separating equilibrium:

**LEMMA 8.** Given the asset pool Y, there is a unique separating equilibrium with  $\Gamma^*(x) = (1-\delta) E[\min(d(x),x+Z)], P^*(d(x)) = E[\min(d(x),x+Z)], \text{ and } d^*(x) \text{ determined by the differential equation:}$ 

$$\frac{\partial}{\partial x}d^*(x) = -\frac{1}{(1-\delta)} \frac{\Pr(Z < d^*(x) - x)}{\Pr(Z > d^*(x) - x)},\tag{11}$$

together with the boundary condition,  $d^*(x_0) = \infty$ .

The equilibrium described by **Lemma 8** depends on two parameters:  $x_0$ , which affects the boundary condition, and the distribution of Z, written  $\sim Z$ , which affects the differential equation (11). Thus, to compare equilibria across environments, define  $\Gamma^*(x;x_0,\sim Z)$  and  $d^*(x;x_0,\sim Z)$  to represent the solutions of **Lemma 8** for the corresponding parameter values. The next result establishes properties of  $\Gamma^*$  analogous to **Lemma 5**:

**LEMMA 9.** The intermediary's payoff is homogeneous of degree 1; that is,  $a\Gamma^*(x;x_0,\sim Z) = \Gamma^*(ax;ax_0,\sim aZ)$ . In addition,  $\Gamma^*(x;x_0,\sim Z) = (1-\delta)x_0 + \Gamma^*(x-x_0;0,\sim Z)$ . Finally,  $\Gamma^*(x;x_0,\sim Z) \leq (1-\delta)x_0$ , and the inequality is strict if  $x > x_0$  and Z is non-degenerate.

Having characterized the optimal security choice for a single asset, we now consider an issuer with *n* assets and compare the issuer's payoff from tranching a pool of assets versus selling and tranching each asset separately. The following result extends **THEOREM II** through **THEOREM IV** to the case of ex-post security design (unlimited tranching):

**THEOREM V.** (i) Suppose  $\eta = 0$  so that the residual risk is idiosyncratic, and that  $Y_i$  have bounded second moments. Then as  $n \to \infty$ , the per-asset payoff from pooling and tranching approaches the theoretical maximum,

$$\Gamma^*\left(\frac{1}{n}\sum_{i=1}^n X_i; \frac{1}{n}\sum_{i=1}^n X_{i0}, \sim \frac{1}{n}\sum_{i=1}^n Z_i\right) \longrightarrow (1-\delta)x_0,$$

so that pooling is optimal for *n* sufficiently large.

- (ii) Suppose  $\varepsilon_i = 0$ , so that the residual risk is common. Then pooling is suboptimal for any n.
- (iii) In the setting of **THEOREM IV** (and the corollary), the issuer's payoff from pooling and tranching is increasing in the number of common factors in the information  $X_i$ .

Thus, with either ex-ante or ex-post security design, pooling and tranching is optimal if the risk diversification effect dominates the information destruction effect.

# 5. Pooling By Uninformed Issuers

Sections 3 and 4 have demonstrated that for an informed intermediary, pure pooling is not optimal, though pooling and tranching may be optimal if there is sufficient diversification within the pool. In this section, I show that for an uninformed seller selling to both informed and uninformed buyers, pure pooling is optimal. This leads to the strong empirical prediction that only uninformed sellers should be observed selling pass-through pools.<sup>25</sup>

Suppose that there exist firms, which I call "originators," that specialize in the marketing and other services associated with originating the assets. These firms do not have a comparative advantage in valuing these assets or holding them to maturity and instead plan to sell the assets at their market price and redeploy the capital for use in further origination projects.

Consider an originator holding an asset with future cash flow Y = X + Z. Assume that the originator does not specialize in valuing the assets, and thus does not know the

21

\_

<sup>&</sup>lt;sup>25</sup> See also Subrahmanyam (1991), Gorton and Pennachi (1993) and Axelson (1999) for related models in which pooling can help reduce adverse selection problems when uninformed and informed buyers compete.

information  $X^{26}$ . As before, there are many potential risk-neutral uninformed buyers for the asset, who also do not know X, and who share the market discount rate of zero.

There are also potential informed investors who do know the realization of X. The informed investors have a higher cost of capital than uninformed, and so have valuation  $\delta$  X for the assets, for some  $\delta < 1$ . I assume that buyers are anonymous, so that it is impossible for the seller to completely exclude these informed buyers from the market.

When the seller lists an asset for sale, uninformed buyers bid some price p for the asset. This price is such that the uninformed break-even on average. Because the uninformed compete with informed buyers who know X, they face an adverse selection problem. This leads to under-pricing, p < E[X], as in Rock's (1986) model of IPO's. The seller is also hurt by the adverse selection, since the valuation of informed buyers is strictly below that of uninformed buyers.

To determine the equilibrium degree of under-pricing, it is necessary to specify an allocation rule. In general, this will depend on the precise mechanism used to sell the asset. Rather than specify a particular mechanism, I assume a reduced form that is general enough to handle a variety of mechanisms. In particular, let the total allocation of the uninformed buyers be given by

$$Q^{u}(X/p, p)$$
.

It is natural to assume that  $Q^u$  is decreasing in X/p; that is, the informed buyers get a larger fraction of the issue the greater the percentage of the under-pricing. It is also natural to assume that  $Q^u$  is increasing in the second argument, p. This is so because p measures the size of the asset, and the larger the issue the more likely the informed buyers are constrained in the amount they purchase. Effectively, this is an assumption that the uninformed have "deeper pockets" than the informed buyers.

As an example, suppose the allocation mechanism is a first price auction and informed buyers have discount factor  $\delta$  and no cash constraint. Then the informed buy if  $\delta X \ge p$ , so that  $Q^u(X/p, p) = 1[\delta X/p < 1]$ , where  $1[\cdot]$  is the indicator function. For a more complex example, if there is a single informed buyer with a cash constraint C, and if the issuer can screen out informed orders with probability  $(1-\theta) > 0$ , then  $C^{2\theta}$ 

Thic

 $<sup>^{26}</sup>$  This does not require that the originator have no private information – indeed, the originator might have private information about the component Z of the cash flows from its knowledge of the original source of the assets. It could then signal this information through its issuance decision in the ways described in Sections 3 and 4. What is relevant for our purposes is that the intermediary possesses some information not known to the originator. In the interest of simplicity, I therefore assume the originator is uninformed.

In general this  $\delta$  is endogenous based on their ability to identify under-priced assets elsewhere. This precise relationship is not important here, but will be analyzed in Section 6.

There is some ambiguity when  $\delta X = p$ , since the informed are indifferent. Since X is assumed continuously distributed, this is inconsequential.

To see this, note that the informed buyer does not buy if  $\delta X < p$ , and so uninformed receive the entire allocation. If  $\delta X > p$ , then the informed buyer will buy the fraction C/p, leaving the remaining shares for the uninformed.

$$Q^{u}(X/p, p) = 1[\delta X/p < 1] + (1 - \theta \min(1, C/p)) 1[\delta X/p \ge 1].$$

Given  $Q^u$ , the equilibrium bid of the uninformed investors is the largest bid such that they earn non-negative expected profits:

**LEMMA 10.** Suppose X is continuous with support  $[x_0, x_1]$ , and that  $Q^u$  is decreasing with only finitely many discontinuities in its first argument and continuous and increasing in its second argument. Also assume there exists a minimal level of underpricing,  $X/p > \beta > 1$ , before the informed will purchase, so that  $Q^u(\beta, x_0) = 1$ . Then the equilibrium uninformed bid is given by

$$P^*[X] = \max \left\{ p \, \middle| \, E[Q^u(X/p,p)(X-p)] \ge 0 \right\}.$$

Also,  $P^*[X] = E[X]$  if and only if  $Q^u(X/E[X], E[X]) = 1$  almost surely; otherwise,  $x_0 < P^*[X] < E[X]$ .

