THE PROBABILITY CHARACTERISTICS OF WAVES AND WAVE PRESSURES AT A VERTICAL BREAKWATER

Huang Peiji* and Zhao Binglai**

ABSTRACT

In this paper, under the condition of waves in front of a breakwater being not broken, studies were made on the characteristcs of probability distribution of waves and wave pressures, the regularity of the spectral component attenuation with depth and the constitution of the high frequency band of wave pressure spectrum.

The distributions of wave heights in front of a vertical breakwater, the range of wave pressure fluctuation at different subsurface levels, and the wave periods have shown that they are practically invariable with depth and can be determined theoretically. The spectral constitution of wave pressure field and the regularity of attenuation of spectral components were analyzed at the vertical breakwater, and a new expression describing the equilibrium range of wave pressure spectrum was obtained.

1. INTRODUCTION

Along with the exploitation of the offshore oil fields and marine minerals as well as with the advance of marine navigation and transportation, it is necessary to build up much more marine structures or wharfs. As for those structures, the vertical-wall type marine structure is still adopted widely in the engineering construction at present. In order to select the optimum conditions for structure of this kind, we must research on the probability characteristics of the waves and wave pressures in front of it. This work directly concerns the calculations of the stability and the strength for such structure. It is very important to investigate, under natural conditions, the probability characteristics of the waves and the wave pressures in front of it.

Since the late 1960's, the effects of sea wave on the vertical breakwater have been studied and observed abroad(9, 10,12). In recent years, we have observed the wave and its action on the vertical breakwater at a certain harbour(1,3). The breakwater of this harbour is situated in the open sea ,

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*Associate Researcher . **Assistant Researcher .
First Institute of Oceanography, National Bureau of Oceanography, P.O. Box 98, Qingdao, China

and its body which is a caisson structure was placed on the basement. The sea bottom is comparatively even. The altitude of the basement is -6.0m and the tidal range 3--4m. The depth of water in front of the breakwater may reach about 10m at high tide. The observation station was built 40m from the head of this breakwater. The sensors which are used for measuring the wave pressure were mounted at different heights of the breakwater side facing the sea (its respectively altitudes are -5.2, -2.2, -0.2, +1.8, +2.8, and +3.8m, the altitude is regared as zero at the tidal datum plane and is positive on the above and negative in the below). The resistence wire staffs are mounted on the breakwater side and on a location where it is 450m far from the breakwater in the open sea. The wave pressures at different heights on the breakwater side , the water surface elevations in front of it and the incident waves are synchro-recorded altogether continuously.

In this paper, as to the successive recordings of the waves and the wave pressure, we have carried on statistical and spectrum analyses on the basis of actually measured data in front of the vertical breakwater in this harbour. Meanwhile, we also investigated, without the wave breaking condition in front of the breakwater, the probability distribution characteristics of the wave and the wave pressure, the characteristics of the component of the wave pressure spectra attenuated with the depth, and the constitution of the wave pressure spectra in front of the breakwater.

2. THE PROBABILITY DISTRIBUTION OF WAVES AND WAVE PRESSURES IN FRONT OF THE BREAKWATER

The waves are random phenomena, the forces resulted from them also exhibit the stochastic behaviour. Therefore, when the interaction between the waves and the breakwater takes place, the wave height and the total pressure acted upon the breakwater, the intensity of pressure at any point on it and other dynamic parameters which characterize the action of wave on the breakwater, all these may be regarded as being of random quantities with the deffinite distribution law. In the case of deep water, this distribution law has ever been investigated by some researchers. They thought that the wave height in front of the breakwater, the range of wave pressure fluctuation at different heights on the breakwater, the crest value and the trough value of the total pressures all follow the Rayleigh Distribution(9,11). In the case of shallow water , this problem has not been studied yet.

