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The preferential formation of high-mass stars in shocked interstellar gas layers

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ABSTRACT

Gravitationally unstable, shocked layers of interstellar gas are produced by cloud-cloud collisions and by expanding nebulæ around massive stars. We show that the resulting fragments are likely to be of high mass ($\gtrsim 7\ \mathrm{M}_\odot$), and initially well separated (i.e. weakly bound to one another, if at all).

This result may explain why dynamically active regions tend to have a high efficiency of massive star formation, and why they tend to relax quickly into a self-propagating mode which generates sequences of OB subgroups. These tendencies are manifested on many scales, from local star-forming regions like Orion, through regions like 30 Doradus in the LMC, to the most IR-luminous starburst galaxies.

We also show that, for a wide range of input parameters, gravitational fragmentation of a shocked layer occurs when the column density of hydrogen nuclei through the accumulating layer reaches a value $\sim 6 \times 10^{21}$ cm^{-2}. This may be one reason for the mass-radius relation for molecular cloud clumps first noted by Larson.

Key words: instabilities – shock waves – stars: formation – ISM: bubbles – ISM: clouds.

1 INTRODUCTION

Giant molecular clouds (GMCs) in which massive stars are forming are observed to be in a highly agitated dynamical state (e.g. Genzel & Stutzki 1989). By massive stars we mean those with masses $\gtrsim 7\ \mathrm{M}_\odot$, i.e. the progenitors of supernovæ. Therefore this high degree of agitation may be both a consequence of massive star formation (due to the injection of kinetic energy into the surrounding gas by the newly formed massive stars), and a cause of massive star formation (in the sense that dynamical agitation predisposes the gas to form massive stars). Elmegreen & Lada (1977) have proposed a self-propagating mode of star formation, in which the massive stars in one generation excite an H II region; this then expands, sweeping up a shell of shocked cool neutral gas, and eventually this shell fragments and condenses to produce a new generation of stars.

This proposal explains rather elegantly the sequences of subgroups seen in OB associations like Orion (Blaauw 1991). However, it requires that a significant fraction of the stars condensing out of a shell around an expanding H II region be of high mass. We have found in the literature no compelling explanation of why this should be the case. This paper goes some way towards furnishing such an explanation, by demonstrating, in a very general and robust manner, that shocked layers generated by a wide variety of mechanisms (colliding clouds, expanding H II regions, stellar wind bubbles and supernova remnants) are none the less equivalent in three key respects. (i) Shocked layers have roughly the same column density, $\sim 4-8 \times 10^{21}$ cm^{-2}, at the stage in their accumulation when they start to fragment gravitationally; this may be an important ingredient in the explanation of Larson's empirical relations between mass, radius and velocity dispersion for GMC clumps (Larson 1981). (ii) Gravitational fragmentation of a shocked layer produces fragments which are massive, even in the worst case $\gtrsim 7\ \mathrm{M}_\odot$. The precise relation between fragment mass and final stellar mass depends on the subsequent evolution of the fragment, which may involve further fragmentation and/or agglomeration, but is outside the scope of this paper. However, we suggest that massive fragments are likely sometimes to engender massive stars, and numerical simulations of protostars condensing out of shocked layers (Chapman et al. 1994a,b) indicate that this is so. (iii) In all cases, the mean fragment mass increases as the sound speed in the shocked layer to a power in the range (3.5, 4.0).

The fragmentation of shocked layers has been considered by several authors, e.g. Elmegreen & Elmegreen (1978), Ostriker & Cowie (1981), Vishniac (1983), Bertschinger (1986), McCray & Kafatos (1987), Elmegreen (1989b) and Lubow & Pringle (1993). Much of this work has been concerned with linear stability analysis and the dynamical instabilities which develop in an accumulating layer long before it becomes gravitationally unstable and spawns protostellar condensations. Vishniac (1983) suggests that these dynamical instabilities may lead to the layer disintegrating before it can become gravitationally unstable. However, a less extreme (and we believe more likely) outcome of dynamical instability is that it generates weak turbulence and thereby sows the mild density structure which acts as a seed for subsequent gravitational instability. Simulations performed by Mac Low & Norman (1993) indicate that this is the case. The other effect of weak (i.e. only very mildly supersonic) turbulence is to increase the effective sound speed a_s in the shocked gas. This would substantially strengthen the conclusions we draw.

Alternatively, the seed for gravitational instability in a shocked layer may be provided by pre-existing density structure in the unshocked gas: colliding clouds are likely to have internal structure before they collide, and the medium into which a nebula expands is likely to be very inhomogeneous. We do not consider this aspect of shocked layers in the current paper, but again we note that in numerical simulations (Chapman et al., in preparation) the masses of fragments condensing out of a shocked layer are not well correlated with the masses of pre-existing clumps; the fragment masses are determined largely by the thermodynamic processes which control the temperature of the gas, and this is also what we find here.

Elmegreen & Elmegreen (1978) derive an expression for the fastest-growing gravitationally unstable mode in a shell around an expanding H II region, but do not identify a particular moment at which non-linear fragmentation gets going. Ostriker & Cowie (1981) and McCray & Kafatos (1987) adopt an approach very similar to that used here, but apply it to different problems. Ostriker & Cowie (1981) are concerned with the propagation of a detonation wave of supernovæ in the early Universe, as a means of generating voids and galaxy clusters. McCray & Kafatos (1987) are concerned with the inflation of supershells by multiple supernovæ, their influence on the overall structure of galactic discs, and the systematic differences between supershells in different parts of spiral galaxies and in Magellanic irregulars.

In the context of star formation, Larson (1985) has considered the fragmentation of quiescent layers, i.e. layers that are not in the process of accumulating, are not confined by ram pressure and are not shocked. He shows that fragments have mass $M_{\rm fragment} \sim a^4/G^2\Sigma \propto T^2/N$ (where a is the sound speed in the layer, Σ is the surface density of the layer, T is the temperature in the layer, and N is the column density of hydrogen nuclei through the layer). He also presents observational evidence for a correlation between the observed sound speeds in GMCs and the mean masses of the stars forming in them. However, layers are unlikely to be quiescent. We have already shown, using very general arguments (Whitworth, et al. 1993; hereafter Paper I), that layers will tend to fragment whilst they are still accumulating. In the present paper we show that the outcome of fragmentation is insensitive to the precise process that sweeps up the layer. This is because an accumulating layer fragments when its column density reaches a rather well-defined value $\sim 4-8\times10^{21}$ cm^{-2}, almost irrespective of the mechanism leading to its formation. Consequently the fragment mass $M_{\rm fragment}$ is mainly determined by a.

