

# The power spectrum of SUSY-CDM on subgalactic scales

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## ABSTRACT

The formation of large-scale structure is independent of the nature of the cold dark matter (CDM), however the fate of very small-scale inhomogeneities depends on the microphysics of the CDM particles. We investigate the matter power spectrum for scales that enter the Hubble radius well before matter–radiation equality, and follow its evolution until the time when the first inhomogeneities become non-linear. Our focus lies on weakly interacting massive particles (WIMPs), and as a concrete example we analyse the case when the lightest supersymmetric particle is a bino. We show that collisional damping and free-streaming of WIMPs lead to a matter power spectrum with a sharp cut-off at about  $10^{-6} M_{\odot}$  and a maximum close to that cut-off. We also calculate the transfer function for the growth of the inhomogeneities in the linear regime. These three effects (collisional damping, free-streaming and gravitational growth) are combined to provide a *WMAP* normalized primordial CDM power spectrum, which could serve as an input for high-resolution CDM simulations. The smallest inhomogeneities typically enter the non-linear regime at a redshift of about 60.

**Key words:** cosmology: theory – dark matter – early Universe.

## 1 INTRODUCTION

Analysis of the anisotropies in the cosmic microwave background (CMB) radiation (Spergel et al. 2003) shows that the relative matter density  $\Omega_m = 0.29 \pm 0.07$  is significantly larger than the relative baryon density  $\Omega_b = 0.047 \pm 0.006$ . These results are consistent with the observed abundances of light elements and primordial nucleosynthesis (see e.g. Tyler et al. 2000) and the power spectrum found from galaxy redshift surveys (Percival et al. 2001), and indicate that the Universe contains a significant amount of non-baryonic cold dark matter (CDM).

Weakly interacting massive particles (WIMPs) are attractive CDM candidates, since a stable relic from the electroweak scale generically has an interesting present-day density,  $\Omega_{\text{cdm}} \sim \mathcal{O}(1)$  (Dimopoulos 1990). The argument goes as follows: annihilation processes cease and the WIMP number density  $n$  becomes fixed when the annihilation rate  $\Gamma_{\text{ann}}$  drops below the expansion rate  $H$  (usually referred to as chemical decoupling or freeze-out). The temperature of chemical decoupling is thus defined by  $\Gamma_{\text{ann}} = \langle \sigma_{\text{ann}} v \rangle n(T_{\text{cd}}) \sim H(T_{\text{cd}})$ , where  $\sigma_{\text{ann}}$  denotes the annihilation cross-section and  $v$  the relative velocity of the annihilating particles. For a typical weakly interacting particle one finds  $T_{\text{cd}} \sim m/25$ ,  $m$  being

the WIMP mass. The present-day relative WIMP density can be estimated as

$$\Omega_{\text{wimp}} = \frac{mn_{\text{cd}}(a_{\text{cd}}/a_0)^3}{(3H_0^2/8\pi G)} \sim 0.2 \frac{(m/T_{\text{cd}})/25}{\langle \sigma_{\text{ann}} v \rangle / 1 \text{ pb}}, \quad (1)$$

where  $a$  denotes the scale factor and we have set  $h = 0.7$  and assumed 90 relativistic degrees of freedom at chemical decoupling. A cross-section of about 1 picobarn (pb) ( $\equiv 10^{-40} \text{ m}^2$ ) is typical for the annihilation of supersymmetric WIMPs since  $1 \text{ pb} \sim \alpha^2/(100 \text{ GeV})^2$ , where  $\alpha \approx 1/100$  is the electroweak coupling and supersymmetric particles typically have mass of the order of the electroweak scale (100 GeV).