2.1 The cumulative probability distribution function for both the wave height in front of the vertical breakwater and the range of wave pressure fluctuation at different heights on the breakwater

In the case of deep water, both the wave height in front of the breakwater and the range of wave pressure fluctuation at any point on it follow the Rayleigh Distribution. In the case of shallow water, the wave height is no more in agreement with the Rayleigh Distribution. According to reference(2), we may obtain the following expressions:

$$F(h) = \exp\left[-\Gamma^{K(H^*,B)}\left(\frac{1}{K(H^*,\,B)}+1\right)\left(\frac{h}{\bar{h}}\right)^{K(H^*,B)}\right], \qquad (1)$$

$$K(H^*,\,B) = (5-B)/(2-1.5H^*), \qquad (2)$$

$$\left.\begin{array}{ll} B = \begin{cases} \bar{C}/U = \bar{\tau}g/2\pi U, & \bar{C} \leqslant U \\ 1, & \bar{C} > U \end{cases} & \\ H^* = \bar{h}/d, & 0 \leqslant H^* \leqslant 0.5 \end{array}\right\}, \qquad (3)$$

here $\bar{h}$ is the average wave height, $\bar{\tau}$ the average wave period, d the water depth, U the wind velocity, g the gravitational accelaration, H^* the shallow water factor, B the stage factor of wind-wave growth. When B=1, the wind wave appears in a fully developed state. It is evident that the expression (1) is the Rayleigh Distribution, while $H^*=0$, and B=1.

In order to find out which kind of distributions will be reflected in the actually measured data, first of all, we have respectively computed the cumulative curve of the nondimensional wave height($h/\bar{h}$) and the nondimensional wave pressure range($p/\bar{p}$) at every recording, and then determined the values of the nondimensional wave height and the nondimensional wave pressure range which correspond to the cumulative probabilities as 1,5,10,20,30,50,70, and 90%. By making use of the values of every cumulative probability mentioned above, we have computed the points corresponding to the value K by the expression(2), and then plotted them on Fig. 1.

Fig. 1a illustrates the distribution of the actually measured wave heights on the vertical breakwater and Fig. 1b illustrates that of the range of wave pressure fluctuation measured at the basement layer of the breakwater. Other mediate layers have similar figures, too.

As seen from the above figures, the distances between the points of the actually measured data quite approximate the corresponding values of the theoretical cumulative probability in expression(1). Thus we may think that, in the case of shallow water, the wave height at the vertical breakwater and the wave pressure range at different heights on it are in agreement with the distribution law in expression(1). The distribution of wave pressure range, as a matter of fact, will not vary with the altitude of the measured points.

2.2 The cumulative distribution function of the surface wave period in front of the vertical breakwater and the wave pressure period at the different heights on it

Regarding the distribution of the wave period, analyses (6,8) of the researchers and results observed indicate that, under the first order approximation, the distribution curves

