IOP Publishing
Journal of Geophysics and Engineering

J. Geophys. Eng. **15** (2018) 2150–2158 (9pp)
https://doi.org/10.1088/1742-2140/aac009

The precursory information of acoustic emission during sandstone loading based on critical slowing down theory

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Received 11 April 2018
Accepted for publication 25 April 2018
Published 26 June 2018

Abstract

In order to study the critical slowing down characteristics of acoustic emission signals produced by the failure of the sandstone, the YAW4306 computer controlled electro-hydraulic servo press and the CTA-1 type acoustic emission data acquisition system were used to carry out the sandstone uniaxial compression experiment. The experiment results show that the acoustic emission signals produced by the destruction of sandstone have obvious stage characteristics and the cumulative acoustic emission count has good correspondences with the damage of sandstone. The increase of slope of the cumulative count curve of the acoustic emission indicates that the destruction of sandstone is coming. The autocorrelation coefficient and the variance of the critical slowing down characteristics are studied in different lag step lengths and window lengths. It can be found that different window lengths and lag step lengths have influences on the autocorrelation coefficient and the variance. When the window length is equal, the autocorrelation coefficient cures corresponding to different lag step lengths are messy and are consistent only in some parts, and the corresponding variance cures are basically coincident. Before the sample is destroyed, the small increases of the variance curve represent the occurrence of large cracks. The sudden and large-scale increase of the autocorrelation coefficient and the variance of the acoustic emission count can be used as the precursor signal of the destruction of the sandstone sample. Compared with the autocorrelation coefficient, the precursor signal of variance is more obvious. This is of great theoretical and guiding significance for enriching and improving the acoustic emission monitoring technology of rock mass.

Keywords: acoustic emission, precursory information, sandstone, critical slowing down

(Some figures may appear in colour only in the online journal)

1. Introduction

Mutation is widespread in nature. The evolution of complex chaotic systems often faces transitions between different states. This shift is huge and catastrophic. In recent years, the critical phenomena of complex dynamic systems have received widespread attention in different subject areas (Scheffer *et al* 2009, Scheffer 2009, Gopalakrishnan *et al* 2016). For example, there shows certain critical transition characteristics before the transition from the stable state to the chaotic state in the ecosystem, the forests, lakes, deserts, and biological populations (Scheffer *et al* 2001, Anderson *et al* 2008, Vasilakopoulos and Marshall 2015, Beck *et al* 2018). In climate dynamic systems, abrupt shifts showed that they were all preceded by a characteristic slowing down of the fluctuations starting well before the actual shift (Dakos *et al* 2008, Lenton *et al* 2008). In seismic dynamic systems, the fractal and critical point characteristics of seismic activity have also been recognized by many scholars (Bak and Tang 1989, Godano *et al* 1993, Yan *et al* 2011). In addition, some major diseases in medicine and major events in the financial market have also shown critical transition characteristics (Mcsharry *et al* 2003, Venegas *et al* 2005, May *et al* 2008, Gidea 2017, Wang and Zou 2017). It can be seen that the phenomenon of critical slowdown has a certain degree of universality before sudden catastrophes. At the same time, it shows great potential in explaining sudden catastrophic events in complex chaotic systems and finding mutation precursors.

The geological activities of rock masses have chaotic characteristics in space and time, and the evolution process of activities often shows complex dynamic characteristics. During the loading failure process of rock, with the change of the external load, the internal strain energy of the rock gradually accumulates, and eventually it loses its stability and the energy is released. In this process, the energy accumulation can be regarded as one phase state. When the sample is destroyed, the large release of energy is regarded as another phase state. The process of phase transition occurs from the accumulation of energy to its release. The released energy includes surface potential energy, electromagnetic radiation energy, thermal energy, acoustic emission energy, etc in the process (Eccles *et al* 2005, Wang *et al* 2011, Kong *et al* 2017, Li *et al* 2018). The acoustic emission energy contains a large amount of information on the evolution of rock damage (Zhang *et al* 2016). Studying and analyzing it can provide insights into the process of rock failure. Many scholars have found that a series of critical slowing down features occur when rock is close to breaking, including significant increases in the count and density values of acoustic emission per unit time, as well as a long period of time (Lei *et al* 2004, Kong *et al* 2015, Yan-Hua *et al* 2016). These phenomena all indicate that there is a critical shift in the acoustic emission signals generated during rock failure.

As a non-destructive monitoring technology, acoustic emission technology has been widely used in the monitoring and early warning of related rock geological activities (Unander 2004, Shiotani 2006, Agioutantis *et al* 2016). Whether it is in the ore mining, tunnel excavation, or natural mountain landslide landslides, abnormal rock burst failure will cause a series of disaster accidents. Precise forecasting and effective preventive measures before the occurrence of geological disasters are crucial. Rock fracture experiment is an important physical simulation experiment to study related geological disasters. The acoustic emission phenomenon caused by the destruction of the rock by the external load has been studied by many scholars. The characteristics of spatio-temporal evolution of acoustic emission signals generated by rock failure (Li *et al* 2010, Moradianz *et al* 2012), the relationship between damage and acoustic emission signals (Moradian *et al* 2010, Khazaei *et al* 2015), and the processing and conversion of acoustic emission signals (Hirata *et al* 1987, Rao and Lakshmi 2005, He *et al* 2010, Kong *et al* 2016, Siracusano *et al* 2016, Kong *et al* 2017) were introduced by many scholars. However, in the conclusions drawn by the predecessors, the research on the critical slowing down characteristics of acoustic emission signals generated by rock destruction is rarely mentioned. Based on previous studies, we carried out sandstone uniaxial compression experiments and obtained acoustic emission counts. Based on the theory of critical transitions, the acoustic emission signals generated by rock mass destruction are analyzed. The autocorrelation coefficient and variance that characterize the critical transition phenomenon show sudden and large-scale increases before the rock fracture. That can be used as precursors to rock failure. It has certain theoretical and guiding significance for further enriching the early warning information of rock mass destruction and instability.