In Paper I we analysed two different mechanisms which produce gravitationally unstable shocked layers of interstellar gas, namely (i) colliding clouds, and (ii) expanding nebulæ (H II regions, stellar wind bubbles and supernova remnants). We showed that layers produced by these mechanisms become gravitationally unstable after a time $t_{\rm fragment} \sim (G\rho_{\rm o}\mathscr{M})^{-1/2}$, and break up into fragments with mean mass $M_{\rm fragment} \sim a_s^3(G^3\rho_{\rm o}\mathscr{M})^{-1/2}$ and mean separation $2r_{\rm fragment} \sim a_s(G\rho_{\rm o}\mathscr{M})^{-1/2}$. Here a_s is the isothermal sound speed in the shocked gas, i.e. in the layer that forms between the colliding clouds and in the shell that is swept up by the expanding nebula; $\rho_{\rm o}$ is the mass density of the unshocked gas, i.e. the cloud gas before it enters the layer or the ambient gas before it is swept up by the expanding nebula; $\mathscr{M}$ is the Mach number (relative to a_s) of the shock that bounds the layer or shell, and G is the gravitational constant.

We now use these results to constrain the properties of the stars and star clusters which form from shocked layers. In Section 2, we consider the parameters of the cloud ensemble in the Milky Way, and in Section 3 we use these parameters to evaluate the properties of the protostellar fragments condensing out of layers resulting from cloud-cloud collisions. In Section 4, we consider the parameters of a representative massive star, and in Sections 5 to 7 we use these parameters to evaluate the properties of protostellar fragments condensing out of shells swept up by the expansion of H II regions, stellar wind bubbles and supernova remnants, respectively, using the formulæ derived in Appendix B. In each case we normalize the parameters characterizing the process so as to minimize the fragment mass $M_{\rm fragment}$, in order to demonstrate how tightly the parameters have to be squeezed to form fragments with $M_{\rm fragment} < 7$ M$_\odot$. In this way we show that protostellar fragments condensing out of shocked layers tend to have high masses and are therefore presumably more likely to spawn high-mass stars. This tendency may explain why high-mass stars appear to form mainly in dynamically agitated molecular clouds and galaxies, and why the formation of high-mass stars is often rapid and self-propagating. In Sections 8 and 9, we discuss our results and summarize our conclusions.

Throughout this paper, we assume that the shocked gas in the layer has uniform and constant isothermal sound speed a_s, where typically $a_s \sim 0.2$–0.6 km s^{-1}, unless the gas is ionized, in which case $a_s \sim 10$ km s^{-1}. This assumption will hold provided that, at the appropriate phases in the evolution, the cooling time-scale is always much smaller than the dynamical time-scale. We check this assumption retrospectively in Appendix C.

Values of the density are given in terms of the number density of hydrogen nuclei (in all forms) n, and we assume that the fraction of hydrogen by mass is $X \approx 0.70$. Therefore

$$\frac{\rho}{n} \equiv m = \frac{m_{\rm H}}{X} \approx 2.4\times10^{-24}\ {\rm g},$$

where $m_{\rm H}$ is the mass of a hydrogen atom.

2 LARSON'S RELATIONS AND THE PARAMETERS FOR CLOUD-CLOUD COLLISIONS

We consider two clouds, each of mass $M_{\rm o}$, which subscribe to Larson's relations (Larson 1981). These relations can be interpreted as meaning that all clouds are in approximate virial equilibrium, and that all clouds have roughly the same column density of hydrogen nuclei, $N_{\rm L}$, and hence the same visual optical depth due to dust, $\tau_{\rm L}$. There is some dispersion amongst the different observational measures of $N_{\rm L}$ and $\tau_{\rm L}$ (Myers 1983; Dame et al. 1986; Falgarone & Puget 1986; Solomon et al. 1987); the reasons for this dispersion are discussed by Elmegreen (1989a). We shall adopt values of $N_{\rm L} \approx 6\times10^{21}$ cm^{-2} and $\tau_{\rm L} \approx 3$ for reference. Hence a typical cloud has radius $R_{\rm o}$, mean internal density $\rho_{\rm o}$ and internal turbulent velocity dispersion $\sigma_{\rm o}$, given by the following approximate relations:

$$R_\circ \sim 0.68 \text{ pc} \quad \tau_3^{-1/2} M_{100}^{1/2},$$

$$\rho_\circ \sim 5.2 \times 10^{-21} \text{ g cm}^{-3} \quad \tau_3^{3/2} M_{100}^{-1/2},$$

$$\sigma_\circ \sim 0.80 \text{ km s}^{-1} \quad \tau_3^{1/4} M_{100}^{1/4},$$

where $\tau_3 \equiv [\tau_L/3]$ and $M_{100} \equiv [M_\circ/100\,\mathrm{M}_\odot]$. Strictly speaking, $\sigma_\circ$ represents all the forms of energy supporting the cloud against self-gravity, i.e. turbulence, magnetic fields, rotation and thermal pressure; therefore it is probably about twice the observed line-of-sight velocity dispersion.

In order to estimate the bulk velocity v_c with which the clouds collide, we assume that they are part of a giant molecular cloud (GMC) with mass in the range $10^5 - 10^6$ $\mathrm{M}_\odot$. v_c will then be less than, or of order, the internal turbulent velocity dispersion in the GMC, i.e.

$$v_c \lesssim 8 \text{ km s}^{-1}.$$

In what follows, we shall omit the dependence on impact parameter and treat all collisions as if they were head-on. This omission is justified in Appendix A. We shall see that inclusion of the dependence on impact parameter would actually strengthen our conclusions, by reducing the effective collision speed.

