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Tobias Gybel Hovgaard, Kristian Edlund, John Bagterp Jørgensen

Abstract— Economic Model Predictive Control is a receding horizon controller that minimizes an economic objective function rather than a weighted least squares objective function as in Model Predictive Control (MPC). We use Economic MPC to operate a portfolio of power generators and consumers such that the cost of producing the required power is minimized. The power generators are controllable power generators such as combined heat and power generators (CHP), coal and gas fired power generators, as well as a significant share of uncontrollable power generators such as parks of wind turbines. In addition, some of the power consumers are controllable. In this paper, the controllable power consumers are exemplified by large cold rooms or aggregations of super markets with refrigeration systems. We formulate the Economic MPC as a linear program. By simulation, we demonstrate the performance of Economic MPC for a small conceptual example.

I. INTRODUCTION

The United States' and Europe's development for future intelligent electricity grid is called GridWise and SmartGrid, respectively. GridWise and SmartGrid are intended to be the smart electrical infrastructure required to increase the amount of green energy (solar and wind) significantly. To obtain an increasing amount of electricity from intermittent energy sources such as solar and wind, we must not only control the production of electricity but also the consumption of electricity in an efficient, agile and proactive manner. In contrast to the current rather centralized power generation system, the future electricity grid is going to be a network of a very large number of independent power generators. To address such problems there has been an increasing interest in hierarchical and distributed control [1].

In this paper we introduce Economic MPC to control a number of independent dynamic systems that must collaborate to minimize the overall cost in satisfying the cooling demand for some goods. Power producing companies must minimize the cost of producing enough power to meet the market demand and respect their contracts with transmission system operators. Minimizing the cost of operation and providing supply security, becomes increasingly difficult as a larger share of intermittent stochastic power generating sources such as solar and wind are introduced in the power system. To balance demand and supply of electricity in a flexible and cost efficient manner, we consider using large power consumers such as cold rooms to adjust the power

demand profile to the power supply. Due to the large thermal capacity of cold rooms, they can to some degree shift the consumption of electricity to periods of the day at which there is a surplus production capacity. The thermal capacity in the refrigerated goods can be utilized to store "coldness" such that the refrigeration system can cool extra when the energy is free (i.e. there is an over production from the generators). Thereby a lower than normally required cooling capacity can be applied later, for a period of time when the energy prices are above zero again. The demands to the temperature in the cold room are not violated at any time since the same total cooling capacity is applied though shifted in a more optimal way. We exploit that the dynamics of the temperature in the cold room are rather slow while the power consumption can be changed rapidly. This, of course, imposes a constraint on the time constant of the temperature in the cold room. If e.g. no goods are loaded into the cold room the dynamics will be must faster reducing the positive effects gained from load shifting.

Our control strategy is an economic optimizing model predictive controller, Economic MPC. Model Predictive Control (MPC) for constrained systems has emerged during the last 30 years as the most successful methodology for control of industrial processes [2]–[4]. MPC is increasingly being considered for refrigeration systems [5]–[7] and for power production plants [8], [9]. Traditionally, MPC is designed using objective functions penalizing deviations from a given set-point. MPC based on optimizing economic objectives has only recently emerged as a general methodology with efficient numerical implementations and provable stability properties [10]–[13]. The idea of utilizing load shifting capabilities to reduce total energy consumption is slowly gaining acceptance (see [14], [15]). However in this paper it is assumed that both power plants and refrigeration systems are owned by the same stakeholder since we are trying to optimize the combined operation.

This paper is organized as follows. Section II introduces Economic MPC. Section III describes the models used for our case study, and the results are provided in Section IV. We give conclusions in Section V.

II. ECONOMIC MPC FOR LINEAR SYSTEMS

In this section we describe the Economic Model Predictive Controller (MPC) for linear systems. The Economic MPC minimizes an economic cost directly as opposed to minimizing the deviation from a set-point in some norm. We consider continuous variables only and the resulting optimal control problem is formulated as a linear program. The solution of this program is implemented on the system in a receding

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horizon manner. The Economic MPC is implemented for a linear distributed system with independent dynamics that must collaborate to meet a common goal.

