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THE PRODUCTION AND INVENTORY BEHAVIOR OF THE AMERICAN AUTOMOBILE INDUSTRY

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ABSTRACT

Understanding inventory movements is central to an understanding of business cycles. This paper presents an empirical study of the behavior of inventories in the automobile industry. It finds that inventory behavior is well explained by the assumption of intertemporal optimization with rational expectations. The underlying cost structure appears to have substantial costs of changing production as well as substantial costs of being away from target inventory, the latter being a function of current sales. Given this cost structure, whether inventory behavior is stabilizing or destabilizing depends on the characteristics of the demand process. In the automobile industry, inventory behavior is destabilizing: the variance of production is larger than the variance of sales.

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Abstract

Understanding inventory movements is central to an understanding of business cycles. This paper presents an empirical study of the behavior of inventories in the automobile industry. It finds that inventory behavior is well explained by the assumption of intertemporal optimization with rational expectations. The underlying cost structure appears to have substantial costs of changing production as well as substantial costs of being away from target inventory, the latter being a function of current sales. Given this cost structure, whether inventory behavior is stabilizing or destabilizing depends on the characteristics of the demand process. In the automobile industry, inventory behavior is destabilizing: the variance of production is larger than the variance of sales.

Introduction

There is substantial agreement in macroeconomics about the importance of inventory behavior in the business cycle. There is however little agreement beyond that, for example on the issue of whether inventory behavior is stabilizing or destabilizing. Although there exists a widely accepted standard inventory equation (Lovell, 1961), it has been shown (Feldstein and Auerbach, 1976) that the empirical estimates obtained in these equations partly contradict the theory from which these equations are derived.

The goal of this study is to learn more about inventory and production behavior. This study makes two choices.

The first is to attempt to recover structural parameters, i.e. the parameters characterizing the technology rather than to estimate reduced form equations. The reason for doing so is well understood (Lucas, 1976):

the knowledge of structural parameters is both conceptually useful and necessary to answer most questions of interest, such as the effect of a particular sales process on inventory behavior or the conditions under which inventory behavior may be stabilizing or destabilizing. Reduced form coefficients are functions of both these structural parameters and of the environment in which the firm or the industry operates. They do not by themselves allow to answer the above questions. The approach used is therefore to assume that production and inventory decisions are the results of a dynamic optimization problem; the empirical work amounts to solving econometrically an inverse problem, i.e. to recover the function being maximized from the observed behavior.

The implication is that the dynamic optimization problem has to be formulated as a linear-quadratic problem, as this is the only case in which we know how to solve econometrically the inverse problem. Starting with the work of Holt, Modigliani, Muth and Simon (1960), the linear quadratic framework has often been used to characterize inventory behavior. Nevertheless, to use it implies doing violence to some facts and excluding from the outset explanations based on nonconvexities. The econometric methods used are extensions of the methods developed by Hansen and Sargent (1980). Because the linear-quadratic formalization is at best an approximation, we should be under no illusion that the estimated structural parameters are truly invariant to all changes in the environment; they are however surely less affected by such changes than reduced form coefficients.

The second choice follows from the first. This type of estimation imposes a very tight structure on the data. It is likely to give reasonable results only if it is reasonable to assume that the data used are indeed generated by the assumed optimization problem. This is less reasonable, the higher the degree of aggregation, the worse the quality of the data used--because of index number problems for inventory, for example. For this reason, this study concentrates only on the behavior of the automobile industry. Excellent data, weekly and by model, can be obtained for production, sales to and by dealers for a long period of time.

I believe that the results obtained in this study are interesting in two respects:

The first is that the use of this technique is overall a success. Previous attempts to use a similar technique, for example on aggregate consumption (Sargent, 1978) have usually yielded negative results, i.e. a rejection of the hypothesis that observed behavior could be generated by the assumed optimization problem. This is not the case here. Observed behavior is well explained as maximizing behavior and the estimated parameters are usually in accordance with prior beliefs. This suggests that this approach can be used successfully.

The second and main respect is that the empirical findings are somewhat at variance with the prevailing view. Heuristically, the conclusion is that inventory behavior is well explained by the combination of two costs, a cost of moving production and a cost of being away from a desired inventory level. The first effect leads to production smoothing and is "stabilizing," although its implications differ from the implications of a convex cost function. The second effect is "destabilizing" because of a high marginal desired inventory to current sales ratio. Which one dominates depends on the characteristics of the demand process.

This paper is organized as follows. Section I describes briefly the relevant aspects of the automobile industry. Section II gives descriptive statistics about production and inventory behavior in the industry. Section III formalizes the model and derives the equations to be estimated. Sections IV and V report the estimation results. Section VI shows the economic implications of the estimation results. Section VII relates these results to the literature on inventory and production behavior.

