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The posterity of Zadeh's 50-year-old paper — Source link

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The posterity of Zadeh's 50-year-old paper:

A retrospective in 101 Easy Pieces – and a Few More

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Abstract—This article was commissioned by the 22nd IEEE International Conference of Fuzzy Systems (FUZZ-IEEE) to celebrate the 50th Anniversary of Lotfi Zadeh's seminal 1965 paper on fuzzy sets. In addition to Lotfi's original paper, this note itemizes 100 citations of books and papers deemed "important (significant, seminal, etc.)" by 20 of the 21 living IEEE CIS Fuzzy Systems pioneers. Each of the 20 contributors supplied 5 citations, and Lotfi's paper makes the overall list a tidy 101, as in "Fuzzy Sets 101".

This note is *not* a survey in any real sense of the word, but the contributors did offer short remarks to indicate the reason for inclusion (e.g., historical, topical, seminal, etc.) of each citation. Citation statistics are easy to find and notoriously erroneous, so we refrain from reporting them – almost. The exception is that according to Google scholar on April 9, 2015, Lotfi's 1965 paper has been cited 55,479 times.

Keywords—fuzzy pattern recognition, fuzzy control fuzzy systems, fuzzy models, list of 101 fuzzy citations

I. WHAT WE TRIED TO DO

We begin with a number of disclaimers about what this article is, and is not. First of all, we recognize that any list such as this is completely arbitrary, probably biased, certainly subjective, and open to argument for any number of valid reasons. Our 101 list is presented in the same spirit as lists such as " the 10 best retirement cities in Europe," the "5 greatest guitar players of all time,", "the 20 best Australian beers," and so on, that are easily found in popular newspapers, magazines and websites. For example, a Google search for "10 best vacations" returned *About 128,000,000 results (0.51 seconds)* on October 15, 2014. The travel channel lists Cancun, London, Miami, Myrtle Beach, New York, Orlando, Paris, Rome, San Francisco as the top 10. National Geographic publishes a book titled "The 10 best of everything: The Ultimate Guide for Travellers." And so on. Recognizing the obvious, we have added some supplemental citations and additional remarks in the last section of this article to compensate for the obvious deficiencies of this – or indeed *any* - such list.

You may ask "the 101 best books and papers according to whom?" And in fact one of our pioneers *did* ask this very question, and refused to participate because he felt such lists were completely arbitrary and therefore entirely useless. Well, perhaps they are - *is there* any value to such a list at all? If you believe that history is important – that the way forward is in some sense better understood if presaged by an understanding of the road already travelled, our list may be helpful.

In the age of internet search, we know that all these references are at your fingertips – as long as you ask the right question or know what to look for. Our hope is that the citations given here encourage you to move in a direction you may not have been interested in before seeing them.

II. HOW THE LIST WAS BUILT

Table I lists the 23 pioneers, arranged in the chronological order in which they received the award.

2000	Lotfi Zadeh
2000	Michio Sugeno
2001	Jim Bezdek
2002	Didier Dubois
2002	Henri Prade
2003	Ebrahim J. Mamdani (D)
2004	Ronald Yager
2005	Enric Trillas
2006	Janusz Kacprzyk
2007	James M. Keller
2007	George Klir
2008	Jerry M. Mendel
2008	Takeshi Yamakawa
2009	Enrique H. Ruspini
2009	Tomohiro Takagi
2010	Hideo Tanaka (D)
2011	Hans J. Zimmermann
2012	Piero P. Bonissone
2012	Abraham Kandel
2013	Witold Pedrycz
2014	Masaharu Mizumoto
2015	Nikhil R. Pal
2015	Dimitar Filev

TABLE I. THE IEEE CIS FUZZY SYSTEMS PIONEERS: D~DECEASED

Here is our collection algorithm. Each of the 21 living pioneers was invited to submit up to five citations subject to these constraints: (i) no more than three self-citations; (ii) no more than one citation involving another pioneer; and (iii) at

least one citation for a non-pioneer. The response was hardly uniform! Indeed, Abe Kandel, one of the 2012 pioneers, refused to participate at all. Here is his statement of declination, reproduced verbatim from his email to us dated September 7. 2014: Abe wrote:

"I am very sorry but I will decline this invitation due to the following reasons: 1) The concept of an Important publication is not really well defined. Important to whom, the author? His friends ? His students ? World peace ? Applications to improve society ? Etc. 2) why as fuzzy logicians we select a binary number of 100 ? What about the paper in location 101 ? And why not 1000 or just 3 "most important"? 3) who made US [eds: "US" is not the USA here]- the fuzzy pioneers the "God of Fuzziness " to make these kinds of decisions ? Why not to include also other very good and promising researchers in the field. Just because we were on this bus does not imply anything as evaluators in this entirely fuzzy process. I think that we should all consider this Idea and not just spend 5 minutes as recently suggested."

And who's to say Abe is wrong? Some pioneers supplied five citations, some supplied less than five citations, and of course the two deceased pioneers supplied none. We exercised our editorial prerogative to fill in the empty slots. Some of the explanatory comments supplied to us were too long or seemed confusing, so in a few cases, we edited them for brevity and/or clarity. Finally, there is little value in knowing which pioneer suggested which citations, so that information is not reported here.

Section III contains the 101 citations and remarks, ordered alphabetically and within author, chronologically, by the last name of the first author. The references are given in a modified form of the standard IEEE format which we think is self-explanatory, brief, and enables alphabetization. Abbreviations for commonly occuring journals in the citations are listed in Table II.

TABLE II. ABBREVIATIONS USED IN THE 101 LIST

FSS	Fuzzy Sets and Systems
IJAR	International Jo. of Approximate Reasoning
IJGS	International Jo. of General Systems
IJIS	International Jo. of Intelligent Systems
IJMMS	International Jo. of Man-Machine Studies
JMAA	Jo. Math Analysis and Applications
TC	IEEE Transactions on Computers
TCS	IEEE Transactions on Circuits and Systems
TEC	IEEE Transactions on Evolutionary Computation
TFS	IEEE Transactions on Fuzzy Systems
TNN	IEEE Transactions on Neural Networks
TPAMI	IEEE Transactions on Pattern Analysis and Machine Intelligence
TSMC	IEEE Transactions on Systems, Man and Cybernetics

III. THE 101 CITATIONS IN ALPHABETICAL ORDER

Our list of 101 begins with the root paper:

Zadeh, L. A., "Fuzzy sets," *Information and Control*, 8(3), 1965, 338-353.