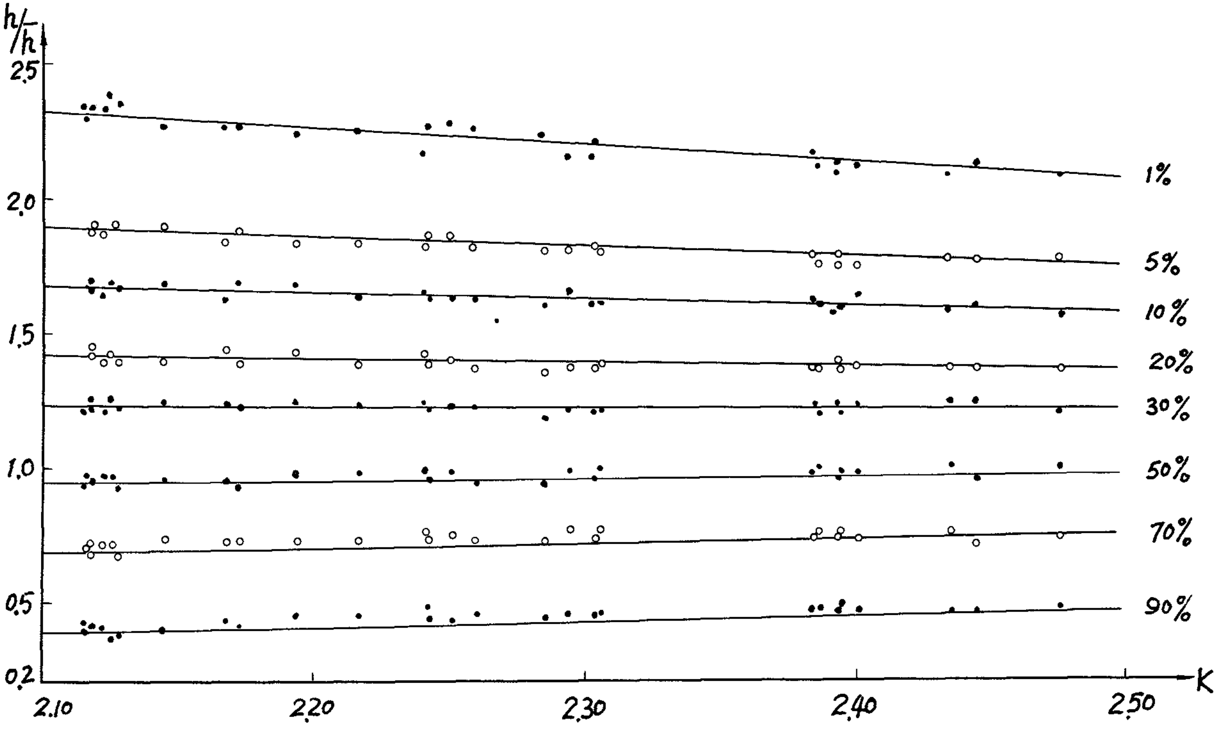

Fig. 1a Comparison of the empirical cumulative distribution(dots) of wave height with the value(line) calculated by eg.(1).