3 THE SHOCKED LAYER BETWEEN TWO COLLIDING CLOUDS

We should distinguish two cases.

In the first case (that treated in Paper I), cloud material is still flowing into the shocks which delimit the layer when it starts to fragment; it is therefore confined mainly by ram pressure. The surface density is given by $\Sigma \approx 2\rho_\circ v_c t$. For a small, roughly circular element of the layer having radius r, the inward acceleration is

$$g \approx G\Sigma - \frac{a_s^2}{r},$$

where the two terms represent self-gravity and internal pressure. The time-scale on which this element starts to condense out is therefore

$$t_g \approx \left(\frac{r}{g}\right)^{1/2} \approx \left\{\frac{G\Sigma}{r} - \left(\frac{a_s}{r}\right)^2\right\}^{-1/2},$$

and the fastest-growing element has $r_{\rm fastest} \approx 2a_s^2/G\Sigma$, and $t_{\rm fastest} \approx 2a_s/G\Sigma \approx a_s/G\rho_\circ v_c t$. Fragmentation starts when $t_{\rm fastest} \approx t$, i.e. at

$$t_{\rm fragment} \approx \left(\frac{a_s}{G\rho_\circ v_c}\right)^{1/2} \sim 0.27 \text{ Myr} \quad a_{.2}^{1/2} v_8^{-1/2} \tau_3^{-3/4} M_{100}^{1/4},$$

where $a_{.2} \equiv [a_s/0.2 \text{ km s}^{-1}]$ and $v_8 \equiv [v_c/8 \text{ km s}^{-1}]$.

The masses and initial separations of the resulting fragments are

$$M_{\rm fragment} \approx 2\pi a_s^{7/2}(G^3\rho_\circ v_c)^{-1/2} \sim 3.2 \text{ M}_\odot \quad a_{.2}^{7/2} v_8^{-1/2} \tau_3^{-3/4} M_{100}^{1/4}, \quad (1)$$

$$2r_{\rm fragment} \approx 2a_s^{3/2}(G\rho_\circ v_c)^{-1/2} \sim 0.11 \text{ pc} \quad a_{.2}^{3/2} v_8^{-1/2} \tau_3^{-3/4} M_{100}^{1/4}.$$

In the second case, fragmentation does not start until all the cloud material has been shocked. The time-scale for all the cloud material to be shocked is $\sim 4R_\circ/3v_c$, and so the second case arises when

$$v_c > 1.37 \text{ km s}^{-1} \quad a_{.2}^{-1} \tau_3^{1/2} M_{100}^{1/2}, \quad (2)$$

i.e. for fast collisions between low-mass clouds. The layer now has mean surface density $\bar{\Sigma} \sim 8\rho_\circ R_\circ/3$. It is quiescent, i.e. it is no longer accumulating matter and is no longer confined by ram pressure. Consequently it relaxes towards a configuration in which it is held together by self-gravity. As shown by Larson (1985), the masses and initial separations of the fragments spawned by a quiescent layer are

$$M_{\rm fragment} \approx \frac{4\pi a_s^4}{G^2\bar{\Sigma}} \approx \frac{3\pi a_s^4}{2G^2\rho_\circ R_\circ} \sim 7.7 \text{ M}_\odot \quad a_{.2}^4 \tau_3^{-1}, \quad (3)$$

$$2r_{\rm fragment} \approx \frac{4a_s^2}{G\bar{\Sigma}} \approx \frac{3a_s^2}{2G\rho_\circ R_\circ} \sim 0.26 \text{ pc} \quad a_{.2}^2 \tau_3^{-1}.$$

Equation (3) gives a minimum fragment mass, in the sense that, if condition (2) is not obeyed, equation (1) gives a higher fragment mass, all other things being equal.

We emphasize that the values $a_s = 0.2$ km s^{-1}, $v_c = 8$ km s^{-1}, $\tau_L = 3$ and $M_\circ = 100$ $\mathrm{M}_\odot$, used to normalize the dimensionless variables $a_{.2}$, v_8, τ_3 and M_{100}, are extremes, chosen to reduce the mass of the fragment to a minimum. Even if the gas kinetic temperature in the layer is as low as 10 K, dynamical instabilities occurring before the layer becomes gravitationally unstable will generate weak turbulence so that the effective sound speed a_s exceeds ~ 0.2 km s^{-1}. v_c is likely to be less than 8 km s^{-1} in the majority of collisions. Observational estimates of τ_L tend to fall below 3. The likelihood of the layer fragmenting to produce multiple protostellar condensations must decrease rapidly for $M_\circ \ll 100$ $\mathrm{M}_\odot$, and the dependence of $M_{\rm fragment}$ on $M_\circ$ is in any case extremely weak.

4 PARAMETERS OF MASSIVE STARS AND THEIR ENVIRONMENTS

In the next three sections, we consider shells swept up by expanding nebulæ around massive stars, i.e. H II regions, stellar wind bubbles and supernova remnants. For reference, we use an isolated star with initial mass $M_\star \sim 40$ $\mathrm{M}_\odot$ (main-sequence spectral type O5; Allen 1973), which for a lifetime of $t_\star \sim 4$ Myr emits hydrogen-ionizing photons at a steady rate $\dot{\mathcal{N}}_{\rm LyC} \sim 10^{49}$ s^{-1}, and during the last 1 Myr has a wind with steady mechanical luminosity $L_w \sim 10^{37}$ erg s^{-1}. At the end of its lifetime it undergoes a supernova explosion, releasing radially directed kinetic energy $\mathscr{E}_\star \sim 10^{51}$ erg.

In reality, $\dot{\mathcal{N}}_{\rm LyC}$ and L_w change with time, but it is appropriate to avoid such refinements in an exploratory calculation of the type presented here. A wind with $L_w \sim 10^{37}$ erg s^{-1} corresponds to a mass-loss rate $\sim 10^{-5}$ $\mathrm{M}_\odot$ yr^{-1} — which would strip the star down to ~ 30 $\mathrm{M}_\odot$ after ~ 1 Myr — and a terminal speed ~ 1800 km s^{-1}.