There is not much we can say about this paper that has not already been said. Without it, there is no 101 list, and many of us would be herding cows, painting houses, riding motorcycles, drinking beer (ok, some of us would be doing that anyway) or playing guitars in seedy juke joints. So, on to the subsequent 100 papers and books supplied by the 20 pioneers.

[1] Atanassov, K. T., "Intuitionistic fuzzy sets," *FSS*, 20, 87-96, 1986.

In this paper Atanassov introduced his ideas about intuitionistic fuzzy sets to the fuzzy set community, and the basic definitions.

[2] Baldwin, J. F., "A new approach to approximate reasoning using fuzzy logic," *FSS*, 2, 1979, 309-325.

This is one of the first papers, that focused on the extension of fuzzy logic to approximate reasoning on the basis of logical considerations. In contrast to fuzzy control, Baldwin used human argumentation rather than the control of artificial systems (machines etc.). It is still computationally simple and efficient and eventually led to the development of the fuzzy computer language ***Fril***.

[3] Bellman, R. E. and L. A. Zadeh, "Decision-making in a fuzzy environment", *Management Sciences*, 17, 1970, 141-154.

Presumably the most influential paper in the entire fuzzy sets literature, this article provides a simple yet extremely powerful fuzzy setting for all kinds of decision problems. It has inspired research in fuzzy decision making, control, optimization, and in a multitude of problems in which a choice is to be made under fuzzy goals, conditions, intentions, etc.

[4] Bezdek, J. C. *Pattern Recognition with Fuzzy Objective Function Algorithms*, Plenum Press, 1981.

One of the first textbooks to present classical pattern recognition problems (clustering and classifier design) in the framework of fuzzy sets and models. Special emphasis on algorithms that use alternating optimization as a means for approximating solutions of fuzzy objective function problems.

[5] Bezdek, J. C., "On the relationship between neural networks, pattern recognition, and intelligence, *IJAR*, 6(2), 1992, 85-107.

Perhaps the first publication to define and use the term "Computational Intelligence," subsequently adopted by the *Neural Networks Council* (NNC). The NNC attached the term to its triumvirate of flagship conferences (WCCI), and eventually changed their name to the IEEE Computational Intelligence Society. For more information on the history of this term and its relationship to the Canadian journal *Computational Intelligence* published by Wiley, visit ieee-cis.sightworks.net/documents/History/Bezdek-eolss-CI-history.pdf.

[6] Bezdek, J. C. and Harris, J. D., "Fuzzy partitions and relations: an axiomatic basis for clustering," *FSS*, 1, 1978, 111-127.

This paper derives a hierarchy of fuzzy similarity relation spaces (FSRs) whose minimal member is the set of crisp equivalence relations, and whose maximal member is the set of $\max-\Delta$ transitive FSRs. A transformation of fuzzy partitions based on sum-min matrix multiplication is shown to induce a pseudo metric on the data.

[7] Bezdek, J. C. and R. J. Hathaway, "Clustering with relational c-means partitions from pairwise distance data, *Math. Modelling*, 9(6), 1987, 435-439.

This paper introduced the idea of relational duals for the hard and fuzzy c-means algorithms. It is the basis for the branch of soft clustering that includes possibilistic and non-Euclidean versions of relational c-means.

[8] Bonissone, P. "Soft computing: the convergence of emerging reasoning technologies", *Soft Computing*, 1(1), 1997, 6-18.

One of the first studies of Hybrid Soft Computing, jointly using fuzzy logic (FL), neural networks (NN) and genetic algorithms (GA). The paper presents several cases studies of hybridization of two or more soft computing techniques, such as the use of FL to control GAs and NNs parameters, the application of GAs to evolve NNs topologies or weights, or to tune FL controllers, and the implementation of FL controllers as NNs tuned by back-propagation type algorithm. This paper has inspired many other subsequent works in hybrid soft computing.

[9] Bonissone, P. and K. Decker, "Selecting uncertainty calculi and granularity: An experiment in trading-off precision and complexity", in *Uncertainty in Artificial Intelligence*, L. Kanal, and J. Lemmer (Eds.), 217-247, North-Holland, 1986.

This paper is the first study of term sets granularity and triangular norms distinguishability. In the paper it is noted that, when using term sets typical for knowledge elicitation, many t-norms collapse into a small number of similarity classes. As a result, five t-norms are enough to cover most situations.

[10] Bosc, P. and O. Pivert, "SQLf: a relational database language for fuzzy querying," *TFS*, 3(1), 1995, pp. 1-17.

This paper describes how to extend well-known languages and algorithms for handling queries to relational databases, when queries involve preferences described in terms of fuzzy sets.

[11] Bouchon-Meunier, B., Rifqi, M. and S. Bothorel, "Towards general measures of comparison of objects," *FSS* 84 (2), 1996, pp. 143-153.

This paper is an extensive study on indices of similarity between fuzzy sets, that bridges the gap between the fuzzy set literature and the mathematical psychology literature on similarity.

[12] Buckles, B. P. and Petry, F. E., "A fuzzy representation of data for relational databases," *FSS*, 7(3), 1982, 213-226.

One of the earliest and most influential papers on the use of fuzzy sets and models in the context of relational databases.

[13] De Luca, A. and S. Termini, "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory", *Inf. and Control*, 20(4), 301-312, 1972.

One of the earliest papers to consider the concept of entropy, defined in the context of fuzzy information.

[14] Dubois, D. and H. Prade, "Operations on fuzzy numbers," *Int. J. Systems Science*, 9(6), 1978, pp. 613-626.

An influential paper in the arithmetic of fuzzy intervals, studying the four operations, as well as the maximum and the minimum. While the basic definitions were proposed by Zadeh and had been studied by some scholars in Japan and the United States, this paper proposed a parametric representation (LR-fuzzy numbers) of fuzzy intervals and showed how to compute practical results with it. It also proved a general shape-invariance result for the addition of fuzzy numbers.