Section I. The Industry

This section first justifies the choice of the level of aggregation adopted in the study, namely the division level, and of the time unit, namely the month. It then describes how a typical division is actually organized and how, because of data limitations and other considerations, it is assumed to be organized in the rest of the study.

The automobile industry was chosen because of the availability of weekly data on sales and production at a disaggregated level. Working at the level of the model is however not desirable, mainly for two reasons. As many plants produce more than one model, production decisions for different models are interrelated and the interrelation is hard to formalize. For most of the econometric work, the assumption that the sales process is stationary is extremely helpful but cannot be made at the model level: models go through a life cycle. The analysis will be done at the division level, for which interrelations between models can be forgotten and for which the assumption of stationarity will be shown to be acceptable for the period of estimation. There are ten divisions considered in this paper: Five are parts of General Motors; they are Chevrolet (denoted in what follows CV), Pontiac (PT), Oldsmobile (OD), Buick (BK) and Cadillac (CD). Two are parts of Ford; they are Ford (FD) and Mercury Lincoln (ML). Two are parts of Chrysler, Chrysler-Plymouth (CP) and Dodge (DG). The last one is American Motors (AM).

The second choice to be made is that of the time unit. If there is such a period as the decision period — in which decisions are taken for the duration of the period and not changed until the next period —, it is very useful to use the same period as a sampling period. As many production decisions and production and sales forecasts are made on a monthly basis, this has led to the choice of the month as the assumed decision period and the time unit for estimation. The period of estimation chosen is 1966-1 to 1979-12. The first date was chosen so as to have no major reorganization of divisions during the sample period.

The Actual Organization of a Division.

Because of the Canadian automobile agreement signed in 1965 which removed most tariff barriers between the U.S. and Canada and has led to an idiosyncratic allocation of production across both countries, it would make little sense to consider the U.S. without Canada. Production for all divisions takes place both in Canada and the U.S., at least for part of the sample period (except for Cadillac). Plants in the U.S. and Canada then ship cars to U.S. dealers, Canadian dealers and the rest of the world, with substantial flows both from U.S. plants to Canadian dealers, and from Canadian plants to U.S. dealers.

Manufacturers do not, except for Chrysler, hold inventories other than cars in transit. As a result, most inventories (91% for divisions other than Chrysler, 87% for Chrysler) are held by dealers. Data on production and flows from plants to dealers are all available. Sales by U.S. dealers are available but sales by Canadian dealers are not; as a result I have no data on Canadian dealer inventories.

The Assumed Organization of a Division.

Production in the U.S. and Canada is aggregated to give American production, Y_t. Producers are assumed to hold no inventories: actual manufacturers' inventories are added to dealers' inventories. Production is sold to two groups: the first is the sum of exports and shipments to Canadian dealers and is denoted Z_t. The second is shipments to U.S. dealers, denoted D_t. Thus $Y_t = D_t + Z_t$.

U.S. dealers receive shipments D_t, hold inventories I_t and sell to U.S. customers S_t. I_t denotes end of month t inventories and therefore satisfy:

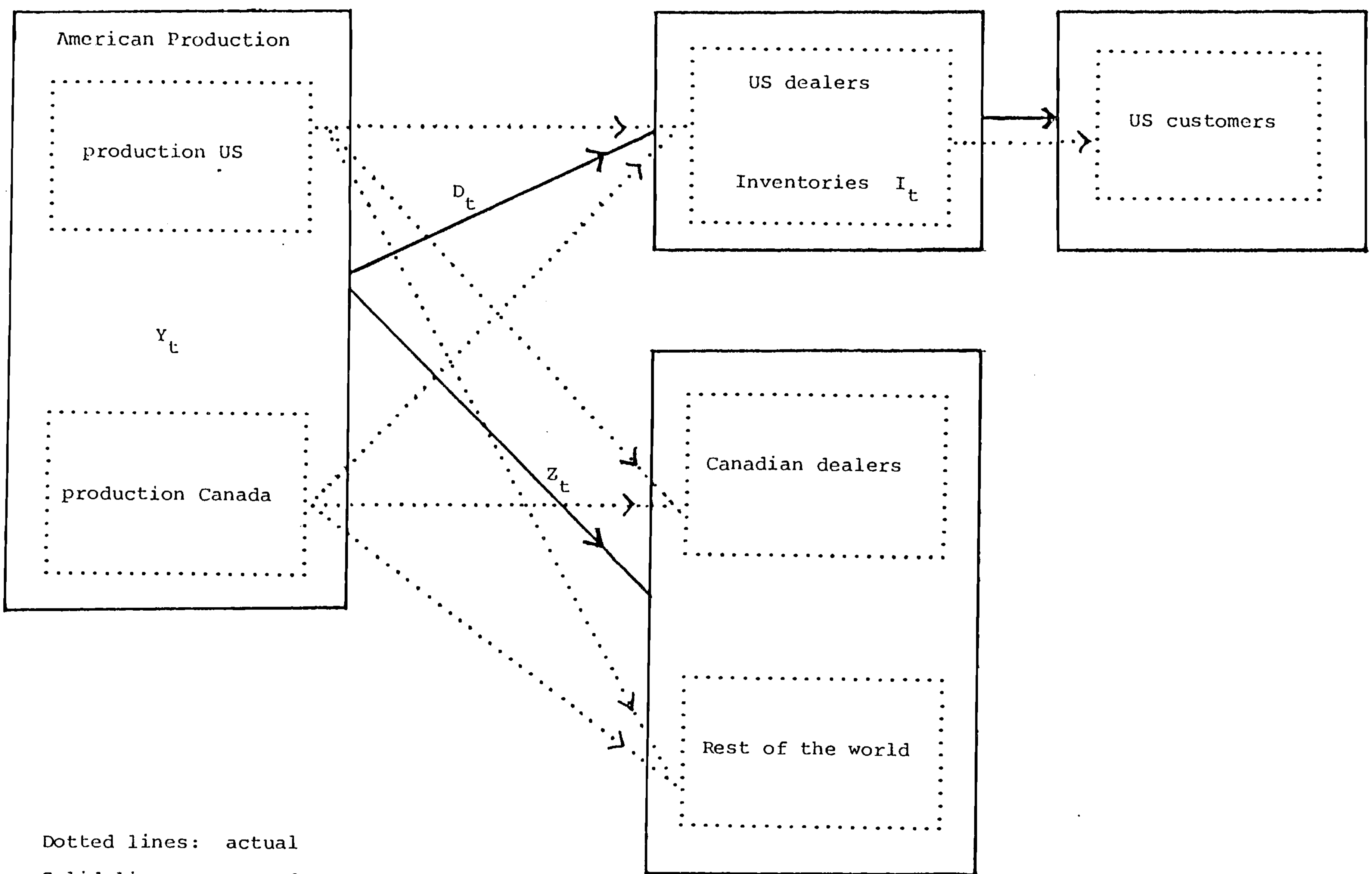
$$I_t = I_{t-1} + D_t - S_t$$

Figure 1 shows the relation of the assumed to the actual structure.

The Manufacturer-Dealers Market.

Each division is composed of a manufacturer and a large number of dealers. There is no actual market in the usual sense and the reported price charged by manufacturers to dealers is approximately the list price minus some constant amount and moves little. One extreme interpretation is that dealers face a perfectly elastic supply of cars at that constant price and that production adjusts passively. The other extreme interpretation is that manufacturers in fact use both forcing and rationing, the "true" price adjusting so that inventory adjusts passively. (See White, 1971, for a description of dealer-manufacturer relations.) None of these two extreme interpretations is consistent with the empirical evidence. The assumption will therefore be one of a shadow market, in which manufacturers have a supply curve, dealers a demand curve, the shadow market clearing competitively. The implication of this assumption (to be formally proven in Section III) is the equivalence of

Figure 1. Actual and Assumed Organization of a Division



Dotted lines: actual

Solid lines: assumed

the decentralized organization of a division with manufacturer and dealers to a centralized firm making both production and inventory decisions. Therefore in most of what follows, a "division" can be thought of as one firm taking both decisions.

To summarize, a division is assumed to be equivalent to a centralized firm. It produces both in the U.S. and Canada, selling cars both to U.S. customers and the rest of the world, holding inventories against sales to U.S. customers only. We have data on production Y_t and both types of sales S_t and Z_t.

Section II. Basic Factors about Production and Sales

This section has two goals. The first is to give basic facts before any elaborate econometrics are applied to the data. The second is to discuss and justify some choices made in formalizing the behavior of a division in the next section.

Table 1 gives the mean values for production, sales and inventories by division. The size of divisions varies in the ratio of approximately 10 (Chevrolet) to 1 (Cadillac). The last column gives the coefficient on a linear time trend with value one in 1966-1 in a regression of production on a constant and this time trend: Total North-American production shows practically no trend; the trend is small compared to average production for most divisions.