Again for reference, the undisturbed number density of hydrogen nuclei in all forms in the surrounding interstellar medium is taken to be $n_\circ \sim 1000$ cm^{-3}. This is intended to be an upper limit to the mean undisturbed density in an extended region around the star (i.e. several parsecs across, since we are concerned here with shells around mature nebulæ). In reality the surrounding interstellar medium will be inhomogeneous.

Since, in all three cases, the expansion of the nebula is described by a power law of the form $R_s \approx Kt^\alpha$, we have derived the basic equations describing the gravitational fragmentation of such a shell in Appendix B.

5 SHELLS AROUND EXPANDING H II REGIONS

In this section we concentrate on the effects of the ionizing radiation from a massive star. Following Dyson & Williams (1980), the initial Strömgren sphere has radius $\sim (3\dot{\mathcal{N}}_{\rm LyC}/4\pi\beta_* n_o^2)^{1/3} \sim 0.74$ pc $\dot{\mathcal{N}}_{49}^{1/3} n_3^{-2/3}$, and is established on a time-scale $\sim (\beta_* n_o)^{-1} \sim 0.0002$ Myr n_3^{-1}, where $\beta_* \sim 2\times 10^{-13}$ cm^3 s^{-1} is the recombination coefficient for atomic hydrogen (into excited states only), $\dot{\mathcal{N}}_{49} \equiv [\dot{\mathcal{N}}_{\rm LyC}/10^{49}\ {\rm s}^{-1}]$ and $n_3 \equiv [n_o/10^3\ {\rm cm}^{-3}]$.

Thereafter the H II region expands and sweeps up the ambient gas into a thin, dense shell, bounded on the inside by an ionization front, and on the outside by a shock front. Again following Dyson & Williams (1980), the radius of an evolved H II region and hence also the radius of the shell it sweeps up are given by

$$R_s \approx \left\{ \frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_* n_o^2} \left(\frac{7a_i t}{4}\right)^4 \right\}^{1/7} \sim 4.5\ {\rm pc}\ \dot{\mathcal{N}}_{49}^{1/7} n_3^{-2/7} t_{\rm M}^{4/7}, \qquad (4)$$

where $a_i \approx 10$ km s^{-1} is the isothermal sound speed in the ionized gas, t is the elapsed time since the massive star 'turned on', and we have defined $t_{\rm M} \equiv [t/{\rm Myr}]$.

For an evolved H II region, after a time $\approx (\dot{\mathcal{N}}_{\rm LyC}/\beta_* n_o^2 a_i^3)^{1/3} \sim 0.12$ Myr $\dot{\mathcal{N}}_{49}^{1/3} n_3^{-2/3}$, most of the swept-up gas is in the thin neutral shell, rather than the H II region, and so the surface density of the shell is $\Sigma_s \approx \rho_o R_s/3$. Until a time $\approx (G\rho_o)^{-1/2} \sim 2.5$ Myr $n_3^{-1/2}$, the shell is confined mainly by the ram pressure of the gas being swept up at the front edge, and the thermal-plus-recoil pressure of the gas departing through the ionization front at the back edge.

Importing the results from Appendix B, and substituting for K and α from equation (4), we obtain the time at which fragmentation starts, the radius of (and column density through) the shell when fragmentation starts, and the mean mass and initial separation of the resulting fragments:

$$t_{\rm fragment} \approx \left(\frac{6a_s\sqrt{65}}{7Gm}\right)^{7/11} \left(\frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_*}\right)^{-1/11} \left(\frac{7a_i}{4}\right)^{-4/11} n_o^{-5/11},$$
$$\sim 1.56\ {\rm Myr}\ a_{.2}^{7/11}\dot{\mathcal{N}}_{49}^{-1/11} n_3^{-5/11};$$

$$R_{\rm fragment} \approx \left(\frac{6a_s\sqrt{65}}{7Gm}\right)^{4/11} \left(\frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_*}\right)^{1/11} \left(\frac{7a_i}{4}\right)^{-4/11} n_o^{-6/11},$$
$$\sim 5.8\ {\rm pc}\ a_{.2}^{4/11}\dot{\mathcal{N}}_{49}^{1/11} n_3^{-6/11}; \qquad (5)$$

$$N_{\rm fragment} \approx \frac{1}{3}\left(\frac{6a_s\sqrt{65}}{7Gm}\right)^{4/11} \left(\frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_*}\right)^{1/11} \left(\frac{7a_i}{4}\right)^{-4/11} n_o^{5/11},$$
$$\sim 6.0\times 10^{21}\ {\rm cm}^{-2}\ a_{.2}^{4/11}\dot{\mathcal{N}}_{49}^{1/11} n_3^{5/11}; \qquad (6)$$

$$M_{\rm fragment} \approx \frac{12\pi a_s^4}{G^2 m}\left(\frac{6a_s\sqrt{65}}{7Gm}\right)^{-4/11} \left(\frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_*}\right)^{-1/11}$$
$$\times \left(\frac{7a_i}{4}\right)^{4/11} n_o^{-5/11},$$
$$\sim 23\ {\rm M}_\odot\ a_{.2}^{40/11}\dot{\mathcal{N}}_{49}^{-1/11} n_3^{-5/11};$$

$$2r_{\rm fragment} \approx \frac{12a_s^2}{Gm}\left(\frac{6a_s\sqrt{65}}{7Gm}\right)^{-4/11} \left(\frac{3\dot{\mathcal{N}}_{\rm LyC}}{4\pi\beta_*}\right)^{-1/11}$$
$$\times \left(\frac{7a_i}{4}\right)^{4/11} n_o^{-5/11},$$
$$\sim 0.83\ {\rm pc}\ a_{.2}^{18/11}\dot{\mathcal{N}}_{49}^{-1/11} n_3^{-5/11}.$$

Again we emphasize that the values $a_s = 0.2$ km s^{-1}, $\dot{\mathcal{N}}_{\rm LyC} = 10^{49}$ s^{-1} and $n_o = 10^3$ cm^{-3}, used to normalize the dimensionless variables $a_{.2}$, $\dot{\mathcal{N}}_{49}$ and n_3, are extremes chosen to minimize the masses of the fragments. a_s is likely to be larger, both due to turbulence generated by dynamical instabilities and due to the extra heating from intense sub-Lyman continuum radiation in the vicinity of a massive star. n_o is likely to be smaller, say 100 cm^{-3}. For a cluster of OB stars, $\dot{\mathcal{N}}_{\rm LyC}$ is likely to be larger, but the results are in any case very insensitive to $\dot{\mathcal{N}}_{\rm LyC}$.