[15] Dubois, D. and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, 1980.

This is the first extensive monograph describing the state of the art of the field after 15 years of fuzzy set research. It covers all aspects of the theory and its applications and contains a very extensive list of references on fuzzy sets at the time. Moreover it provides for the first time extensive accounts on topics such as the arithmetic of fuzzy intervals and fuzzy analysis, possiblity theory and its relation to the theory of evidence, fuzzy linear programming and fuzzy logic control.

[16] Dubois, D. and H. Prade, "*Possibility Theory – An Approach to Computerized Processing of Uncertainty*", New York, London, Plenum Press, 1988.

Possibility theory, independently outlined by the economist G. L. S. Shackle, and reintroduced on another basis by L. A. Zadeh (Fuzzy sets as a basis for a theory of possibility, *FSS* 1(1), 3-28, 1978), is an approach to the processing of epistemic uncertainty. This book describes and explains possibility theory from the underlying mathematics to database applications in a very concise and understandable way (with the collaboration of H. Farreny, R. Martin-Clouaire, and C. Testemale)

[17] Dubois, D. and H. Prade, "Rough fuzzy sets and fuzzy rough sets," *IJGS,* 17(2-3), 1990, pp. 191-209.

This paper shows that fuzzy sets and rough sets address different issues and are complementary. It applies for the first time the machinery of rough sets to fuzzy sets, thus yielding upper and lower fuzzy approximations, and replaces the equivalence relation underlying rough sets by a fuzzy similarity relation in the sense of Zadeh.

[18] Dubois, D., Lang, J. and H. Prade, "Possibilistic logic," in: *Handbook of Logic in Artificial Intelligence and Logic Programming*, D. M. Gabbay, C. J. Hogger, J. A. Robinson, D. Nute, eds., Oxford University Press, 3, 1994, pp. 439-513.

This paper defines an extension of classical logic to the case where propositions have various levels of certainty. It is based on the old principle that the validity of a reasoning chain is the validity of its weakest link. The model-theoretic semantics is in terms of fuzzy sets of models. This logic is inconsistency-tolerant. This work demonstrates a close connection between fuzzy sets and the literature on nonmonotonic reasoning and belief revision in artificial intelligence.

[19] Dunn, J. C., "A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters, *Jo. Cyber.*, 3(3), 1974, 32-57.

The first paper to derive a fuzzy version of the classical batch hard c-means (aka k-means) clustering model and alternating optimization algorithm, which was subsequently generalized as described in [4].

[20] Filev, D. "Fuzzy modeling of complex systems", *IJAR*, 5(3), 1991, 281-290.

This paper introduces state space and polytopic Takagi-Sugeno type models as alternative to the conventional dynamic state space models of nonlinear systems.

[21] Fodor, J. C. and M. R. Roubens, *Fuzzy Preference Modelling and Multicriteria Decision Support*, Springer Theory and Decision Library, 14, 1994.

This monograph achieved a breakthrough in the study of fuzzy relations meant to model the idea of preference, a topic pioneered by Sergei Orlowski in the 1970's. It relied on the state of the art in fuzzy aggregation operations, especially t-norms and co-norms, and applied it to the decomposition of a preference relation into its strict part, its equivalence part and its incomparability part. It shows the difficulty of carrying preference modeling techniques over to the valued case in the max-min setting, indicating the need for algebraic structures like MV-algebras.

[22] Goguen, J. A., "L-fuzzy sets," *JMAA*, 18, 1967, 145-174.

This paper laid bare the mathematical nature of fuzzy sets as mappings from a set to a complete lattice. It can be considered as the seminal work that motivated much of the mathematical literature on fuzzy sets.

[23] Goguen, J. A., "The logic of inexact concepts," *Synthese*, 19, 1968/69, 325-373.

This paper develops a remarkably sophisticated foundation for this new logic. The results provide the framework for both the development of Zadeh's agenda of fuzzy logic in the broad sense as well for the parallel development on the agenda of fuzzy logic in the narrow sense.

[24] Grabisch, M. and Labreuche, Ch. "Bi-capacities, Part I: definition, Möbius transformation and interaction", *FSS*, 151, 211-236, 2005.

Bi-capacities arise as a natural generalization of capacities (or fuzzy measures) in a context of decision making where underlying scales are bipolar. They are able to capture a wide variety of decision behaviours, encompassing models such as Kahneman and Tversky' Cumulative Prospect Theory. The paper extends all familiar notions used for fuzzy measures in this more general framework, and introduces the interaction index for bi-capacities, generalizing the Shapley value in a cooperative game theoretic perspective.

[25] Gustafson, D. E. and Kessel, W. C. "Fuzzy clustering with a fuzzy covariance matrix," *Proc. IEEE CDC*, 1979, 761-766.

This is the first fuzzy clustering model with an objective function that attempts to match cluster shapes by adapting the individual norm associated with each cluster. As alternating optimization proceeds, the norm associated with each cluster adapts to fit the local structure of the cluster via the fuzzy covariance matrix.

[26] Hajek, P. *Metamathematics of Fuzzy Logic*, Kluwer, Dordrecht, 1998.

This book is the culmination of seminal contributions by Peter Hajek to fuzzy logic in the narrow sense. The book is the first comprehensive axiomatic presentation of important fuzzy logics, each based on a distinct t-norm and its residuum, with the rigorous proofs that these fuzzy logics are both sound and complete. Contrary to other contributors to fuzzy logic in the narrow sense, Hajek has always considered advances in fuzzy logic in the broad sense as an important source of inspiration for research in fuzzy logic in the narrow sense.

[27] Hall, L. O.; Ozyurt, I. B. and J. C. Bezdek, "Clustering with a genetically optimized approach," *TEC,* 3(2), 1999, 103-112.

This paper introduces a new way to optimize a fuzzy objective function for clustering. The evolutionary approach is shown to provide partitions with a better optimized objective function value than the classical alternating optimization scheme.

[28] Herrera, F., Herrera-Viedma, E. and J. L. Verdegay. "A model of consensus in group decision making under linguistic assessments, *FSS*, 78 (1), 1996, 73-87.

