

The power spectrum of a videosignal

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The power spectrum
of a videosignal

by
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November 1968

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Summary

The first part of this article consists of a derivation of the power spectrum of a video signal. The calculation starts from the "elementary" autocorrelation function of the brightness as a function of time as found when a picture is scanned along a straight line. The "total" autocorrelation function, resulting from the systematic way of scanning a T.V. picture, is then derived.

The typical power spectrum of such a video signal is related to this autocorrelation function and can be derived from it by applying the Wiener-Khintchine theorem. This process is described, special attention being paid to a power spectrum resulting from a fine-structured picture.

In the second main part it is proved that it is impossible to suppress the so-called "idle" parts or "gaps" in the spectrum, without loss of information. The idea of using these "gaps" in the spectrum in some way or another for the transport of additional information cannot therefore be realised without loss of information in the video-signal.

Introduction.

There are two different ways of calculating the spectrum of a video signal. One can analyse the brightness as a function of the position in the picture as a double Fourier series, which leads to a video signal that is also expressed in the form of a double Fourier series. This first method is followed for instance by the authors Mertz and Gray {1}.

One of the conclusions reached by Mertz and Gray is that the power of the video signal is concentrated around multiples of the line frequency. About 50% of the region half-way in between these maxima of power is "idle" and can be filtered out without affecting the reproduced picture. This conclusion is quoted by several authors {2} {6}.

A second method starts with the autocorrelation function of a picture {7}. The power spectrum is then calculated by applying the Wiener-Kintchine relation.

In this article the last mentioned method will be followed. This distinguishes itself from other similar methods by giving a simple mathematical relation between the autocorrelation function of the picture and the total autocorrelation function of the video signal.

This theory gives a clear insight into the relationship between the power spectrum and the structure of the particular scene given the method of scanning and the available bandwidth. This is demonstrated using the European 625-lines system as an example.

The elementary autocorrelation function.

The brightness in the scanning point of a picture, linearly scanned in a random direction with constant velocity is assumed to be a stationary process. Subtracting the mean value gives a brightness-function $b(t)$ with autocorrelation function $R_e(\tau)$.

$$R_e(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T b(t) \cdot b(t-\tau) \cdot dt \quad (1)$$

We will call $R_e(\tau)$ the "elementary" autocorrelation function as distinct from the total autocorrelation function resulting from the scanning along successive parallel lines.

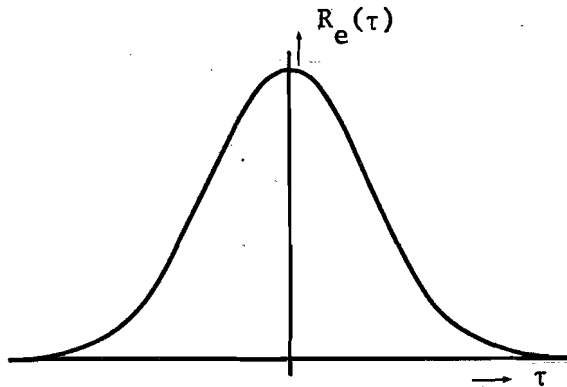


fig. 1

Although (1) assumes an infinitely extensive picture,

$$R_e(\tau) = \frac{1}{2T} \int_{-T}^T b(t) \cdot b(t-\tau) \cdot dt \quad (2)$$

will be a good approximation as long as T is sufficiently large.

It will be clear that the structure and the contrast of the picture are qualitatively related to the autocorrelation function in the following way :

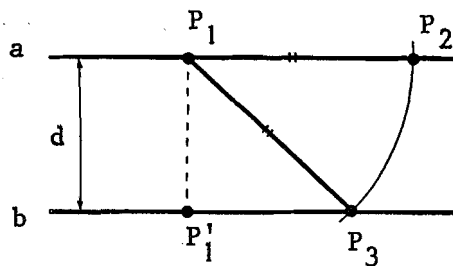
- a) If R_e diminishes rapidly with increasing τ , the picture will be of a fine structure and if R_e falls off slowly, the structure will be coarse.
- b) A high maximum of R_e corresponds with high contrast, a low maximum with low contrast.

The contributions to the total autocorrelation function caused by scanning the picture via parallel lines belonging to the same frame.

In the usual television systems, for instance the European system, the scanning path consists of a series of successive parallel lines.

Taking t_1 as the line duration, we assume that $R_e(\tau)$ is negligibly small for $|\tau| > \frac{1}{2} t_1$.

Except at the upper and lower marginal area of the picture, the scanning point will be in the same small picture region at the beginning and the end of a time interval t_1 . This gives a contribution to the total autocorrelation function, which can be derived in the following manner :



Imagine that the two lines a and b (fig. 2) are scanned successively. The time interval between the passage of the point P_1 on line a and the projection P_1' of P on line b equals the line duration t_1 . If τ is the time interval between the passage of P_1 and P_3 , the time interval between P_1' and P_3 equals $\tau - t_1$. If the distance P_1P_3 equals the distance P_1P_2 , the correlation between points P_1 and P_2 must be the same as the correlation between P_1 and P_3 , because $R_e(\tau)$ is assumed to be independent of the scanning direction. The resulting contribution to the autocorrelation function R_1^0 is obtained by substituting $\{\tau_d^2 + (\tau - t_1)^2\}^{\frac{1}{2}}$ for τ in the expression $R_e(\tau)$:

$$R_1^0 = R_e(\{\tau_d^2 + (\tau - t_1)^2\}^{\frac{1}{2}}) \quad (3)$$

with :

$$\tau_d = \frac{v}{d}$$

v = scanning speed

d = distance between successive lines.

The upper index (zero) of R_1^0 indicates that the contribution arises from scanning lines belonging to the same frame; the lower index (one) refers to one time interval t_1 . After a time interval nt_1 the scanning point arrives again in the same region, causing a contribution :

$$R_n^0 = R_e \left(\{ (n\tau_d)^2 + (\tau - n t_1)^2 \}^{\frac{1}{2}} \right) \quad (4)$$

The contributions caused by scanning the picture via lines belonging to different frames.

About one frame-duration (t_f) later the scanning point again makes repeated traverses of the same picture region, causing a new set of contributions to the total autocorrelation function.

The system followed to indicate the different contributions will be clear from fig. 3. The T.V. picture is assumed to be composed of two interlacing frames.