2. Experiment

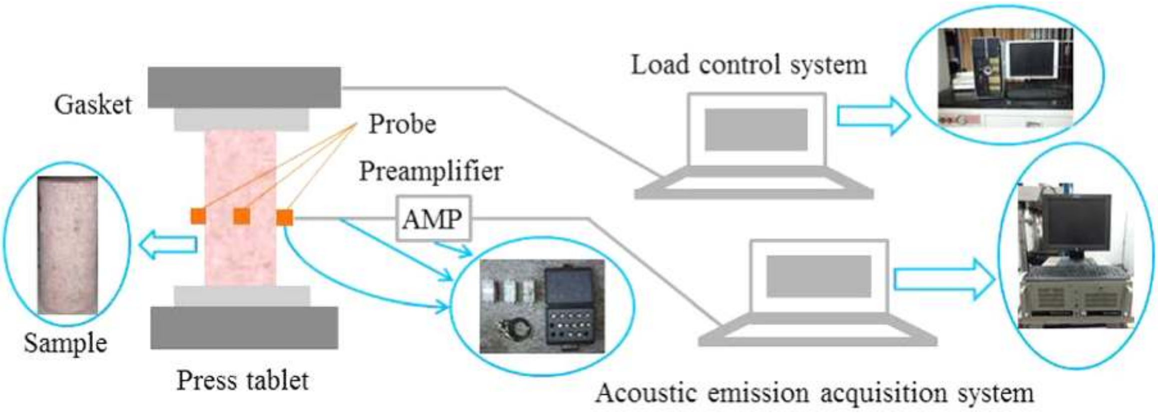
2.1. Experimental sample

The test samples were taken from the Tongjialiang Mine, Shanxi, China. Through the processing of the sandstone obtained in the field, the standard sandstone samples with 50 mm in diameter and 100 mm in height were prepared. The parallelisms at both ends of the specimen satisfy the experimental requirements. Through measuring, the density of the experimental samples was about 2.62–2.71 $\mathrm{g\,cm^{-3}}$ and the porosity was about 4.34%–5.26%. In the paper, five sandstone samples were performed under uniaxial loading with force control of 300 $\mathrm{N\,s^{-1}}$.

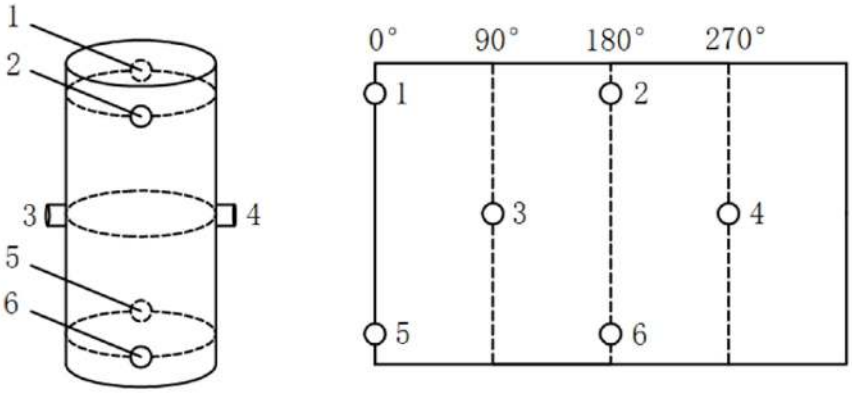
2.2. Experimental system and method

The diagram of the experimental device is shown in figure 1(a). The experimental system includes loading control system and acoustic emission acquisition system. The loading control system adopts YAW4306 computer control electro-hydraulic servo press. The maximum load is 3 MN and the experimental force resolution is 1/300 000. The relative error of the displayed value of the experiment force is ±1% and the loading rate rang is 0–60 $\mathrm{kN\,s^{-1}}$. Mechanical tests such as uniaxial compression, tension, cyclic loading and creep can be carried out by means of force and displacement control methods. The experimental loading method is force-controlled loading. The loading rate was 300 $\mathrm{N\,s^{-1}}$.

The acoustic emission acquisition system adopts the CTA-1 type acoustic emission data acquisition system produced by the United States Physical Acoustics company. The system can simultaneously collect acoustic emission data from 8 channels at high speed. There are six acoustic emission sensors on the surface of the sandstone sample and they are divided into three groups. The first group of acoustic emission probes is 10 mm from the upper plane, the second group is 50 mm, and the third group is 90 mm. The first group is parallel to the third group. The first group and the third group are orthogonal to the second group, respectively. The



(a) Experimental device



(b) Location of acoustic emission probes

Figure 1. The experimental system diagram.

position of the acoustic emission probe is shown in figure 1(b). The threshold values of the preamplifier and the acoustic emission acquisition system are both set to 45 dB. The sampling frequency is 1 MHz. Before the beginning of the experiment, the instruments were debugged. The experiment was carried out after the debugging.

3. Critical slowing down theory

Critical slowing down is the concept of statistical physics. When the system changes from one phase to another, the phenomenon of dispersion and fluctuation which is favorable for the formation of new phase will appear near the critical point (Scheffer 2009). The phenomenon of dispersion and fluctuation not only shows an increase in amplitude, but also shows the phenomenon that the fluctuation time is lengthened, the disturbance recovery rate becomes slow, and the ability to return to the old phase becomes smaller. This phenomenon that time is lengthened, the rate of recovery slows down and the recovery ability becomes smaller are called slowing down (Wissel 1984, van Nes Egbert and Scheffer 2007, Brock 2009, Scheffer 2009). When complex dynamical systems are approaching the critical point, such as ecosystems, climate systems, the critical slowing down often leads to the increase of autocorrelation coefficient and variance of one parameter of the system (Brock 2006, Dakos *et al* 2008, Scheffer 2009). Here refers to the acoustic emission count value of sandstone samples during uniaxial compression failure.