In supersymmetry every standard model particle has a supersymmetric partner and in most models there is a conserved quantum number (R-parity), which makes the lightest supersymmetric particle stable. Supersymmetry models have a large number of free parameters, however in most models the lightest supersymmetric particle is the lightest neutralino (which is a mix of the supersymmetric partners of the photon, the  $Z$  and the Higgs bosons; see e.g. Jungman, Kamionkowski & Griest 1996), furthermore in large regions of parameter space the lightest neutralino is mainly the bino (Roszkowski 1991). Accelerator searches place a lower limit on the neutralino mass of  $m > 37 \text{ GeV}$  (see e.g. Hagiwara et al. 2002), while the *WMAP* measurement of the matter density leads to an upper limit of  $m < 500 \text{ GeV}$  (Ellis et al. 2003).

In CDM cosmologies structure forms hierarchically; galaxy haloes form from the merger and accretion of subhaloes (which

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themselves formed from smaller subhaloes). The internal structure of galaxy haloes is determined by the dynamical processes that act on the accreted components. Dynamical friction causes subhaloes with mass  $M > 10^9 M_\odot$  to spiral toward the centre of a Milky Way mass parent halo within a Hubble time, while the tidal field of the parent halo can strip material from a subhalo. Numerically simulated galaxy haloes contain large numbers of subhaloes that have not been destroyed (Klypin et al. 1999; Moore et al. 1999) and the subhalo mass function varies roughly as  $N(M) \propto M^{-1.8}$  down to the resolution limit of the simulations,  $M \sim 10^6 M_\odot$  (Stoeckl et al. 2003). Furthermore the anomalous flux ratios of multiply imaged QSOs (Mao & Schneider 1998; Schechter 2003) may be due to microlensing by subhaloes, providing tentative observational support for the existence of substructure in galaxy haloes.

Substructure on scales far smaller than those resolved by numerical simulations has potentially significant consequences for WIMP direct (Drukier, Freese & Spergel 1986) and indirect (see e.g. Bergström 2000) detection. WIMP direct detection (using terrestrial detectors) probes the dark matter distribution on submilliparsec scales (Silk & Stebbins 1993; Moore et al. 2001; Green 2002, 2003). WIMP indirect detection via WIMP annihilation products (gamma-rays, antiprotons and neutrinos) is most sensitive to the highest density regions of the Milky Way (e.g. Silk & Stebbins 1993; Bergström et al. 1999; Bergström, Edsjö & Gunnarsson 2001; Calaneo-Roldan & Moore 2000) and clumping will also enhance the extragalactic gamma-ray signal (Ullio et al. 2002; Taylor & Silk 2003).

In this Letter we present the matter power spectrum close to the end of the linear regime, for scales that enter the horizon well before matter–radiation equality. Three effects are important: after chemical decoupling the primordial CDM density perturbations are collisionally damped, owing to elastic interactions with other species, and then after kinetic decoupling free-streaming leads to further damping. CDM inhomogeneities on physical scales larger than the damping scale grow logarithmically during the radiation-dominated epoch and then roughly proportional to the scalefactor during the matter-dominated epoch.

The small-scale CDM power spectra for the density, velocity and potential perturbations are an essential input for attempts to estimate analytically the size and mass of the very first gravitationally bound objects. Ultimately, very high-resolution CDM simulations will be necessary to make detailed predictions for the fate of the very first objects, however the very different characteristic smallest scales that have been predicted for different CDM candidates [ $10^{-6} M_\odot$  for neutralinos (Hofmann, Schwarz & Stöcker 2001; Schwarz, Hofmann & Stöcker 2001; Berezhinsky, Dokuchaev & Eroshenko 2003) compared with  $10^{-13} M_\odot$  for axions (Kolb & Tkachev, 1996)] could have observable astrophysical consequences [such as femtolensing and picolensing of gamma ray bursts (Gould 1992)]. This would open up the exciting possibility of discriminating between CDM candidates astronomically.