A. Centralized System

The linear system in continuous time may be represented as

$$Y(s) = G_{yu}(s)U(s) + G_{yd}(s)D(s) \quad (1a)$$

$$Z(s) = G_{zu}(s)U(s) + G_{zd}(s)D(s) \quad (1b)$$

in which the transfer functions are multi-input-multi-output. $U \in \mathbb{C}^{n_u}$ is the manipulable variables, $D \in \mathbb{C}^{n_d}$ is known disturbances, $Y \in \mathbb{C}^{n_y}$ is the outputs associated with a cost, and $Z \in \mathbb{C}^{n_z}$ is the outputs associated with output constraints. G_{yu}, G_{yd}, G_{zu}, and G_{zd} are transfer function matrices of compatible size. Using a zero-order-hold discretization of the inputs, $u(t)$ and $d(t)$, that are related to $U(s)$ and $D(s)$, (1) may be represented as the discrete-time state space model

$$x_{k+1} = Ax_k + Bu_k + Ed_k \quad (2a)$$

$$y_k = Cx_k + Du_k + Fd_k \quad (2b)$$

$$z_k = C_z x_k + D_z u_k + F_z d_k \quad (2c)$$

Using this linear model we may formulate the Economic MPC as the linear program

$$\min_{\{x,u,y,z\}} \quad \phi = \sum_{k \in \mathcal{T}} c'_y y_k + c'_u u_k \quad (3a)$$

$$\text{s.t.} \quad x_{k+1} = Ax_k + Bu_k + Ed_k \quad k \in \mathcal{T} \quad (3b)$$

$$y_k = Cx_k + Du_k + Fd_k \quad k \in \mathcal{T} \quad (3c)$$

$$z_k = C_z x_k + D_z u_k + F_z d_k \quad k \in \mathcal{T} \quad (3d)$$

$$u_{\min} \le u_k \le u_{\max} \quad k \in \mathcal{T} \quad (3e)$$

$$\Delta u_{\min} \le \Delta u_k \le \Delta u_{\max} \quad k \in \mathcal{T} \quad (3f)$$

$$z_{\min} \le z_k \le z_{\max} \quad k \in \mathcal{T} \quad (3g)$$

with $\mathcal{T} \in \{0, 1, \ldots, N\}$. The cost of the Economic MPC is a linear function of the manipulable inputs, u_k, and the outputs, y_k. Typically, the cost is only dependent on the manipulable inputs, u_k, and $c_y = 0$. The manipulable inputs, u_k, are constrained by the input constraints (3e) and (3f). (3e) is a bound constraint on the inputs while (3f) is a constraint on the rate of movement ($\Delta u_k = u_k - u_{k-1}$). The outputs, z_k, are limited by the output constraints (3g). We assume that the Economic MPC (3) is feasible, i.e. that the initial state, x_0, and the disturbances, $\{d_k\}_{k=0}^N$, are such that the feasible manipulable variables, $\{u_k\}_{k=0}^N$, can bring the system to satisfy the output constraints (3g). If this is not the case, the output constraints must be formulated as soft constraints with a large penalty associated with violating the output limits, $z_{\min}$ and $z_{\max}$.

By state elimination, the Economic MPC (3) may be expressed as the linear program

$$\min_x \quad \psi = c'x \quad (4a)$$

$$\text{s.t.} \quad Ax \ge b \quad (4b)$$

and algorithms for linear programs (4) may be used for computing the solution of the Economic MPC.