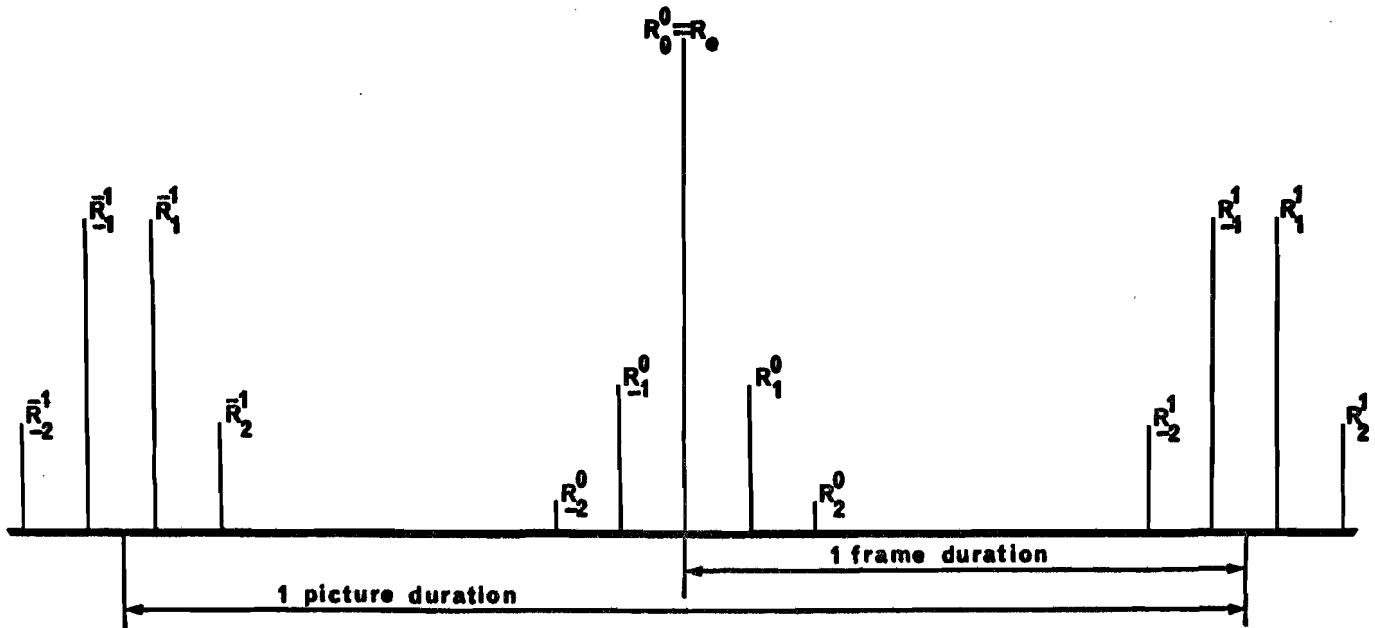


fig 3

With m the difference between the orders of the frames concerned, the time interval between the successive passages of the points P and P' (fig. 4) belonging to the contribution R_n^m equals :

$$mt_f + nt_1 \quad \text{for } m \text{ even} \quad (7)$$

$$mt_f + nt_1 - n \frac{t_1}{2|n|} \quad \text{for } m \text{ odd} \quad (8)$$

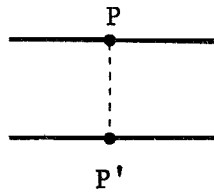


fig. 4

The corresponding contributions to the total autocorrelation functions are then given by :

$$R_n^m = R_e(\{(n\tau_d)^2 + (\tau - mt_f - nt_1)^2\}^{\frac{1}{2}}) \quad \text{for } m \text{ even} \quad (9)$$

$$R_n^m = R_e(\{(|n| - \frac{1}{2})^2 \cdot \tau_d^2 + (\tau - mt_f - nt_1 + n \frac{t_1}{2|n|})^2\}^{\frac{1}{2}}) \quad \text{for } m \text{ odd} \quad (10)$$

The power spectrum of the video signal.

The power spectrum of a signal is the Fourier transform of the autocorrelation function, according to the Wiener-Kintchine-relation.

Therefore the sum of the Fourier transforms of all the contributions R_n^m gives the total power spectrum.

The European 625-lines system as an example.

We will now apply the derived results to a video signal resulting from scanning according to the European system. This system has the following properties :

- line duration $t_1 = 64 \mu\text{sec.}$
- height of the picture h
- width of the picture $w \quad w = \frac{4}{3} h$
- 2 frames per picture
- frame duration $t_f = 20 \text{ msec.}$

There are about 294 lines per frame or 588 lines per picture. The remaining 37 lines are blanked during the frame fly-back and do not contribute to the transmission of information.

The picture-width w is scanned in $52.5 \mu\text{sec.}$; 18% of the line duration is used for synchronisation.

The scanning speed $v = \frac{w}{52.5}$ units of length per $\mu\text{sec.}$ (11)

$$\tau_d = \frac{3}{4} \cdot \frac{w}{294} \cdot \frac{52.5}{w} = 0.134 \mu\text{sec.} \quad (12)$$

The elementary autocorrelation function and the related power spectrum.

Imagine $R_e(\tau)$ to be given by (fig. 5) :

$$R_e(\tau) = R_o \epsilon^{-\left(\frac{\tau}{\tau_o}\right)^2} \quad (13)$$

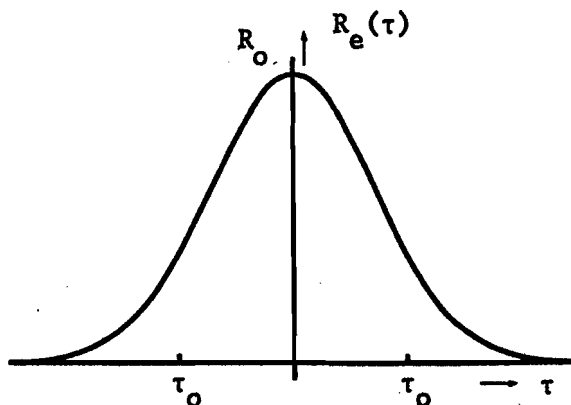


fig. 5

According to the relations stated before, R_o will be proportional to the contrast and τ_o will decrease as the structure of the picture becomes finer. Let us take τ_o on the one hand as small as possible, but on the other hand still so large that the major part of the power remains concentrated in the frequency band from 0 - 5 MHz.

With $\tau_o = 0.1$ μ sec. we find the related power spectrum (fig. 6) :

$$W_e(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_o \epsilon^{-\left(\frac{\tau}{0.1}\right)^2} \cdot \epsilon^{-j\omega\tau} d\tau = \frac{0.1 R_o}{2\sqrt{\pi}} \epsilon^{-(0.1 \pi f)^2} \quad (14)$$

(f in MHz)

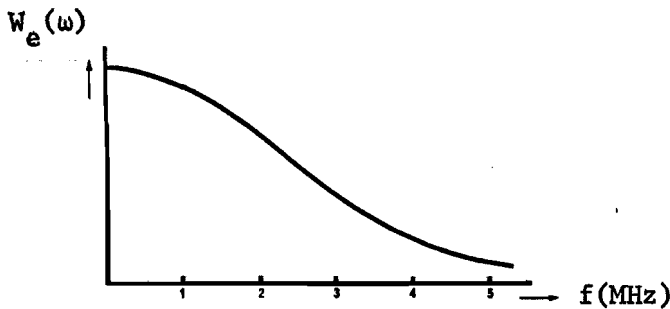


fig. 6

98% of the total power is concentrated in the frequency band from 0 - 5 MHz. The correlation now corresponds to the finest structure that can be transmitted with a 5 MHz system.

The total autocorrelation function.

Combining (9) and (13) gives R_n^m for m even :

$$R_n^m = R_o \epsilon^{-\left(\frac{n\tau_d}{\tau_o}\right)^2} \cdot \epsilon^{-\left(\frac{\tau - mt_f - nt_1}{\tau_o}\right)^2} \quad (15)$$

These contributions, resulting from lines in similar frames, are of the same shape as $R_e(\tau)$, but they are shifted by an amount $mt_f + nt_1$ in time and become a factor

$\epsilon^{-\left(\frac{n\tau_d}{\tau_o}\right)^2}$ smaller.