Given this specification of equilibrium in the origination market, consider next the incentives for pooling by an originator. Recall from Section 3 that it is never optimal for the informed intermediary to sell a pure pass-through pool of assets. The following result shows, however, that pure pass-through securities can be optimal for an uninformed originator, and can approach the first-best outcome.

**THEOREM VI.** Suppose  $X_i$  are independent, the assumptions of **LEMMA 10** hold, and  $\frac{1}{n}\sum_i E[X_i] \to x > 0$  as  $n \to \infty$ . Then  $\frac{1}{n}P^*\left[\sum_i X_i\right] \to x$ .

Thus, as the size of the pool grows large, the per-asset payoff to the originator approaches the assets' expected value. The intuition for this is straightforward. The adverse selection problem comes from the informed buyers' ability to purchase the "best" assets. Pooling reduces the precision of the selection the informed can make, as even the best pools will likely contain poor assets. This result provides an important motivation for pooling (in addition to transactions costs) in markets where traders other than the asset originators are likely to have the greatest expertise in valuing the assets.

I conclude this section with a numerical illustration. Suppose  $X_i$  are i.i.d. with distribution  $90 + \xi_i$ , where  $\xi_i$  is exponential with mean 10. Thus,  $E[X_i] = 100$ . Figure 3 shows equilibrium uninformed bid,  $\frac{1}{n}P^*\left[\sum_i X_i\right]$ , when  $Q^u = 1[\delta X < p]$ ; that is, the informed buy the entire issue whenever their valuation  $\delta X$  exceeds p.

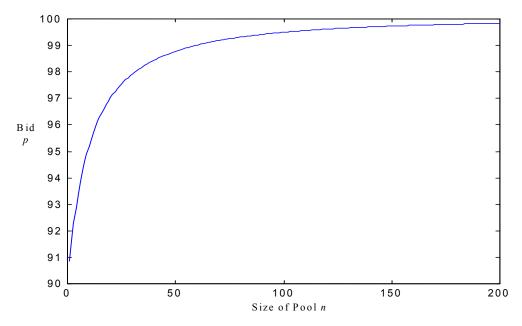
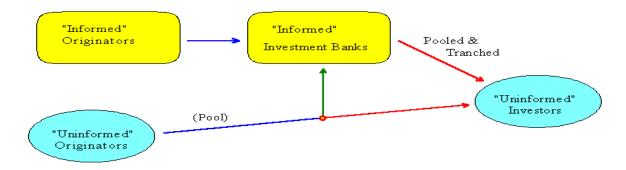


Figure 3: Per-Asset Payoff of Uniformed Issuer Based on Size of Pool

# 6. A Dynamic Model of Informed Intermediation

The results of the previous sections demonstrate the following. While uninformed sellers can benefit by pooling assets prior to sale, informed sellers do not gain from pure pooling. On the other hand, informed sellers can benefit by pooling the assets and selling a debt tranche. Combining these results in the context of a dynamic model leads to a theory of informed intermediation.



A simple motive for intermediation is implied by the returns to scale in the pooling and tranching process. Consider an informed "originator" or holder of the asset. This originator could sell these assets (or a tranche backed by them) directly to uninformed investors. Informed investment banks can add value, however, by acquiring the assets from informed originators and forming even larger pools prior to tranching.

The second channel for intermediation begins with uninformed originators who attempt to sell assets to investors. Given their superior information, informed investment banks will purchase the best (i.e., the most underpriced) of these assets. To minimize this adverse selection, uninformed originators will pool the assets if possible prior to sale. Once the investment bank acquires the assets, it again has an incentive to pool them further and sell a senior tranche to investors in order to raise new capital for additional asset purchases.

In this section I build a simple dynamic model of this second channel for intermediation. While highly stylized, it demonstrates that the model is a consistent story of intermediation. Unlike standard models of informed trading which assume a buy and hold strategy for the informed, the model developed here illustrates the benefits of asset resale through securitization. In particular, I show how the ability to repackage assets allows the intermediary to leverage its capital and increase the returns from its information. The model also reveals how the key parameters of the static model, the worst case information  $x_0$  and the discount factor  $\delta$ , arise endogenously from more primitive features of the market.

#### 6.1. The Timing

Consider a dynamic setting with the following timing. There is a single intermediary with access to a technology yielding private information. At the start of period t, the intermediary holds a portfolio of cash  $C_t$  and "old" securities with value  $O_t$ . At the start of period t, the origination market opens. The intermediary can use its available cash and superior information to purchase assets in the origination market that are under-priced.

After purchasing assets in the origination market, the intermediary holds both old securities (with value  $O_t$ ) plus any new securities just acquired. Denote the full-information value of the new securities by  $N_t$ . In addition, the intermediary might have unused cash, denoted  $U_t$ , if the supply of new under-priced securities did not exceed its original cash balance  $C_t$ .

Once the origination market closes, the intermediary then has the opportunity to sell assets from its portfolio in the secondary market. By selling assets, the intermediary raises cash it can use in period t + 1.

I assume that any private information the intermediary had regarding securities purchased prior to period t becomes public by the start of period t. Thus, in the sale phase, the intermediary will sell all the old securities  $O_t$  that were held at the start of the period for their full-information value. The new assets  $N_t$  will either be sold for cash or retained until the next period. Here there is a lemon's problem since the intermediary holds private information about these securities. However, the intermediary also has a motive to trade to raise additional cash to purchase assets next period. Thus, the signaling models of Sections 3 and 4 will determine the fraction of the value of the newly acquired assets that are sold and retained. Recall that in the separating equilibria of the previous analysis, the fraction sold will be sold for its full information value, denoted  $S_t$ .

Thus, the fraction retained has value  $N_t - S_t$ . The intermediary then begins the next period with a portfolio of cash  $C_{t+1} = U_t + S_t + O_t$  and old securities worth  $O_{t+1} = N_t - S_t$ . See Figure 4.

	Start	Purchase assets in		Start
Asset	Period <i>t</i>	origination market	Sell assets to investors	Period <i>t</i> +1
Cash	$C_t$	$\rightarrow U_t$	$\begin{array}{ccc} & & & & \\ & & & & \\ & & & & \\ & & & & $	$\left. ight\} C_{t+1}$
Securities	$O_t$	$ \begin{array}{c} N_t \\ O_t \end{array} $	$N_t - S_t$	$=$ $O_{t+1}$

Figure 4: The Timing of the Model

In the following sections, I employ the models of this paper to determine the dynamics of this intermediation process.

### 6.2. Asset Acquisition With Cash Constraints

Each period, uninformed originators sell assets in the origination market. As demonstrated in Section 5, uninformed originators have an incentive to pool assets prior to sale. Thus, each asset can be thought of as pool of even smaller assets. The pool sold by issuer i in period t has payoff  $Y_{it} = X_{it} + Z_{it}$ .

In order to provide for the simplest characterization of the equilibrium in the origination market, I suppose that there is a continuum of originators  $i \in [0,M_t]$  with associated measure  $\mu$ . I assume that the intermediary's private information about each asset  $X_{it}$  is independent and identically distributed. In this case, it is natural to assume that

$$\mu\{i \in [0,M_t] : X_{it} \le x\} = M_t \Pr(X_t \le x),$$

where  $X_t$  has the same distribution as each of the  $X_{it}$ . I assume that  $X_t$  is continuously distributed. To simplify notation in what follows, I drop the time subscripts except where necessary.