## 2 COLLISIONAL DAMPING AND FREE STREAMING

Primordial CDM inhomogeneities on the smallest scales are smeared out by collisional damping and free streaming, if the CDM has once been in thermal contact with the hot component of the Universe (as is the case for WIMPs). These damping processes give rise to a sharp cut-off in the primordial CDM power spectrum. In order to make quantitative predictions, we assume the CDM particle to be a bino with mass  $m$ . The bino abundance is fixed at

chemical decoupling, which happens at typically  $T_{\text{cd}} \sim m/25$ , i.e. above 1 GeV. Below that temperature, elastic scattering of binos and fermions (leptons and quarks from the radiation component) can still occur and the bino decouples kinetically at significantly lower temperatures (Schmid, Schwarz & Widerin 1999):  $T_{\text{kd}} = 10$  MeV to 40 MeV, depending on the parameters of the supersymmetric model.

Between chemical and kinetic decoupling the CDM particles interact with the cosmic heat bath, which consists of all the relativistic particles in the Universe, and this leads to collisional damping of the CDM density perturbations. Since CDM is non-relativistic at this epoch, the dominant processes are bulk and shear viscosity. Hofmann et al. (2001) calculated the effect of these processes on the primordial density perturbations and found exponential damping with a characteristic comoving wavenumber

$$k_d \approx 1.8 \left( \frac{m}{T_{\text{kd}}} \right)^{1/2} \frac{a_{\text{kd}}}{a_0} H_{\text{kd}}, \quad (2)$$

$$\approx \frac{3.8 \times 10^7}{\text{Mpc}} \left( \frac{m}{100 \text{ GeV}} \right)^{1/2} \left( \frac{T_{\text{kd}}}{30 \text{ MeV}} \right)^{1/2}.$$

This corresponds to a length-scale of  $\sim 10^{-2}/H$  at kinetic decoupling. The total CDM mass contained in a sphere with radius  $\pi/k_d$  is  $M_d \sim 10^{-10} M_\odot$ .

For a bino, the kinetic decoupling temperature depends on  $m$  and on the various sfermion masses  $m_{\tilde{f}}$ . Here we make the simplifying assumption that all sfermions have the same mass, which is not correct in more realistic models, but is a reasonable assumption for a first estimate. We consider two fiducial sets of parameters: model A (B) has  $m = 100(150)$  GeV and  $m_{\tilde{f}} = 230(190)$  GeV. For these parameters we find, neglecting coannihilations with other supersymmetric particles, that chemical decoupling happens at  $T_{\text{cd}} = 4.0(5.8)$  GeV and  $\Omega_{\text{bino}} = 0.31(0.17)$ , thus models A and B have CDM densities at the high and low end respectively of the range of values found by *WMAP*. For these models kinetic decoupling occurs at  $T_{\text{kd}} = 33(21)$  MeV and the damping mass-scale is  $M_d = 9 \times 10^{-11}(6 \times 10^{-11}) M_\odot$ .

After kinetic decoupling, the CDM particles enter the free-streaming regime. We calculate the effect of free streaming by solving the collisionless Boltzmann equation in Fourier space on subhorizon scales, to first order in the perturbed thermodynamics quantities and neglecting metric perturbations, starting immediately after kinetic decoupling. We include the spectrum of CDM density perturbations at this time, and also take into account the matter and radiation components of the Universe. The net result is again exponential damping with a characteristic scale that is inversely proportional to the (time-dependent) free-streaming length-scale ( $k_{\text{fs}} \equiv 2\sqrt{3}/l_{\text{fs}}$ ), multiplied by a polynomial term that arises from the ratio of the CDM particle kinetic energy to the thermal averaged kinetic energy. The free-streaming scale becomes approximately constant soon after matter–radiation equality:

$$k_{\text{fs}} \approx \left( \frac{m}{T_{\text{kd}}} \right)^{1/2} \frac{a_{\text{eq}}/a_{\text{kd}}}{\ln(4a_{\text{eq}}/a_{\text{kd}})} \frac{a_{\text{eq}}}{a_0} H_{\text{eq}}, \quad (3)$$

$$\approx \frac{1.7 \times 10^6}{\text{Mpc}} \frac{(m/100 \text{ GeV})^{1/2} (T_{\text{kd}}/30 \text{ MeV})^{1/2}}{1 + \ln(T_{\text{kd}}/30 \text{ MeV})/19.2}.$$