Divisions look similar in other respects. The ratio of manufacturers' inventories to total inventories is small, varying between 3.1% for Ford and 17.1% for Oldsmobile. No inventories are held against Z_t, sales to Canadian dealers and the rest of the world. Therefore, the relevant inventory sales ratio is the ratio of total inventories, I_t, to sales by U.S. dealers, S_t. This ratio varies

Table 1. Basic Statistics. Means. 1966-1 to 1979-12

	American Production, Y_t	Sales by US dealers, S_t	Sales to rest of the world, Z_t	Manu-facturers' inventories	Dealers' inventories	Total inventories, I_t	I_t/S_t (in months)	Time trend of production (cars p/month)
CV	194,995	177,543	17,057	19,741	422,258	441,999	2.49	141 (2.8) t statistic
PT	70,516	61,924	8,036	7,861	122,358	130,219	2.10	-21 (-.79)
OD	65,062	60,304	4,485	20,980	101,097	122,077	2.02	252 (10.51)
BK	57,299	53,114	3,895	5,786	102,977	108,763	2.04	89 (4.35)
CD	22,615	21,698	726	2,192	28,303	30,495	1.40	81 (10.23)
FD	166,501	152,523	13,568	11,578	351,809	363,387	2.38	-115 (-2.43)
ML	47,166	40,971	5,766	9,255	111,224	120,479	2.94	213 (11.60)
AM	24,698	21,798	3,157	7,924	58,515	66,439	3.04	-46 (-3.77)
DG	47,869	41,553	6,772	28,396	156,690	185,086	4.45	-90 (-5.04)
CP	77,377	67,821	9,787	34,566	242,600	277,166	4.08	-138 (-5.29)

between 1.40 and 3.04 months, except for the two divisions of Chrysler, Dodge and Chrysler-Plymouth for which it is 4.45 and 4.08 months respectively.

Since the work of Holt, Modigliani, Muth and Simon, it is generally accepted that production and inventory behavior depend on three types of costs: the first is simply the cost of producing, which under the standard assumption of decreasing returns is convex and therefore leads — if factor prices are constant — to production smoothing; the second is the cost of moving production, which also clearly leads to production smoothing. The two types of costs differ slightly in their implications, the second one implying that current production levels depend on past production, the first one not having this implication. The third type of cost is the cost of being away from some target level of inventory, the target level being either constant (as in Blinder, 1982, for example), or a function of current — or depending on the exact formalization, next period expected — sales. (To avoid semantic discussions, it should be pointed out that this target level is not the "desired inventory" level found in reduced form empirical inventory equations). Only if this last cost is large and inventory is a function of current sales can inventory behavior lead not to production smoothing but to larger movements in production than in sales.

It is therefore interesting to compare the variance of production and the variance of sales. The results are shown in Table 2. To have variables of the same magnitude, the table compares the variance of production (Y_t) to the variance of total sales, which is the sum of sales by U.S. dealers and sales to Canada and the rest of the world, $S_t + Z_t$. The results are striking. The first three columns give the standard deviations of the raw series and the ratio of the standard deviations. This ratio varies between 1.23 (ML) and 1.43 (PT, CP). To see whether this is due possibly to factors such as August holidays affecting production, the production and sales series are then regressed on two sets of

Table 2. Standard Deviations of Production (Y) and Sales (S±Z)

	Raw Time Series			Seasonal Component			Cyclical Component		
	1	2	3	4	5	6	7	8	9
	$\sigma_{production}$	σ_{sales}	$\frac{\sigma_{production}}{\sigma_{sales}}$	$\sigma_{production}$	σ_{sales}	$\frac{\sigma_{production}}{\sigma_{sales}}$	$\sigma_{production}$	σ_{sales}	$\frac{\sigma_{production}}{\sigma_{sales}}$
CV	54,571	38,443	1.42	40,778	24,736	1.65	30,700	24,900	1.23
PT	25,013	17,433	1.43	17,635	9,997	1.76	17,100	13,300	1.29
OD	25,288	19,817	1.27	15,584	8,834	1.76	15,000	11,800	1.30
BK	19,626	14,160	1.38	13,253	8,143	1.63	12,900	9,860	1.30
CD	9,039	6,670	1.35	6,403	3,297	1.94	4,960	3,620	1.37
FD	42,768	30,082	1.42	25,718	18,603	1.38	28,500	20,700	1.37
ML	17,196	13,998	1.23	7,637	5,836	1.31	11,000	8,320	1.30
AM	9,680	7,227	1.34	5,830	2,974	1.96	7,720	6,370	1.22
DG	12,838	9,502	1.35	7,902	5,961	1.33	9,740	6,750	1.44
CP	21,290	14,807	1.43	12,984	8,847	1.47	16,400	10,700	1.53