B. Distributed Independent System

In this paper, we consider a distributed independent system

$$Y_i(s) = G_{yu,i}(s)U_i(s) + G_{yd,i}(s)D_i(s) \quad i \in \mathcal{P} \quad (5a)$$

$$Z_i(s) = G_{zu,i}(s)U_i(s) + G_{zd,i}(s)D_i(s) \quad i \in \mathcal{P} \quad (5b)$$

with $i \in \mathcal{P} = \{1, 2, \ldots, P\}$ being an index referring to each plant. The dynamically independent plants must collaborate to meet a common objective i.e. satisfy the market demand for the goods they produce. This representation may be related to (1) by $Y = [Y_1; Y_2; \ldots; Y_P]$, $Z = [Z_1; Z_2; \ldots; Z_P]$, $U = [U_1; U_2; \ldots; U_P]$, $D = [D_1; D_2; \ldots; D_P]$, $G_{yu}(s) = \text{diag}\{G_{yu,1}(s), G_{yu,2}(s), \ldots, G_{yu,P}(s)\}$, $G_{yd}(s) = \text{diag}\{G_{yd,1}(s), G_{yd,2}(s), \ldots, G_{yd,P}(s)\}$, $G_{zu}(s) = \text{diag}\{G_{zu,1}(s), G_{zu,2}(s), \ldots, G_{zu,P}(s)\}$, and $G_{zd}(s) = \text{diag}\{G_{zd,1}(s), G_{zd,2}(s), \ldots, G_{zd,P}(s)\}$. The representation (5) is useful because it may be used in Dantzig-Wolfe solution procedures for systems with a large number of plants, P [9], [16]. The set of plants, $\mathcal{P}$, consists of controllable producers (e.g. conventional power plants), $\mathcal{S}_C$, non-controllable producers (e.g. farms of wind turbines), $\mathcal{S}_{NC}$, and controllable consumers (e.g. large industrial facilities or cooling houses as in this paper), $\mathcal{D}$. We denote the producing plants by $\mathcal{S} = \mathcal{S}_C \cup \mathcal{S}_{NC}$.

The plants must collaborate such that the supply of goods exceed the demand of goods at all times

$$\sum_{i \in \mathcal{S}} y_{i,k} \ge \sum_{i \in \mathcal{D}} y_{i,k} + r_k \quad k \in \mathcal{T} \quad (6)$$

r_k is the demand from non-controllable consumers at time $k \in \mathcal{T}$.

The optimal control problem defining the Economic MPC for (5) may be stated as the block-angular linear program:

$$\min_{\{x,u,y,z\}} \quad \phi = \sum_{i \in \mathcal{S}} \left(\sum_k c'_{u,i} u_{i,k} + c'_{y,i} y_{i,k} \right) \quad (7a)$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{S}} y_{i,k} - \sum_{i \in \mathcal{D}} y_{i,k} \ge r_k \quad (7b)$$

$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + E_i d_{i,k} \quad (7c)$$

$$y_{i,k} = C_i x_{i,k} + D_i u_{i,k} + F_i d_{i,k} \quad (7d)$$

$$z_{i,k} = C_{z,i} x_{i,k} + D_{z,i} u_{i,k} + F_{z,i} d_{i,k} \quad (7e)$$

$$u_{\min,i} \le u_{i,k} \le u_{\max,i} \quad (7f)$$

$$\Delta u_{\min,i} \le \Delta u_{i,k} \le \Delta u_{\max,i} \quad (7g)$$

$$z_{\min,i} \le z_{i,k} \le z_{\max,i} \quad (7h)$$

with $i \in \mathcal{P}$ and $k \in \mathcal{T}$. The objective function (7a) says that the total cost of production from all the power plants in the time horizon considered must be minimal. (7b) couples the independent plants by requiring that the supply exceeds the demand. (7c)-(7e) is a discrete-time state space realization of (5). (7f) and (7g) constitute the input constraints. The output constraints are represented by (7h).

The supply-demand constraint (7b) and the output constraints (7h) may not be feasible for every disturbance and initial state scenario. In such situations (7) may be modified to a feasible linear program by representing (7b) and (7h) as soft constraints with large constraint violation penalties.