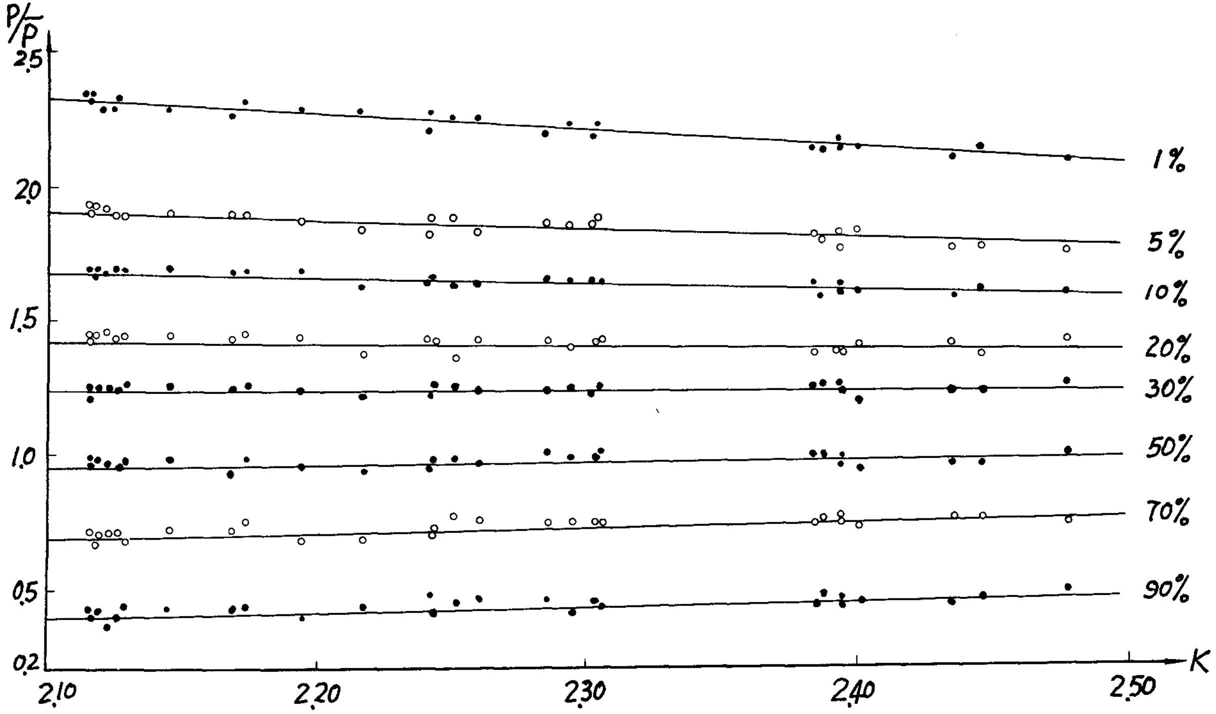

Fig. 1b Comparison of the empirical cumulative distribution(dots) of range of wave pressure fluctuation with the value(line) calculated by eg.(1).

of the nondimensional wave periods for the wind waves and the swell before the breaking area are, in fact, a constant no matter how deep the water is. Its distribution function is given as follows:

$$F(\tau) = \exp\left[-\Gamma^4\left(\frac{5}{4}\right)\left(\frac{\tau}{\bar{\tau}}\right)^4\right]。 \qquad (4)$$

To analyse the distributions of both the wave period in front of the breakwater and the wave pressure period at different heights on it, we have plotted the experimental cumulative distribution on the special coordinate paper such as Fig.2, whose curves denote the expression(4).

In Fig. 2, Z=0 represents the distributions of the surface wave period in front of the vertical breakwater, and Z=-5.2m, the distributions of the wave pressure periods at the basement layer of the breakwater. There are also similar distribution for the mediate layers.

After analysing the experimental cumulative distributions of periods for the wave and wave pressure, it shows that, in the case of shallow water, the distribution of wave periods and wave pressure periods in front of the vertical breakwater may be calculated by expression(4), and that the distribution of period for the wave pressure does not vary with the altitudes of the measuring points practically.

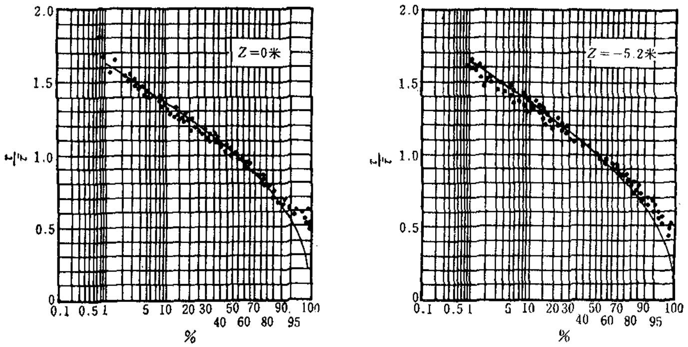

Fig. 2a,b The comparison of the experimental cumulative distribution of wave period and wave pressure period(points) with the theoretical cumulative distribution(curves).

2.3 The cumulative distribution function of the total pressure

The total pressure is one of the most important characteristic quatities for the wave acted on the vertical breakwater. At present, research on the total pressure is mainly based on the analysis to the values of crest and trough of single wave hitted against the breakwater.