\_

<sup>&</sup>lt;sup>30</sup> The intermediary might also have an incentive to issue securities backed by its *future* profit stream, rather than just its existing portfolio. Modeling this alternative is beyond the scope of this paper. However, it is natural that such securities would be subject to an even more extreme asymmetric information problems than the asset-backed securities considered. (For example, consider the distinction between secured and unsecured debt.) Serious moral hazard considerations would also be introduced. Thus, the intermediary would rely on asset-backed securities as a primary source of capital. This is consistent with actual practice: investment banks raise almost all of the cash used for acquiring new assets by selling or borrowing against existing assets in their portfolio.

Since the assets are ex-ante identical, the uninformed bid a common price p for all assets. Thus, the intermediary would like to purchase assets with  $X_i \ge p$ . This may violate the intermediary's budget constraint. The intermediary optimally purchases the best securities (those with the highest  $X_i/p$ ) first, up to the budget constraint. <sup>31</sup>

I assume that originators can screen out informed trades with probability  $1-\theta > 0$ , so that  $\theta$  is the probability that the intermediary purchases an asset that it has bid on.<sup>32</sup> The quantity of assets the intermediary would purchase absent a budget constraint is  $\theta$  M Pr(X > p), which violates the budget constraint if  $\theta$  M Pr(X > p) p > C. Thus, the optimal purchase for the intermediary is to buy all assets above a critical quality  $x^c$  (i.e., the intermediary purchases  $\{i \in [0,M]: X_i > x^c\}$ ), where

$$x^{c} = \min x' \text{ s.t. } x' \ge p \text{ and } \theta M Pr(X > x') p \le C.$$
 (12)

Anticipating this, uninformed investors realize that they receive assets disproportionately more of the lowest quality assets. Hence, the equilibrium bid p of the uninformed is the largest p satisfying the zero profit condition

$$E\left[\left(1-\theta 1[X>x^{c}]\right)(X-p)\right]=0. \tag{13}$$

Together, equations (12) and (13) determine the equilibrium values of  $x^c$  and p given  $\theta$  and C/M, the amount of cash held by the intermediary relative to the size of the market. This extends the model of Section 5 to the continuum case.<sup>33</sup> Note the following comparative statics for p and  $x^c$  as a function of the cash available to the intermediary.

**LEMMA 11.** There exists  $\overline{C} < \theta ME[X]$  such that for  $C \le \overline{C}$ , both  $x^c$  and p strictly decrease with C. For  $C \ge \overline{C}$ ,  $x^c = p = p_0$ .

As an example, suppose  $\theta = 0.90$ , E[X] = 100 and X is lognormal with a volatility of  $\sigma = 2\%$ . Figure 5 shows the values of  $x^c$  and p as a function of C/M. Note that for  $C/M \ge 72$ , the cash constraint no longer binds and  $x^c = p = 98.2$ . However, for C/M < 72, the intermediary earns a positive return from additional cash balances since the marginal security is under-priced ( $x^c > p$ ).

<sup>32</sup> Equivalently, informed traders may simply be unaware of some fraction of the assets traded in the market. The assumption  $\theta < 1$  is technically useful to prevent p from dropping discontinuously (and potentially without bound) as the intermediary's expenditures approach the size of the market.

27

This generalizes to the case in which there is more than one "type" of security that can be distinguished ex-ante. In that case, for each type j there is an associated  $X^j$  and uninformed bid  $p^j$ . In that case it is optimal for the intermediary to purchase those assets with the highest ratio  $X^j_i / p^j$  first, until the budget constraint binds. The analysis for this case is more cumbersome but qualitatively identical.

<sup>&</sup>lt;sup>33</sup> If a single infinitesimal originator were to enter this market and place assets X' up for sale, the informed intermediary would purchase these assets if  $X'/p' > x^c/p$ . If we define  $\delta = p/x^c$ , this becomes  $\delta X' > p'$ , and  $O^u = 1 - \theta 1 [\delta X' > p']$ . The originator thus faces the same problem as in Section 5.

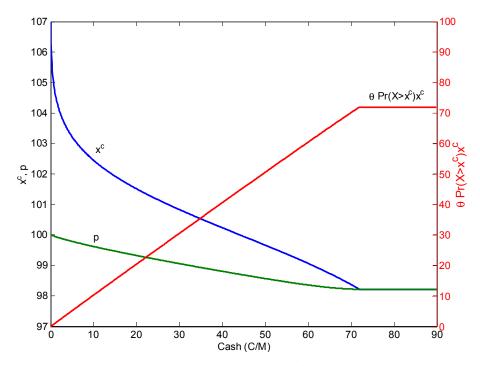


Figure 5: Equilibrium Price p and Informed Cutoff  $x^c$  in the Origination Market

Figure 5 also shows value of the purchased portfolio,  $\theta Pr(X>x^c) x^c$ , when valued at the minimum quality level  $x^c$ . I assume, as is true in this case, that this amount increases as the amount of cash spent increases and  $x^c$  decreases:

**Assumption B.**  $\theta Pr(X > x^c) x^c$  is decreasing in  $x^c$  for  $x^c > p$ .

Assumption B is a distributional assumption on the private information X. It is satisfied by standard distributions if the volatility of X is not too large. <sup>34</sup> It is important because it implies that the minimal resale value of the intermediary's portfolio is increasing in its size.

#### 6.3. Asset Resale

At the end of the acquisition stage, the intermediary holds old assets (acquired in previous periods), new assets (just acquired), as well as any unused cash in the event  $x^c = p$ . The amount of unused cash is given by

$$U = C - \theta M Pr(X > x^{c}) p. \tag{14}$$

The value of the new assets just purchased by the intermediary is given by

<sup>&</sup>lt;sup>34</sup> For example, it is satisfied if X is uniform on  $[x_0, x_1]$  with  $x_1 \le 2x_0$ , or for  $X = x_0 + \xi$  with  $\xi$  exponential and  $x_0 \ge E[\xi]$ . In the case of a lognormal distribution, it holds as long as  $\theta$  and  $\sigma$  are not too large. In the example here, we could raise  $\theta$  to 0.999 or  $\sigma$  to 42% before the assumption is violated.

$$N = \theta M E[X 1[X > x^{c}]] = C - U + M(E[X] - p).$$
 (15)

The first expression is simply the integral of the value of the assets purchased. The second expression follows from (13): since the uninformed earn zero profits, the losses of the sellers M(E[X] - p) must correspond to the profits of the intermediary.

After acquiring new assets, the intermediary may resell them immediately or hold them until they are old and then resell them. The advantage of immediate resale is that the cash raised can be used to purchase new securities in the next period. The disadvantage is that the intermediary faces a lemons problem due to its private information.

First, consider the resale problem in the absence of asset securitization / tranching. The results in Section 3 establish that an informed intermediary should sell the assets individually. In that case there is a unique separating equilibrium, given in **LEMMA 2**, in which an asset with value x is priced correctly and issued in quantity

$$Q^*(x;x_0,\delta) = \left(\frac{x}{x_0}\right)^{\frac{-1}{1-\delta}},$$

where  $x_0$  is the worst case information of the intermediary, and  $\delta$  reflects the intermediaries preference for cash. Note that in this case,  $x_0 = x^c$ , the lowest quality asset purchased by the intermediary in the acquisition stage.