The variance is the characteristic quantity describing the degree of deviation of the sample data to the mean $\bar{x}$, denoted as s^2:

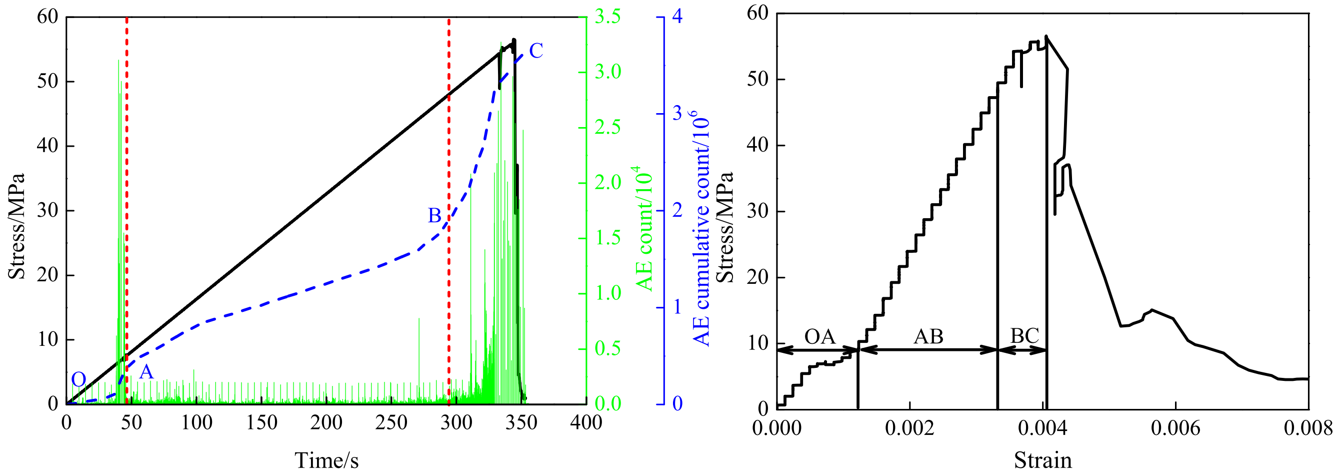
$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2, \tag{1}$$

where x_i represents the ith data and n is the number of data produced in the sample.

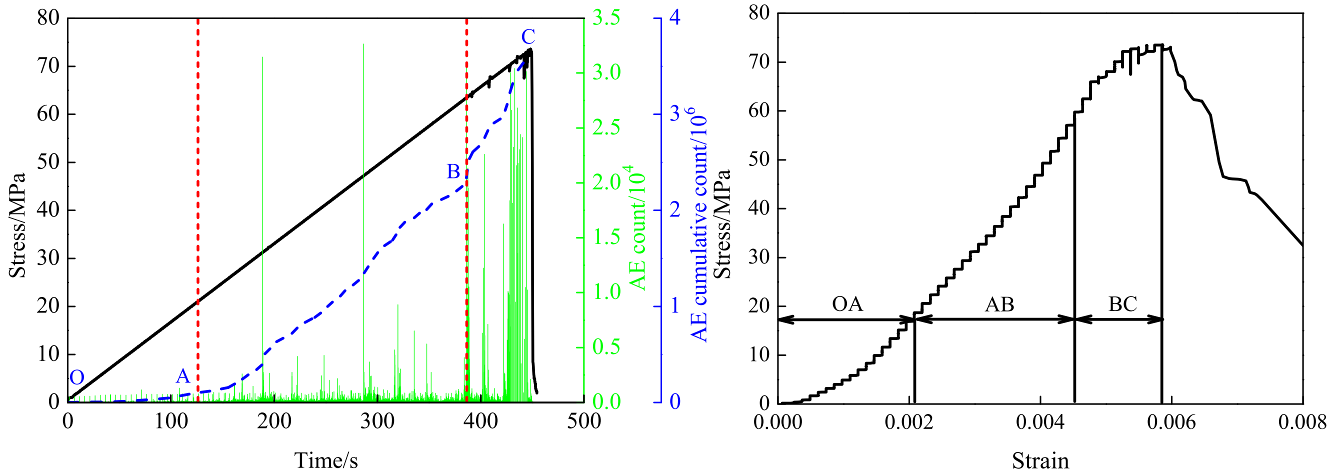
The autocorrelation coefficient is the statistic describing the correlation between different moments of the same variable, and the autocorrelation coefficient that the variable x lags length j is denoted by $\alpha(j)$:

$$\alpha(j) = \sum_{i=1}^{n-j}\left(\frac{x_i - \bar{x}}{s}\right)\left(\frac{x_{i+j} - \bar{x}}{s}\right). \tag{2}$$

It is assumed that the state variable has a forced perturbation of period Δt, which is approximately exponential in the process of disturbance, and the recovery rate is λ. In the



（a）Sample 2 （b）Sample 2



（c）Sample 5 （d）Sample 5

Figure 2. Characteristics of acoustic emission signals.

regression model, it can be described as:

$$y_{n+1} = \mathrm{e}^{\lambda\Delta t} y_n + s\varepsilon_n, \tag{3}$$

where y_n is the deviation from the system variable to the equilibrium state, ε_n is the random quantity that matches the normal distribution. If λ and Δt do not depend on y_n, the process can be simplified as:

$$y_{n+1} = \alpha y_n + s\varepsilon_n, \tag{4}$$

where the autocorrelation coefficient $\alpha = \mathrm{e}^{\lambda\Delta t}$. Equation (4) is analyzed by variance:

$$\mathrm{Var}(y_{n+1}) = E({y_n}^2) + (E(y_n))^2 = \frac{s^2}{1 - \alpha^2}. \tag{5}$$

When the system is approaching the critical point, the recovery rate of small amplitude disturbances is getting slower (van Nes Egbert and Scheffer 2007). When the recovery rate λ approaches 0, the autocorrelation coefficient α will approach 1 and the variance will approach infinity. Therefore, the autocorrelation coefficient and variance can be regarded as the precursor signal of the system approaching the critical point (Scheffer 2009, Brock 2009).