Period: 1966-1 to 1979-12

variables. The first is a set of 12 monthly dummies which therefore accounts for additive seasonality; the second includes a time trend and individual division dummies accounting for strikes (see the data appendix for exact definitions). Columns 4 to 6 in Table 2 report the "standard deviations" of the seasonal component of the production and sales series; as this series is deterministic, it does not have a standard deviation in the statistical sense. The value reported is simply $\left[\frac{\sum_{i=1}^{12}(\hat{\beta}_i-\bar{\hat{\beta}})^2}{11}\right]^{1/2}$ where $\hat{\beta}_i$ is the estimated coefficient of the i^{th} dummy. Columns 7 to 9 report the standard deviations of the cyclical component, which are simply the standard errors of the regressions. For both the seasonal and the cyclical components and for all divisions, the variance of production is larger than the variance of sales. The ratio is somewhat larger for the seasonal component, a result which may be surprising as the seasonal component of sales is better anticipated than the cyclical component and allows for better production planning. Table 2 suggests strongly that production smoothing is not the dominant element of inventory behavior and that target inventory is probably a function of current sales.

Can we also say something about the relative importance of the <u>convexity of the cost function versus the cost of moving production</u>? If there are costs of moving production, the formal model and the associated econometrics are substantially more complex; it is therefore worth getting some prior evidence.

Intuition (supplemented in the next section by a formal proof) suggests that this can be done. If there were no costs of moving production and the only two costs were the cost of producing and the cost of being away from target inventory, the only variable from the past directly affecting decisions would be last period end of period inventories. Lagged production would not itself affect current decisions. If, instead, the cost function is linear but there are costs of moving production, the only relevant variable from the past will be lagged production. Formally, if we write

$$Y_t = \alpha Y_{t-1} - \beta I_{t-1} + \xi_t'$$

where ξ_t' presumably depends on current and future expected sales, in the first case we have $\alpha = 0$ and in the second case $\beta = 0$. If both costs are relevant $\alpha \neq 0$ and $\beta \neq 0$. This can also be written, using the identity $I_{t-i} = I_{t-i-1} + Y_{t-i} - S_{t-i} - Z_{t-i}$, as:

$$I_t = (1+\alpha-\beta)I_{t-1} - \alpha I_{t-2} + \xi_t \text{ with}$$

$$\xi_t = \xi_t' - S_t - Z_t + \alpha S_{t-1} + \alpha Z_{t-1}$$

If expected sales depend only on past S's and past Z's, ξ_t can be replaced by a distributed lag of S and Z and the following reduced form regressions can be run, allowing to test $\alpha = 0$ and/or $\beta = 0$:

$$I_t = (1+\alpha-\beta)I_{t-1} - \alpha I_{t-2} + \sum_{i=0}^{n} a_i S_{t-i} + \sum_{i=0}^{n} b_i Z_{t-i} + \varepsilon_t$$

$\hat{\alpha}$, $\hat{\beta}$ are consistent and the tests valid only if no variables other than past S's and Z's were used to forecast sales and if ε_t is white. Even if these assumptions are not exactly satisfied, these regressions appear to be a simple and useful first step: They do not allow us to estimate any structural parameters, nor can we interpret the coefficients on current and lagged S and Z; they help however in choosing the appropriate model.

The results are given in Table 3. A set of monthly dummies is included in each regression. n is chosen equal to 3. Two regressions are given for each division. The first includes I_{t-1} and I_{t-2}. The second includes also I_{t-3}; only the coefficients on lagged I's are reported for the second one to avoid cluttering the table.

Table 3. Reduced Form Estimates

I_t	I_{t-1}	I_{t-2}	(I_{t-3})	S_t	$\sum_1^3 S_{t-i}$	Z_t	$\sum_1^3 Z_{t-i}$	R^2	SSR	D.W.	$\hat{\alpha}$	$\hat{\beta}$
CV	0.99 (11.43)	- .33 (-3.85)		- .26 (-2.90)	.42	3.20 (6.17)	-2.00	.89	6.93 xE10	1.98	.33 (3.85)	.34 (5.58)
	1.02	- .40	.06 (.74)									
PT	1.23 (15.15)	- .38 (-4.85)		- .43 (-4.27)	.54	2.02 (4.32)	-1.35	.91	1.33 xE10	2.09	.38 (4.85)	.15 (4.69)
	1.18 (13.31)	- .23 (-1.69)	- .11 (-1.31)									
OD	1.24 (14.83)	- .37 (-4.49)		- .48 (-5.11)	.57	2.82 (4.58)	-2.65	.91	1.23 xE10	2.07	.37 (4.49)	.13 (3.97)
	1.19	- .21	- .12 (-1.35)									
BK	1.28 (15.29)	- .38 (-4.73)		- .36 (-3.72)	.45	2.03 (3.77)	- .63	.95	9.28 xE9	2.04	.36 (3.72)	.10 (3.74)
	1.25	- .29	- .06 (- .72)									
CD	1.12 (12.63)	- .26 (-3.00)		- .28 (-2.79)	.17	6.00 (5.12)	- .18	.94	1.97 xE9	2.11	.26 (3.00)	.14 (4.04)
	1.06	- .03	- .18 (-2.08)									

continued on following page

Table 3 (continued)