The Economic MPC (7) may be expressed as the block-angular linear program

$$\min_{\{x_i\}_{i \in \mathcal{P}}} \quad \psi = \sum_{i \in \mathcal{P}} c_i' x_i \qquad (8a)$$

$$s.t. \qquad \sum_{i \in \mathcal{P}} A_i x_i \geq b \qquad (8b)$$

$$B_i x_i \geq d_i \quad i \in \mathcal{P} \qquad (8c)$$

which may be solved efficiently using Dantzig-Wolfe decomposition. (8) is an instance of a linear program (4) with

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_P \end{bmatrix} \quad c = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_P \end{bmatrix} \quad A = \begin{bmatrix} A_1 & A_2 & \dots & A_P \\ B_1 & & & \\ & B_2 & & \\ & & \ddots & \\ & & & B_P \end{bmatrix} \quad b = \begin{bmatrix} b \\ d_1 \\ d_2 \\ \vdots \\ d_P \end{bmatrix}$$

C. Linear Programs and Control

The optimum of a linear program is an extreme point as illustrated in Fig. 1. This property of linear programs leads to either dead-beat or idle control when linear programs are used for solving model predictive control problems with an ℓ_1-penalty [17]. For Economic MPC the fact that the optimum is an extreme point implies that even small perturbations in the data or the disturbances may change the optimal solution dramatically. In practice, to handle this situation one often backs off a bit from the boundaries of the feasible region to leave some room for robustness. For the purpose of revealing the potential of Economic optimizing MPC for the combined control of both energy producing and consuming plants, we will use the Economic MPC in its basic form as described above.
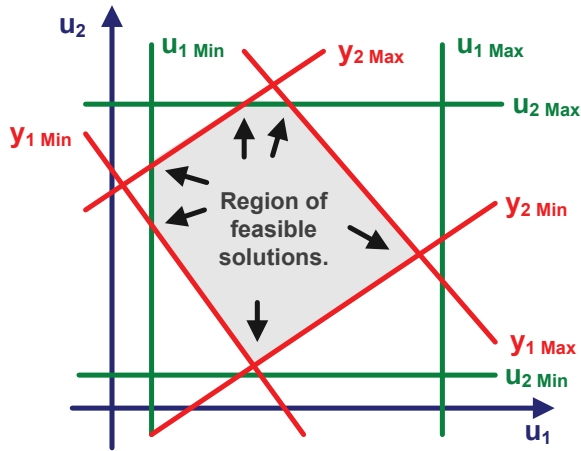

Fig. 1. Example of LP with two inputs and two outputs. Boundaries of the feasible region are illustrated with green for input constraints and red for output constraints. The arrows indicate possible optimal solutions which are always found at one of the vertexes depending on the objective function.

III. MODELS FOR A POWER SYSTEM

The case study used in this paper includes two controllable power generators and one power consumer. The power consumer is a cold room for which we provide a simple model. This case study is used to illustrate the properties and potential of Economic MPC in managing the power production and consumption in a distributed energy system. Compared to the studies in [8], [9], [16], the novelty in this paper is inclusion of a controllable power consumer to shed the power load.

A. Controllable Power Generators

[18] provides simple models for power generators. In this paper we used the models of the form

$$\phi_i = \sum_{k \in \mathcal{T}} c_i' u_{i,k} \qquad (9a)$$

$$Y_i(s) = G_i(s) U_i(s) \qquad G_i(s) = \frac{1}{(\tau_i s + 1)^3} \qquad (9b)$$

$$u_{\min,i} \leq u_{i,k} \leq u_{\max,i} \qquad (9c)$$

$$\Delta u_{\min,i} \leq \Delta u_{i,k} \leq \Delta u_{\max,i} \qquad (9d)$$

to model two conventional power generators where u_i is the power set-point for the i-th generator. (9a) represents the costs of producing power from a given power generator. Power generator 1 is cheap and slow, $(c_1, \tau_1, u_{\min,1}, u_{\max,1}, \Delta u_{\min,1}, \Delta u_{\max,1}) = (1, 20, 0, 15, -1, 1)$. Power generator 2 is expensive and fast, $(c_2, \tau_2, u_{\min,2}, u_{\max,2}, \Delta u_{\min,2}, \Delta u_{\max,2}) = (2, 10, 0, 12, -3, 3)$. The model in Eq. (9) describes the closed-loop system with internal controllers and is therefore quite simple without the lower level complexity of the generators. The model has been validated versus experimental data at DONG Energy, Denmark.