4. Results and analysis

4.1. Characteristics of acoustic emission signals during sandstone destruction

The acoustic emission signals generated during the destruction process of the rock showed obvious stage characteristics (Mogi 1962, Lei *et al* 2004, Kong *et al* 2015, Yan-Hua *et al* 2016). The sandstone samples were subjected to uniaxial compression until failure, and the time-stress curve and acoustic emission count series were obtained. Due to the space limitation of the paper, only the experimental results of the sample 2 and the sample 5 were shown in the paper. Figures 2(a) and (c) show the relationship between acoustic emission signals and the time in the process of sandstone sample failure. Figures 2(b) and (d) show the stress–strain curve generated during the destruction of the sample. According to the relationship between the cumulative count of acoustic emission and time, combined with the stress–strain curve, it can be seen that the characteristics of acoustic emission signals generated by different stages of sandstone failure process under uniaxial loading are different. In general, before the destruction of the sample, the acoustic emission signals can be divided into three stages: initial compaction stage, slow growth stage, and rapid growth stage.

Initial compaction stage (OA): sandstone, as a porous medium, contains many primary cracks and pores. In the initial stage of loading, the original cracks and pores in the sandstone were compacted under external loads. Strain energy accumulated at this stage. As shown in figures 2(a) and (c), the number of acoustic emission events generated during this phase was small and the growth was slow. The acoustic emission counts were low at the initial compaction stage.

Slow growth stage (AB): with the loading, the original cracks and the pores in the sandstone became compacted. Then the sample entered the elastic phase and new cracks began to steadily develop. This stage was called the slow growth stage. At this stage, acoustic emission signals were more active than the previous stage. The number of acoustic emission counts increased and the curve of acoustic emission cumulative counts presented a slow growth trend.

Rapid growth stage (BC): at the end of the loading, the sandstone sample entered the yielding stage and the failure stage. Under the external load, the new micro-cracks generated in the sandstone constantly expanded, accumulated and penetrated, and finally formed large cracks in the direction of the principal stress, resulting in a large number of acoustic emission events. The curve of acoustic emission cumulative counts increased rapidly and then the sandstone samples were destroyed.

In summary, it can be seen that the acoustic emission signals generated in the sandstone failure process showed obvious phase characteristics. In the process of loading, the acoustic emission cumulative count corresponded well to failure process of sandstone sample. The samples were destroyed after the sudden increase of the slope of the curve of the acoustic emission cumulative counts.

4.2. Analysis of critical slowing down characteristics of sandstone failure

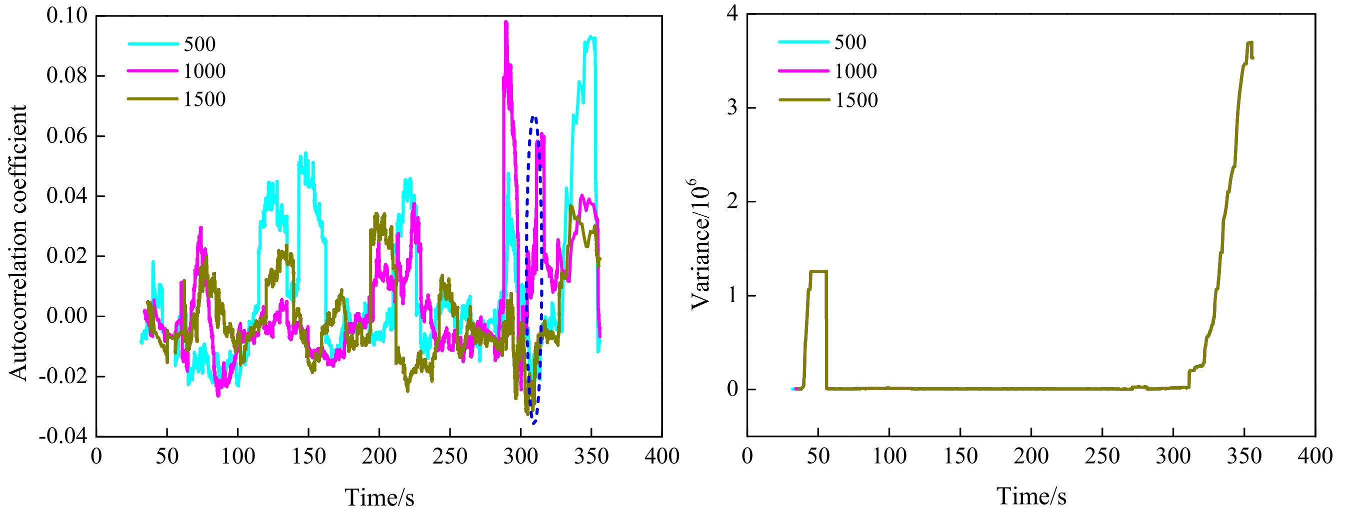
4.2.1. Critical slowing down window length and lag step length. The window length and lag step length were related to the stability of the autocorrelation coefficient and the variance in the critical transitions (Dakos *et al* 2008, Scheffer 2009, Dakos *et al* 2012). Thus, the acoustic emission count sequences obtained from the uniaxial compression experiments of sandstone were studied with different window lengths and lag step lengths. The window length represented the basic unit for sequence computation. The lag step length represented the length of the lag sequence from the sequence of selected window length to another identical sequence. The variance was the variance of the new sequence that was obtained by lagging the fixed step length of the selected window length. The autocorrelation coefficient was the correlation between the sequence of the selected window length and the new sequence obtained by that the selected window length was lagged by the fixed step length. First, the window length was taken as 3000 and the lag step length was taken as 500, 1000, 1500 respectively. The effects of different lag step lengths on the autocorrelation coefficient and variance were compared under the same window lengths. Then, the effects of different lag step lengths under the same window length on the autocorrelation coefficient and variance were compared with the lag step length of 1500 and the window length of 2000, 2500, 3000 respectively. Due to the space limitation, only the experimental results of the sample 2 and the sample 5 were analyzed in the paper. The remaining groups are basically consistent with the two groups listed.