I_t	I_{t-1}	I_{t-2}	(I_{t-3})	S_t	$\sum_1^3 S_{t-i}$	Z_t	$\sum_1^3 Z_{t-i}$	R^2	SSR	D.W.	$\hat{\alpha}$	$\hat{\beta}$
FD	1.23 (14.06)	- .37 (-4.32)		- .29 (-2.75)	.26	4.55 (8.22)	-2.52	.89	5.09 xE10	2.00	.37 (4.32)	.14 (3.96)
	1.22	- .34	- .02 (- .29)									
ML	1.35 (15.49)	- .41 (-4.91)		- .38 (-3.56)	.54	2.34 (6.26)	-2.35	.97	5.63 xE9	1.95	.41 (4.91)	.06 (2.50)
	1.37	- .49	.05 (.63)									
AM	1.06 (12.75)	- .17 (-2.24)		- .44 (-3.99)	.38	2.25 (7.00)	- .97	.89	2.04 xE9	2.01	.17 (2.24)	.11 (3.30)
	1.06	- .18	.00 (.02)									
DG	1.15 (13.21)	- .17 (-1.98)		- .44 (-3.06)	.55	1.42 (2.92)	-1.18	.97	7.30 xE9	2.01	.17 (1.98)	.02 (.82)
	1.16	- .22	.04 (.48)									
CP	1.01 (11.47)	- .13 (-1.44)		- .32 (-2.45)	.42	1.64 (2.83)	- .53	.82	2.03 xE10	1.98	.13 (1.44)	.12 (3.12)
	1.02	- .22	.09 (1.05)									

- t statistics in parentheses. Estimation period 1966-4 to 1979-12
- Only the sums of coefficients are given for lagged S and lagged Z

The results are very clear. Both $\hat{\alpha}$ and $\hat{\beta}$ are significant for nearly all divisions. The coefficient on I_{t-2}, $(-\hat{\alpha})$ is always significant. This suggests that costs of moving production are important and must be incorporated in the model. The coefficient on I_{t-3} is usually insignificant; this suggests that the costs described above may be sufficient to explain production behavior.

The last set of facts given in this section relates to seasonality. Mainly for reasons of convenience, seasonality will be assumed to be additive rather than multiplicative. Table 2 shows that the seasonal component is large, with a standard deviation about as large as that of the cyclical component, both for sales and production. The question is whether there is useful information in the seasonal movements of production and sales. If the costs themselves had no seasonal component, then seasonal movements in production would be entirely due to seasonal movements in sales. As these movements are anticipated, there would be substantial information in seasonal movements. The other extreme is the case where costs themselves had a seasonal component. If the movements of this seasonal were left unconstrained, there would be no information to be obtained from seasonal movements in production and sales. Table 4 gives the values of the seasonals for two representative divisions. The pattern for sales shows two peaks, one in March to June and one in October-November corresponding to the introduction of the new models. The pattern of production shows two lows, one in July-August, the other in December. It appears very unlikely that the seasonal pattern of production can be explained only by the seasonal pattern of sales; the lows are due partly to holidays, partly to the technical modifications needed to change the models. Rather than constrain a priori the seasonal pattern of cost, I shall leave it unconstrained. This is equivalent to not using the information contained in seasonal components.

I now turn to the formal model.

Table 4. Seasonal Components in Production, (Y) and Sales, (S+Z)

	Chevrolet		Cadillac	
	Y	S + Z	Y	S + Z
January	1.96 xE5	1.50 xE5	1.94 xE4	1.46 xE4
February	1.76	1.65	1.73	1.47
March	2.08	1.98	1.99	1.91
April	2.02	1.90	1.83	1.72
May	2.14	2.14	1.89	1.74
June	2.21	2.20	1.87	1.51
July	1.45	1.86	.72	1.28
August	.80	1.56	.03	1.05
September	1.84	1.65	1.53	1.42
October	2.27	2.14	2.29	2.23
November	2.04	1.95	1.85	2.02
December	1.59	1.59	1.54	1.71

Section III. The Model

We have, for each division, data on production and sales but not on final prices. (List prices bear only a vague resemblance to transaction prices, especially cyclically. The only transaction price series, the "new automobile" component of the CPI, is available only for the industry as a whole.) We must therefore construct a model which, once estimated, allows us to recover the technology even in the absence of price data. We can choose one of two formalization strategies:

The first is to specify, in addition to the technology, the demand system and the nature of market equilibrium (competitive, monopolistic or otherwise). We can then derive equilibrium price, sales and production. Even if prices are unobservable, it may still be possible to recover some or all of the parameters of the technology (Blanchard, 1982). They will depend however on the assumptions made about demand, and market equilibrium.