This paper proposes the use of linguistic preferences to represent individuals' opinions, and a definition of fuzzy majority of consensus, represented by means of a linguistic quantifier. Several linguistic consensus degrees and linguistic distances are defined to indicate how far a group of individuals is from the maximum consensus, and how far each individual is from current consensus labels over the preferences.

[29] Inuiguchi, M. and Ramık, J. "Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problems, *FSS*, 111 (1), 2000, 3-28.

This survey reviews the application of possibility theory to fuzzy optimization, augmenting flexible constraints in fuzzy linear programming with uncertainty about coefficients, represented by fuzzy numbers. Then degrees of possibility and necessity of satisfying constraints can be used in the spirit of chance-constrained programming.

[30] Jang, J. S. R. "ANFIS: Adaptive-network-based-fuzzy-inference-system," *TSMC*, 23, 1993, 665–685.

This paper describes a very useful algorithm which is a staple in the Matlab™ Fuzzy Toolbox. The author presents a Takagi-Sugeno (TS) fuzzy system in network form and combines it with a back-propagation like learning algorithm to provide automated tuning of membership functions and polynomial coefficients. This algorithm enabled a large number of useful applications during the 1990s.

[31] Kacprzyk, J. Group decision making with a fuzzy majority, *FSS,* 18, 1986, 105-118.

Introduction of a fuzzy majority – equated with fuzzy linguistic quantifiers and dealt with in terms of a calculus of linguistically quantified propositions. One of the primary references for fuzzy models of group decisions, social choice, and voting schemes.

[32] Kacprzyk, J., *Multistage Fuzzy Control: A Model-Based Approach to Control and Decision-Making*, Wiley & Sons, 1997.

The first comprehensive coverage of multistage optimal fuzzy control, viz., fuzzy dynamic programming, for deterministic, stochastic and fuzzy systems under control. Real world applications include socio-economic regional development, and power systems planning.

[33] Kacprzyk, J., Zadrozny S., Linguistic database summaries and their protoforms: towards natural language based knowledge discovery tools. *Information Sciences*, 173 (4), 2005, 281-304.

This paper puts together an approach to linguistic summaries of databases after Yager and Zadeh's notion of protoform, in connection with the handling of queries in fuzzy databases.

[34] Kandel, A. *Fuzzy Techniques in Pattern Recognition*, John Wiley & Sons, New York, 1982.

One of the first comprehensive and pioneering treatises of the subject of pattern recognition in the framework of fuzzy sets. The fundamentals of fuzzy sets are discussed in the framework of constructive ways to use this technology to formulate and solve certain pattern recognition problems.

[35] Karnik, N., J. M. Mendel and Q. Liang, "Type-2 fuzzy logic systems," *TFS*, 7, 1999, 643-658.

This is a foundational paper that established many of the basic concepts in the field of type-2 fuzzy logic systems.

[36] Kasabov N. and Qun Song, "DENFIS: Dynamic Evolving Neural-Fuzzy Inference System and its application for time-series prediction," *TFS*, 10(2), 2002, 1-37.

This paper introduces a new type of fuzzy inference system, DENFIS, for adaptive on-line and off-line learning, and shows how to apply it to dynamic time series prediction.

[37] Kasabov, N., *Foundations of Neural Networks, Fuzzy Systems and Knowledge Engineering*, MIT Press, 1996.

This book provides an understandable approach to knowledge-based systems for problem solving by combining different methods of AI, fuzzy systems, and neural networks.

[38] Kaufmann A. *Introduction to the Theory of Fuzzy Subsets,* Academic Press, 1975.

This book is the English translation of the first monograph ever written (in French) on fuzzy set theory. It contains elementary definitions of fuzzy sets and related topics, covering the first papers by Zadeh, with special emphasis on max-min-transitive fuzzy similarity relations. This book is tutorial and contains many examples and exercises.

[39] Keller, J., and Hunt, D., "Incorporating fuzzy membership functions into the perceptron algorithm," *TPAMI*, 7(6), 1985, 693-699.

This paper develops a fuzzy perceptron model and algorithm that (unlike the classical crisp perceptron) terminates on non-linearly separable data sets. The article includes a proof of convergence for iterative optimization of the fuzzy perceptron objective function.

[40] Klement, E. P., Mesiar, R. and E. Pap "*Triangular Norms"*, Springer, 2000

This book gathers many mathematical results concerning fuzzy set connectives in an organized ways, with a stress on solving functional equations.

[41] Klir, G. J. and B. Yuan, "*Fuzzy Sets and Fuzzy Logic: Theory and Applications"*, Prentice-Hall, 1995.

A very complete and comprehensive textbook that covers the basic elements of fuzzy models from a mathematical point of view.

[42] Kosko, B. *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence*. Prentice Hall, 1992.

This book is of historical significance due to its important role in the genesis of neurofuzzy systems.

[43] Krishnapuram, R. and J. M. Keller, "A possibilistic approach to clustering", *TFS*, 1(2), 1993, 98-110.

This paper generalized (hard and fuzzy) c-means clustering by eliminating the constraint that the sum of cluster memberships for any object must equal 1. It also introduced the idea of a possibilistic partition as one consisting of typicalities.

[44] Kruse, R. and K. D. Meyer, *Statistics with Vague Data*, Springer, 1987.

This monograph develops an approach to fuzzy random variables originally proposed by Huibert Kwakernaak in the late 1970's. A fuzzy random variable is viewed as an ill-known random variable in contrast with the Madan Puri - Dan Ralescu approach.

[45] Lee, S. C. and E. T. Lee, "Fuzzy neural networks," *Math. Biosciences*, 23, 1975, 151-177.

This was the first paper to define the idea of a fuzzy neuron as a generalization of the McCulloch-Pitts neuron. Although cast in the more formal language of automata theory, it is the first paper about a fuzzy neural network.

[46] Lin, C. T. and George Lee, C. S. , "Neural-network-based fuzzy logic control and decision system," *TC*, 40(12), 1991, 1320-1336.

This paper introduces an innovative five-layer neural architecture for realizing a fuzzy rule based system for control and other decision making applications. It uses a hybrid learning scheme involving an unsupervised phase for defining the membership functions and a supervised phase for refining neuro-fuzzy system is proposed.