6 SHELLS SWEPT UP BY EXPANDING STELLAR WIND BUBBLES

Following Dyson & Williams (1980), the stellar wind bubble has radius and expansion speed

$$R_s \approx \left(\frac{125 L_w t^3}{154\pi\rho_o}\right)^{1/5} \sim 10\,{\rm pc}\ L_{37}^{1/5} n_3^{-1/5} t_{\rm M}^{3/5}, \qquad (7)$$

$$\dot{R}_s \approx \left(\frac{243 L_w}{3850\pi\rho_o t^2}\right)^{1/5} \sim 6.2\ {\rm km\ s^{-1}}\ L_{37}^{1/5} n_3^{-1/5} t_{\rm M}^{-2/5},$$

where $L_{37} \equiv [L_w/10^{37}\ {\rm erg\ s^{-1}}]$. The shell is preceded by a shock which compresses the gas by a factor $\sim (\dot{R}_s/a_s)^2$.

In the early stages, the shell is predominantly ionized with $a_s \sim 10$ km s^{-1}, and so the number density of hydrogen nuclei in the shell is given by

$$n_s \approx \left(\frac{\dot{R}_s}{a_s}\right)^2 n_o \sim 380\ {\rm cm}^{-3}\ L_{37}^{2/5} n_3^{3/5} t_{\rm M}^{-4/5}.$$

Hence the net recombination rate in the shell is

$$\mathcal{R}_r \approx \frac{4\pi R_s^3 n_o \beta_* n_s}{3} \sim 1.0\times 10^{52}\ {\rm s}^{-1}\ L_{37} n_3 t_{\rm M}, \qquad (8)$$

and the shell will start to recombine once this rate exceeds $\dot{\mathcal{N}}_{\rm LyC}$, i.e. at time

$$t_r \sim 0.001\ {\rm Myr}\ L_{37}^{-1} n_3^{-1}.$$

Once the gas in the shell recombines, its sound speed drops rapidly to below 1 km s^{-1}, and its density rises rapidly to

$$n_s \approx \left(\frac{\dot{R}_s}{a_s}\right)^2 n_o \sim 9.5\times 10^5\ {\rm cm}^{-3}\ a_{.2}^{-2} L_{37}^{2/5} n_3^{3/5} t_{\rm M}^{-4/5}.$$

Gravitational fragmentation does not get under way until after recombination. By this stage, most of the swept-up matter is in the shell, so its surface density is $\Sigma_s \approx \rho_o R_s/3$.

Importing the results from Appendix B, and substituting for K and α from equation (7), we obtain estimates for the time at which fragmentation occurs, the radius of (and the column density through) the shell at this time, and the mean mass and initial separation of the fragments:

$$t_{\text{fragment}} \approx \left(\frac{6a_s\sqrt{34}}{5G}\right)^{5/8}\left(\frac{125L_w}{154\pi}\right)^{-1/8}\rho_o^{-1/2},$$
$$\sim 0.9 \text{ Myr } a_{.2}^{5/8}L_{37}^{-1/8}n_3^{-1/2};$$

$$R_{\text{fragment}} \approx \left(\frac{6a_s\sqrt{34}}{5G}\right)^{3/8}\left(\frac{125L_w}{154\pi}\right)^{1/8}\rho_o^{-1/2},$$
$$\sim 9.6 \text{ pc } a_{.2}^{3/8}L_{37}^{1/8}n_3^{-1/2};$$

$$N_{\text{fragment}} \approx \frac{1}{3m}\left(\frac{6a_s\sqrt{34}}{5G}\right)^{3/8}\left(\frac{125L_w}{154\pi}\right)^{1/8}\rho_o^{1/2},$$
$$\sim 9.9\times10^{21}\text{ cm}^{-2}\ a_{.2}^{3/8}L_{37}^{1/8}n_3^{1/2}; \quad (9)$$

$$M_{\text{fragment}} \approx \frac{12\pi a_s^4}{G^2}\left(\frac{6a_s\sqrt{34}}{5G}\right)^{-3/8}\left(\frac{125L_w}{154\pi}\right)^{-1/8}\rho_o^{-1/2},$$
$$\sim 9.8\text{ M}_\odot\ a_{.2}^{29/8}L_{37}^{-1/8}n_3^{-1/2};$$

$$2r_{\text{fragment}} \approx \frac{12a_s^2}{G}\left(\frac{6a_s\sqrt{34}}{5G}\right)^{-3/8}\left(\frac{125L_w}{154\pi}\right)^{-1/8}\rho_o^{-1/2},$$
$$\sim 0.31\text{ pc } a_{.2}^{13/8}L_{37}^{-1/8}n_3^{-1/2}.$$

Once again these results illustrate how tightly the parameters have to be squeezed to give low-mass fragments. Once again the least important parameter is the one characterizing the energy input from the central star, in this case the mechanical luminosity of the wind.

7 SHELLS SWEPT UP BY EXPANDING SUPERNOVA REMNANTS

Following Dyson & Williams (1980), we divide the evolution of a supernova remnant into an adiabatic phase and a snowplough phase.