B. Simple Cold Room

The energy balance for the cold room is

$$mc_p \frac{dT_{cr}}{dt} = Q_{load} - Q_e \qquad (10)$$

with

$$Q_{load} = (UA)_{amb-cr}(T_{amb} - T_{cr}) \qquad (11a)$$

$$Q_e = (UA)_{cr-e}(T_{cr} - T_e) \qquad (11b)$$

T_{cr} is the temperature in the cold room which must be kept within certain bounds, $T_{cr,\min} \leq T_{cr} \leq T_{cr,\max}$. T_e is the evaporation temperature of the refrigerant. It can be controlled by the compressor work and must satisfy $T_{cr} \geq T_e$. T_{amb} is the ambient temperature. UA is the heat transfer coefficient. m and c_p are the mass and the overall heat capacity of the refrigerated goods, respectively. The energy consumed by the refrigeration system is work performed by the compressors: $W_C = \eta Q_e$. η is the coefficient of performance. In this work η is assumed to be constant and

independent of the temperatures. Consequently

$$W_C(s) = \frac{a - bs}{\tau s + 1} T_e(s) + \frac{\alpha K_d}{\tau s + 1} T_{amb}(s) \qquad (12a)$$

$$T_{cr}(s) = \frac{K_u}{\tau s + 1} T_e(s) + \frac{K_d}{\tau s + 1} T_{amb}(s) \qquad (12b)$$

with $Y_3 = W_C$, $Z_3 = [T_{cr}; T_{cr} - T_e]$, $U_3 = T_e$, $D_3 = T_{amb}$. The parameters are

$$K_u = \frac{(UA)_{cr-e}}{(UA)_{cr-e} + (UA)_{amb-cr}} \qquad (13a)$$

$$K_d = \frac{(UA)_{amb-cr}}{(UA)_{cr-e} + (UA)_{amb-cr}} \qquad (13b)$$

$$\tau = \frac{mc_p}{(UA)_{cr-e} + (UA)_{amb-cr}} \qquad (13c)$$

$$\alpha = \eta(UA)_{cr-e} \qquad (13d)$$

$$a = \alpha(K_u - 1) \qquad (13e)$$

$$b = \alpha\tau \qquad (13f)$$

and the constraints are

$$T_{cr,min} \le T_{cr} \le T_{cr,max} \qquad (14a)$$

$$0 \le T_{cr} - T_e \le \infty \qquad (14b)$$

In addition to these constraints, we enforce the evaporation temperature (T_e) to be between specified limits and to respect some rate of change constraints. Therefore, the cooling system can be modeled in a form compatible with the Economic MPC for linear systems.

The model here is quite simplified, especially the assumption for (12). However the resulting dynamics are well suited for illustrating the conceptual case in this paper.

C. Supply and Demand

The production by the power generators, $y_{1,k} + y_{2,k}$, must exceed the demand for power by the cooling house and the other consumers

$$y_{1,k} + y_{2,k} \ge y_{3,k} + r_k \quad k \in \mathcal{T} \qquad (15)$$

We model farms of wind turbines as instantaneously changing systems and include the effect of their power production in the exogenous net power demand signal, r_k.

IV. RESULTS

The Economic Optimizing MPC as described above has been implemented in Matlab and simulations are presented in this section. Fig. 2 visualizes a simulation. In this scenario, the power demand from all other consumers than the cold room increases slowly, then stays at a steady state and eventually drops significantly. This sudden drop could for instance be seen as an increase in wind speed that changes the demand to the power generators drastically. The ambient temperature is assumed to be constant in this scenario.

If the cold room was a non-controllable load from the power producers' point of view but of course still had to consume as little power as possible then, intuitively, the evaporation temperature T_e would stabilize at a level sufficient for keeping the temperature T_{cr} just below

the upper constraint. Thus, with a constant load on the refrigeration system the power demand W_C that should be added to the reference r would simply be a constant over the entire scenario. Among other things, the result is that a great amount of surplus electricity is produced after the sudden drop in demand. However, when the cold room is considered a controllable consumer it is able to absorb the majority of this otherwise redundant energy, as seen in Fig. 2. This causes the temperature in the cold room to decrease from the upper constraint to the lowest feasible level. Due to the thermal capacity in the refrigerated goods the "pre-cooling" applied when the power is "free" makes it possible to entirely shut down the cooling and thereby limit power consumption at a time where the production cost has increased. Other positive effects can be noticed. A slight pre-cooling occurs up to time= 160 such that the refrigeration system can be shut off just before the power demand reaches its maximum, thereby limiting the overshoot in the production. Also at time= 275 it is seen how the power consumed by the refrigeration system momentarily goes to zero allowing the decrease in the slow power generator to be initiated earlier without causing an underproduction.