[47] Mamdani, E. H. and Assilian, S. "An experiment in linguistic synthesis with a fuzzy logic controller," *IJMMS*, 7, 1975, 1-13.

The starting point of fuzzy control whose continuation was a turning point for the acceptance of fuzzy logic in engineering.

[48] Marinos, P. N., "Fuzzy logic and its applications to switching systems," *TC*, 18(4), 1969, 343-348.

This is the first paper that presents a technique for analysis and synthesis of fuzzy logic functions with implementation in terms of logic gates. This paper led to the implementation of real fuzzy information processing hardware systems such as high-speed fuzzy logic controllers.

[49] Mendel, J. M., *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, Prentice-Hall, 2001.

This textbook offers comprehensive coverage of both type-1 and type-2 fuzzy sets and rule-based systems for singleton and non-singleton fuzzifications.

[50] Mendel, J. M. and R. John, "Type-2 fuzzy sets made simple," *TFS*, 10, 2002, 117-127.

This paper provides a representation theorem that shows a new way to represent a type-2 fuzzy set in terms of simpler embedded type-2 fuzzy sets.

[51] Mizumoto, M., "Fuzzy controls by product-sum-gravity method dealing with fuzzy rules of emphatic and suppressive types," *Int. Jo. of Uncertainty, Fuzziness and Knowledge-Based Systems,* 2(3), 1994, 305-319.

This paper shows that emphatic or suppressive effects on fuzzy inference results are observed under the product-sum-gravity method by using fuzzy control rules whose consequent part is characterized by a membership function whose grades are greater than 1, or a negative-valued membership function. The use of negative-valued membership functions is beneficial to the construction of fuzzy control rules.

[52] Mizumoto, M. and Tanaka, K., "Some properties of fuzzy sets of type 2," *Inf. and Control*, 31(4), 1976, 312-340.

This paper investigates the algebraic structure of Type 2 fuzzy sets under set operations defined by means of the extension principle on fuzzy numbers on the unit intervals serving as fuzzy membership grades.

[53] Murofushi. T and Sugeno, M. “An interpretation of fuzzy measures and the Choquet integral as an integral with respect to a fuzzy measure”, *FSS*, 29, 1989, 201-227.

This paper first showed with concrete examples that (1) a non-additive measure (capacity in the sense of Choquet or fuzzy measure in the sense of Sugeno) represents interactions among subsets and (2) the Choquet integral is a reasonable integral with respect to such a non-additive measure.

[54] Negoita, C. V. and Ralescu, D. A., *Application of Fuzzy Sets to Systems Analysis,* Wiley, 1975.

This is the first book written on the basics of fuzzy sets, fuzzy theories (categories, topologies, etc.) and fuzzy logic, and also their possible applications to systems, automata, clustering, etc.

[55] Nguyen H. T. "A note on the extension principle for fuzzy sets," *JMAA*, 64, 1978, 369-380.

This pioneering paper describes the connection between the extension principle and the calculation of functions with set-valued arguments using alpha-cuts. It shows that fuzzy number calculations commute with cuts.

[56] Pal, N. R. and J. C. Bezdek, "Measuring fuzzy uncertainty." *TFS*, 2(2), 1994, 107-118.

This paper introduces two new classes, additive and multiplicative classes, of measures of fuzziness, which satisfy the five axioms of such measures. This paper also introduces the concept of weighted fuzziness to incorporate subjectivity in measures of fuzziness.

[57] Pedrycz, W., "Algorithms of fuzzy clustering with partial supervision," *Pattern Recognition Letters*, 3, 1985, 13 - 20.

This paper introduced the concept of partial supervision for fuzzy clustering and proposed algorithms that used it to do clustering in presence of partially labeled data.

[58] Pedrycz, W., *Fuzzy Control and Fuzzy Systems*, John Wiley. 1991

This research monograph is one of the first comprehensive and innovative publications that focuses on fuzzy control and fuzzy systems within a framework of fuzzy relational equations.

[59] Puri, M. L. and D. A. Ralescu, "Fuzzy random variables," *JMAA*, 114 (2), 1986, pp. 409-422.

This seminal paper proposed a mathematical extension of the theory of random sets to fuzzy random sets, 10 years after pioneering but largely ignored works by Robert Féron. In this approach, a fuzzy random variable is viewed as a mapping from a probability space to a space of membership functions, equipped with a suitable metric structure. Since then many scholars have followed this line to handle random linguistic variables.

[60] Rodriguez, R. O., Esteva, F., Garcia, P. and Godo, L., "On implicative closure operators in approximate reasoning," *IJAR*, 33, 2003, 159-184.

This paper clarifies the notions of graded implication and, through the imposition of reasonable constraints, characterization of the nature of implication operators.

[61] Ruspini, E. H. ,“A new approach to clustering,” *Inform. Control*, 15(1), 1969, 22–32.

This is the first paper to define the notion of a fuzzy c-partition of data. As such, it is the root paper for the entire field of fuzzy clustering, which is now a very large part of the pattern recognition landscape.

[62] Ruspini, E. H., "On the semantics of fuzzy logic," *IJAR*, 5, 1991, 45-88.

This paper presents a formal characterization of the major concepts and constructs of fuzzy logic in terms of notions of distance, closeness, and similarity between pairs of possible worlds. The similarity logic developed in the paper allows a form of logical "'extrapolation'" between possible worlds. It is shown to have connections with possibility theory, in the setting of metric spaces.

[63] Ruspini, E. H., P. Bonissone, and W. Pedrycz, *Handbook of Fuzzy Computing*, Institute of Physics, 1998.

A handbook on fuzzy sets, systems, and applications that was state of the art in 1998. Co-edited by three fuzzy pioneers, it offers a coherent presentation and notation across multiple entries, which were written by a large number of other fuzzy pioneers.

[64] Saffiotti, A., Konolige, K., and Ruspini, E. H., "A multivalued logic approach to integrating planning and control," *Artificial Intelligence*, 76, 1981, 481-522.