In the adiabatic phase, the radius and expansion speed of the supernova remnant are

$$R_s \approx \left(\frac{25\mathscr{E}_\star t^2}{3\pi\rho_o}\right)^{1/5} \sim 21\text{ pc } \mathscr{E}_{51}^{1/5}n_3^{-1/5}t_M^{2/5},$$

$$\dot{R}_s \approx \left(\frac{32\mathscr{E}_\star}{375\pi\rho_o t^3}\right)^{1/5} \sim 8.2\text{ km s}^{-1}\ \mathscr{E}_{51}^{1/5}n_3^{-1/5}t_M^{-3/5},$$

where $\mathscr{E}_{51}\equiv[\mathscr{E}_\star/10^{51}\text{ erg}]$. The remnant is bounded by a strong adiabatic shock, so the temperature of the newly shocked gas is

$$T \approx \frac{3\bar{m}\dot{R}_s^2}{16k} \sim 900\text{ K } \mathscr{E}_{51}^{2/5}n_3^{-2/5}t_M^{-6/5},$$

where $\bar{m}\sim10^{-24}$ g is the mean gas-particle mass, and k is Boltzmann's constant. The number density of hydrogen nuclei in the outer part of the remnant is $n_s\approx4n_o$.

The adiabatic phase ends when the bubble has become cool enough by expansion to start cooling significantly by radiation. Using equation (C1) for the radiative cooling time (on the assumption — to be justified a posteriori — that the shell is expanding faster than 320 km s^{-1}), and substituting $v=\dot{R}_s$, we conclude that radiative cooling becomes effective after a time

$$t_{\text{transition}} \sim 0.0006\text{ Myr } \mathscr{E}_{51}^{3/14}n_3^{-4/7},$$

when the shell has radius and expansion speed

$$R_{\text{transition}} \sim 1.0\text{ pc } \mathscr{E}_{51}^{2/7}n_3^{-3/7},$$

$$\dot{R}_{\text{transition}} \sim 730\text{ km s}^{-1}\mathscr{E}_{51}^{1/14}n_3^{1/7}.$$

Hence the choice of equation (C1) for the cooling time is indeed appropriate.

At this juncture, rapid cooling of the shocked gas causes the outer layers of the bubble to collapse into a thin shell, Thereafter the dynamics of the shell are largely determined by momentum conservation. This is the snowplough phase. The radius of the shell is given by

$$R_s \approx R_{\text{transition}}\left(\frac{8t-3t_{\text{transition}}}{5t_{\text{transition}}}\right)^{1/4} \sim 7.2\text{ pc } \mathscr{E}_{51}^{13/56}n_3^{-2/7}t_M^{1/4} \quad (10)$$

(Dyson & Williams 1980), where the final expression in equation (10) assumes that $t \gg t_{\text{transition}}$. Protostars do not condense out of the shell until the snowplough phase is well established. By this stage, most of the swept-up matter is in the shell, so its surface density is $\Sigma_s\approx\rho_oR_s/3$.

Importing the results from Appendix B, and substituting for K and α from equation (10), we obtain

$$t_{\text{fragment}} \sim 1.1\text{ Myr } a_{.2}^{4/5}\mathscr{E}_{51}^{-13/70}n_3^{-4/7},$$

$$R_{\text{fragment}} \sim 7.4\text{ pc } a_{.2}^{1/5}\mathscr{E}_{51}^{13/70}n_3^{-3/7}, \quad (11)$$

$$N_{\text{fragment}} \sim 7.6\times10^{21}\text{ cm}^{-2}\ a_{.2}^{1/5}\mathscr{E}_{51}^{13/70}n_3^{4/7}, \quad (12)$$

$$M_{\text{fragment}} \sim 13\text{ M}_\odot\ a_{.2}^{19/5}\mathscr{E}_{51}^{-13/70}n_3^{-4/7},$$

$$2r_{\text{fragment}} \sim 0.42\text{ pc } a_{.2}^{9/5}\mathscr{E}_{51}^{-13/70}n_3^{-4/7}.$$

Again the parameters have to be squeezed to give low-mass fragments; again the least important parameter is the one characterizing the energy input from the central star, here the energy released by the supernova explosion.

8 DISCUSSION

We have shown that gravitational instability in a shocked layer will normally produce fragments of high mass. Fragments with masses below $\sim 7\text{ M}_\odot$ can only be produced if extreme values are given to all the parameters characterizing the mechanism that generates the layer. This does not of itself prove (but does at least suggest) that some of the stars that form from such fragments will also be massive.

If this is the case, then it is clear that the process can be self-propagating. Given a sufficient reservoir of material, the massive stars condensing out of one shocked layer can, by exciting H II regions, blowing stellar wind bubbles and undergoing supernova explosions, sweep up another layer and thereby spawn another generation of stars. The time-scale between successive generations will be a few megayears.

In all the scenarios considered here the most critical parameter is the effective sound speed in the shocked gas, i.e. the velocity dispersion on scales much smaller than the putative fragments. The outcome is apparently very insensitive to the parameter characterizing the bulk energy that drives

Table 1. Column 1 gives the mechanism invoked to produce the (shocked) layer. Columns 2, 3 and 4 give the exponent characterizing the dependence of the fragment mass $M_{\rm fragment}$ on – respectively – the effective sound speed a_s in the shocked gas, the density n_o in the unshocked gas, and the third parameter ξ ($\equiv \tau_L, v_c^2/2, \mathcal{N}_{\rm LyC}, L_w, \mathcal{E}_*$) which represents the energy source driving the shock. Column 5 identifies ξ for each mechanism.

Scenario	$\frac{d\ln[M_{\rm fragment}]}{d\ln[a_s]}$	$-\frac{d\ln[M_{\rm fragment}]}{d\ln[n_o]}$	$-\frac{d\ln[M_{\rm fragment}]}{d\ln[\xi]}$	ξ
Quiescent layer (Larson 1985)	4	—	(1)	(τ_L)
Shocked layer due to cloud-cloud collision	$\frac{7}{2} = 3.50$	$\frac{1}{2} = 0.50$	$\frac{1}{4} = 0.25$	$v_c^2/2$
Shocked layer due to expanding H II region	$\frac{40}{11} = 3.64$	$\frac{5}{11} = 0.46$	$\frac{1}{11} = 0.09$	$\mathcal{N}_{\rm LyC}$
Shocked layer due to stellar wind bubble	$\frac{29}{8} = 3.63$	$\frac{1}{2} = 0.50$	$\frac{1}{8} = 0.13$	L_w
Shocked layer due to supernova remnant	$\frac{19}{5} = 3.80$	$\frac{4}{7} = 0.57$	$\frac{13}{70} = 0.19$	$\mathcal{E}_*$

the shock which compresses the layer: the collision speed v_c for cloud-cloud collisions, the Lyman continuum output $\mathcal{N}_{\rm LyC}$ for H II regions, the mechanical luminosity L_w blowing a stellar wind bubble, or the energy $\mathcal{E}_*$ injected into a supernova remnant.