Given the parameter  $\delta$  (we will see how  $\delta$  is determined in the next section), the intermediary will raise cash from the resale of

$$S^{1} = \theta M E[X Q^{*}(X; x^{c}, \delta) 1[X > x^{c}]], \tag{16}$$

immediately by reselling the assets. The remaining fraction  $1-Q^*$  will be held for sale in the following period when the private information X is publicly known.

Alternatively, suppose tranching is possible. In this case, the diversifiable risk  $Z_i$  becomes relevant. I assume that this risk is diversifiable, so that it is eliminated in large pools. Thus, the results of Section 4 demonstrate that the optimal security design for the intermediary is to form a pool of the purchased assets, and issue debt with face value  $x_0 = x^c$  per asset. This debt is riskless, and so will sell for its face value of  $x^c$ . Since the number of assets in the pool is equal to the quantity purchased,  $\theta M Pr(X > x^c)$ , the issuer will raise total cash of

$$S^* = \theta M Pr(X > x^c) x^c. \tag{17}$$

Again, the remaining junior tranche of the pool is sold in the following period, for the amount  $N - S^*$ . Comparing (16) and (17), pooling and tranching benefits the intermediary by allowing it to raise more cash through immediate resale. That is, because  $x O^*(x; x^c, \delta) < x^c$  for  $x > x^c$ , we have  $S^* > S^1$ .

## 6.4. Growth Through Securitization

The analysis above leads to the following specification for the dynamic evolution of the intermediary. From equations (12) and (13), we have

$$x_{t}^{c} = x_{t}^{c}(C_{t}, M_{t}), p_{t} = p(C_{t}, M_{t}).$$

Combining this with (14) and (15), it is possible to write

$$U_t = U(C_t, M_t), N_t = N(C_t, M_t).$$

Finally, using either (16) (no tranching,  $S = S^1$ ) or (17) (tranching,  $S = S^*$ ), we have

$$S_t = S(\delta_t, C_t, M_t),$$

where  $S = S^1$  or  $S = S^*$  depending on the setting. This leads to the subsequent portfolio for the intermediary,

$$C_{t+1} = U_t + S_t + O_t, \quad O_{t+1} = N_t - S_t.$$
 (18)

The only endogenous parameter not identified by the above system is  $\delta_t$ , the intermediary's preference for cash. Suppose the intermediary sells an additional fraction of one of the assets in period t. The incremental cash raised can be used to purchase additional securities in the acquisition stage of period t+1. This will lead the issuer to purchase assets with value  $x^c_{t+1}$  for price  $p_{t+1}$ , as long as  $x^c_{t+1} \ge p_{t+1}$ . Since the incremental purchase is of the lowest quality asset, the intermediary can immediately resell the asset for price  $x^c_{t+1}$ . Thus, an incremental infinitesimal dollar of cash generates a return of  $x^c_{t+1}/p_{t+1}$  that would be missed if the securities were not sold. Hence, the intermediary's preference for cash in period t is given by

$$\delta_t = p_{t+1} / x^c_{t+1} \le 1. \tag{19}$$

Equation (19) can be combined with the above to yield the following fixed-point problem for  $\delta_t$ ,

$$\delta_{t} = \frac{p(C_{t+1}, M_{t+1})}{x^{c}(C_{t+1}, M_{t+1})} = \frac{p(U(C_{t}, M_{t}) + S(\delta_{t}, C_{t}, M_{t}) + O_{t}, M_{t+1})}{x^{c}(U(C_{t}, M_{t}) + S(\delta_{t}, C_{t}, M_{t}) + O_{t}, M_{t+1})}.$$
(20)

The solution to (20) implies that<sup>36</sup>

$$\delta_t = \delta(C_t, O_t, M_t, M_{t+1}).$$

<sup>&</sup>lt;sup>35</sup> This holds with or without tranching, since in a separating equilibrium the worst type bears no signaling cost.

<sup>&</sup>lt;sup>36</sup> With tranching,  $S = S^*$  does not depend on δ, hence the solution to (20) is immediate. Without tranching,  $S = S^1$  is decreasing in  $\delta_t$ , and the fixed-point problem is non-trivial.

Combining all of the above, given some exogenous growth of the origination market, we may characterize the growth of the intermediary

$$C_t, O_t \xrightarrow{M_t, M_{t+1}} C_{t+1}, O_{t+1},$$

and derive the dynamics of market prices and pooling and tranching activity. The following result demonstrates the benefit of pooling and tranching for the intermediary:

**THEOREM VII.** Given initial assets,  $(C_0, O_0)$ , the size of the intermediary,  $C_T + O_T$ , on date T > 2 will be larger when asset securitization (pooling and tranching) is possible than when assets are sold individually or there is no resale. The comparison is strict if  $\delta_t < 1$  for some t < T-1.

I conclude by illustrating the increased growth from pooling and tranching for the numerical example of Figure 5. Figure 6 shows the growth rate of the intermediary with pooling and tranching, individual resale, and no resale (buy and hold). Also shown is the marginal return on the worst assets purchased,  $x^c/p$ , showing the intermediary's preference for cash.

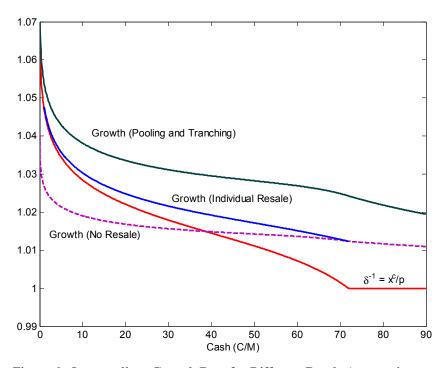


Figure 6: Intermediary Growth Rate for Different Resale Assumptions

For example, Figure 6 implies that if the market growth rate is 3%, with pooling and tranching the intermediary will grow faster than the market until its size reaches a steady state of about 36% of the total market. In contrast, with individual resale, the

intermediary grows to about 10% of the market, and without resale, the steady state size of the intermediary is below 1% of the total market.<sup>37</sup>

#### 7. Conclusion

The results of this paper can be viewed as a theory of financial intermediation based upon intermediaries having private information regarding asset values. Due to its information advantage, the intermediary has the ability to identify high quality and therefore underpriced assets in the origination market. The intermediary can therefore profit by buying and holding these assets. Of course, this creates an adverse selection problem in the origination market, implying that in equilibrium these assets will be priced at a discount. In order to mitigate this problem, originators have an incentive to pool the assets prior to selling them. Pooling reduces the intermediary's ability to purchase the assets that are most under-priced.

Once the intermediary has purchased assets in the origination market, it can hold the assets to maturity. The intermediary, however, would prefer to liquidate the assets at their true value to raise cash to use for future asset purchase opportunities. That is, the intermediary wishes to leverage its available capital to exploit its information for as many deals as possible. Unfortunately, because the intermediary has private information regarding the assets, it faces a lemons problem if it attempts to sell the assets for cash. This lemons problem leads to a natural signaling equilibrium in which the intermediary signals the value of the assets by its willingness to retain some portion of the cash flows. I show that if only pure pass-through securities can be sold, the intermediary finds it optimal to sell the assets individually rather than as a pool. However, if non-linear asset-backed securities can be issued, and if the intermediary holds enough assets, it may be optimal to pool the assets and issue a debt-like security that is backed by the pool. Asset securitization allows the intermediary to leverage its capital more efficiently and increase the returns associated with its private information.