This paper presents the first significant application of fuzzy logic methods to the planning and control of autonomous robots. This approach led to the SAPPHIRA architecture, which, until recently, was employed in many commercial autonomous mobile robots. The multilevel hierarchical, supervisor-controller, architecture introduced in this paper has been widely applied to other control systems.

[65] Sanchez, E. "Resolution of composite fuzzy relation equations," *Inf. and Control*, 30, 1976, 38-48.

This is the first, highly original and influential publication in the area of fuzzy relational equations. It is the root paper for a large ongoing research effort in relational theory.

[66] Seki, H. and Mizumoto, M., "On the equivalence conditions of fuzzy inference methods -part 1: Basic concept and definition," TFS, 19(6), 2011, 1097-1106.

This paper addresses equivalence conditions of a number of fuzzy inference methods such as the product-sum-gravity method, simplified fuzzy inference method, fuzzy singleton-type inference method, SIRMs inference method, and SIC inference method.

[67] Sugeno, M. *Theory of Fuzzy Integrals and Its Applications*, Ph.D. Thesis, Tokyo Institute of Technology, 1974.

Starting point of the fertile subject of fuzzy measures and the so-called Sugeno's integral. This paper introduced a family of measures (the lambda ones) that are either additive, or subadditive, or superadditive.

[68] Sugeno, M., and Yasukawa, T., "A fuzzy-logic-based approach to qualitative modeling. " *TFS*, 1(1), 1993, 7-31.

This paper proposes a two-step process, fuzzy modelling and its linguistic approximation, to generate qualitative models of systems based on input-output numerical data. Although the primary emphasis

of this is on qualitative modelling, it also introduces another very important concept, the use of clustering to find fuzzy rules from numerical data, which drastically reduces the complexity of fuzzy rule generation.

[69] Tahani, H. and J. Keller, "Information fusion in computer vision using the fuzzy integral", *TSMC*, 20(3), 1990, 733-741.

This was the first journal paper (preceded by 2 conference papers) that framed the pattern recognition problem in terms of fuzzy integral fusion of information.

[70] Takagi, T. and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control", *TSMC*, 15, 1985, 116-132.

This paper established a link between conventional and fuzzy systems models and paved the way for the use of machine learning and control theory in fuzzy systems.

[71] Tanaka, H., Uejima, S. and Asai, K., "Linear regression analysis with fuzzy model," *TSMC*, 12(6), 1982, 903-907.

This paper was the first to propose a study of *fuzzy linear regression* (FLR), by adding fuzziness to regression analysis. It considered parameter estimation of FLR models under two factors: (i) the degree of fit; and (ii) the vagueness of the model. This paper inspired many subsequent works in fuzzy linear regression models.

[72] Tanaka, K and Wang, H., *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, 2004.

This book offers a systematic approach to the analysis and synthesis of stable fuzzy control systems based on Takagi-Sugeno type models.

[73] Trillas, E. and Riera, T. "Entropies in finite fuzzy sets", *Inf. Sciences,* 15(2), 1978, 159-168.

This paper first studied fuzzy entropies which are different from a Shannon-type (employed by De Luca and Termini) and considered relations between entropies and fuzzy integrals.

[74] Trillas, E. and Valverde, L., "On mode and implication in approximate reasoning," In *Approximate Reasoning and Expert Systems* (M. M, Gupta, A. Kandel, W, Bandler, and J. B. Kiszka, eds.), 1985, 157-166.

Trillas and Valverde's paper clarifies the nature of fuzzy implication - a central concept in fuzzy logic - while producing representation theorems that clearly define the proper structure of implication operators.

[75] Valverde, L. , "On the structure of F-indistinguishability operators," *FSS*, 17, 1985, 313-328.

Valverde's paper on the structure of fuzzy similarities brought clarity, through a principled approach, to the structure of fuzzy preorders and fuzzy similarity equations. Furthermore, this paper clarified the relationship between the notions of fuzzy preference and fuzzy similarity.

[76] Wang, L-X., and Mendel, J. M., "Generating fuzzy rules by learning from examples," *TSMC*, 22(6), 1992, 1414-1427.

This paper proposes a useful scheme for generating a fuzzy rule based system from numerical data for function-approximation type systems. It also proves that such a fuzzy rule based system has the universal approximation capability, which can approximate any nonlinear continuous function on a compact set to an arbitrary accuracy.

[77] Wang, L.-X. , "Fuzzy systems are universal approximators," *Proc. FUZZ-IEEE*, 1992.

This paper provided the first rigorous proof that a Mamdani fuzzy logic system is a universal approximator. cf. E. P Klement, "Are fuzzy systems universal approximators? *IJGS*, 28(2/3), 1999, 259-282.

[78] Wang, X., De Baets, B. and E. Kerre. "A comparative study of similarity measures," *FSS*, 73 (2), 1995, 259-268.

A systematic study of the notion of similarity between fuzzy sets and the properties of such similarity indices. This is used as a basis for defining a notion of approximate equality between fuzzy sets.

[79] Yager, R. R., "On a general class of fuzzy connectives," *FSS*, 4, 1980, 235-242.

One of the earliest papers to provide a generalization of the union and intersection operators used in fuzzy sets.

[80] Yager, R. R., "A procedure for ordering fuzzy subsets of the unit interval,", *Inf. Sci*., 24, 1981, 143-161.

This early work deals with the issue of comparing fuzzy sets of the unit interval, with an approach compatible with fuzzy arithmetics.

[81] Yager, R.R., "A new approach to the summarization of data", *Inf. Sci.*, 28, 1982, 69-86.

A breakthrough paper that introduces the concept of a linguistic data summary, which is equated to a linguistically quantified proposition with a fuzzy linguistic quantifier. As opposed to linguistic summarization of data (previously known for many years), this scheme accounts for imprecision in data and enables us to grasp the very essence of data in a human consistent way.

[82] Yager, R.R., "On Ordered Weighted Averaging aggregation operators," *TSMC*, 18, 1988, 183-190.

Perhaps the central paper that fueled many subsequent studies of aggregation functions in fuzzy logic. OWA operators were, and are, widely used in various applications.

[83] Yager, R. R., "Quantifier guided aggregation using OWA operators, *IJIS*, 11, 1996, 49-73.