It is interesting to compare our results with those obtained by Larson (1985), who considered the fragmentation of a relaxed layer held together by self-gravity. The masses of the fragments condensing out of such a layer are given by equation (3). In Table 1 we compare the dependences of fragment mass $M_{\rm fragment}$ on (i) the effective sound speed a_s in the shocked gas, (ii) the pre-shock density n_o, and (iii) whatever else it depends on (τ_L, $v_c^2/2$, $\mathcal{N}_{\rm LyC}$, L_w, $\mathcal{E}_*$), for the different scenarios considered in this paper. We note that the dependence of $M_{\rm fragment}$ on a_s is very similar for all the scenarios considered: $d\ln[M_{\rm fragment}]/d\ln[a_s] \approx 3.5-4.0$. For the scenarios involving shocked layers, the dependence of $M_{\rm fragment}$ on n_o is also very similar: $d\ln[M_{\rm fragment}]/d\ln[n_o] \approx 0.46-0.57$. For the scenarios involving shocked layers, the dependence of $M_{\rm fragment}$ on ξ, the parameter characterizing the energy that drives the shock, is in all cases weak. Note that for cloud-cloud collisions we have used the specific kinetic energy of the collision $\xi \equiv v_c^2/2$.

Another interesting aspect of the results presented here concerns specifically the shells swept up by expanding nebulæ. The column density $N_{\rm fragment}$ of hydrogen through these shells, when they undergo gravitational fragmentation, is given by equations (6), (9) and (12), for the three different scenarios considered. We see that in all three cases $N_{\rm fragment}$ is only weakly dependent on any of the parameters; and in all cases it appears to have a preferred value of $4-8\times10^{21}$ cm^{-2}. This corresponds to an optical depth due to dust in the visible of $\tau_V \sim 2-4$. These are precisely the values of N and τ_V that correspond to Larson's relations. Therefore, if the density structure in giant molecular clouds is continuously regenerated by the expansion of nebulæ, the inference is that higher values of N and τ_V are pre-empted by gravitational fragmentation. Smaller values of N and τ_V are less commonly observed because the growth of the shell traverses small values of N relatively quickly, and in the early stages fragments are not yet well defined and therefore hard to identify. The range of clump masses subscribing to Larson's relations must then be due primarily to variations in the effective sound speed a_s.

9 CONCLUSIONS

By analysing various scenarios which produce shocked layers of interstellar gas, and the subsequent gravitational fragmentation of such layers, we have shown that fragmentation develops on a short time-scale and produces predominantly massive fragments which are likely to engender a high proportion of massive stars. If this is the case, there must be a strong tendency for star formation in dynamically agitated regions (for instance, giant molecular clouds overrun by a galactic shock, or giant molecular cloud complexes in interacting galaxies) to relax to a cyclic self-propagating mode in which the massive stars in one generation excite nebulæ whose expansion sweeps up the dense shell out of which the next generation of stars condenses. This tendency may explain how IR-luminous starburst galaxies concentrate their star formation into high-mass stars, thereby releasing large amounts of energy without exhausting their reservoirs of interstellar matter (e.g. Rieke et al. 1993).

This cycle may also help to explain the relation between the masses and radii of clumps in molecular clouds first noted by Larson (1981). This relation implies that there is a preferred column density $N_L \sim 6\times10^{21}$ cm^{-2} through clumps. The analysis in this paper suggests that this column density is selected by nature because, as a shell is swept up by an expanding nebula, smaller column densities are traversed relatively quickly and the fragments are not yet well defined, while larger column densities are never realized because the layer undergoes gravitational fragmentation before they are reached. Alternative explanations are offered by Franco & Cox (1986), Chièze (1987), Maloney (1988), Fleck (1988), Elmegreen (1989a), and Hartquist et al. (1993).

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APPENDIX A: SHEAR IN THE LAYER BETWEEN TWO COLLIDING CLOUDS

In Paper I we showed (a) that, for cloud-cloud collisions at finite impact parameter b_c, the effective collision speed is reduced by a factor

$$\beta \approx \left\{1-\left(\frac{b_c}{2R_o}\right)^2\right\}^{1/2},$$

and (b) that fragmentation of the shocked layer which results from the collision is inhibited by shear, unless $b_c < \sqrt{2}R_o$. In fact, numerical simulations (Chapman et al., in preparation) show that a coherent layer only forms if $b_c < \sqrt{2}R_o$. Therefore we can put

$$\langle\beta\rangle \sim \int_{b=0}^{b=\sqrt{2}R} \beta(b,R)\frac{b\,\mathrm{d}b}{R^2} \approx 0.86.$$

Since this factor is close to unity, and given the much greater sensitivity of our results to other parameters, we are justified in neglecting it and thereby treating all collisions as if they were head-on.

APPENDIX B: FRAGMENTATION OF AN EXPANDING SHELL

We consider a shell driven outwards by an expanding nebula, whose radius is given by

$$R_s = Kt^\alpha,$$

so that $\dot{R}_s = \alpha K t^{\alpha-1}$.