The second is closer to the traditional empirical approach to inventory behavior, which regresses production (equivalently, inventory) on sales. The above simultaneous determination of price, sales and production can be equivalently recast as a two-level decision problem. First a division derives its optimal price as a function of exogenous variables and shocks; if the assumptions about demand and market equilibrium are the same as above, the price rule will also be the same. Given the price rule and demand disturbances, sales are determined; the firm then solves the second decision problem, that of scheduling production given sales. As we have no data on prices, we may formalize and concentrate only on the second decision. A further assumption is however needed to allow us to estimate the technology by looking only at the response of production to sales. It is that prices be uncorrelated with current and lagged cost disturbances. If they were correlated, sales would also be correlated with these disturbances, making identification and estimation difficult or impossible, depending on other assumptions of the model.

This paper adopts the second strategy, which does not require specification of demand and market equilibrium but requires the assumption that prices be uncorrelated with technological shocks. This assumption is unlikely to hold exactly but is hopefully not too strongly violated. It cannot be tested by itself but only as a joint assumption; we shall return to this issue in Section V. The alternative strategy is pursued in another paper (Blanchard and Melino, 1981).

A division consists of a manufacturer and dealers; we now describe the behavior of the manufacturer and dealers, the manufacturer-dealer market equilibrium and derive the equation to be estimated.

Time is formalized as follows. At time t, dealers have inventories I_{t-1} from which they satisfy sales S_t. At time t_+, production, Y_t, takes place, dealers demand D_t, the rest of the world demands Z_t and the manufacturer-dealer "market" clears at shadow price p_t. The two implications are that monthly sales are known when monthly production decisions are taken and that dealers cannot use current shipments to satisfy current sales.

The Problem of the Manufacturer.

The manufacturer at time t has revenues $p_t\ Y_t$ and two types of costs. The first is the cost of producing Y_t:

$$C_t \equiv \frac{1}{2}\, c(Y_t + \varepsilon_t)^2 \quad ; \quad c \geq 0.$$

This cost is quadratic in Y_t; although there could be a linear term, it plays no important role and is deleted for notational simplicity. ε_t is a disturbance. It can represent either technological disturbances or factor price disturbances. It is the sum of a stochastic component $\tilde{\varepsilon}_t$ and a seasonal component $\Delta_t \beta_\varepsilon$, where Δ_t is a set of twelve monthly dummies.

The second is the cost of changing production and is given by:

$$K_t \equiv \frac{1}{2} k(Y_t - Y_{t-1} + \eta_t)^2 \quad ; \quad k \geq 0.$$

This cost is quadratic in $(Y_t - Y_{t-1})$; again the linear term is deleted. η_t is a disturbance, with stochastic component $\tilde{\eta}_t$ and seasonal component $\Delta_t \beta_\eta$.

At time t the manufacturer maximizes the expected present value of profits. The discount factor $b < 1$ is assumed constant, so that this present value is:

$$E[\Sigma b^i (p_{t+i} Y_{t+i} - C_{t+i} - K_{t+i}) | \Omega_t]$$

The same information set Ω_t is assumed for the manufacturer and for the dealers. It includes at least current and past values for I, Y, Z, S, p and disturbances; some of these variables are obviously redundant and some of them irrelevant either to the manufacturer or to dealers. The set of first-order conditions is, in addition to the transversality condition: $\forall\, i \geq 0$,

$$(1) \quad E[p_{t+i} = (c+k(b+1))Y_{t+i} - kY_{t+i-1} - bkY_{t+i+1} + c\varepsilon_{t+i} + k\eta_{t+i} - bk\eta_{t+i+1} | \Omega_t]$$

This set of conditions is easily understood by considering for i=0, the cases where $c = 0$ or $k = 0$. If $k = 0$:

$$Y_t = c^{-1} p_t - \varepsilon_t$$

In this case, behavior reduces to the standard condition that marginal cost equals price. Given p_t, neither the past nor the expected future matters. If $c = 0$:

$$Y_t = (b+1)^{-1} [k^{-1} p_t + bE(Y_{t+1} | \Omega_t) + Y_{t-1}] + (b+1)^{-1} [-\eta_t + bE(\eta_{t+1} | \Omega_t)].$$

In this case, production depends positively on the price, but also on lagged and expected future production, with weights adding to unity. The larger the cost of changing production, k, the smaller the effect of the current price.

The Problem of Dealers.