In this paper Yager provides an approach for going from a linguistic specification of an aggregation imperative to its manifestation in terms of an OWA operator. It gives us an example of the concept of computing with words applied to aggregation.

[84] Yager, R. R. and Filev, D. P., "*Essentials of Fuzzy Modeling and Control",* John Wiley, 1994.

A textbook containing a systematic approach to fuzzy models and control, methods for developing and learning fuzzy models from data, and their applications.

[85] Yamakawa, T., "High-speed fuzzy controller hardware system : The mega-FIPS machine," *Inf. Sci*., 45, 1988, 113-128.

This article describes a high-speed fuzzy controller hardware system which facilitates approximate reasoning at 1,000,000 FIPS (fuzzy

inferences per second). This was the first step in an approach to a fuzzy computer.

[86] Yamakawa, T., "A fuzzy inference engine in nonlinear analog mode and its application to a fuzzy logic control," *TNN*, 4(3), 1993, 496-522.

This is a tutorial on the utility of fuzzy systems that provides a broad scope overview of analog mode hardware.

[87] Yamakawa, T., "Silicon implementation of a fuzzy neuron," *TFS*, 4(4), 1996, 488-501.

This paper describes a fuzzy neuron chip which modifies an ordinary neuron model by fuzzy logic and facilitates high speed recognition (less than 0.5 microseconds) of handwritten characters.

[88] Zadeh, L. A., "Similarity relations and fuzzy orderings," *Inf. Sci*., 1971, 177-200.

The first paper that showed how to decompose a fuzzy similarity relation to discover cluster substructure in a partition tree on relational (usually dissimilarity) data. Also introduced the idea of transitive closures for fuzzy similarity relations.

[89] Zadeh, L. A., "Fuzzy logic and approximate reasoning," *Synthese* 30, 1975, 407-428.

This paper introduces two basic formalisms: fuzzy logic and approximate reasoning. Basically, fuzzy logic is a system of reasoning and computation in which the objects of reasoning and computation are classes with unsharp (fuzzy) boundaries. Fuzzy logic is much more than a logical system.

[90] Zadeh, L. A. "Outline of a new approach to the analysis of complex systems and decision processes", *TSMC*, 3(1), 1973, 28-44.

This paper introduces to the concepts of fuzzy systems, algorithms, models, and optimization from the perspective of conventional systems theory. It is the genesis of the fuzzy logic control literature.

[91] Zadeh, L. A. "The concept of a linguistic variable and its application to approximate aeasoning," Parts 1-3, *Inf. Sci*., p1: 8, 1975, 199-249; 1975, p2: 8, 301-357; 1976; p3: 9, 43-80.

This three part publication develops the definition, theory and applications of linguistic variables for use in approximate reasoning. It is a superb treatment of an integral component of all subsequent work in fuzzy logic, linguistic data processing, and computing with words.

[92] Zadeh, L. A., "A theory of approximate reasoning," in *Machine Intelligence,* 9, Hayes, J., Michie, D., and Mikulich, L. I., Eds., ed New York: Halstead Press, 1979, 149-194.

In this paper Zadeh very elegantly puts together many of his ideas on approximate reasoning in a wholistic framework. It provides the basis of much of Zadeh subsequent work on computing with words.

[93] Zadeh, L. A., "Fuzzy sets and information granularity," in Advances in Fuzzy Set Theory and Applications, eds. M. Gupta, R. Ragade and R. R. Yager, North Holland, 1979, 3-18.

This paper introduces the concept of granularity and relates it to information. Fuzzy granularity is a concept which is unique to fuzzy logic. The linguistic variable is a granular variable. A concept which is introduced in this paper is that of a fuzzy-set-value random variable, with fuzzy probabilities. This concept provides a basis for a generalization of the Dempster-Shafer Theory of Evidence.

[94] Zadeh, L. A., "Precisiation of meaning via translation into PRUF," In *Cognitive Constraints and Communication*, Vaina, L. and Hintica, J. (eds.), D. Reidel, Boston, 1984, 372-402.

This is the best paper that clearly and completely describes one of Lotfi Zadeh's greatest ideas - the one of precisiating the meaning of utterances in natural language by translating them into the meaning representation language PRUF. The language is based on a fuzzy-set interpretation of the theory of graded possibilities, whose expressive power is comparable to that of natural languages.

[95] Zadeh, L. A., "Fuzzy logic = computing with words," *TFS*, 2, 1996, 103-111.

Prof. Zadeh led the fuzzy community with innovative ideas that possessed deep insights. There were two phases: 1) propose fuzzy sets and their mathematical foundations and 2) propose of the idea "computing with words," which had significant value in expanding fuzzy logic from a scientific tool to the liberal arts. This was the first paper in that direction.

[96] Zadeh, L. A., "Generalized theory of uncertainty (GTU) - Principal concepts and ideas," *Comp. Stat. and Data Analysis*, 51, 2006, 15-46.

A basic premise in this paper is that there are many different kinds of uncertainty. The three principal kinds are possibilistic uncertainty, probabilistic uncertainty and bimodal uncertainty. GTU addresses the three principal kinds and others. GTU is a challenge to the Bayesian doctrine which posits that any kind of uncertainty can and should be dealt with through the use of probability theory. GTU has a unique capability--the capability to compute with probabilities, possibilities, events and relations which are described in natural language.

[97] Zadeh, L. A., "Towards a restriction-centered theory of truth and meaning (RCT), *Information Sciences,* 248, 2013, 1-14.

This paper is a radical departure from traditional approaches to representation of meaning and definition of truth. The meaning of a proposition is expressed as a restriction. A proposition is associated with two truth values: internal truth value and external truth value.

[98] Zimmermann, H.-J., "Fuzzy programming and linear programming with several objective functions". *FSS*, 1, 45-55, 1978.

This paper paved the way for many developments and applications in Operations Research. For example, classical linear programming requires crisp constraints that are often unrealistic. This paper showed how to soften the constraints, obtaining a more realistic model.

[99] Zimmermann, H.-J.. and Zysno, P., "Latent Connectives in Human Decision Making," *FSS*, 4, 1980, 37-51.