As mentioned the potential savings depend on the time constant and the temperature limits of the cold room and thereby its ability to store coldness. Fig. 3 is the result of running a series of simulations on both a system with the cold room made controllable by the power producer and one where it is non-controllable. The simulations are performed for a range of mc_p, i.e. different amounts of goods in the cold room but identical loads on the system, and the savings for each pair of simulations are calculated in percentages and plotted. As expected larger time constants entails larger savings. Furthermore the savings tend to go asymptotically towards some maximum value. The maximum is clearly dependent on the chosen scenario since the amount of "free" power available sets an upper limit on the potential savings.

Another possibility for utilizing the combined control scheme for controllable power producers and controllable consumers lies in the daily variations. For instance the outdoor temperature is usually higher, causing a higher load on the refrigeration system, during the day than it is at nighttime. Also power demands are known to vary over the day, e.g. due to industries and domestic users shutting down most of their consumption at night while the wind turbines are still producing roughly the same amount of energy. The potential savings by controlling some of the loads in a scenario with varying outdoor temperature and power demands are investigated in the two simulations seen in Fig. 4. In Fig. 4.(a)-(b) it is observed how the behavior of the refrigeration system is as expected when the cold room is non-controllable. When the outdoor temperature is high a lot of cooling has to be applied in order to keep the temperature in the cold room at the maximal limit. Unfortunately this coincides with a time where the demand from all other
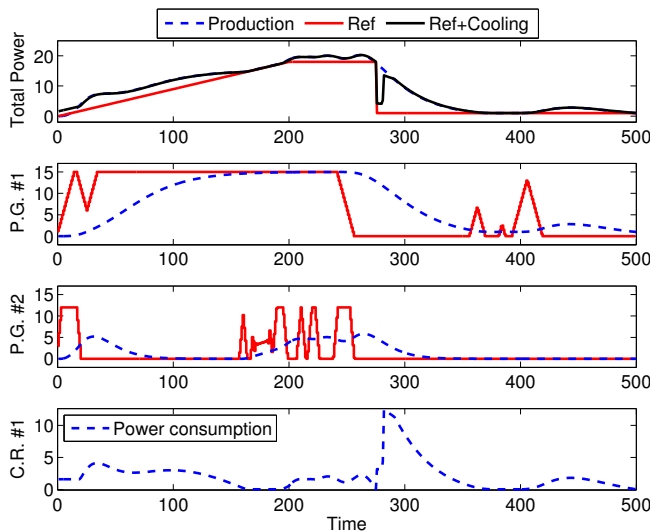
(a) Power productions / consumption. P.G. #1 and 2 show the power productions from the two power plants (dotted blue) and their power set-points (solid red). C.R. #1 is power consumption in the cold room and "Total Power" shows total power production (dotted blue) versus the reference consumption (with (solid black) and without (solid red) the consumption for refrigeration included.)
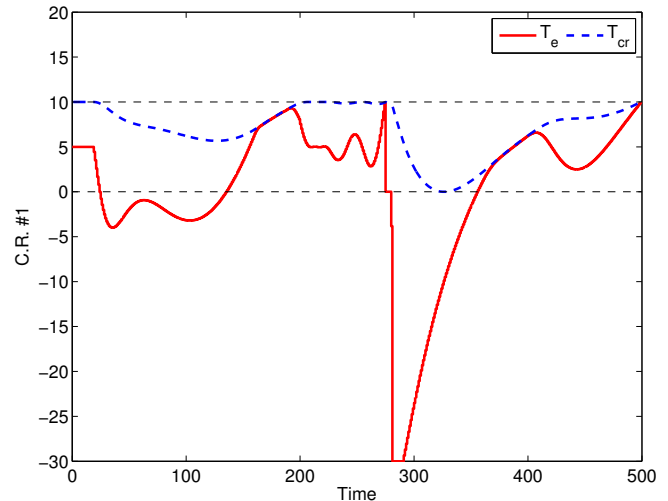
(b) Temperature in the cold room T_{cr} and the control signal for the refrigeration system T_e. $T_{cr,min}$ and $T_{cr,max}$ are shown with dotted black.