The incentive for pooling and tranching by the intermediary is shown to depend on several factors. First, pooling has an information destruction effect that is costly for the intermediary. This effect is reduced if the intermediary's private information is positively correlated across the assets. Second, the gains from issuing a debt tranche are enhanced if the pool has lower residual risk. This risk diversification benefit is reduced therefore if the residual risks of the assets are positively correlated. Thus, pooling and tranching is most effective for assets for which the private information is general and the residual risks are specific.

<sup>&</sup>lt;sup>37</sup> Note that without resale, even though the marginal return is above 5% when cash is below 1%, this return is realized over 2 periods (assets are not sold until information is public). Thus the annualized grow rate is below the marginal return. With either type of resale, since the intermediary can always sell for at least the worst case value  $x^c$ , the growth rate cannot fall below the marginal return  $\delta^{-1}$ . Note also that the growth rate of with no resale and individual resale coincide for  $\delta = 1$  since then the intermediary sells only the worst asset (which have measure zero). With pooling and tranching, however, the intermediary can extract  $x^c$  from each asset (even those with higher value).

# 8. Appendix

#### 8.1. Pooling By A Risk Averse Intermediary

Here I generalize the results of Section 3 to an intermediary whose motivation to sell the assets is risk aversion. Applying the signaling model of Leland and Pyle (1977) (L&P) again leads to a downward sloping demand for the assets due to asymmetric information. Thus, the intermediary must tradeoff the price at which it can sell for the amount of risk it must bear. I demonstrate that this equilibrium shares the basic convexity property of the intermediary's payoff that was crucial in the previous section. As there, this convexity implies that the intermediary is better off selling the assets individually rather than as a pool.

Specifically, suppose the intermediary has CARA utility over final wealth, with risk aversion parameter r; that is,

$$U(W) = -e^{-rW}.$$

Assume that conditional on X, the residual risks  $Z_i$  are independent<sup>38</sup> and normally distributed with variance  $\sigma_i^2$ . This implies that the intermediary's wealth is conditionally normally distributed. Recall that for CARA utility, if W is normal then

$$E[U(W)] = U(E[W] - \frac{1}{2}r \operatorname{Var}[W]).$$

This allows us to evaluate payoffs in terms of their "certainty equivalents,"  $E[W] - \frac{1}{2}r \text{Var}[W]$ .

First consider the intermediary's incentive to sell shares in the entire pool. Suppose the intermediary sells a fraction q of the entire pool of assets to investors at a market price of p for the pool. Then the payoff to the intermediary is given by

$$E[U((1-q)Y^n + qp)|X] = U((1-q)X^n + qp - \frac{1}{2}r(1-q)^2\sigma^2),$$

where  $\sigma^2$  is the conditional variance of the pool; i.e.,  $\sigma^2 = \sum_i \sigma_i^2$ .

\_

<sup>&</sup>lt;sup>38</sup> The assumption of independence is a "neutral" one regarding the decision to pool or not. One could also consider the correlated case, but this raises certain difficulties. If the intermediary plans to sell each asset sequentially, the cost of holding the initial asset depends upon how much of the other assets will be sold. This leads to incentives for cross-signaling. Instead, one could consider an intermediary who intends to buy the assets  $Y_i$  from separate informed holders in order to pool them. In this case, the incentive to pool would be reduced (increased) by positive (negative) correlation.

If the intermediary anticipates a market demand schedule given by P(q) for the pool, then the intermediary with conditional value  $X^n = x$  will choose a quantity  $Q_A^P(x)$  that maximizes the certainty equivalent payoff,

$$\max_{q} (1-q)x + qP(q) - \frac{1}{2}r(1-q)^{2} \sigma^{2}$$

$$= x + \max_{q} q(P(q) - x) - \frac{1}{2}r(1-q)^{2} \sigma^{2}.$$
(21)

Equation (21) reveals that the total certainty equivalent payoff to the intermediary is the intermediary's value of the asset plus any "profit" from the sale of the security net a risk premium for the fraction retained. Thus, the intermediary's asset sale decision is to choose a quantity to sell that maximizes this "risk-adjusted profit":

$$R^{P}(x) = \max_{q \in [0,1]} q(P(q) - x) - \frac{1}{2}r(1 - q)^{2} \sigma^{2}.$$
 (22)

This yields the following immediate consequence:

**LEMMA 12.** For any demand function P, the intermediary's payoff  $R^P$  is decreasing and convex in x. Also, the fraction  $Q_A^P(x)$  that the intermediary sells is decreasing in x.

**PROOF:** For fixed q, the intermediary's objective is decreasing and linear in x with slope -q. The rest is identical to the proof of **LEMMA 1**. •

Leland and Pyle (1977) characterized the signaling equilibrium for this model:

**LEMMA 13.** There is a unique separating equilibrium, given by  $Q_A^*(x) = q_A \left( \frac{x - x_0}{r\sigma^2} \right)$  and  $P_A^*(q) = x_0 - r\sigma^2 \left[ \log(q) + 1 - q \right]$ , where the monotone decreasing function  $q_A$  is defined implicitly by

$$s = - \left\lceil \log \left( q_{\scriptscriptstyle A}(s) \right) + 1 - q_{\scriptscriptstyle A}(s) \right\rceil.$$

Let  $\rho(x) = -\frac{1}{2}(1 - q_A(x))^2$ . The equilibrium payoff function for the intermediary is given by

$$R^*(x) = \rho \left(\frac{x - x_0}{r\sigma^2}\right) r\sigma^2.$$

**PROOF:** See L&P. The solution follows from differentiation of (21) and (3), and the boundary condition that  $Q_A^*(x_0) = 1$ . Note that  $\log(q) + 1 - q$  is negative and increasing in q; thus  $q_A$  is well-defined.  $\bullet$ 

Note that without any further calculation, **LEMMA 12** implies that  $\rho$  is decreasing and convex.

Next consider the intermediary's payoff if the assets are sold separately, rather than as a single pool. Suppose the intermediary sells quantity  $q_i$  of asset i for price  $p_i$ . Then the intermediary's certainty equivalent payoff is given by

$$\sum_{i=1}^{n} (1-q_i) x_i + q_i p_i - \frac{1}{2} r (1-q_i)^2 \sigma_i^2 = x^n + \sum_{i=1}^{n} q_i (p_i - x_i) - \frac{1}{2} r (1-q_i)^2 \sigma_i^2,$$

using the conditional independence of the payoffs  $Y_i$ .

Thus, for each asset i, the intermediary faces the same problem as in (21) above with  $x_i$  in place of x and  $\sigma_i^2$  in place of  $\sigma^2$ . As in the previous section, any information that investors may have learned from prior asset sales only affects the distribution of  $X_i$ , but not  $x_{i0}$ . Hence the characterization of the equilibrium is unchanged.

**LEMMA 14.** If the intermediary sells each asset *i* separately, the intermediary's total certainty equivalent payoff is given by

$$\sum_{i=1}^{n} \rho \left( \frac{x_i - x_{i0}}{r\sigma_i^2} \right) r\sigma_i^2$$

**PROOF:** Follows by the same argument as **LEMMA 3**. •

This allows the following comparison of the intermediary's payoff from a separate or a pooled sale of the assets, which shows that pooling is again not optimal for the informed intermediary.

**THEOREM VIII.** In the Leland and Pyle setting, the intermediary prefers a separate sale of the assets to a pooled sale; that is,

$$\sum_{i=1}^{n} \rho \left( \frac{X_i - x_{i0}}{r\sigma_i^2} \right) r\sigma_i^2 \ge \rho \left( \frac{X^n - x_0^n}{r\sigma^2} \right) r\sigma^2,$$

where the inequality is strict if  $(X_i - x_{i0})/r\sigma_i^2$  is not equal for all *i*.