To analyse the distribution characteristics of the total pressure, we have calculated the crest value, R^+, and trough value, R^-, of the total pressure corresponding to the phases of every wave crest and wave trough based on 27 successive recordings. Then we obtained the cumulative distribution curves of the total pressures for the nondimensional crest($R_i^+/\bar{R}^+$) and trough($R_i^-/\bar{R}^-$). Furthermore, we determined the values of the total pressures of the nondimensional crest and trough corresponding to cumulative probability of 1,5,10,20,30,....., 90%. According to the following expression(5)

$$F(R^{\pm}) = \exp\left[-\Gamma^{\frac{5-B}{2-1.5H^*}}\left(\frac{2-1.5H^*}{5-B}+1\right)\left(\frac{R_i^{\pm}}{\bar{R}^{\pm}}\right)^{(5-B)/2-1.5H^*}\right], \tag{5}$$

we computed the above mentioned total pressures of the nondimensional crest value and trough value for every cumulative probability as well as the relative differences between them and the actually measured value. Thus we have respectively made statistics of the occurrence probability for the relative differences within the given range(-5—+5%, -10—+10%). Then the mean absolute difference was calculated and the maximum relative differences corresponding to cumulative probability were obtained. Their results are shown in table 1.

Table 1

relative error		cumulative probability %							
		1	5	10	20	30	50	70	90
-5~+5%	R^+	100	100	100	100	100	100	100	81.48
	R^-	96.30	100	100	100	100	100	96.30	74.07
-10~+10%	R^+	100	100	100	100	100	100	100	96.30
	R^-	100	100	100	100	100	100	100	100
maximum relative error	R^+	-4.109	3.84	4.32	4.96	4.09	4.211	4.34	10.25
	R^-	7.272	3.261	3.012	3.546	4.065	-4.211	5.633	-8.888
average absolute error	R^+	0.030	0.022	0.0237	0.0200	0.0248	0.014	0.0155	0.014
	R^-	0.035	0.0229	0.0203	0.0174	0.0181	0.0155	0.0155	0.0163

Table 1 shows that the occurrence probability corresponding to every cumulative probability is quite high within the relative differences at -5—+5%, while the lowest is 74.07%. As for the mean absolute difference, its maximum is not more than 0.035. Therefore, it is deemed that the distribution function of the total pressures for the crest and trough of the wave acted upon the vertical breakwater in shallow water may be calculated by expression(5) and may be of identical distribution law of wave heights in front of the vertical breakwater.

As for the above cumulative distribution function of every parameter, the analysed results are of importance. In the case of shallow water, making use of these distribution law, we might obtain the required values of the arbitrarily given cumulative probability by the average wave height known in front of the vertical breakwater, the mean range of wave pressure fluctuation at different heights on it, the mean wave period and the mean total pressure. They are very useful to the engineering calculations.

3. ATTENUATION OF WAVE PRESSURE WITH DEPTH

The problem of the wave and the wave pressure attenuated with depth was first investigated on the open sea and ocean(6,13) or in the offshore area without any obstacles(6). Since the beginning of the 1970's, there were research results of both the waves and the wave pressures attenuated with the depth in front of the vertical breakwater under the deep water condition(9,11). On the basis of the actually measured data, we have analysed and investigated the regularity of the wave pressure spectral component attenuated with the depth and the constitution of the high frequency portion of the wave pressure spectrum in front of the vertical breakwater.

3.1 The spectral component of wave pressure attenuated with depth

According to the ordinate data picked out by the same time interval(t=0.75sec.) for both the wave profile in front of the breakwater and the wave pressure fluctuations at different heights on it, we have obtained their spectrum densities respectively.

We regarded the frequency spectrum of the wave pressure at the levels -0.2, -2.2, and -5.2m as original data used for analysing the spectral component of the wave pressure attenuated with the depth, then we compared the actually measured results with that of the wave theory for small amplitude at the limited depth. The attenuation coefficient of spectral component varied with the depth may be obtained, within the limited depth, by the wave theory

$$r(\omega, Z, d) = \mathrm{ch}k(d + Z)/\mathrm{ch}k(d + Z_0), \qquad (6)$$

where d stands for water depth in front of the vertical break-water, k stands for wave number; it has to be satisfied with the expression as follows:

$$\omega^2 = gk \,\mathrm{th}\, kd_0 \qquad (7)$$

Fig.3 shows the comparison of the measured attenuation coefficient of wave pressure spectral component at the level of water layers being -2.2, and -5.5m respctively with the values calculated(curve) by the expression(6).