The damping scale from free streaming depends on  $\omega_m \equiv \Omega_m h^2$  only via the logarithm; we therefore set it equal to *WMAP*'s best-fitting value,  $\omega_m = 0.14$  (Spergel et al. 2003). The corresponding length-scale at matter–radiation equality  $\sim 10^{-8}/H$  and the total mass contained in a sphere with radius  $\pi/k_{\text{fs}}$  is  $M_{\text{fs}} \sim 10^{-6} M_\odot$ . More precisely, for model A (B) we find  $M_{\text{fs}} = 9 \times 10^{-7} (6 \times 10^{-7}) M_\odot$ .

We can now put together all the damping factors for the CDM mass density perturbation  $\Delta \equiv \delta\rho/\rho$ . Well after equality (our approximations are valid for  $a/a_{\text{eq}} \geq 10$ ), when the comoving free streaming length is frozen, density inhomogeneities with large wavenumbers are suppressed by a damping factor

$$D(k) = \left[ 1 - \frac{2}{3} \left( \frac{k}{k_{\text{fs}}} \right)^2 \right] \exp \left[ - \left( \frac{k}{k_{\text{fs}}} \right)^2 - \left( \frac{k}{k_{\text{d}}} \right)^2 \right]. \quad (4)$$

The present approximation is valid for  $k/k_{\text{fs}} < 1$ , as terms of order  $(k/k_{\text{fs}})^4$  have been neglected in the polynomial.

The mass density contrast as function of redshift and wavenumber can now be written

$$\Delta(k, z) = \Delta(k, z_i) T_{\Delta}^{1/2}(k, z) D(k), \quad (5)$$

where  $\Delta(k, z_i)$  is the primordial density perturbation and  $T_{\Delta}(k, z)$  is the transfer function which encodes the gravitational evolution of  $\Delta(k, z)$ .

### 3 GRAVITATIONAL GROWTH ON SUBHORIZON SCALES

Following Weinberg (2002), we model the Universe as a two component (non-relativistic matter and radiation) fluid. Here we utilize the zero shear (or longitudinal or conformal Newtonian) gauge. It is useful to define for each fluid (the index ‘a’ stands for radiation,  $p_r = \epsilon_r/3$ , or non-relativistic matter,  $p_{\text{cdm}} = 0$ ) the energy density perturbation

$$\Delta_a = \frac{\delta\epsilon_a}{\epsilon_a + p_a}, \quad (6)$$

(note that  $\Delta_{\text{cdm}} \equiv \Delta$ ) and the velocity perturbations  $v_a$  by

$$v_a = \frac{ik}{k} v_a, \quad (7)$$

where  $v_a$  is the peculiar velocity.

The Newtonian gravitational potential  $\phi$  is defined via the metric, which is in the zero shear gauge given by

$$ds^2 = -(1 + 2\phi) dt^2 + a^2 (1 - 2\phi) \delta_{ij} dx^i dx^j, \quad (8)$$

in the absence of anisotropic stress, where cosmic time is denoted by  $t$ . Another useful quantity is the hypersurface-independent quantity  $\zeta (= -\mathcal{R}) \equiv \Delta/3 - \phi$  (Bardeen 1980, 1989), which is conserved on superhorizon scales. We use this definition to make contact with the *WMAP* normalization, which is given in terms of  $\mathcal{R}$  (Verde et al. 2003).