Fig. 2.   Simulation of Power Generation problem

consumers is high too, causing the needed cooling capacity to be rather expensive to deliver. If we instead take a look at Fig. 4.(c)-(d) an evaporation temperature trajectory that would have been hard to come up with by intuition is seen. The system now uses the ability to pre-cool when excess power is available and thereby saves a lot of power by reducing the cooling capacity when the energy is in high demand. The temperature of the cold room is varying between the maximal limit and almost down to the lower constraint. In this particular scenario the savings amount to 17 % for a system with $mc_p = 60$.
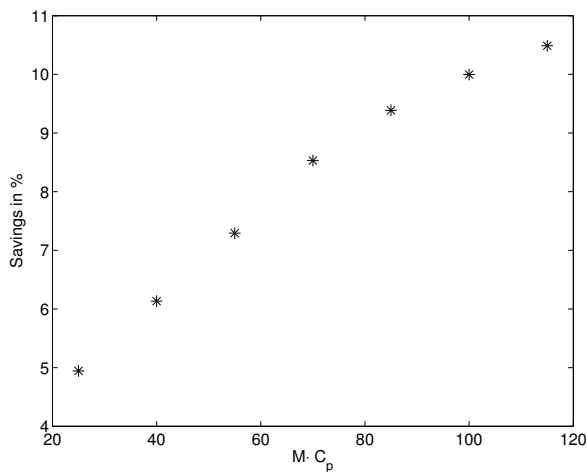

Fig. 3.   Savings compared to non-controllable load for different values of mc_p

V. Conclusion

We have presented Economic MPC and demonstrated its use on a conceptual example with a portfolio of power producers (power generators) and a power consumer (a cold room). Economic MPC provides the most cost efficient production plan to make supply exceed demand while observing plant limitations. For the conceptual example used in this paper, Economic MPC can utilize the thermal capacity in the cold room such that significant cost savings are obtained. The purpose of this paper was to present the concept of Economic MPC for a set of independent dynamic systems that must be coordinated to minimize a common objective and motivate this type of controllers in energy systems engineering. Future extensions include demonstration of Economic MPC for large scale systems using Dantzig-Wolfe decomposition.