**PROOF:** By the convexity of  $\rho$ , and the fact that  $x_0^n = \sum_i x_{i0}$  and  $\sigma^2 = \sum_i \sigma_i^2$ , we have

$$\sum_{i=1}^{n} \frac{r\sigma_{i}^{2}}{r\sigma^{2}} \rho \left( \frac{X_{i} - x_{i0}}{r\sigma_{i}^{2}} \right) \ge \rho \left( \sum_{i=1}^{n} \frac{r\sigma_{i}^{2}}{r\sigma^{2}} \frac{X_{i} - x_{i0}}{r\sigma_{i}^{2}} \right) = \rho \left( \frac{X^{n} - x_{0}^{n}}{r\sigma^{2}} \right)$$

The strict inequality follows from that fact that  $Q_A^*$  is strictly decreasing so that  $\rho$  is strictly convex.  $\bullet$ 

#### 8.2. Other Proofs

**PROOF OF LEMMA 3:** Given  $X_{-n}$ , consider the sale of asset n. By the initial assumptions, the conditional support of  $X_n$  is an interval with greatest lower bound  $x_{n0}$ . Thus, from the previous results, there is a unique separating equilibrium for the sale of asset n, with the intermediary's profit given by  $\pi(X_n/x_{n0})x_{n0}$ . Now consider the sale of asset n-1. Since the intermediary's profit in the sale of asset n does not depend on the outcome of the sale of asset n-1, the intermediary's problem is unchanged. The proof thus proceeds by induction.  $\bullet$ 

**PROOF OF LEMMA 5:** For any security design F for Y, define the security  $\hat{F}$  for aY by  $\hat{F}(y) = a F(y/a)$ . Then  $\hat{F}(aY) = a F(Y)$ . Hence  $\hat{f} = af$ , and homogeneity follows immediately from (4).

Next note that since  $\pi$  is decreasing,  $\pi(f/f_0)f_0 \le \pi(1)f_0 = (1-\delta)f_0$ . The inequality then follows since  $f = E[F(Y) \mid X] \le E[Y \mid X] = X$ , so that  $f_0 \le x_0$ .

For strictness, suppose  $f_0 = x_0$ . Then  $F(x_0 + Z) = x_0 + Z$  almost surely. Thus, F is strictly increasing on the support of  $x_0 + Z$ . Since F is everywhere non-decreasing, if  $x > x_0$  then  $F(x+Z) > F(x_0+Z)$  for Z in the interior of its support, or almost surely. Hence  $f/f_0 > 1$  almost surely. Since  $\pi$  is strictly decreasing, the result follows.  $\bullet$ 

**PROOF OF THEOREM II:** Define  $H_n(d,x) = E\left[\min\left(d,x + \frac{1}{n}\sum_{i=1}^n \varepsilon_i\right)\right]$ .  $H_n$  is continuous and increasing, and by the Law of Large Numbers,  $H_n(d,x) \to \min(d,x)$  as  $n \to \infty$ .

First consider  $d=x_0$ . Then  $f_0^d=H_n\left(d,x_0^n/n\right)\to \min\left(d,x_0\right)=x_0$ . Since  $d\geq f^d=H_n\left(d,X^n/n\right)\geq H_n\left(d,x_0^n/n\right)$ , we also have  $f^d\to x_0$  almost surely. Hence,  $E\left[\pi\left(f^d\big/f_0^d\right)f_0^d\right]\to \pi(1)\,x_0=(1-\delta)\,x_0$ . Since this is an upper bound for G by **LEMMA** 5, we conclude that  $G\left[\frac{1}{n}\sum_{i=1}^n Y_i\right]\to (1-\delta)\,x_0>\frac{1}{n}\sum_{i=1}^n G\left[Y_i\right]$ .

To show that  $D^*$  must converge to  $x_0$ , consider  $d < x_0$ . In that case,  $f_0^d \to d$ , so that  $\overline{\lim}_{n \to \infty} E\Big[\pi\Big(f^d\big/f_0^d\Big)\,f_0^d\Big] \le (1-\delta)d$ , which is sub-optimal. Next consider  $d > x_0$ . In that case,  $f_0^d \to x_0$ , so it remains to show that  $\overline{\lim}_{n \to \infty} E\Big[\pi\Big(f^d\big/f_0^d\Big)\,\Big] < (1-\delta)$ .

Since  $G[Y_i] \ge E[\pi(X_i/x_{i0})x_{i0}] \ge \pi(E[X_i]/x_{i0})x_{i0}$ , (7) implies that  $\lim_{n\to\infty} \frac{1}{n} \sum_i E[X_i] > x_0$ . Thus, there exists N and  $\gamma \in (0,d-x_0)$  such that for all n > N,  $E[X^n]/n > x_0 + \gamma$ . By hypothesis,  $X^n/n$  has bounded second moment, so that  $^{39} \Pr(X^n/n > x_0 + \gamma/2) > \lambda$  for some  $\lambda > 0$ . Also, there exists N' > N such that for all n > N',  $H_n(d, x_0 + \gamma/2) \ge x_0 + \gamma/4$  and  $H_n(d, x_0^n) \le x_0 + \gamma/6$ . Thus, for all n > N',  $\Pr\left(\pi\left(f^d / f_0^d\right) \le \pi\left(\frac{x_0 + \gamma/4}{x_0 + \gamma/6}\right)\right) > \lambda$ , which proves the result.  $\bullet$ 

**PROOF OF THEOREM III:** Fix an n and let  $d = D^*[Y^n/n]$ , the optimal debt level if the assets are pooled. Next define  $h(x) = E[\min(d, x+\eta)]$ , and note that h is concave. Then for any realization  $x_i$ , Jensen's inequality implies  $h(\sum_i x_i/n) \ge \sum_i h(x_i)/n$ . Since  $\pi$  is decreasing, this implies

$$\pi\left(\frac{h\left(\frac{1}{n}\sum_{i}x_{i}\right)}{h\left(x_{0}\right)}\right) \leq \pi\left(\frac{1}{n}\sum_{i}\frac{h\left(x_{i}\right)}{h\left(x_{0}\right)}\right) \leq \frac{1}{n}\sum_{i}\pi\left(\frac{h\left(x_{i}\right)}{h\left(x_{0}\right)}\right),$$

where the last inequality follows from the convexity of  $\pi$ . Thus,

$$G\left[\frac{1}{n}\sum_{i}Y_{i}\right] = E\left[\pi\left(\frac{h\left(\frac{1}{n}\sum_{i}X_{i}\right)}{h\left(X_{0}\right)}\right)h\left(X_{0}\right)\right] \leq \frac{1}{n}\sum_{i}E\left[\pi\left(\frac{h\left(X_{i}\right)}{h\left(X_{i0}\right)}\right)h\left(X_{i0}\right)\right] \leq \frac{1}{n}\sum_{i}G\left[Y_{i}\right],$$

where the last inequality follows since  $d \neq D^*[Y_i]$  in general.  $\bullet$ 