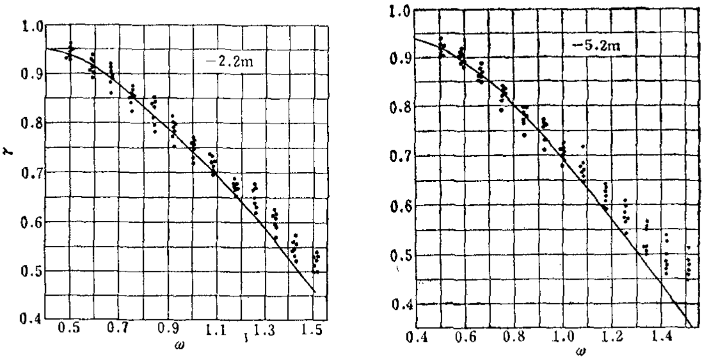

Fig.3 Comparison between the measured attenuation coefficient(dots) of wave pressure spectral components and the result(curve) of small amplitude wave theory

The above comparison shows that, within the whole range of $0.5 < kd < 1.15$, the low frequency components of the wave pressure spectra attenuated with the depth may be considered coincidence with the results of small amplitude wave theory for a limited depth. while $kd > 1.15$, the spectral component(corresponding to the high frequency portion of the wave pressure spectra) attenuated with the depth has a more even trend . This is due to the non-linear effect of the interaction bet between the wave and the vertical breakwater.

3.2Equilibrium range of wave pressure spectrum varied with the depth

It is well known that there is a most important and specific hypothesis for the wind wave spectrum, namely, while the frequency is high enough, there is a range agreed with Phillips' equilibrium range(4,5,7). Within the equilibrium range, the wave energy is only related to the physical parameters characterized the mechanism of the wave crest breaking , but is independent of external factors of wind wave generation. If we only consider this range, in which the sea-water surface tension and sea-water molecular viscosity have no effect on the wave frequency, the constitution of the equilibrium range only depends on the frequency and the gravitational acceleration. Basing ourselves on the dimensional analysis , we may obtain:

$$S(\omega) = \beta g^2 \omega^{-5}, \qquad (8)$$

where β is nondimensional constant, adopted as 7.8×10^{-3}(7).

At present, most measured wind wave spectra have primarly verified the expression(8) of the equilibrium range worked out by Phillips(5,7). Until now we have studied very little of constitution of high frequency portion for the wave pressure spectra at different depth below the sea surface. In 1973, Цыплухин and Попов (14) analysed the wave pressure spectra at different depths on the vertical breakwater in the harbours of Сочи and Шесхарис They pointed out that, in the expression(8), the power of frequency varied with the depths from 5.0 of Z=-0.6m to 8.0 of Z=-11.2m. Such a reasult show that it is not proper to analyse by the dimension because the dimension of the spectrum density is varied with the power of the frequency in the expression(8).

We intend to investigate the constitution of the high frequency portion of the spectrum for both the wave in front of the breakwater and the wave pressure at different depth on it. We have plotted the points of the spectrum density values actually measured for the wave in front of the breakwater and the wave pressures at different depth on it onto the logarithmic paper, such as Fig. 4a shows the wind wave spectra in front of the breakwater, and Fig. 4b the wave pressure spectra at the level of -2.2m.

Fig. 4a, we see that the high frequency portion of the wind wave spectra in front of the vertical breakwater may be described by the expression(8)(rough inclined line in the Fig.). From Fig. 4b, we see that the high frequency portion of wave pressure spectra almost parallely deviates from the rough lines with the increase in depth. From this we may think that within the constitution of the equilibrium range of wave pressure spectrum, the power of the frequency is still eqeal to 5, and does not vary with the increase in depth, but its spectrum density values may get smaller with the increase in depth. From the expression(8), we may obtain:

$$\beta = S(\omega) \cdot \omega^5/g^2, \qquad (9)$$

Now we calculate the actually measured β values by expression(9), and then plot it (Fig. 5).