Take the window length 3000 and compare the influence of different lag step lengths on autocorrelation coefficient and variance. As shown in figure 3, it can be seen that when the window length was the same, the curves of autocorrelation coefficients corresponding to different lag step lengths were messy. The autocorrelation coefficient curves did not show certain regularities with changes of the lag step lengths, but only partially in the same trend. Compared with the autocorrelation coefficient, the variance curves with different lag steps under the same window length were basically coincident and the trends of curve changes are the same. That was, the variance curve did not change with the change of the lag step length.

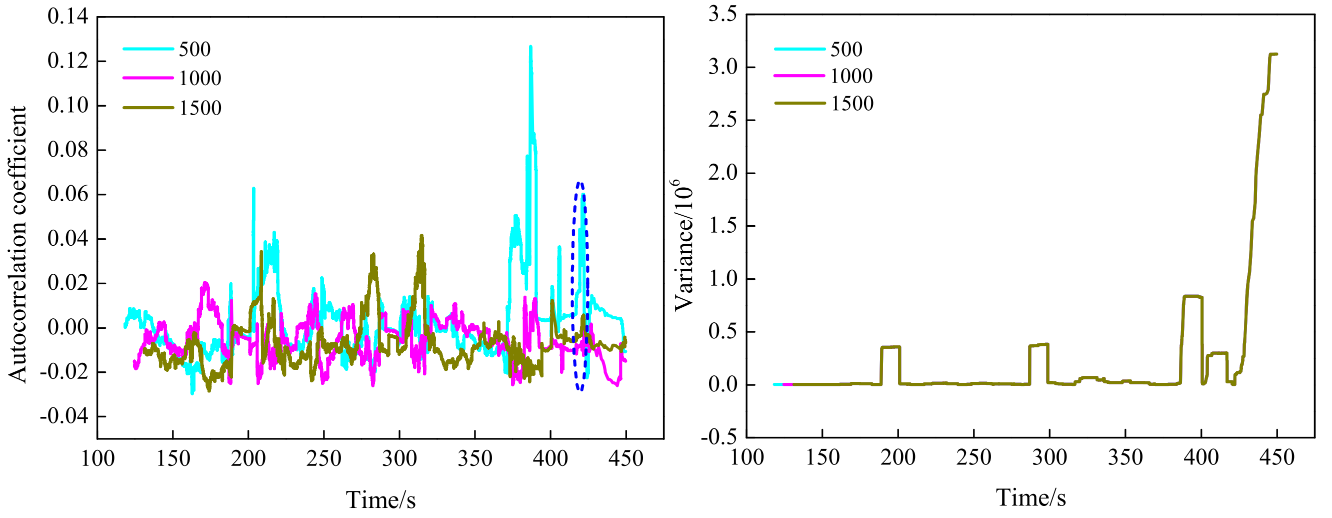
Take the lag step length 1500 and compare the influence of different window lengths on autocorrelation coefficient and variance. As shown in figure 4, it can be seen that the fluctuation trends of the autocorrelation coefficient curves corresponding to the lengths of different windows under the same lag step length were the same, and the fluctuation was gradually stabilized with the increase of window length. The trends of the variance curves with the change of the window length were the same. The amplification of the variance curve at the inflection point decreased with the increase of the window length.

Through the comparison, it can be found that the different window lengths and lag step lengths had influences on the stability of the autocorrelation coefficient cure and the variance cure. When the window length was equal, the autocorrelation coefficient curves of different lag step lengths were messy, and the variance curves were basically coincident. When the lag step length was equal, the autocorrelation coefficient cures corresponding to the different window lengths were in the same trend and the fluctuation tends to be stable with the increase of the window length. The trends of the variance curves were consistent and the amplification at the inflection point decreased with the increase of the length of the window. There were more peaks in the autocorrelation coefficient curve through the comparison between the autocorrelation coefficient curve and the variance curve. That was, more spurious signals were generated in the autocorrelation coefficient curve. Therefore, compared with the autocorrelation coefficient, the change of variance was more obvious in characterizing the precursor signals of sandstone failure.

4.2.2. Precursory characteristics of critical slowing down in sandstone failure. In the process of studying the autocorrelation coefficient and the variance of critical transition characteristics in the dynamics system, the autocorrelation