Combining (10) and (13) gives R_n^m for m odd :

$$R_n^m = R_o \epsilon \left[-\left(\frac{|n|\tau_d - \frac{1}{2}\tau_d}{\tau_o}\right)^2 - \left(\frac{\tau - mt_f - nt_1 + nt_1 \frac{1}{2|n|}}{\tau_o}\right)^2 \right] \quad (16)$$

The contributions (15) and (16) are drawn in fig. 7. In this figure, the τ -axis is divided into elements of 64 μ sec each, and these elements are then rearranged in the same manner as a T.V. picture is composed of scanning lines. Thus two successive parts of the τ -axis are placed parallel to each other, with a distance between them equalling the distance τ_d on the τ -axis. This kind of display offers two advantages :

1. The parts of the τ -axis where the autocorrelation function is practically zero can be omitted; the parts of the τ -axis where R_n^m is of importance can be expanded.
2. When $R_e(\tau)$ is known, every R_n^m can be constructed, according to the method shown in fig. 7.

Note : This construction is clearly general and not limited to the special example.

The contributions arising from similar frames are drawn with a full line, those resulting from dissimilar frames are represented by a broken line. For every even m only the contributions with $n = 0$ and $n = \pm 1$ have appreciable value. For m odd, there are only non negligible contributions if $n = \pm 1$. The total autocorrelation function resulting from a stationary, unchanging picture is a periodic function with a period of 40 msec. In our example one period is described by the five above mentioned non-negligible contributions.

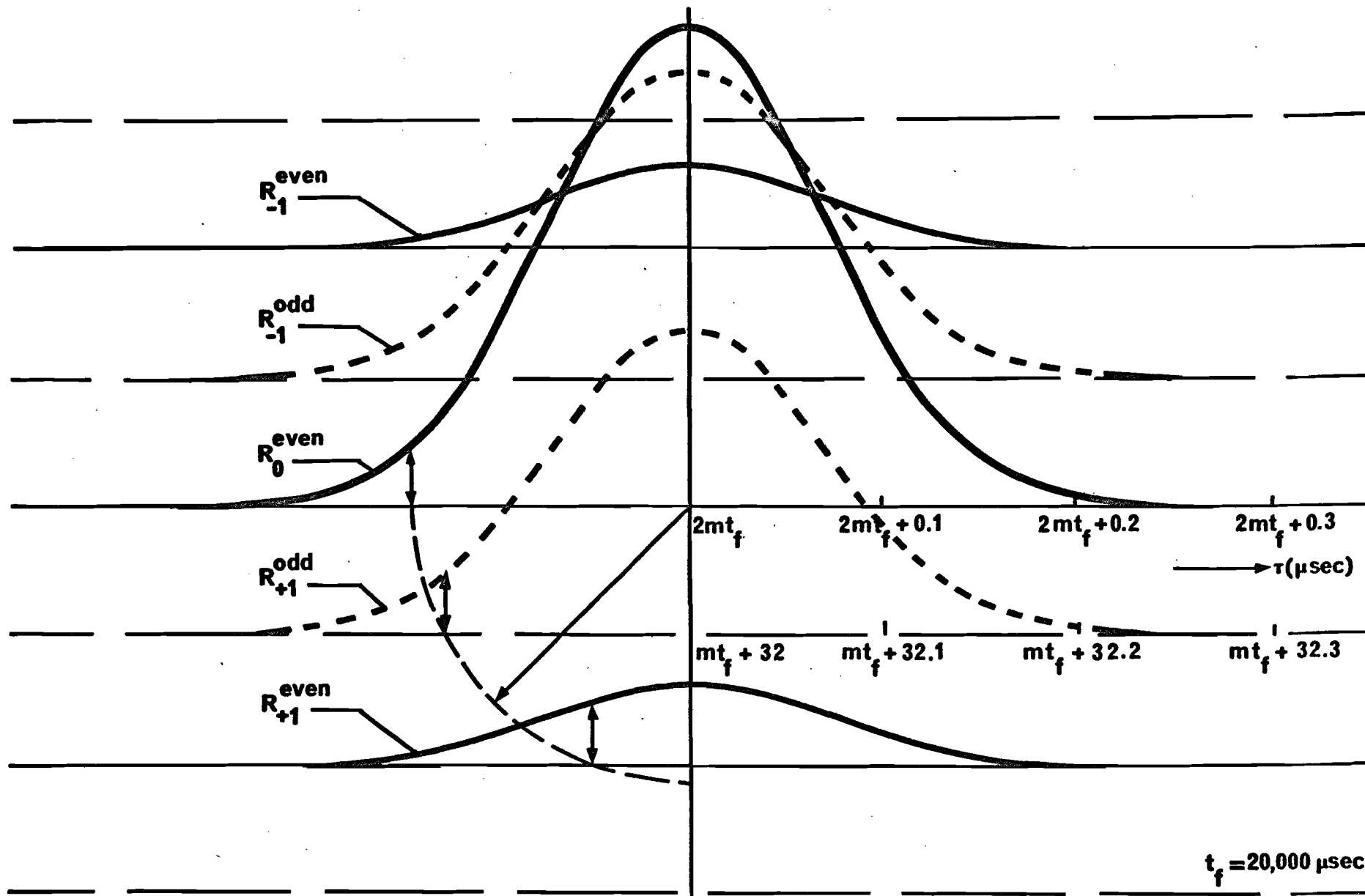


fig 7

The power spectrum.

To derive the power spectrum from the autocorrelation function, we first obtain the Fourier transform of the period of the autocorrelation function running from -20 msec tot +20 msec. This is then multiplied with an infinite row of delta-functionals arising every 25 Hz.

The five non-negligible contributions together with there Fourier transforms are given by (17) to (21) :

$$R_{+1}^{-1} = 0.638 R_o \epsilon^{-\left(\frac{\tau + 19968}{0.1}\right)^2} \quad o-o \quad 0.638 W_e(\omega) \cdot \epsilon^{j19968\omega} \quad (17)$$

$$R_{-1}^o = 0.166 R_o \epsilon^{-\left(\frac{\tau + 64}{0.1}\right)^2} \quad o-o \quad 0.166 W_e(\omega) \cdot \epsilon^{j64\omega} \quad (18)$$

$$R_e = R_o^o = R_o \epsilon^{-\left(\frac{\tau}{0.1}\right)^2} \quad o-o \quad W_e(\omega) = \frac{0.1 R_o}{2\sqrt{\pi}} \epsilon^{-(0.1 \pi f)^2} \quad (19)$$

$$R_{+1}^o = 0.166 R_o \epsilon^{-\left(\frac{\tau - 64}{0.1}\right)^2} \quad o-o \quad 0.166 W_e(\omega) \cdot \epsilon^{-j64\omega} \quad (20)$$

$$R_{-1}^{+1} = 0.638 R_o \epsilon^{-\left(\frac{\tau - 19968}{0.1}\right)^2} \quad o-o \quad 0.638 W_e(\omega) \cdot \epsilon^{-j19968\omega} \quad (21)$$

(τ in μsec)

$\omega = 2\pi f$ (f in MHz)

The total power spectrum is given by :

$$W(\omega) = W_e(\omega) (1 + 0.332 \cos 64\omega + 1.276 \cos 19968\omega) \sum_{k=-\infty}^{k=\infty} \delta(\omega - k\omega_o) \quad (22)$$

$$\omega_o = 50\pi \cdot 10^{-6} \text{ MHz}$$

Some parts of this spectrum are drawn in fig. 8.