Several comments are in order here. First, we work in the zero shear gauge, because all subhorizon quantities can be interpreted in terms of Newtonian physics, which is not the case in the synchronous gauge, where the Newtonian gravitational potential is gauged to zero. Secondly, neutrinos are included in the radiation component in order to allow an analytic treatment, i.e. their anisotropic stress is neglected. This leads to errors of around 10 per cent (Hu et al. 1995). Furthermore we ignore the effect of a non-zero cosmological constant or curvature (which only effect the evolution of the perturbations at very late times) and baryon inhomogeneities. At early times the baryons are tightly coupled to the radiation fluid, and photon diffusion damping rapidly erases small-scale perturbations in the baryon fluid at  $z \sim 10^6$  to  $10^5$ . On small scales the tight coupling breaks down prior to recombination, and the baryon perturbations grow, however  $\Delta_b \ll \Delta_{\text{cdm}}$  still (Yamamoto, Sugiyama & Sato 1997, 1998). Post decoupling on scales  $k > k_b \sim 10^3 \text{ Mpc}^{-1}$ , the residual

electrons allow transfer of energy between the photon and baryon fluids so that thermal pressure prevents the baryon perturbations from growing until  $z_b \sim 150$  (Yamamoto et al. 1997; Padmanabhan 2002). As we are interested in CDM perturbations on small scales at early times, we can neglect the perturbations in the baryon fluid.

The gravitational evolution of CDM inhomogeneities is described by the corresponding transfer functions. For the calculation of the transfer functions the equations of motion of the related perturbation variables have to be solved. Unfortunately, this is not possible exactly for all times, however there are two overlapping regimes for which exact solutions exist. The first of these regimes is radiation domination,  $\epsilon_r \gg \epsilon_{\text{cdm}}$ , for which an exact solution can be found that is valid for all scales (Schmid et al. 1999). In the superhorizon limit ( $k/a \ll H$ ),  $\phi \rightarrow \phi_0$ ,  $\Delta_{\text{cdm},r} \rightarrow -3\phi_0/2$ ,  $v_{\text{cdm},r} \rightarrow -(\phi_0/2)$  ( $k/aH$ ) and  $\zeta \rightarrow -3\phi_0/2$ , while in the subhorizon limit ( $k/a \gg H$ ) with  $x = k/(\sqrt{3}aH)$ :

$$\begin{aligned} \Delta_{\text{cdm}}(x) &= -9\phi_0 \left[ \ln x + \gamma_E - \frac{1}{2} \right], \quad v_{\text{cdm}}(x) = -\frac{3\sqrt{3}\phi_0}{x}, \\ \Delta_r(x) &= \frac{9\phi_0}{2} \cos x, \quad \phi(x) = -3\phi_0 \frac{\cos x}{x^2}, \end{aligned} \quad (9)$$

with  $\gamma_E$  denoting Euler’s constant. We see that on subhorizon scales during radiation domination the matter perturbation  $\Delta_{\text{cdm}}$  grows logarithmically while  $\Delta_r$  oscillates with constant amplitude. Thus for small scales, which enter the horizon sufficiently long before matter–radiation equality, the condition  $\epsilon_{\text{cdm}} \Delta_{\text{cdm}} \gg \epsilon_r \Delta_r$  is satisfied during the radiation-dominated era. As long as the CDM density perturbations  $\epsilon_{\text{cdm}} \Delta_{\text{cdm}}$  dominate the density perturbations of the radiation and baryons, the evolution of  $\Delta_{\text{cdm}}$  is governed by (Hu & Sugiyama 1996)

$$y(1+y) \frac{d^2 \Delta_{\text{cdm}}}{dy^2} + \left( 1 + \frac{3}{2} \right) \frac{d\Delta_{\text{cdm}}}{dy} - \frac{3}{2} (1 - f_b) \Delta_{\text{cdm}} = 0, \quad (10)$$