For a small, roughly circular element of the shell having radius r, the inward acceleration is

$$g \approx G\Sigma - \frac{1}{r}\left\{a_s^2 + \left(\frac{\dot{R}_s r}{R}\right)^2\right\} = G\Sigma - \frac{a_s^2}{r} - \left(\frac{\alpha}{t}\right)^2 r,$$

where the three terms represent self-gravity, internal pressure, and tangential divergence due to the radial expansion of the shell. The time-scale on which this element starts to condense out is therefore

$$t_g \approx \left(\frac{r}{g}\right)^{1/2} \approx \left\{\frac{G\Sigma}{r} - \left(\frac{a_s}{r}\right)^2 - \left(\frac{\alpha}{t}\right)^2\right\}^{-1/2},$$

and the fastest-growing element has $r_{\rm fastest} \approx 2a_s^2/G\Sigma$, and

$$t_{\rm fastest} \approx \left\{\left(\frac{G\Sigma}{2a_s}\right)^2 - \left(\frac{\alpha}{t}\right)^2\right\}^{-1/2}.$$

Fragmentation starts in earnest when $t_{\rm fastest} \approx t$, i.e. when $G\Sigma/2a_s \approx (1+\alpha^2)^{1/2}/t$. Substituting $\Sigma \approx \rho_o R/3 \approx \rho_o K t^\alpha/3$, on the assumption that most of the swept-up matter is in the shell, we find that fragmentation commences at

$$t_{\rm fragment} \approx \left\{\frac{(1+\alpha^2)^{1/2} 6a_s}{G\rho_o K}\right\}^{1/(1+\alpha)},$$

when the shell has radius and column density

$$R_{\rm fragment} \approx K\left\{\frac{(1+\alpha^2)^{1/2} 6a_s}{G\rho_o K}\right\}^{\alpha/(1+\alpha)},$$

$$N_{\rm fragment} \approx \frac{\Sigma_{\rm fragment}}{m} \approx \frac{\rho_o R_{\rm fragment}}{3m};$$

the masses and mean initial separations of fragments are

$$M_{\rm fragment} \approx \pi r^2_{\rm fragment}\Sigma_{\rm fragment} \approx \frac{12\pi a_s^4}{G^2\rho_o R_{\rm fragment}},$$

$$2r_{\rm fragment} \approx \frac{12a_s^2}{G\rho_o R_{\rm fragment}}.$$

APPENDIX C: POST-SHOCK RADIATIVE COOLING

We adopt the following cooling law:

$$\frac{\Lambda}{n^2} \sim \begin{cases} \Lambda_2 T^{3/2}, & \Lambda_2 = 10^{-31}\ \mathrm{erg\ s^{-1}\ cm^3\ K^{-3/2}},\ T < 10^4\ \mathrm{K}, \\ \Lambda_1 T^{-1/2}, & \Lambda_1 = 10^{-19}\ \mathrm{erg\ s^{-1}\ cm^3\ K^{1/2}},\ T \geq 10^4\ \mathrm{K}; \end{cases}$$

which mimics the broad features of radiative cooling of optically thin interstellar gas (cf. Dalgarno & McCray 1972; Raymond, Cox & Smith 1976) but smooths out the details and avoids taking into account the dependence on the metallicity or the degree of ionization.

Material flowing into a shock at highly supersonic speed v_c is initially heated to a temperature $T_1 \sim 3\bar{m}v_c^2/16k$, and compressed to a density $n_1 \sim 4n_o$. In order to estimate the cooling time, we assume that the cooling takes place isobarically. For $T_1 \gg 10^4$ K ($v_c \gg 25$ km s^{-1}), it takes a time

$$t_1 \approx \left(\frac{27\bar{m}}{k}\right)^{1/2} \frac{m v_c^3}{256\Lambda_1 n_o} \sim 0.7 \times 10^{-9} \text{ Myr } n_3^{-1} v_8^3 \tag{C1}$$

to cool down to 10^4 K, and the subsequent cooling to $\ll 10^4$ K (typically to ~ 10 K) takes a time

$$t_2 \approx \frac{20m(k/\bar{m})^2(10^4 \text{ K})^{1/2}}{3\Lambda_2 n_o v_c^2} \sim 0.004 \text{ Myr } n_3^{-1} v_8^{-2}.$$

Thus for shock speeds greater than ~ 180 km s^{-1} the cooling time is $\sim t_1$; whilst for shock speeds between ~ 25 and ~ 180 km s^{-1} the cooling time is t_2.

For shock speeds below ~ 25 km s^{-1}, the cooling time is

$$t_3 \approx \left(\frac{3\bar{m}}{k}\right)^{-3/2} \frac{15m}{\Lambda_2 n_o v_c} \sim 0.0006 \text{ Myr } n_3^{-1} v_8^{-1}. \tag{C2}$$

When two clouds collide at supersonic speed, a compressed layer of hot, shocked gas forms at the interface. The clouds will merge if the gas in this layer cools before it re-expands. Cloud-cloud collisions are very seldom at speeds in excess of 25 km s^{-1}, and so the cooling time is given by equation (C2). The expansion time-scale for the uncooled gas is approximately equal to the collision time-scale,

$$t_x \approx t_c \approx \frac{R_o}{v_c} \sim 0.09 \text{ Myr } \tau_3^{-1/2} M_{100}^{1/2} v_8^{-1},$$

and this is much longer than the cooling time-scale t_3, provided that

$$\tau_L \gg 0.003.$$

We see that instantaneous radiative cooling is an excellent approximation for all cloud-cloud collisions involving clouds that subscribe to Larson's relations with $\tau_L \approx 3$.

When the shells swept up by expanding nebulæ undergo gravitational fragmentation, they are expanding at speeds of order or less than 25 km s^{-1}, and so again the cooling time is given by equation (C2) with v_c replaced by $\dot{R}_{\text{fragment}}$. Comparing t_3 thus obtained with the fragmentation times given by equations (5), (8) and (11), we again find that instantaneous radiative cooling is an excellent approximation for all likely values of the parameters. The precise conditions for the three scenarios treated in Sections 5 to 7 are

H II region: $n_o \gg 6 \times 10^{-4}$ cm^{-3} $a_{.2}^{-4/5} \dot{\mathcal{N}}_{49}^{-1/5}$;

stellar wind bubble: $n_o \gg 7 \times 10^{-4}$ cm^{-3} $a_{.2}^{-3/4} L_{37}^{-1/4}$;

supernova remnant: $n_o \gg 3 \times 10^{-2}$ cm^{-3} $a_{.2}^{-7/20} \mathcal{E}_{51}^{-13/40}$.

We conclude that our a priori assumption of instantaneous radiative cooling is fully justified.

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