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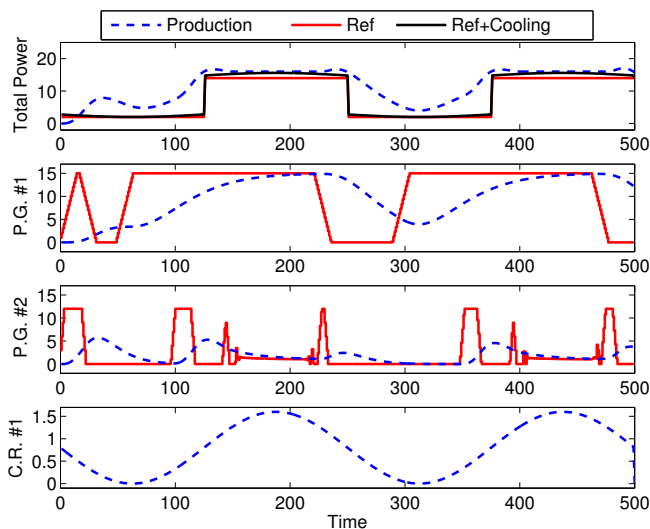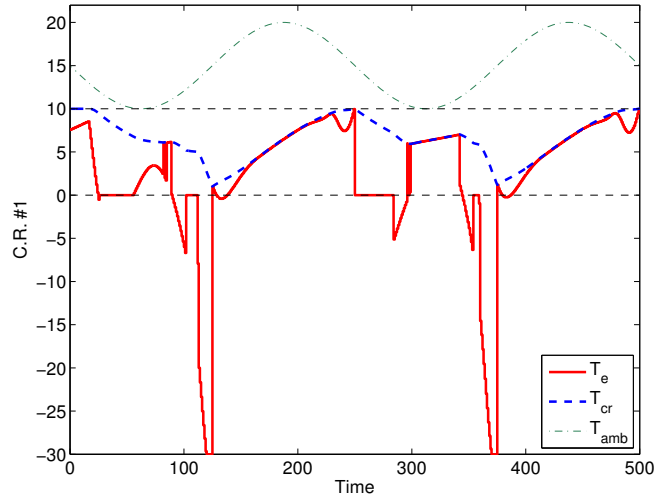
(a) Power productions / consumption. Non-controllable load.
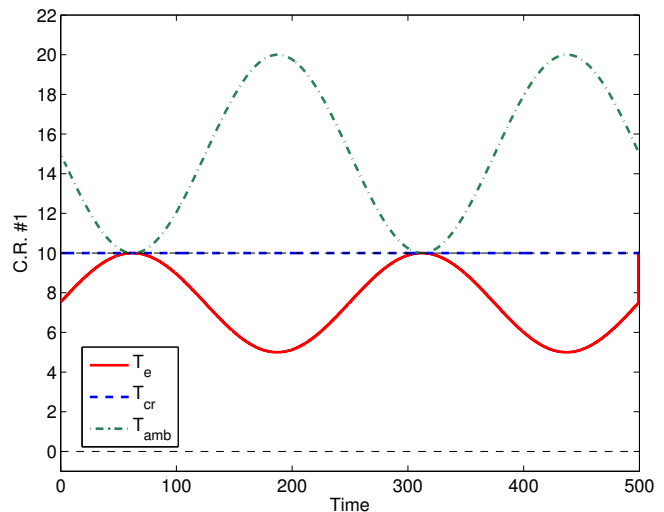
(b) Temperatures. Non-controllable load. $T_{cr,min}$ and $T_{cr,max}$ with dotted black.
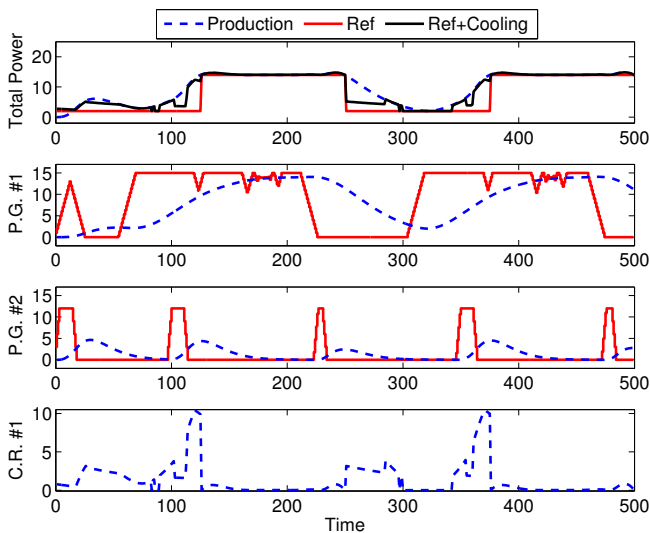
(c) Power productions / consumption. Controllable load.

(d) Temperatures. Controllable load. $T_{cr,min}$ and $T_{cr,max}$ with dotted black.

Fig. 4. Simulation of Power Generation problem with varying outdoor temperature and night/day power usage. For Fig. (a) and (c) P.G. #1 and 2 show the power productions from the two power plants (dotted blue) and their power set-points (solid red). C.R. #1 is power consumption in the cold room and "Total Power" shows total power production (dotted blue) versus the reference consumption (with (solid black) and without (solid red) the consumption for refrigeration included. Fig. (b) and (d) show the temperature in the cold room T_{cr} and the control signal for the refrigeration system T_e.

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