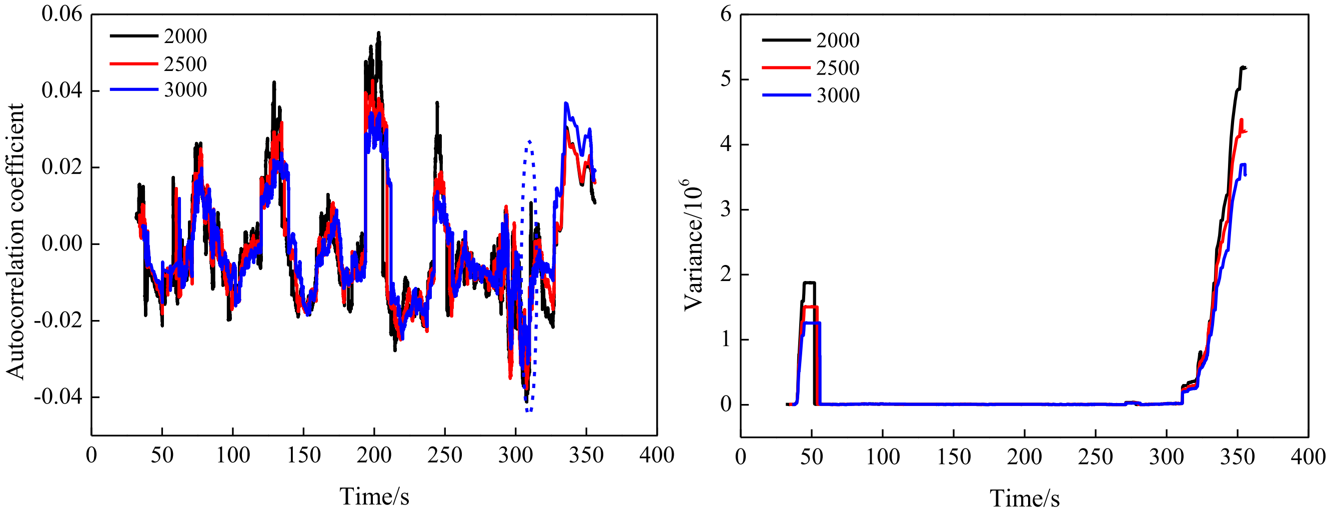
（a） Sample 2 （b） Sample 2



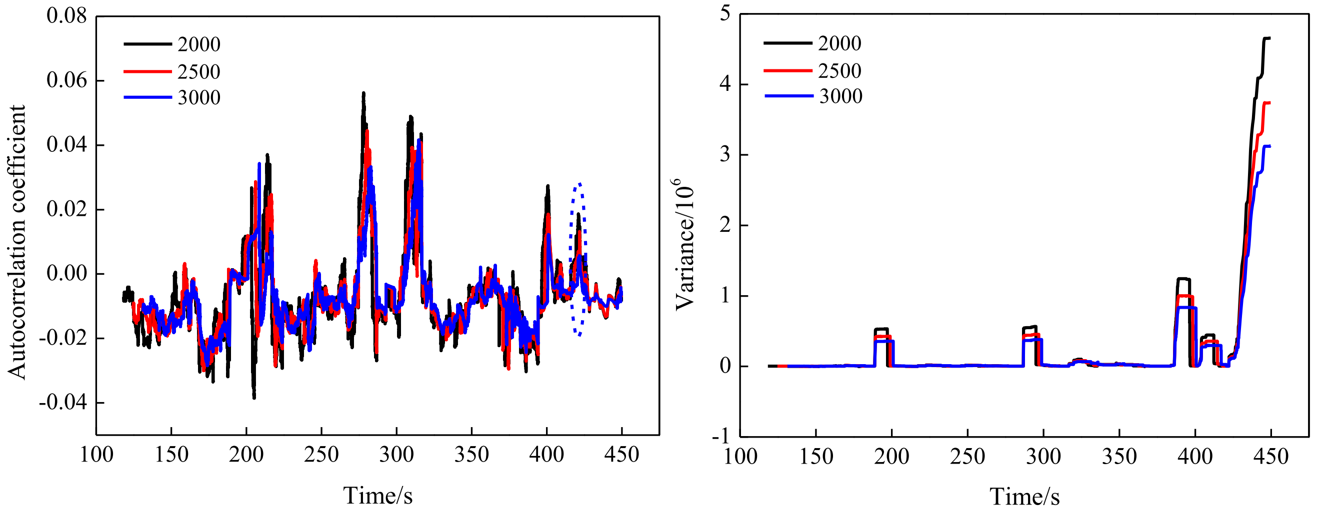
（c） Sample 5 （d） Sample 5

Figure 3. Autocorrelation coefficient and variance curve of different lag step lengths in the same window length.

and variance had their own advantages and disadvantages for different system parameter data (Dakos *et al* 2012, Wu *et al* 2015). The amount of data that characterized system parameters was important. The greater the amount of data selected, the more reliable the early warning signal in the system (Dakos *et al* 2008, Carpenter *et al* 2009, Carpenter *et al* 2011, Seekell *et al* 2011). The number of the data in the lag step length and the window length had certain influences on the stabilities of the autocorrelation coefficient and the variance. Generally, in suitable ranges, the longer the length of the lag step and the window length, the more stable the autocorrelation coefficient and the variance, and the more favorable the discovery of the early warning signal. From the above comparison, it can be seen that there were fewer spurious signals generated by the variance curve than the autocorrelation coefficient curve. Besides, the precursor signal was more obvious in the variance curve. Therefore, we analyzed the relationship between the variance and the acoustic emission counts by using the window length of 3000 and the lag step of 1500. As shown in figure 5, it can be seen that before the sandstone destruction, there was small fluctuations in stress curve at 44 s on sample 2 and 193 s, 292 s, 392 s and 408 s on sample 5, indicating that large cracks are generated at the time. At the same time, the acoustic emission counts produced responses and the corresponding variance curves also produced sudden increases. It can be known that the variation of the variance curve in the process of sandstone destruction had good correspondences with the acoustic emission signals. The small increases in variance curve can indicate the occurrence of larger cracks. It can be seen that the variance of sample 2 suddenly increased at 306 s, 86.1% of the time, 86.3% of the stress. Then the sample was destroyed after 49 s and the variance stop growing. In figures 3(a) and 4(a), the autocorrelation coefficient experienced the same sudden increase at 306 s. Similarly, the variance of sample 5 at 421 s, 93% of the time, 93% of the stress, appeared the same increase. At the same time, the autocorrelation coefficients of sample 5 in figures 3(c) and 4(c) also yielded responses. Then the sample 5 was destroyed after 27 s. At the same time point,