**PROOF OF THEOREM IV:** Define  $h(d,x) = E[\min(d,x+\eta+\sum \epsilon_i/n)]$ , and note that h is concave and increasing. Next define  $Q(d,x,x_0) = \pi(h(d,x)/h(d,x_0))h(d,x_0)$ . Since  $\pi$  is decreasing and convex, Q is also decreasing and convex in x. Therefore,

$$E\left[Q\left(d,\frac{1}{n}\sum_{i}X_{i},\frac{1}{n}\sum_{i}x_{i0}\right)\right] \geq E\left[Q\left(d,\frac{1}{n}\sum_{i}\hat{X}_{i},\frac{1}{n}\sum_{i}x_{i0}\right)\right],$$

which proves the result. •

**PROOF OF LEMMA 8:** In any sequential equilibrium, the "worst type"  $x_0$  behaves according to the first-best, which in the context of this model is to sell all of the assets to the investors. This is equivalent to issuing 100% of the equity interest in the assets, or equivalently debt with a face value equal to or greater than the maximum possible payoff. It is then straightforward to check that the model satisfies the standard single crossing condition and that the above differential equation does indeed determine an equilibrium. Uniqueness follows by similar arguments to Mailath (1987).  $\bullet$ 

**PROOF OF LEMMA 9:** Define  $d(x) = a d^*(x/a;x_0,\sim Z)$ . Then  $d(ax_0) = \infty$  and

<sup>&</sup>lt;sup>39</sup> This follows from the fact that if  $E[A] \ge a$  and  $E[A^2] \le b$ , then for  $c \in [0,a]$ ,  $\Pr(A \ge c) \ge (a-c)^2/b$ .

$$\begin{split} d'(x) &= a \frac{1}{a} d^{*'}(x/a; x_{0}, \sim Z) = -\frac{1}{(1-\delta)} \frac{\Pr\left(Z < d^{*}(x/a; x_{0}, \sim Z) - x/a\right)}{\Pr\left(Z > d^{*}(x/a; x_{0}, \sim Z) - x/a\right)} \\ &= -\frac{1}{(1-\delta)} \frac{\Pr\left(aZ < d(x) - x\right)}{\Pr\left(aZ > d(x) - x\right)}. \end{split}$$

Hence,  $d(x) = d^*(x; ax_0, \sim aZ)$ . Then we have,

$$\Gamma^*(ax; ax_0, \sim aZ) = (1 - \delta) E \left[ \min \left( d(ax), ax + aZ \right) \right]$$
$$= (1 - \delta) E \left[ \min \left( ad^*(x; x_0, \sim Z), ax + aZ \right) \right]$$
$$= a\Gamma^*(x; x_0, \sim Z).$$

Next define  $d_0(x) = d^*(x+x_0;x_0,\sim Z) - x_0$ . Then  $d_0(0) = \infty$  and

$$d_{0}'(x) = d^{*'}(x + x_{0}; x_{0}, \sim Z) = -\frac{1}{(1 - \delta)} \frac{\Pr(Z < d^{*}(x + x_{0}; x_{0}, \sim Z) - (x + x_{0}))}{\Pr(Z > d^{*}(x + x_{0}; x_{0}, \sim Z) - (x + x_{0}))}$$
$$= -\frac{1}{(1 - \delta)} \frac{\Pr(Z < d_{0}(x) - x)}{\Pr(Z > d_{0}(x) - x)}.$$

Thus,  $d_0(x) = d^*(x; 0, -Z)$ . Therefore,

$$\Gamma^*(x - x_0; 0, \sim Z) = (1 - \delta) E \Big[ \min \Big( d_0(x - x_0), x - x_0 + Z \Big) \Big]$$

$$= (1 - \delta) E \Big[ \min \Big( d^*(x; x_0, \sim Z) - x_0, x - x_0 + Z \Big) \Big]$$

$$= \Gamma^*(x; x_0, \sim Z) - (1 - \delta) x_0.$$

For the bound on  $\Gamma^*$ , note that

$$\Gamma^*(x_0; x_0, \sim Z) = (1 - \delta)E[\min(d^*(x_0; x_0, \sim Z), x_0 + Z)] = (1 - \delta)E[x_0 + Z] = (1 - \delta)x_0,$$

and by **Lemma 7**,  $\Gamma^*$  is decreasing in x. For strictness, note that if  $\Gamma^*$  at  $x_0$  does not strictly decrease, by **Lemma 7** it must be constant. Therefore,  $E[\min(d^*(x;x_0,\sim Z),x+Z)] = x_0$ . For  $x > x_0$ , this implies that  $\Pr(x_0+Z > d^*(x;x_0,\sim Z)) > 0$ . But this implies type  $x_0$  would prefer to issue  $d^*(x;x_0,\sim Z)$ , violating incentive compatibility. •

**PROOF OF THEOREM V:** For case (i), suppose the intermediary issues debt with a (per asset) face value of  $d = \frac{1}{n} x_0^n$ . In any sequential equilibrium, this debt will sell for a (per asset) price of at least  $p_n = E[\min(\frac{1}{n} x_0^n, \frac{1}{n} x_0^n + \frac{1}{n} \varepsilon^n)]$ . Therefore,

$$\Gamma^* \left( \frac{1}{n} X^n; \frac{1}{n} x_0^n, \sim \frac{1}{n} Z^n \right) \ge p_n - \delta E[\min(\frac{1}{n} x_0^n, \frac{1}{n} X^n + \frac{1}{n} \varepsilon^n)]$$

$$\ge p_n - \delta \frac{1}{n} x_0^n$$

$$= (1 - \delta) \frac{1}{n} x_0^n + E[\min(0, \frac{1}{n} \varepsilon^n)]$$

$$\ge (1 - \delta) \frac{1}{n} x_0^n - \frac{1}{2\sqrt{n}} \overline{\sigma}$$

where the last inequality follows from an application of Cauchy-Schwarz and  $var(\varepsilon_i) \le \overline{\sigma}^2$ . For an upper bound, recall from **LEMMA 9** that the per asset payoff to the intermediary is bounded above by  $(1-\delta)x^n_0/n$ , which has the same limit.

For case (ii), if the intermediary pools the assets the resulting payoff is  $\Gamma^*(\sum_i X_i/n; \sum_i x_{i0}/n, \sim \eta)$ . From **LEMMA 9**, this is equal to  $\Gamma^*(\sum_i (X_i - x_{i0})/n; 0, \sim \eta) + (1 - \delta)\sum_i x_{i0}/n$ . Next, by **LEMMA 7**,  $\Gamma^*$  is convex, so that

$$\Gamma^* \left( \frac{1}{n} \sum_{i=1}^n (X_i - x_{i0}); 0, \sim \eta \right) + (1 - \delta) \left( \frac{1}{n} \sum_{i=1}^n x_{i0} \right) \le \frac{1}{n} \sum_{i=1}^n \Gamma^* (X_i - x_{i0}; 0, \sim \eta) + (1 - \delta) x_{i0}$$

$$= \frac{1}{n} \sum_{i=1}^n \Gamma^* (X_i; x_{i0}, \sim \eta).$$

Hence, the payoff from the sales of separate tranches exceeds the payoff from tranching a single asset pool. Finally, case (iii) follows immediately from the convexity of  $\Gamma^*$ .

**PROOF OF LEMMA 10:** The expected profits of an uninformed bidder with bid p is given by  $E[Q^u(X-p)]$ . Since  $|Q^u(X-p)| \le |X-p|$ , dominated convergence implies  $E[Q^u(X-p)]$  is continuous in p. Thus  $P^*$  earns zero profits for the uninformed bidders, and is an equilibrium since any higher bid earns negative profits. Finally,  $E[Q^u(X-p)] \le 0$  implies

$$p \le E[X] + \frac{Cov(Q^u, X)}{E[Q^u]} \le E[X]$$

where the second inequality follows since  $Q^u$  is weakly decreasing in X, and the inequality is strict if  $Q^u$  is not constant.  $\bullet$ 

**PROOF OF THEOREM VI:** First note that  $\frac{1}{n}P^*\left[\sum_i X_i\right] = P^*\left[\frac{1}{n}\sum_i X_i\right]$ . For any  $\gamma \in (1,\beta)$ , let  $p_n = \beta^{-1}\gamma E\left[\frac{1}{n}X^n\right]$ . From the Schwarz inequality,

$$E[A|B] \ge E[A] - \sigma_A \sqrt{\frac{1}{\Pr(B)} - 1}$$
.