The term $1.276 \cos 19968\omega$, with maxima at multiples of 50.08 Hz accomplishes, that the mean value of two neighbouring functionals are practically given by :

$$W_e(\omega) \{1 + 0.332 \cos 64\omega\} \quad (23)$$

(23) oscillates between the two values :

$1.332 W_e(\omega)$ and $0.668 W_e(\omega)$ as illustrated in fig. 9, with maxima at multiples of 15.625 kHz. With the time scale as in fig. 9a, these maxima would be spaced about 0.7 mm.

Therefore some parts of fig. 9a are expanded in fig. 9b.

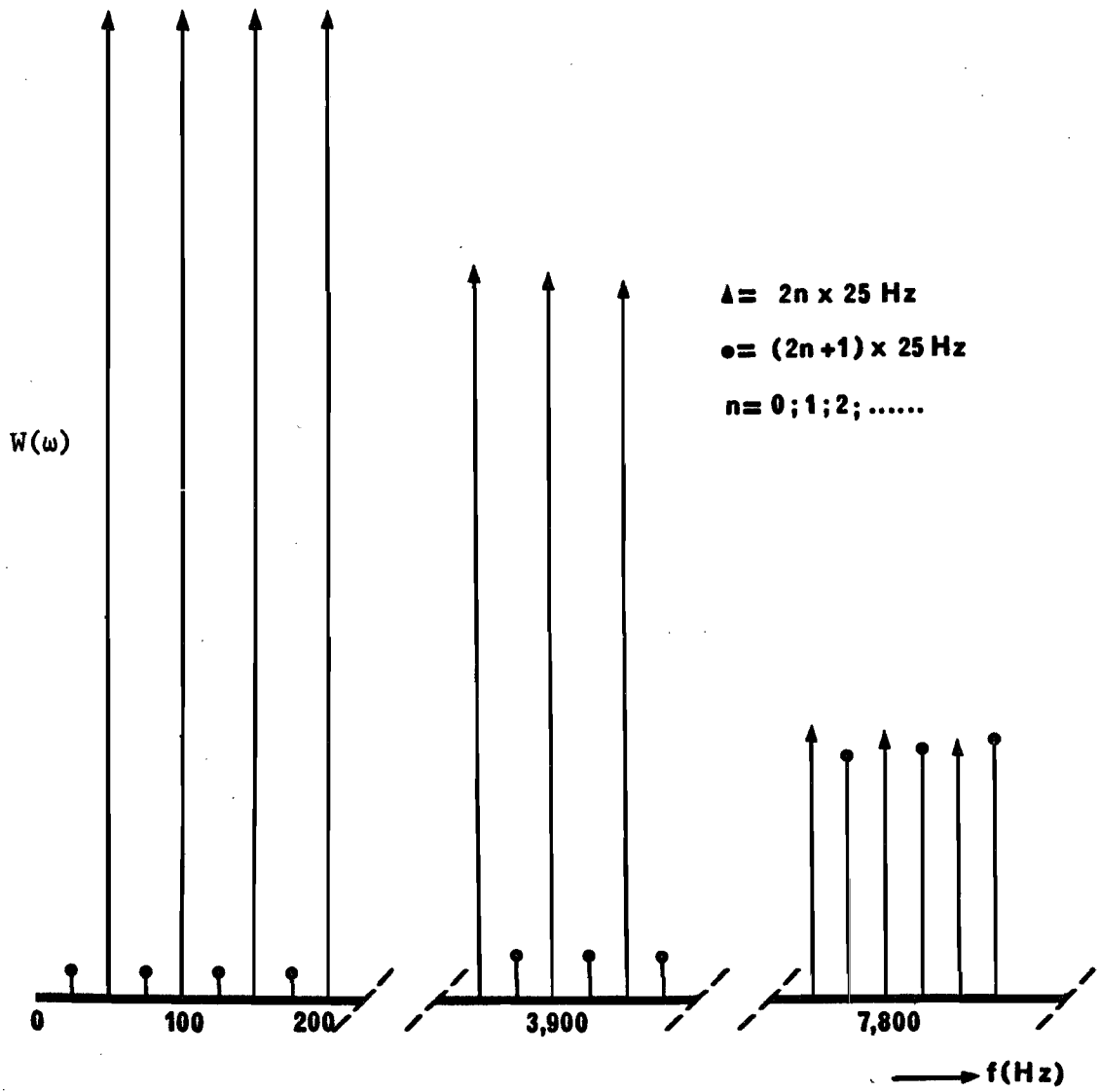


fig 8

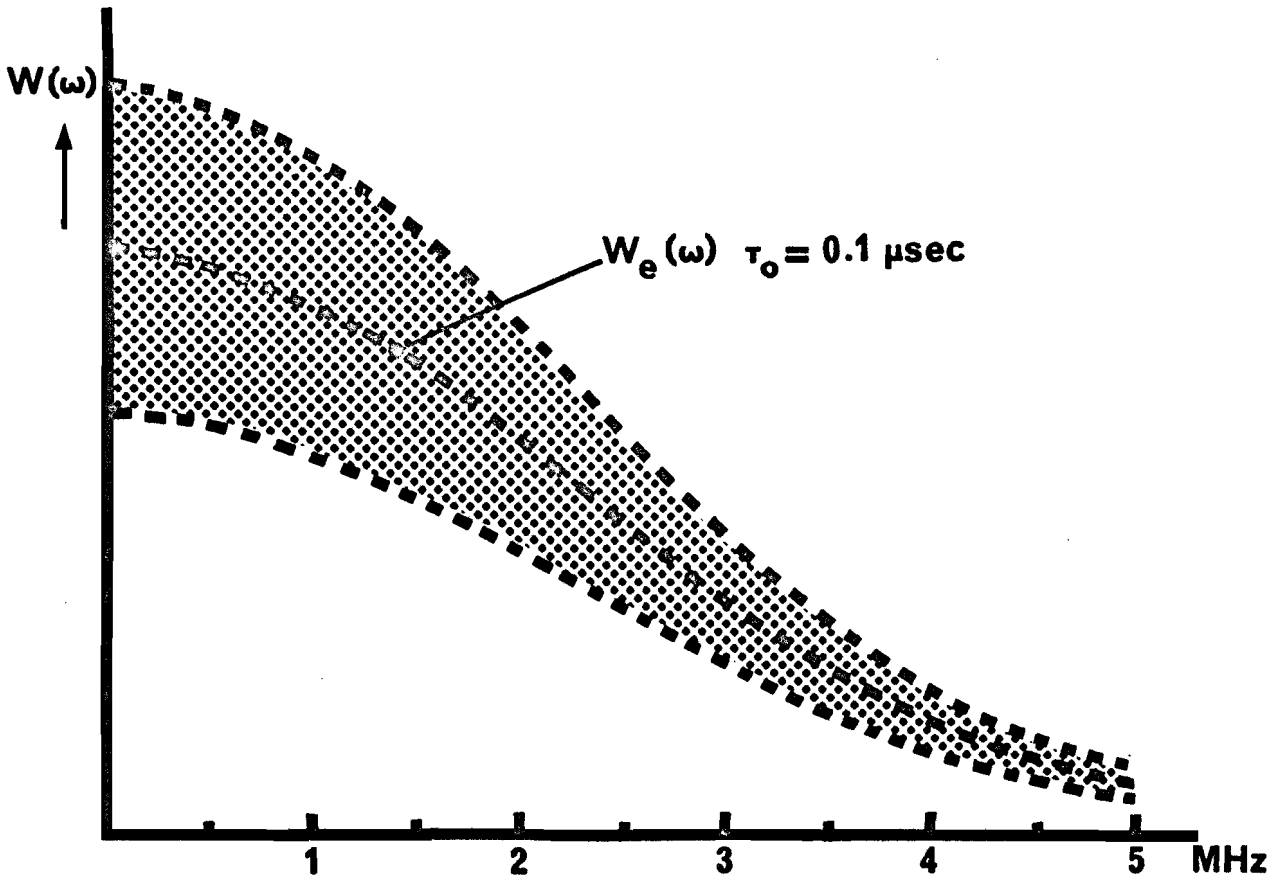


fig 9a

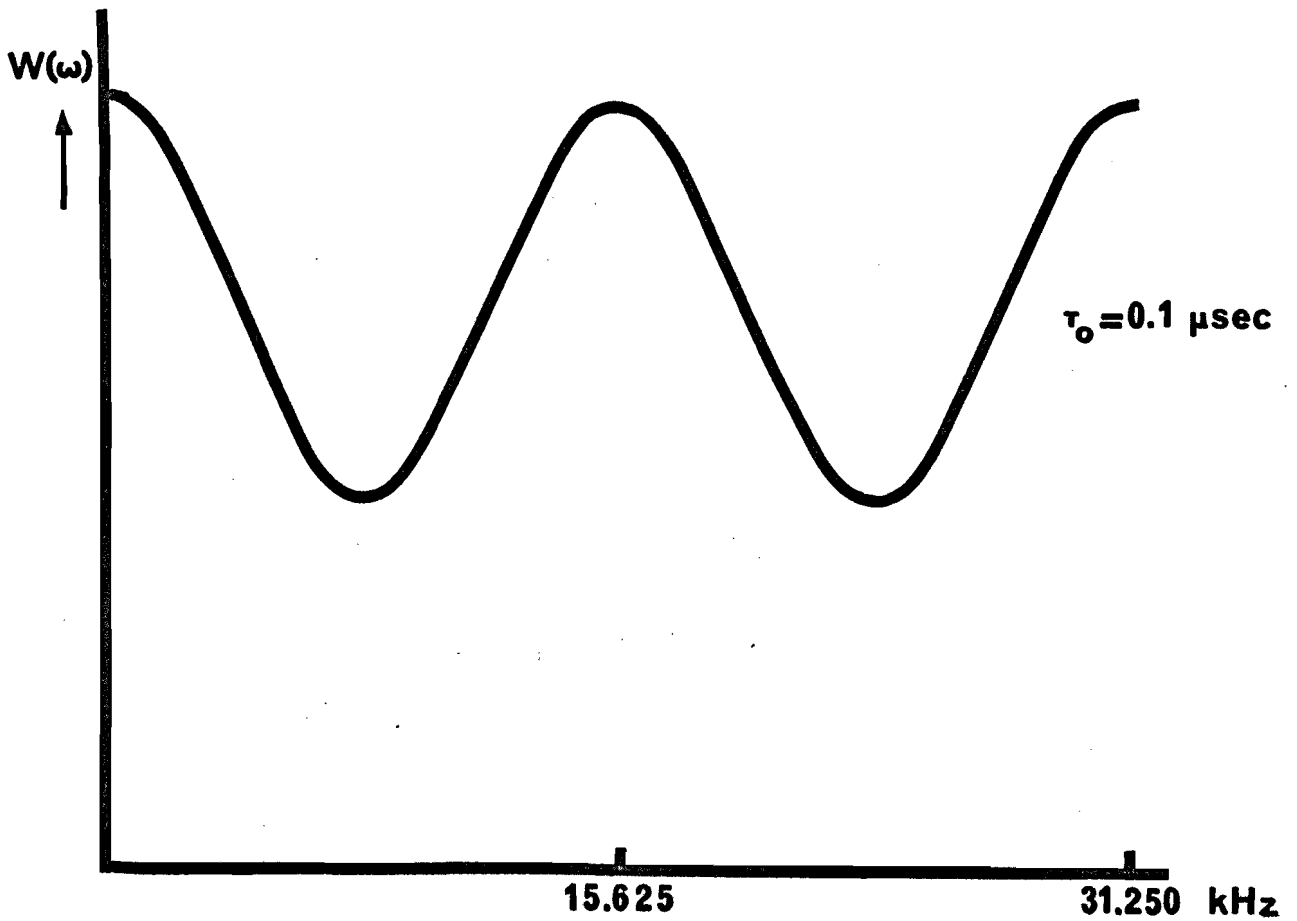


fig 9b

The distribution of the power in the frequency band as a function of the picture structure.

If the structure of the picture is less fine than in the example described above, τ_0 will be greater. The elementary autocorrelation function $R_e(\tau)$ will then be broader and the related elementary power spectrum $W_e(\omega)$ will be narrower. At the same time there are more non-negligible contributions to the total autocorrelation function. Instead of (23) the mean value of the δ -functionals is given by (24) :

$$W_e(\omega) \left\{ 1 + 2 \epsilon \frac{-(0.134)^2}{\tau_0} \cos 64\omega + 2 \epsilon \frac{-(2 \times 0.134)^2}{\tau_0} \cos 2 \times 64\omega + \right. \\ \left. 2 \epsilon \frac{-(3 \times 0.134)^2}{\tau_0} \cos 3 \times 64\omega + \dots \right\} \quad (24)$$

If for instance τ_0 is increased from 0.1 μsec to 0.2 μsec or to 0.4 μsec , the power spectrum as given in fig. 9 changes to the power spectra as given in fig. 10 and fig. 11.

Thus an increasing τ_0 results in a power spectrum that decreases faster with ω . Moreover an increasing number of contributions causes the power to be concentrated more around multiples of the line-frequency.

In a real picture there will in general be a combination of structures from fine to coarse. The resulting power spectrum is the sum of the spectra related to the respective structures. In general, the power half-way between the maxima of power at multiples of the line-frequency will form only a very small percentage of the total power.

Nevertheless this power cannot be eliminated without affecting the fine structure of the picture. This will be shown in the next section.