where  $y = a/a_{\text{eq}}$  and  $f_b = \Omega_b/\Omega_m$  is the baryon fraction, with best-fitting value from *WMAP*  $f_b = 0.16$  (Spergel et al. 2003). The exact solution to this equation is a combination of Legendre functions of first and second kind,  $P_\nu(\sqrt{1+y})$  and  $Q_\nu(\sqrt{1+y})$  with index  $\nu(f_b) = (\sqrt{25 - 24f_b} - 1)/2$ . We smoothly join the small  $y$  expansion of these functions with the radiation domination subhorizon solution (equation 9), to obtain the normalization of the solution to equation (10). Expanding the result for  $y \gg 1$ , we find

$$\Delta_{\text{cdm}}(y) = -9\phi_0 c y^{v/2} \left[ \ln \left( \frac{k}{k_{\text{eq}}} \right) + b \right], \quad (11)$$

where  $k_{\text{eq}} = 1/(aH)_{\text{eq}}$ ,  $c(\nu) = \Gamma[1 + 2\nu]/(2^v \Gamma^2[1 + \nu])$  and  $b(f_b) = 1/2 \ln(2^5/3) - \gamma_E - 1/2 - 2/\nu - 2\Gamma'[\nu]/\Gamma[\nu]$ , e.g.  $\nu(f_b) = 1.80$  (2),  $c[\nu(f_b)] = 1.37$  (3/2) and  $b(f_b) = -1.57$  (−1.74) for  $f_b = 0.16$  (0). Note that before  $z_b$ , CDM density perturbations grow as  $\Delta_{\text{cdm}} \propto a^{v/2}$ . Later the baryons follow the CDM and the matter fluctuations grow as  $a$ . For the peculiar velocity and the Newtonian gravitational potential we obtain

$$\begin{aligned} v_{\text{cdm}}(y) &= \frac{k_{\text{eq}}}{k} \sqrt{\frac{y}{2}} \frac{d}{dy} \Delta_{\text{cdm}}(y), \\ \phi(y) &= -\frac{3}{4} \left( \frac{k_{\text{eq}}}{k} \right)^2 (1 - f_b) \frac{\Delta_{\text{cdm}}(y)}{y}. \end{aligned} \quad (12)$$

In the following, we omit the subscript cdm.

For redshifts  $z_{\text{eq}} > z > z_b$  (between matter–radiation equality and the epoch at which small-scale baryon perturbations start growing) we find the transfer function for the CDM density perturbations for

modes which satisfy  $k > k_b$ :

$$T_\Delta(k, z) = (6c)^2 \left[ \ln \frac{k}{k_{\text{eq}}} + b \right]^2 \left( \frac{1+z_{\text{eq}}}{1+z} \right)^\nu, \quad (13)$$

and the transfer function for the Newtonian gravitational potential on these scales is given by

$$T_\phi(k, z) = \left[ \frac{27(1-f_b)c}{4} \right]^2 \left[ \ln \frac{k}{k_{\text{eq}}} + b \right]^2 \left( \frac{k_{\text{eq}}}{k} \right)^4 \left( \frac{1+z_{\text{eq}}}{1+z} \right)^{\nu-2}. \quad (14)$$

The transfer function for the velocity depends on the initial time and is therefore not a very useful quantity.

#### 4 POWER SPECTRA

In this section we present the dimensionless power spectra (defined as  $\mathcal{P}_X(k, z) = (k^3/2\pi^2) \langle |X(k, z)|^2 \rangle$ ) normalized to the *WMAP* measurements (Spergel et al. 2003; Verde et al. 2003). For simplicity we assume that gravitational waves have a negligible contribution to the CMB anisotropies and that the density perturbations have a Harrison–Zel’dovich primordial power spectrum ( $n = 1$ ). We find, for  $k > k_b$  and  $z_{\text{eq}} \gg z > z_b$ ,

$$\frac{\mathcal{P}_\Delta(k, z)}{10^{-7}A} = 1.06 c^2 \left[ \ln \frac{k}{k_{\text{eq}}} + b \right]^2 D^2(k) \left( \frac{1+z_{\text{eq}}}{1+z} \right)^\nu, \quad (15)$$