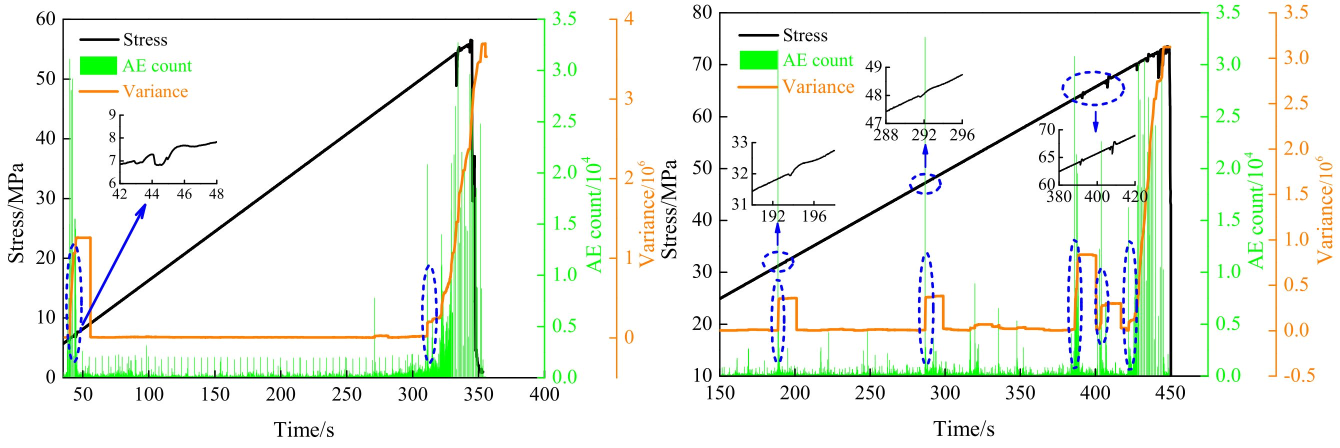


（a） Sample 2 （b） Sample 2



（c） Sample 5 （d） Sample 5

Figure 4. Autocorrelation coefficients and variance curves of different window lengths in the same lag step length.



（a） Sample 2 （b） Sample 5

Figure 5. Precursor signals characteristic.

Table 1. The ratio of variance mutational point of the sample to the failure point in the time and load.

Sample	1	2	3	5
Time	0.819	0.861	0.871	0.93
Load	0.827	0.863	0.869	0.93

the acoustic emission counts and the acoustic emission counts density per unit time of sample 2 and sample 5 produced sudden increases. The rest of the samples also produced basically consistent phenomenon. The ratio of all samples that mutation time and load point of the variance to specimen failure time and load point are shown in table 1 (the data of sample 4 is lost).

As a non-destructive monitoring technology, acoustic emission technology has been widely used in geotechnical engineering monitoring. Based on the theory of critical slowing down, this paper studies the critical slowing down characteristics of acoustic emission in the process of rock mass failure. The autocorrelation coefficient and variance which characterized the critical slowing down of rock mass failure were obtained. The variance was significantly increased before rock failure and the warning characteristics were relatively obvious. This is of great theoretical significance for enriching the acoustic emission monitoring technology of rock mass.

5. Conclusion

In this paper, the experiment of sandstone uniaxial compression was carried out on the basis of previous studies. Based on the theory of critical slowing down, the acoustic emission counts during the failure process of sandstone were analyzed, and the following conclusions were obtained.

(1) During the whole loading process, acoustic emission signals produced by sandstone failure showed obvious phase characteristics. The cumulative count of acoustic emission corresponded well to the damage and failure process of sandstone. The increase of the slope of the acoustic emission cumulative count curve indicated that the sandstone will be destroyed soon.

(2) Different window lengths and lag step lengths had effects on the autocorrelation coefficient and variance of acoustic emission counts that characterize the destruction of sandstone. When the length of the window was equal, the autocorrelation coefficient cures corresponding to different lag step lengths were messy, which were consistent only in some parts, and the corresponding variance cures were basically coincident. When the lag step length was equal, the fluctuation trends of the autocorrelation coefficient cure and variance cure of different window lengths were basically consistent, and the fluctuation tended to be stable with the increase of the window length.

(3) Before the sandstone was destroyed by loading, the small increase of the variance curve of the critical slowing down represented the occurrence of large cracks during the loading process of sandstone. The sudden and large-scale increase of the autocorrelation coefficient cure and the variance cure can be used as the precursor signals of sandstone failure. Compared with the autocorrelation coefficient cure, the variance cure was more obvious.

Acknowledgments

The research described in this paper was financially supported by the National Natural Science Foundation of China (51674254), the State Key Research Development Program of China (Grant No. 2016YFC0801404), State Key Laboratory of Coal Resources and Safe Mining, CUMT (SKLCRSM15X03), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher, Education Institutions (PAPD).

References

Agioutantis Z, Kaklis K, Mavrigiannakis S, Verigakis M, Vallianatos F and Saltas V 2016 Potential of acoustic emissions from three point bending tests as rock failure precursors *Int. J. Min. Sci. Technol.* **26** 155–60

Anderson C N *et al* 2008 Why fishing magnifies fluctuations in fish abundance *Nature* **452** 835–9

Bak P and Tang C 1989 Earthquakes as a self-organized critical phenomenon *J. Geophys. Res. Solid Earth* **94** 15635–7

Beck K K *et al* 2018 Variance and rate-of-change as early warning signals for a critical transition in an aquatic ecosystem state: a test case from Tasmania, Australia *J. Geophys. Res.: Biogeosci.* **123** 495–508

Brock W A 2006 Rising variance: a leading indicator of ecological transition *Ecol. Lett.* **9** 311–8

Brock W A 2009 Early-warning signals for critical transitions *Nature* **6** 53–9

Carpenter S R, Brock W A, Cole J J and Pace M L 2009 Leading indicators of phytoplankton transitions caused by resource competition *Theor. Ecol.* **2** 139–48

Carpenter S R *et al* 2011 Early warnings of regime shifts: a whole-ecosystem experiment *Science* **332** 1079–82

Dakos V, Nes E H V, D'Odorico P and Scheffer M 2012 Robustness of variance and autocorrelation as indicators of critical slowing down *Ecology* **93** 264–71

Dakos V, Scheffer M, Nes E H V, Brovkin V, Petoukhov V and Held H 2008 Slowing down as an early warning signal for abrupt climate change *Proc. Natl. Acad. Sci. USA* **105** 14308–12

Eccles D, Sammonds P R and Clint O C 2005 Laboratory studies of electrical potential during rock failure *Int. J. Rock Mech. Min. Sci.* **42** 933–49

Gidea M 2017 Topological data analysis of critical transitions in financial networks *Int. Winter School and Conf. on Network Science (NetSci-X 2017)* (Cham: Springer) (https://doi.org/10.1007/978-3-319-55471-6_5)

Godano C, Alonzo M L and Caruso V 1993 Self-organized criticality and earthquake predictability *Phys. Earth Planet. Inter.* **80** 117–23