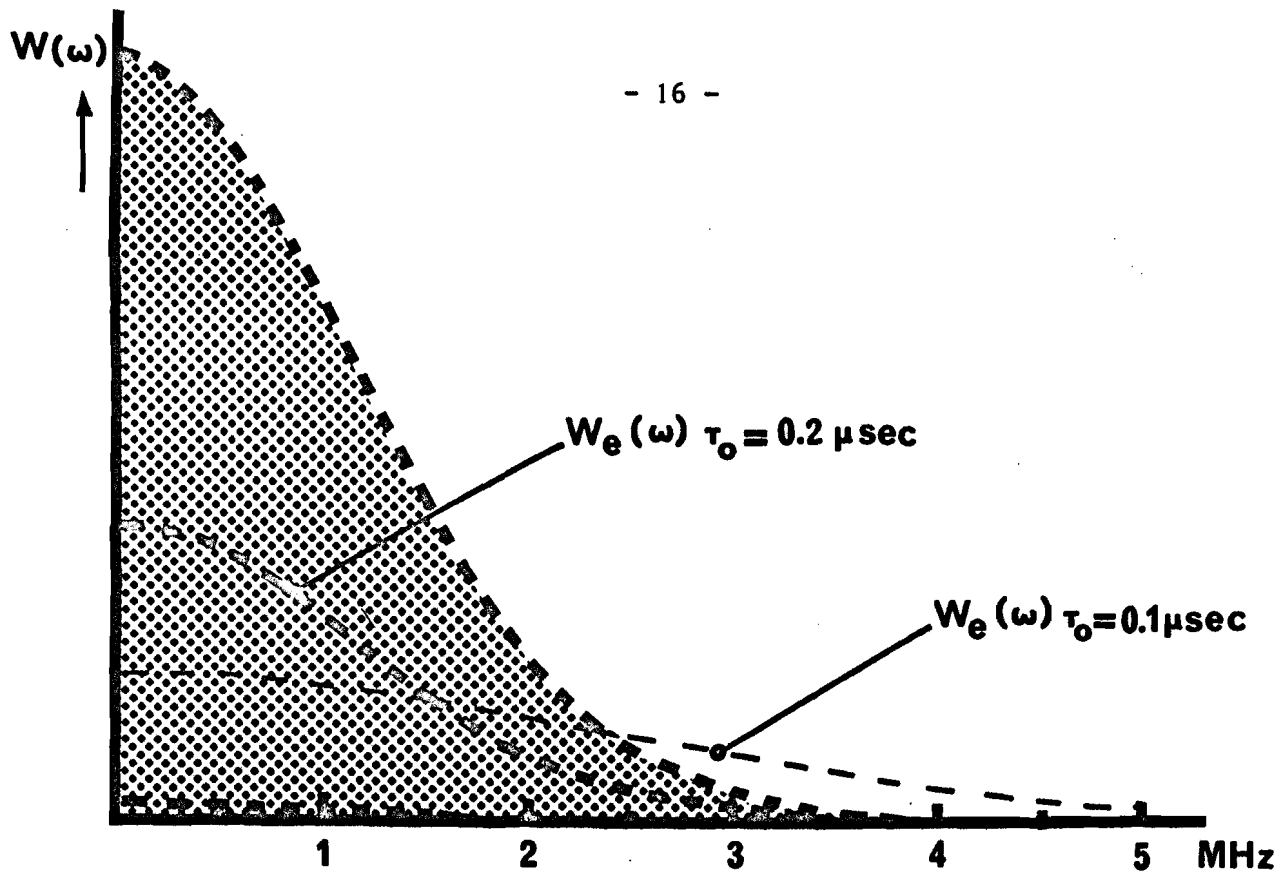


fig 10 a

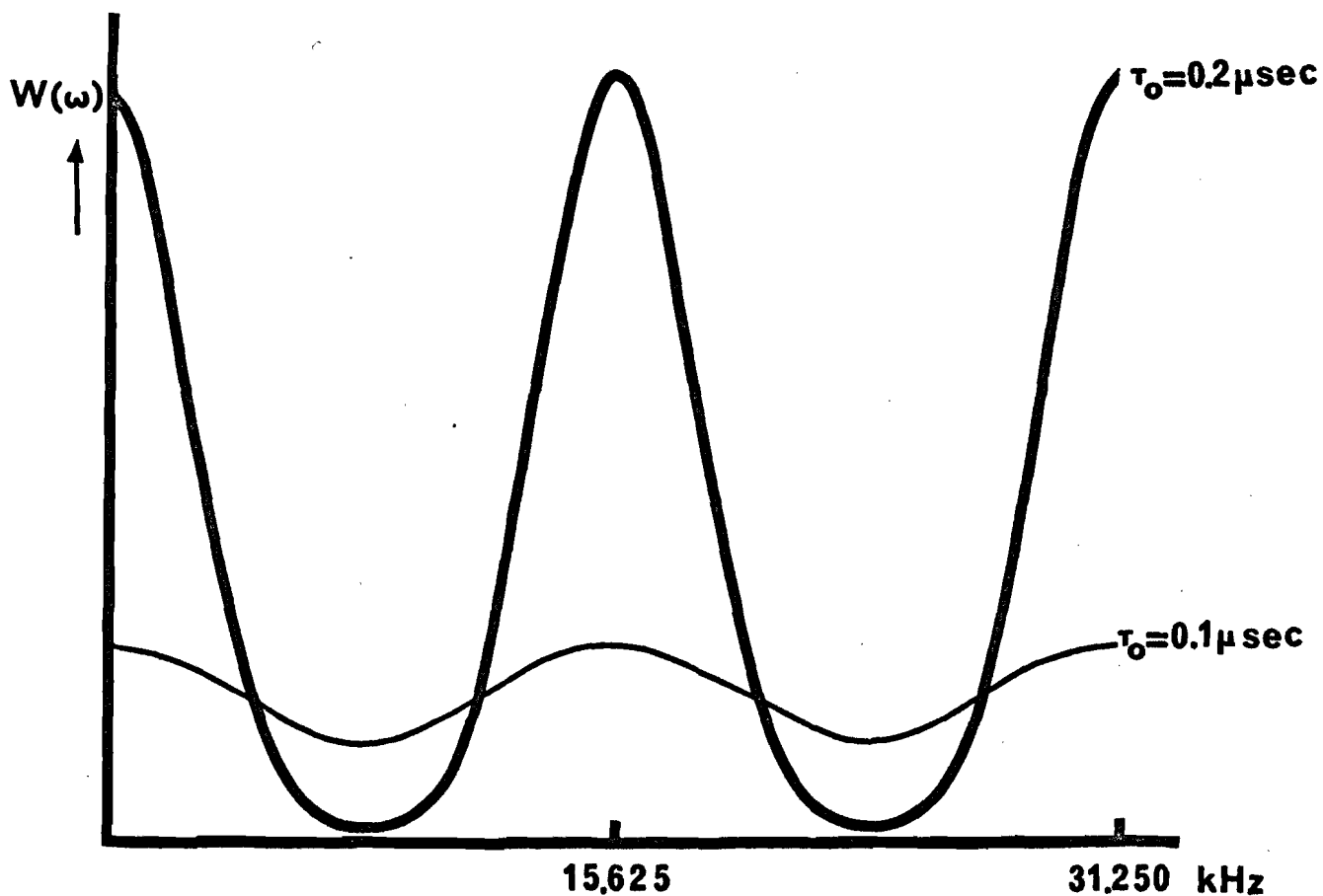


fig 10 b

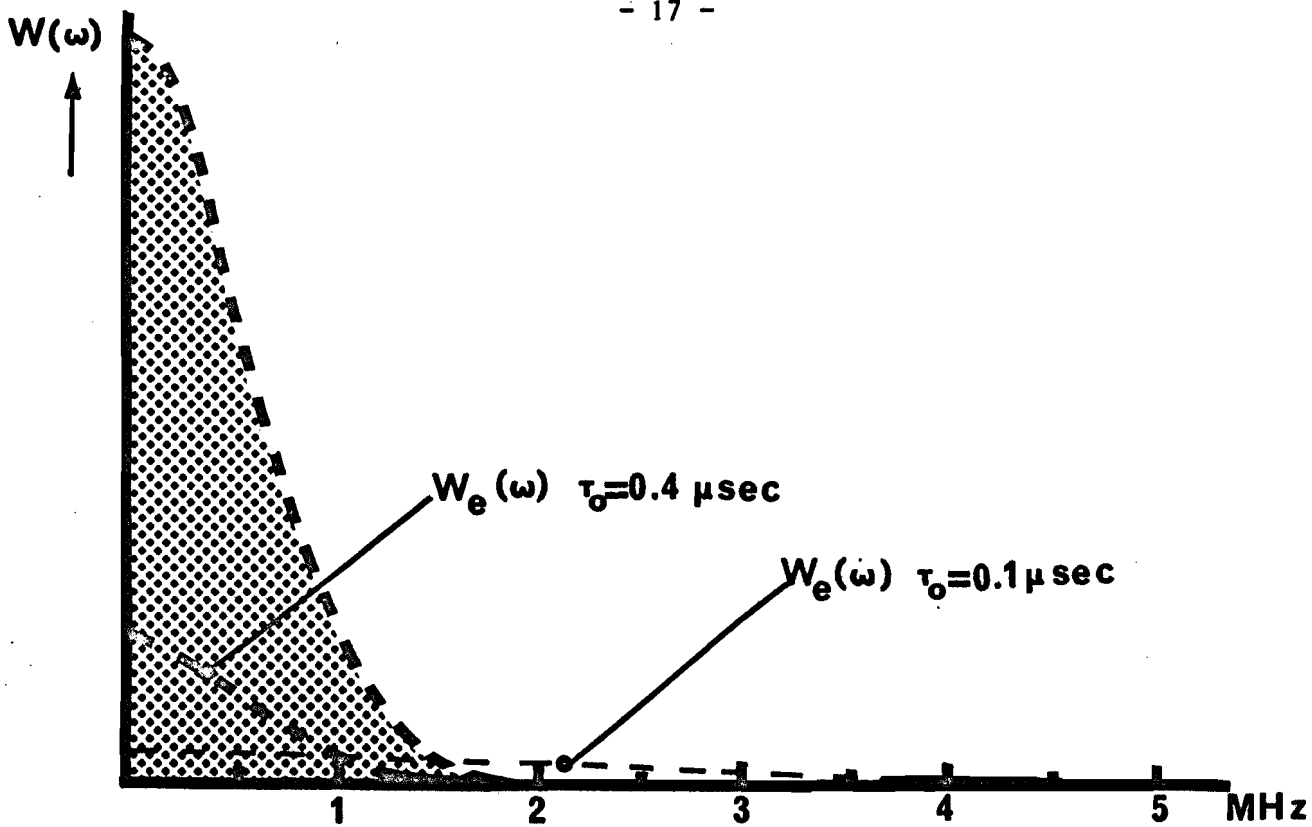


fig 11 a

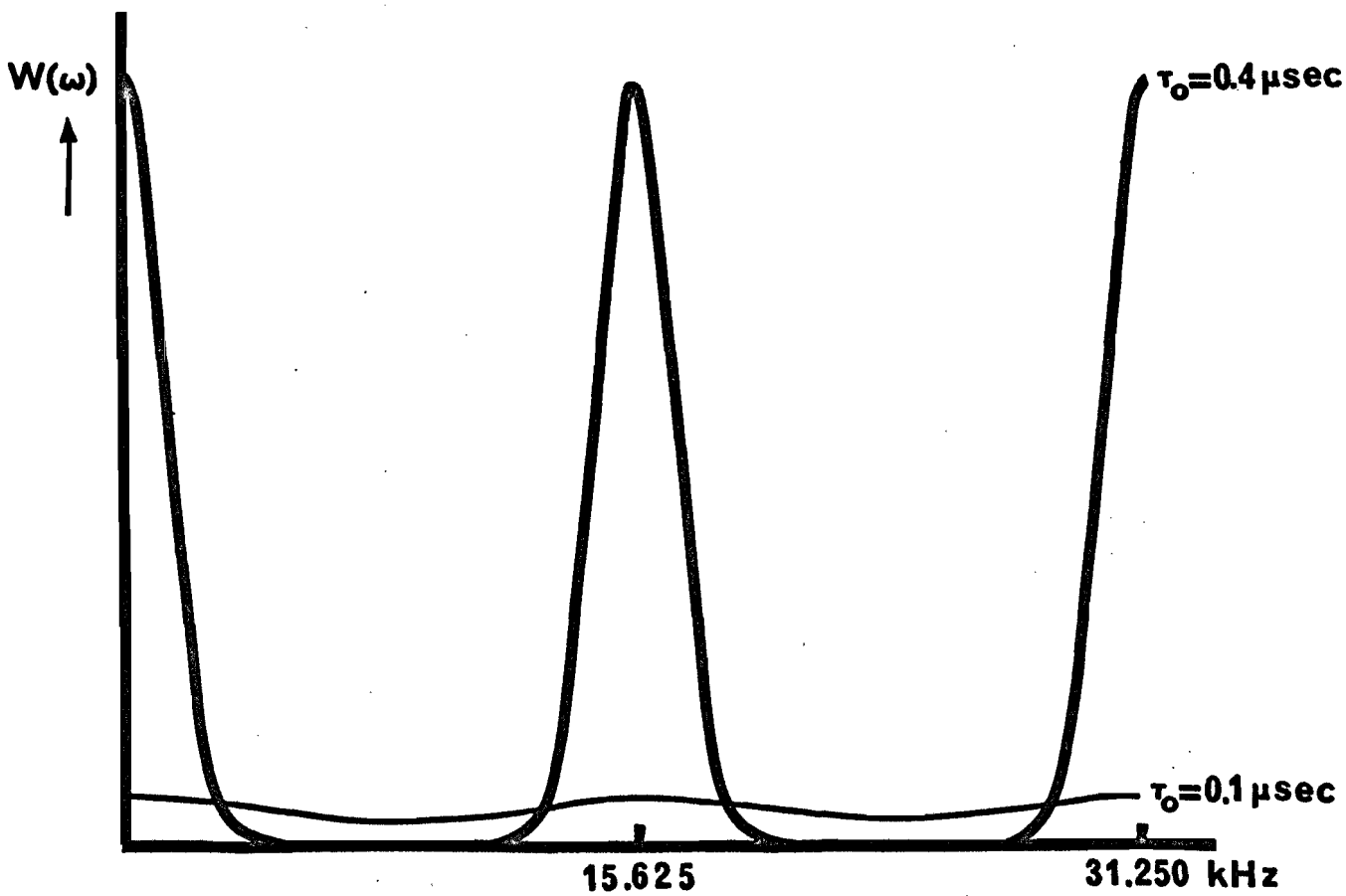


fig 11 b

Elimination of the "idle" parts of the power spectrum and the influence on the structure of the picture.

Imagine for instance the power spectrum as given in fig. 9, corresponding to the value $\tau_0 = 0.1 \mu\text{sec}$.

The parts of $W(\omega)$ between the power maxima around multiples of the line-frequency cannot be eliminated without affecting the good reproduction of the fine picture detail. To enable a study of the distortion resulting when this is done, the power spectrum $W(\omega)$ (fig. 12a) is multiplied by a square wave function $h(\omega)$ that equals 1 around multiples of the line frequency and is zero in the "empty" part of the spectrum (fig. 12b). The result is the modified power spectrum $W_m(\omega)$ (fig. 12c).