As an example, Fig. 5 gives the β values at the level of -0.2m through many observations; we can obtain similar diagram at other water levels, too. Since the tidal levels are not the same at each observation, the actual depths in every observation also are not the same at the level of -0.2m. Thus in Fig. 5, the β values measured in every observation always have some difference. While $\omega > \omega_{max}$, however, the β values actually measured in each time do not vary with the frequency primarily, and vary up and down at some β value. This is due to the errors in the observation and the spectrum analysis. As a result, after evaluating the average value of β actually measured at different depth, Fig. 6 is plotted with the average value of β as ordinate versus the nondimensional parameter $Z\omega_{max}^2/g$ as abscissa. Here ω_{max} is the spectral peak frequency.

Fig. 6 shows that, while the sea surface Z=0, the ave-

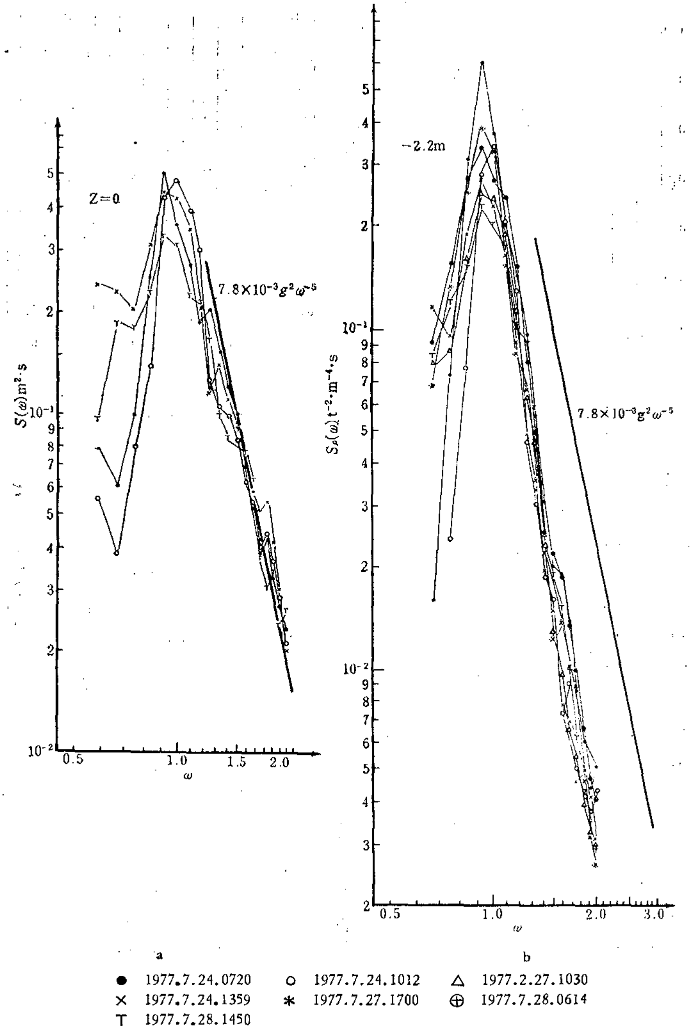

Fig. 4a,b The observed spectra of wave and wave pressure fluctuations at a vertical breakwater, declined line is expression(8).