A paper published before t-norms and t-conorms were broadly used in fuzzy logic. The authors showed that fuzzy connectives cannot belong to universal classes, but should be contextually chosen. It also suggested the use of aggregation functions.

[100] Zimmermann, H.-J., "*Fuzzy Sets, Decision Making, and Expert Systems"*, Kluwer, 1987.

This book was one of the first texts that discusses how modeling with mathematics and empirical findings can be used to turn expert systems based on classical dichotomous logics into fuzzy expert systems.

IV. Discussion and Supplemental Reading

The 101 list was compiled using a very constrained method of sampling (i.e., only IEEE CIS pioneers were consulted). Consequently we feel justified in expanding the list a bit by adding some remarks and citations that might otherwise go unrecognized.

(i) Many important papers have been written in fields that are not directly germane to engineering applications. As you might expect, since our contributors are IEEE pioneers, this 101 list is heavily weighted towards the theory and applications in pattern recognition and control. However, there are very important papers in areas that might be called "pure mathematics, logic, philosophy, etc." such as topology, category theory, etc., that fall outside the natural interests of most members of a professional engineering society. Here are a few early citations, in chronological order, which were overlooked by our IEEE pioneers:

R. Lowen "A comparison of different compactness notions in fuzzy topological spaces," *JMAA*, 64, 1978, 446- 454.

S. Rodabaugh, "The Hausdorff separation axiom for fuzzy topological spaces," *Topology and its Applications*, 11, 1979, 225-233.

Pu Pao-Ming, Liu Ying-Ming, "Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence," *JMAA*, 76, 1980, 571-599.

A. Di Nola, A. G. S. Ventre, "On some chains of fuzzy sets," *FSS*, 4, 1980, 185-191.

S. Gottwald "Fuzzy propositional logics", *FSS*, 3, 1980,181-192.

E. P. Klement "Construction of fuzzy σ-algebras using triangular norms," *JMAA*, 85, 1982, 543-565.

U. Höhle, "Fuzzy measures as extensions of stochastic measures," *JMAA*, 92, 1983, 372-380.

M. Togai and H. Watanabe, "A VLSI implementation of a fuzzy inference engine: Toward an expert system on a chip,", Inf. Sciences, 38(2), 1986, 147-163.

P. Diamond, P. Kloeden "Metric spaces of fuzzy sets", *FSS*, 35(2), 1990, 241-249.

(ii) There are some general papers and books which are not specifically related to fuzzy sets that nevertheless had an important impact on many people within our community. A very few of them are listed here:

G. Choquet, "Theory of capacities", *Annales de l'Institut Fourier*, 5, 1953/54, 131-295.

B. Schweizer and A. Sklar. "Associative functions and abstract semi-groups", *Pub. Math. Debrecen*, 10, 1963, 69-81.

R. Moore "*Interval Analysis*", 1966, Prentice-Hall, Englewood Cliffs N.J..

R. O. Duda and Hart, P. E. "*Pattern Classification and Scene Analysis*, 1973, John Wiley and Sons, NY.

G. Shafer. "*A Mathematical Theory of Evidence*", 1976, Princeton University Press.

(iii) There are also some works that do not appear in the 101 list because they were not necessarily foundational (at least, for the 20 contributing IEEE pioneers). We want to mention three of them here, with historical footnotes of a sort, that explain in part why we wanted to include them.

W. G. Wee and K. S. Fu. "A formulation of fuzzy automata and its application as a model of learning systems," *IEEE Trans. Syst. Science and Cyberns,* 5(3), 1969, 215-223.

K. S. Fu was one of the really important "big guys" in the early history of fuzzy sets. The importance of his interest in the field at a time when it was quite embryonic and survival was a real issue cannot be overstated. He strongly encouraged the publication of the book [15]. He was also the first president of NAFIPS, the *North American Fuzzy Information Processing Society*, which in turn was the first professional society whose primary focus was fuzzy sets and models. As an example of his breadth of interest, this paper was a very early contribution to learning systems – now a hugely important field. Fu's student Bill Wee wrote the first PhD thesis on fuzzy pattern recognition, published just two years after Lotfi's 1965 paper.

A.Rosenfeld. "Fuzzy digital topology", *Information and Contro*l, 40, 1979, 76-87.

Azriel Rosenfeld was a second "big guy" who helped keep the wolves from Lotfi's door in the early days. Rosenfeld, his students, and some of his colleagues produced a number of early papers on fuzzy graph theory, fuzzy geometry, and the use of fuzzy models in image processing. This paper is an early example.

R.L.P. Chang and T. Pavlidis. "Fuzzy decision tree algorithms," *TSMC*, 7(1), 1977, 28-35.

Theodore Pavlidis was a third influential supporter who encouraged scientists and engineers to have an open mind about fuzzy sets. His stewardship of the *IEEE Transactions on Pattern Analysis and Machine Intelligence*, inherited from K. S. Fu, offered an important early repository for emerging research in various fuzzy disciplines. This paper of his about fuzzy decision trees was perhaps the first of its kind, but notice it appeared in another IEEE Transactions, *TSMC*, whose editor at that time was Andrew Sage, yet a fourth patron saint for early workers in fuzzy sets.

(iv) The above lists, mainly oriented towards papers by "IEEE pioneers", do not give much credit to the newer generation of fuzzy set researchers, active in the last twenty years. We could give yet another partially arbitrary list recognizing these newer papers and books, but refrain from doing it. Yet they fully belong to the posterity of Zadeh's 50-year-old paper. Many of them are named on the editorial boards of the numerous fuzzy sets and soft computing journals.

We conclude with this observation. Instead of the method of collection used here, we could have polled the past presidents of IFSA (the *International Fuzzy Systems Association*), all the editors of FSS, or only those researchers working in business, or for a government. Each poll would produce a somewhat different list. The intersection of our 101 list with any of these lists would in all likelihood not be empty. But, for example, a list of the 100 most cited references in fuzzy sets, would certainly not coincide with our 101 list either. But ... which citation engine? – each one would undoubtedly produce a slightly different set of rankings. Carrying this argument to its logical conclusion, there can obviously be an infinite number of lists, no two of which coincide. We can only hope that this list is of some value to readers and attendees at the 2015 FUZZ- IEEE.

That's all, Folks!