$$\frac{\mathcal{P}_v(k, z)}{10^{-7}A} = 0.13 c^2 \nu^2 \times \left[ \ln \frac{k}{k_{\text{eq}}} + b \right]^2 \left( \frac{k_{\text{eq}}}{k} \right)^2 D^2(k) \left( \frac{1+z_{\text{eq}}}{1+z} \right)^{\nu-1}, \quad (16)$$

$$\frac{\mathcal{P}_\phi(k, z)}{10^{-7}A} = 0.60 c^2 (1-f_b)^2 \times \left[ \ln \frac{k}{k_{\text{eq}}} + b \right]^2 \left( \frac{k_{\text{eq}}}{k} \right)^4 D^2(k) \left( \frac{1+z_{\text{eq}}}{1+z} \right)^{\nu-2}, \quad (17)$$

where  $A = 0.9 \pm 0.1$  according to Spergel et al. (2003). Note that from the *WMAP* data  $n = 0.99 \pm 0.04$ , which is consistent with our assumption of  $n = 1$ . The scale of equality is  $k_{\text{eq}} = (0.01/\text{Mpc}) (\omega_m/0.14)$  and  $1+z_{\text{eq}} = 3371(\omega_m/0.14)$ .

Fig. 1 shows the power spectrum for the density contrast at a redshift of 500, close to the end of the linear regime of structure formation. It can be observed that the induced cut-off is indeed very sharp and that the power spectrum has a maximum close to the cut-off.

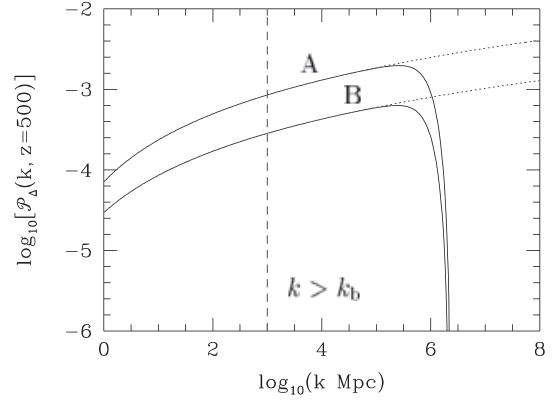
#### 5 DISCUSSION

The mechanisms of collisional damping and free streaming of WIMPs lead to a cut-off in the CDM power spectrum, which sets the typical scale for the first haloes in the hierarchical picture of structure formation. A rough estimate of the redshift at which typical fluctuations on comoving scale  $R$  go non-linear can be made via

$$\sigma(R, z_{\text{nl}}) = 1, \quad (18)$$

where  $\sigma(R, z)$  is the mass variance defined by

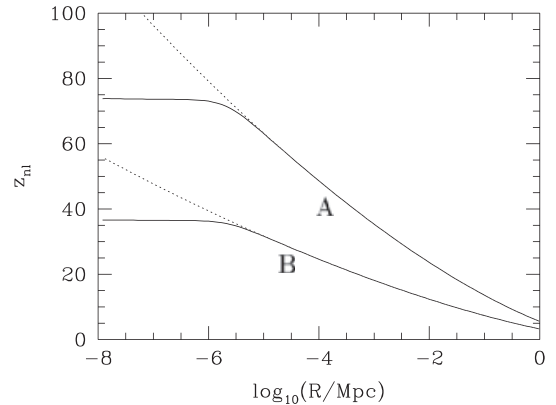
$$\sigma^2(R, z) = \int_0^\infty W^2(kR) \mathcal{P}_\Delta(k, z) \frac{dk}{k}, \quad (19)$$



**Figure 1.** The dimensionless power spectrum of the CDM density contrast at  $z = 500$  for models A and B from the text (full lines). Without the effects of collisional damping and free streaming, the power spectra would be given by the dotted lines.