Since  $Var[X^n/n] = \frac{1}{n^2} \sum_{i=1}^n Var[X_i] \le \frac{1}{n} \overline{\sigma}^2$  this implies,

$$E\left[\frac{1}{n}X^{n}\left|\frac{1}{n}X^{n} \leq \beta p_{n}\right|\right] \geq E\left[\frac{1}{n}X^{n}\right] - \frac{1}{\sqrt{n}}\overline{\sigma}\sqrt{\frac{1}{\Pr\left(\frac{1}{n}X^{n} \leq \beta p_{n}\right)} - 1}.$$
 (23)

Since  $\gamma > 1$ , by the Weak Law of Large Numbers,

$$\Pr\left(\frac{1}{n}X^n \le \beta p_n\right) = \Pr\left(\frac{1}{n}\left(X^n - E\left[X^n\right]\right) \le (\gamma - 1)\frac{1}{n}E\left[X^n\right]\right) \to 1.$$

Thus, (23) implies  $E\left[\frac{1}{n}X^n \middle| \frac{1}{n}X^n \leq \beta p_n\right] \to x$ . Hence, for n sufficiently large, since  $p_n \to \beta^{-1} \gamma x < x$ ,

$$p_n \leq E\left[\frac{1}{n}X^n \middle| \frac{1}{n}X^n \leq \beta p_n\right] \leq \frac{E\left[Q^u \left(\frac{1}{n}X^n, p_n\right)X\right]}{E\left[Q^u \left(\frac{1}{n}X^n, p_n\right)\right]},$$

where the last inequality follows since  $Q^{u}(X/p,p) = 1[X \le \beta p] + 1[X > \beta p]Q^{u}(X/p,p)$ .

Thus, at price  $p_n$ , uninformed investors earn a non-negative profit. This implies that  $E\left[\frac{1}{n}X^n\right] \ge P^*\left[\frac{1}{n}X^n\right] \ge p_n$ , and therefore  $\lim_{n\to\infty} P^*\left[\frac{1}{n}X^n\right] \in (\beta^{-1}\gamma x, x)$ . Since this is true for all  $\gamma \in (1,\beta)$ , we have  $P^*\left[\frac{1}{n}X^n\right] \to x$ .

**PROOF OF LEMMA 11:** Differentiating (13) implies p decreases (increases) as  $x^c$  decreases for  $x^c > (<) p$ . Continuity of p in  $x^c$  follows from the continuity of p. Thus there exists  $p_0$  such that  $p_0 < p_0$  such that  $p_0 < p_0$ . Since the cash constraint in (12) is relaxed with an increase in  $p_0 < p_0$ , both  $p_0 < p_0$  strictly decrease with  $p_0 < p_0$ . This occurs for  $p_0 < p_0 < p$ 

**PROOF OF THEOREM VII:** Given the same initial cash,  $N_0$  is equal in both cases. However, since  $S^* > S^1$ , the intermediary has more cash at date 1 with pooling and tranching. If  $\delta_0 < 1$ ,  $x^c_1 > p_1$ , and so  $p_1$  is decreasing in the amount of cash held by the intermediary from **LEMMA 11**. Thus,  $N_1$  will be higher with pooling and tranching, and thus so will  $C_2 + O_2$ . Also, from **ASSUMPTION B**, the decrease in  $x^c_1$  implies that  $C_2$  will also be higher with pooling and tranching. Thus, on date 2 the intermediary has both higher cash and higher total assets with pooling and tranching. This argument can be repeated at each future date.  $\bullet$ 

## 9. References

Akerlof, G. (1970). The market for "lemons": Qualitative uncertainty and the market mechanism, *Quarterly Journal of Economics*, vol. 89, 488-500.

Allen, F., and Gale, D. (1989). Optimal security design, *Review of Financial Studies*, vol. 1, 229-263.

- Axelson, U. (1999). Pooling, splitting, and security design in the auctioning of financial assets, mimeo.
- Bernardo, A., and Cornell, B. (1997). The valuation of complex derivatives by major investment firms, *Journal of Finance*, vol. 52, 785-798.
- Boot, A., and Thakor, A. (1993). Security design, *Journal of Finance*, vol. 48, 1349-1378.
- Cho, I. K., and Kreps, D. (1987). Signaling games and stable equilibria. *Quarterly Journal of Economics*, vol. 102, 179-221.
- Cho, I. K., and Sobel, J. (1990). Strategic stability and uniqueness in signaling games, *Journal of Economic Theory*, vol. 50, 381-413.
- DeMarzo, P. (2003). Portfolio Liquidation and Security Design with Private Information, mimeo.
- DeMarzo, P., and Duffie, D. (1999). A liquidity based model of security design, *Econometrica*, vol. 67, 65-99.
- Diamond, D. (1984). Financial intermediation and delegated monitoring, *Review of Economic Studies*, vol. 51, 393-414.
- Diamond, D. (1993). Seniority and Maturity of Debt Contracts, Journal of Financial Economics, vol. 33, 341-368.
- Glaeser, E., and Kallal, H. (1997). Thin markets, asymmetric information, and mortgage-backed securities, *Journal of Financial Intermediation*, vol. 6, 64-86.
- Gorton, G., and Pennachi, G. (1990). Financial intermediaries and liquidity creation, *Journal of Finance*, vol. 45, 49-71.
- Gorton, G., and Pennachi, G. (1993). Security Baskets and Index-linked Securities, *Journal of Business*, vol. 66, 1-27.
- Leland, H., and Pyle, D. (1977). Information asymmetries, financial structure and financial intermediaries, *Journal of Finance*, vol. 32, 371-387.
- Mailath, G. (1987). Incentive compatibility in signaling games with a continuum of types, *Econometrica*, vol. 55, 1349-1366.
- Myers, S., and Majluf, N. (1984). Corporate Financing and Investment When Firms Have Information Shareholders Do Not Have, *Journal of Financial Economics*, vol. 13, 197-221.
- Milgrom, P., and Stokey, N. (1982). Information, trade, and common knowledge, *Journal of Economic Theory*, vol. 26, 17-27.

- Prékopa, A. (1973). On logarithmic concave measures and functions, *Acta Sci. Math.*, vol. 33, 335-343.
- Riddiough, T. (1997). Optimal design of asset-backed securities, *Journal of Financial Intermediation*, vol. 6, 121-152.
- Rock, K. (1986). Why new issues are under-priced in the IPO market, *Journal of Financial Economics*, vol. 15, 187-212.
- Subrahmanyam, A. (1991). A theory of trading in stock index futures, *Review of Financial Studies*, vol. 4, 17-51.
- Wallace, N. (2001). Heterogeneity in the Mortgage-Backed Securities Market: Effects on Investor Returns, mimeo.
- Winton, A. (1995). Costly State Verification and Multiple Investors: The Role of Seniority, *Review of Financial Studies*, vol. 8, 91-123.
- Winton, A. (2001). Institutional liquidity concerns and the structure of monitored finance, *Review of Financial Studies*, forthcoming.