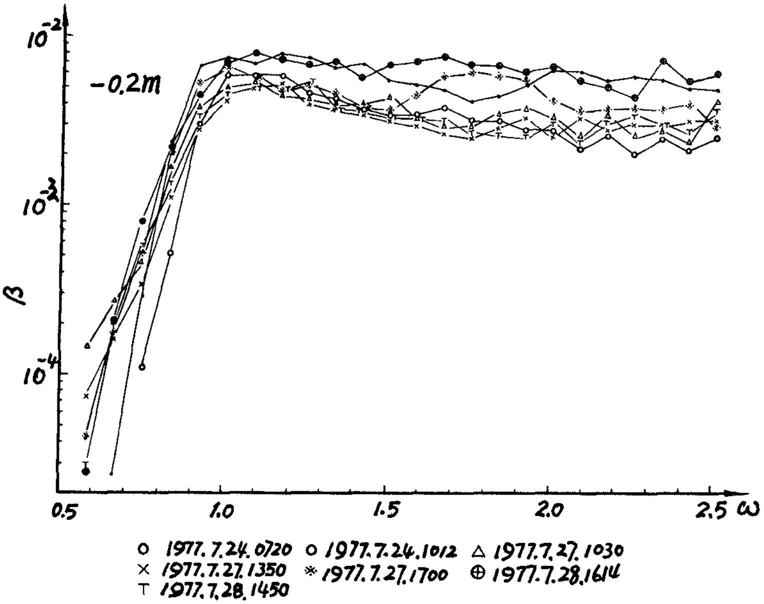

Fig. 5 The values of wave pressure spectra at level Z= -0.2m.

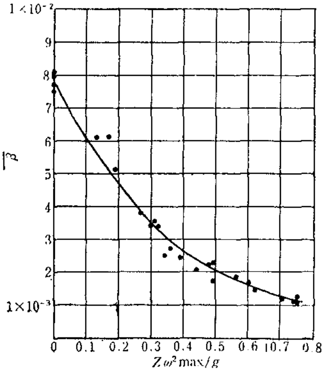

Fig. 6 The relationship between the measured value and the nondimensional parameter , of which the curve from the eg.(10).

rage value of β is 7.8×10^{-3}. Since the value of β get smaller with the increase of $Z\omega_{max}^2/g$, we can obtain:

$$\beta = \beta_0 \exp[-4\pi Z\omega_{\max}^2/5g], \quad Z\omega_{\max}^2/g \leqslant 0.75 \tag{10}$$

where $\beta_0 = 7.8 \times 10^{-3}$.

The above analyses, indicate that the equilibrium range of the wave pressure spectra at different depth on the vertical breakwater varied with depth may be decribed by the following expression:

$$S_P(\omega) = \beta_0 \omega^{-5} g^2 \exp[-4\pi Z\omega_{\max}^2/5g], \tag{11}$$

4. CONCLUSION

The results of this study are concluded as follows:

1. In the case of shallow water, the cumulative distribution of range of wave pressure fluctuation at different heights on the vertical breakwater actually does not vary with the heights of the measuring points on the breakwater, and it may be described by the expression(1). The cumulative distribution of both the crest value and the trough value of total pressure acted upon the breakwater agrees with that of wave height, and it may be calculated by expression(5).

2. The cumulative distribution of wave period also does not vary with the heights of measured points on the breakwater; it just follows the distribution law of expression(4).

3. The analysis of the wave pressure spectra at different altitudes in front of the vertical breakwater shows that within the whole range of $0.5 < kd < 1.15$, the low frequency portion of the wave pressure spectra attenuated with depth may be calculated according to the small amplitude wave theory within a limited depth. When $kd > 1.15$, the spectral component of the wave pressure (corresponding to the high frequency portion of the wave pressure spectrum) attenuated with depth has a more even trend. Thus we may think that it related to the nonlinear effect of the interaction between the wave and the vertical breakwater. In order to obtain aclear explanation, it is necessary to establish a nonlinear model of the interaction between the irregalar wave and the vertical breakwater.

4. The high frequency portion of the spectra for both the wave in front of the vertical breakwater and the wave pressure at different altitudes on it agrees with the Phillips' eqilibrium range. With the increase in depth, the power of the frequency in the constitution of the equilibrium range is basically unchanged, but the value of β may get smaller with the increase in the nondimensional parameter and can be described by the expression(10).

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