Gopalakrishnan E A, Sharma Y, John T, Dutta P S and Sujith R I 2016 Early warning signals for critical transitions in a thermoacoustic system *Nature* **6** 53

He M C, Miao J L and Feng J L 2010 Rock burst process of limestone and its acoustic emission characteristics under true-triaxial unloading conditions *Int. J. Rock Mech. Min. Sci.* **47** 286–98

Hirata T, Satoh T and Ito K 1987 Fractal structure of spatial distribution of microfracturing in rock *Geophys. J. Int.* **90** 369–74

Khazaei C, Hazzard J and Chalaturnyk R 2015 Damage quantification of intact rocks using acoustic emission energies recorded during uniaxial compression test and discrete element modeling *Comput. Geotech.* **67** 94–102

Kong B, Wang E, Li Z, Wang X, Chen L and Kong X 2016 Nonlinear characteristics of acoustic emissions during the deformation and fracture of sandstone subjected to thermal treatment *Int. J. Rock Mech. Min. Sci.* **90** 43–52

Kong X, Wang E, He X, Li D and Liu Q 2017 Time-varying multifractal of acoustic emission about coal samples subjected to uniaxial compression *Chaos Solitons Fractals* **103** 571–7

Kong X *et al* 2015 Critical slowing down on acoustic emission characteristics of coal containing methane *J. Nat. Gas Sci. Eng.* **24** 156–65

Lei X *et al* 2004 Detailed analysis of acoustic emission activity during catastrophic fracture of faults in rock *J. Struct. Geol.* **26** 247–58

Lenton T M *et al* 2008 Tipping elements in the Earth's climate system *Proc. Natl. Acad. Sci. USA* **105** 1786–93

Li Y H, Liu J P, Zhao X D and Yang Y J 2010 Experimental studies of the change of spatial correlation length of acoustic emission events during rock fracture process *Int. J. Rock Mech. Min. Sci.* **47** 1254–62

Li Z H, Lou Q, Wang E Y, Liu S J and Niu Y 2018 Study on acoustic–electric–heat effect of coal and rock failure processes under uniaxial compression *J. Geophys. Eng.* **15** 71–80

May R M, Levin S A and Sugihara G 2008 Complex systems: ecology for bankers *Nature* **451** 893

Mcsharry P E, Smith L A and Tarassenko L 2003 Prediction of epileptic seizures: are nonlinear methods relevant? *Nat. Med.* **9** 241

Mogi K 1962 Study of elastic shocks caused by the fracture of heterogeneous materials and its relations to earthquake phenomena *Bull. Earthq. Res. Inst.* **40** 125–73

Moradianz A, Ballivy G and Rivard P 2012 Correlating acoustic emission sources with damaged zones during direct *Rev. Can. De Géotechnique* **49** 710–8

Moradian Z A, Ballivy G, Rivard P, Gravel C and Rousseau B 2010 Evaluating damage during shear tests of rock joints using acoustic emissions *Int. J. Rock Mech. Min. Sci.* **47** 590–8

Rao M V M S and Lakshmi K J P 2005 Analysis of b-value and improved b-value of acoustic emissions accompanying rock fracture *Curr. Sci.* **89** 1577–82

Scheffer M 2009 *Critical Transitions in Nature and Society* (Princeton, NJ: Princeton University Press)

Scheffer M, Bascompte J, Brock W A, Brovkin V, Carpenter S R, Dakos V, Held H, van Nes E H, Rietkerk M and Sugihara G 2009 Early-warning signals for critical transitions *Nature* **461** 53–9

Scheffer M, Carpenter S, Foley J A, Folke C and Walker B 2001 Catastrophic shifts in ecosystems *Nature* **413** 591–6

Seekell D A, Carpenter S R and Pace M L 2011 Conditional heteroscedasticity as a leading indicator of ecological regime shifts *Am. Naturalist* **178** 442–51

Shiotani T 2006 Evaluation of long-term stability for rock slope by means of acoustic emission technique *NDT E Int.* **39** 217–28

Siracusano G *et al* 2016 A framework for the damage evaluation of acoustic emission signals through Hilbert–Huang transform *Mech. Syst. Signal Process.* **75** 109–22

Unander T E 2004 Analysis of acoustic emission waveforms in rock *Res. Nondestruct. Eval.* **15** 119–48

van Nes Egbert H and Scheffer M 2007 Slow recovery from perturbations as a generic indicator of a nearby catastrophic shift *Am. Naturalist* **169** 738–47

Vasilakopoulos P and Marshall C T 2015 Resilience and tipping points of an exploited fish population over six decades *Glob. Change Biol.* **21** 1834–47

Venegas J G *et al* 2005 Self-organized patchiness in asthma as a prelude to catastrophic shifts *Nature* **434** 777–82

Wang E, He X, Liu X, Li Z, Wang C and Xiao D 2011 A non-contact mine pressure evaluation method by electromagnetic radiation *J. Appl. Geophys.* **75** 338–44

Wang G and Zou X 2017 Qualitative analysis of critical transitions in complex disease propagation from a dynamical systems perspective *Int. J. Bifurcation Chaos* **26** 1650239

Wissel C 1984 A universal law of the characteristic return time near thresholds *Oecologia* **65** 101–7

Wu H, Hou W, Yan P-C, Zhang Z-S and Wang K 2015 A study of the early warning signals of abrupt change in the pacific decadal oscillation *Chin. Phys.* B **24** 662–73

Yan-Hua H *et al* 2016 An experimental study on fracture mechanical behavior of rock-like materials containing two unparallel fissures under uniaxial compression *Acta Mech. Sin.* **32** 442–55

Yan R, Jiang C and Zhang L 2011 Study on critical slowing down phenomenon of radon concentrations in water before the Wenchuan MS8.0 earthquake *Chin. J. Geophys.* **54** 1817–26

Zhang Z, Wang E, Chen D, Li X and Li N 2016 The observation of ae events under uniaxial compression and the quantitative relationship between the anisotropy index and the main failure plane *J. Appl. Geophys.* **134** 183–90