where  $W(kR)$  is the Fourier transform of the window function divided by its volume. In accordance with the usual procedure, we take the window function to be a top hat. For this calculation we need the power spectrum on all scales, however our calculation of the CDM transfer function is only valid for  $k > k_b \sim 10^3 \text{ Mpc}^{-1}$ . We therefore instead use the matter transfer function found neglecting the baryon density (i.e.  $f_b = 0$ ) which is valid for  $k > k_{\text{eq}}$  and take  $\Omega_m = 0.36(0.22)$  for model A (B). This introduces errors at the 10 per cent level, however the criterion for non-linearity (equation 18) is only an order of magnitude estimate. Finally we include the power spectrum on scales  $k \sim k_{\text{eq}}$  by normalizing  $\sigma(R, z)$  to  $\sigma_8 \equiv \sigma(8/h \text{ Mpc}, 0) = 0.9 \pm 0.1$  (Spergel et al. 2003), taking into account the suppression of the growth of  $\Delta$  at late times due to the cosmological constant.

In Fig. 2 we plot  $z_{\text{nl}}$ , as defined by equation (18), as a function of the scale  $R$ . The plateau at  $R < 1 \text{ pc}$  is due to the sharp cut-off in the power spectrum. We can now give a more precise picture of the onset of the hierarchical structure formation process; non-linear structure formation starts at a redshift  $z_{\text{nl}}^{\text{max}}$ , which takes values in the range 30 to 80, and  $z_{\text{nl}}^{\text{max}} \sim 60$  for the best-fitting *WMAP* matter density. To be more specific, this is the epoch when the typical overdense regions go non-linear. Note, that rare fluctuations with large amplitude will go non-linear at much higher redshifts. If the density fluctuations



**Figure 2.** The redshift at which typical fluctuations of comoving scale  $R$  become non-linear, for the two models discussed in the text. The full lines take into account the effects of collisional damping and free streaming, whereas the dashed lines show the behaviour without a cut-off in the power spectrum.

have a Gaussian probability distribution then a  $N\sigma$  fluctuation will go non-linear at roughly  $Nz_{\text{nl}}$ . We leave the discussion of these rare large fluctuations and their cosmological consequences for a forthcoming paper and conclude with some comments on the typical fluctuations.

Let us estimate the size and mass of the first generation of subhaloes that form at  $z_{\text{nl}}^{\text{max}}$  using the spherical collapse model (e.g. Padmanabhan 2002). The mean CDM mass within a sphere of comoving radius  $R$  is  $M(R) = 1.6 \times 10^{-7} M_{\odot} (\omega_m/0.14) (R/\text{pc})^3$ . CDM overdensities that go non-linear have mass twice this value, i.e. roughly equal to the mass of Mars. These WIMP haloes are however much less compact than Mars. The physical size of the first haloes at turnaround (when their evolution decouples from the cosmic expansion) is  $r = 1.05 R/(1 + z_{\text{nl}}^{\text{max}}) \sim 0.02 \text{ pc}$ . The first haloes then undergo violent relaxation, decreasing in radius by a factor of 2 so that their present-day radius is of order tens of milliparsec (mpc) (comparable to the size of the solar system). If some of these first haloes could survive to the present day, their overdensity would be of order  $\Delta_{\text{halo}} = 7(1 + z_{\text{nl}}^{\text{max}})^3 \sim 10^6$ , several orders of magnitude larger than that of galaxies. Rare fluctuations, a non-Gaussian fluctuation distribution, or a blue ( $n > 1$ ) primordial power spectrum could lead to even larger overdensities.

We regard this Letter as a first step towards an ab initio calculation of the small-scale structure of CDM. The resulting power spectra are presented in a form that can be used as input for future very high-resolution simulations. Only once the fate of the first CDM haloes has been understood in detail will it be possible to make robust predictions for the expected signals in direct and indirect dark matter searches.

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