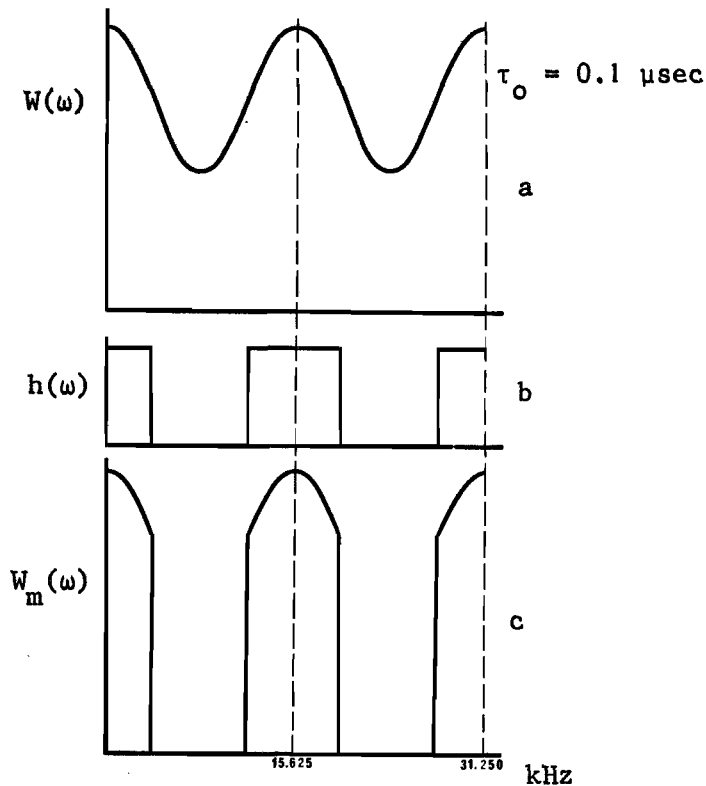


fig. 12

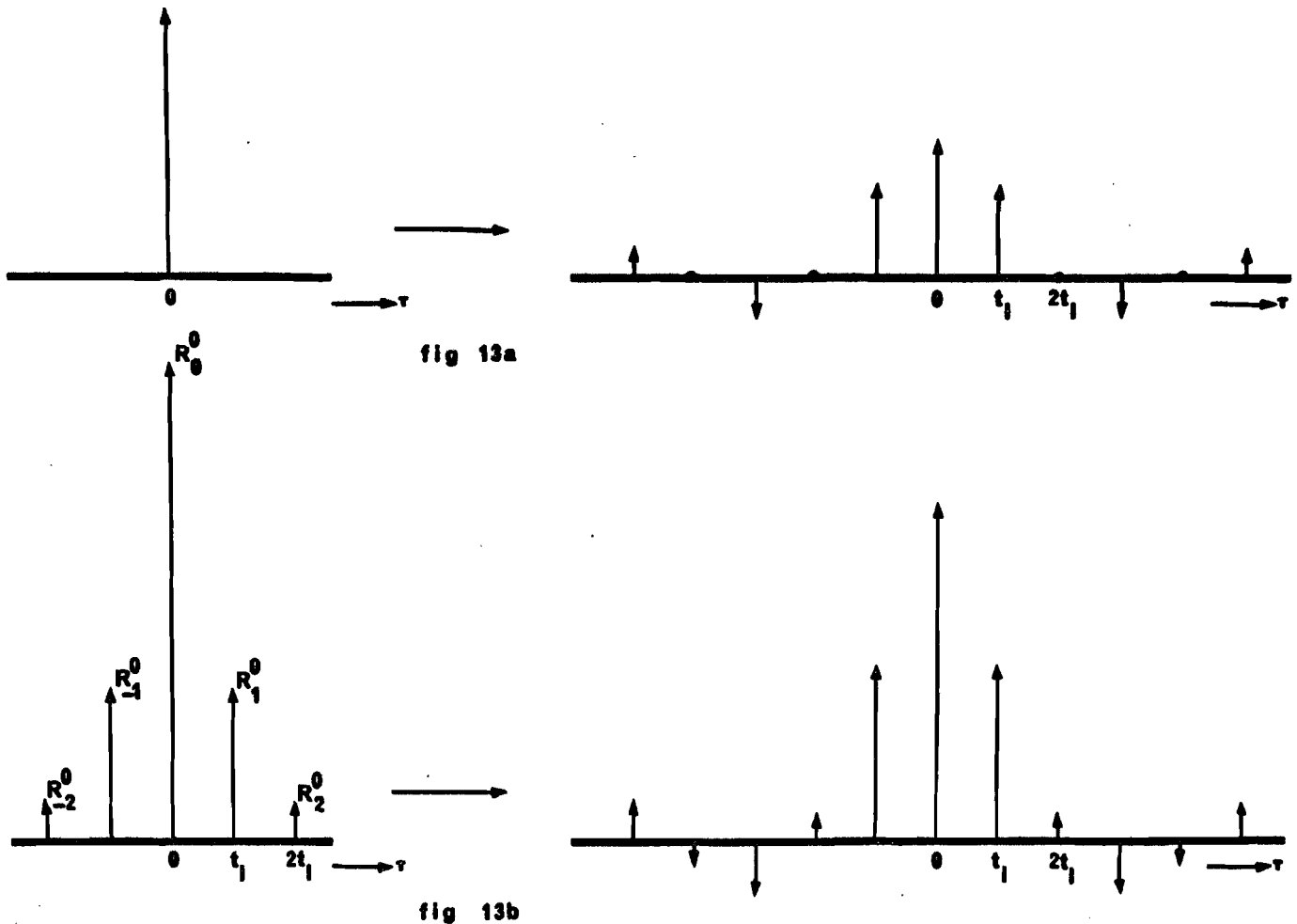
$$W(\omega) \quad \text{o-o} \quad R(\tau) \tag{25}$$

$$h(\omega) \quad \text{o-o} \quad R_h(\tau) = \frac{1}{2} \delta(\tau) + \frac{1}{\pi} \delta(\tau - \tau_1) + \frac{1}{\pi} \delta(\tau + \tau_1) + \dots \tag{26}$$

$$W_m(\omega) = W(\omega) \cdot h(\omega) \quad \text{o-o} \quad R(\tau) * R_h(\tau) \tag{27}$$

The autocorrelation function, belonging to the power spectrum of fig. 12c is the convolution integral of the autocorrelation functions of the functions drawn in fig. 12a and fig. 12b.

Every contribution R_n^m to the autocorrelation function belonging to the original power spectrum is transformed into a number of contributions (fig. 13a).



Corresponding parts of the autocorrelation functions belonging to $W(\omega)$ and $W_m(\omega)$ are drawn in fig. 13b. If the parts of the spectrum which we are discussing are suppressed a correlation not present in the original picture will be created in the reproduced picture.

This means a transformation of the fine structure in the direction perpendicular to the scanning lines into a less fine structure.

Moreover, as is readily seen from fig. 13, the maximum value of the autocorrelation function decreases, resulting in a loss of contrast in the direction of the scanning lines. This of course applies particularly to the fine structured parts of the picture.

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