Given demand conditions and the price rule, dealers face sales S_t. They decide about their shipments from manufacturers, D_t, at shadow price p_t. In addition to their purchases, they have only one type of cost, the cost of deviating from target inventory:

$$G_t \equiv \frac{1}{2} d(I_{t-1}-I^*_{t-1}+u_{t-1})^2 \quad ; \quad I^*_{t-1} = aS_t \quad ; \quad a, d > 0$$

The cost is quadratic in the distance of inventories to target inventories; target inventories are a linear function of sales. The dating of I at t-1 comes from the measurement of I as end of period inventories. u_t is again a technological disturbance, with stochastic component $\tilde{u}_t$ and seasonal component $\Delta_t\beta_u$.

The underlying justification is that this cost function is itself the sum of two cost functions: the first is the physical cost of carrying inventories, which is an increasing function of the level of inventories; the second is the expected cost of stocking out, which is a decreasing function of the level of inventories given sales, as a higher inventory to sales ratio decreases the probability of stocking out. The sum of these two costs reaches a minimum for some level of inventories which is denoted I*. An increase in sales, for any level of inventories, increases the expected cost of stocking out, moves the minimum of the cost function to the right. This description makes clear however that the coefficients a and d are very likely to depend at least on the second moments of the distribution of sales: In this sense, they are not truly structural.

The decision variable of dealers at time t is D_t, the shipments from the manufacturer. At time t, dealers minimize the expected present value of cost:

$$E[\sum_{i=0}^{\infty} b^i (p_{t+i}D_{t+i}+G_{t+i})|\Omega_t]$$

subject to $I_{t+i} = I_{t+i-1} + D_{t+i} - S_{t+i}$.

The set of first order conditions, in addition to the transversality condition is: $\forall i \geq 0$,

(2) $\quad E[p_{t+i} = bp_{t+i+1} - db(I_{t+i}-aS_{t+i+1}+u_{t+i})|\Omega_t]$

For i=0 and rearranging:

$$I_t = aE(S_{t+1}|\Omega_t) + d^{-1}(E(p_{t+1}|\Omega_t)- b^{-1}p_t)- u_t.$$

If $E(p_{t+1}|\Omega_t)- b^{-1}p_t = 0$, dealers'demand is such as to attain target inventory $aE(S_{t+1}|\Omega_t)$. If the (discounted) price is expected to increase, they demand more; the size of the response to expected price changes is inversely proportional to d, the convexity of their cost function.

Manufacturer-Dealer Market Equilibrium.

At any time t, the current and expected price sequence $\{E(p_{t+i}|\Omega_t)\}_{i \geq 0}$ must be such that the dealer-rest-of-the-world-manufacturer market is expected to clear, i.e. that $\forall i \geq 0$,

(3) $\quad E[Y_{t+i} = D_{t+i} + Z_{t+i}|\Omega_t]$

where Y_t, D_t are given by (1) and (2) respectively. This set of market equilibrium conditions can be solved as follows: Production Y is eliminated using the identity $Y_t = I_t - I_{t-1} + Z_t + S_t$ and p_t is eliminated using (1). Tedious manipulations give:

(4) $\quad E[B_1(L)I_{t+i} = B_2(L)Z_{t+i} + B_3(L)S_{t+i} + \xi_{t+i}|\Omega_t]$

where L is the lag operator, $L^{-1} = F$ and

$$B_1(L) \equiv b^2kL^{-2} - b(c+2(b+1)k)L^{-1}$$
$$+ [c + (2b+1)k + b(c+(b+2)k) + db]$$
$$- (c+2(b+1)k)L + kL^2$$

$$B_2(L) \equiv - b^2kL^{-2} + b(c+(b+2)k)L^{-1} - (c+(2b+1)k) + kL$$

$$B_3(L) \equiv - b^2kL^{-2} + b(ad+c+(b+2)k)L^{-1} - (c+(2b+1)k) + kL$$

$$\xi_{t+i} \equiv - dbu_{t+i} - (1-bL^{-1})(c\varepsilon_{t+i}+k\eta_{t+i}-bk\eta_{t+i+1})$$

This is still only a first order condition, giving a relation between endogenous variables. The following remarks can however be made: In addition to expected, current and past sales, inventory depends on itself lagged once and twice and itself led once and twice. As k goes to zero, the effect of I lagged or led twice also goes to zero, substantially reducing the complexity of (4). The effect of sales to U.S. customers S_t or other sales Z_t on inventory behavior is the same except for the term $abdL^{-1}$ for S_t: by assumption inventory is held against S_t and not against Z_t. The term abd represents the direct effect of S on target inventory, the others the effects of S and Z on the scheduling of production. The last remark is on the effect of b: the smaller b, the larger the rate at which the future is discounted, the smaller the effect of the expected future.

Equation (4) has been derived as a market equilibrium equation. Not surprisingly, it could have been derived as the first order condition of a centralized problem, namely the solution to:

$$\min_{\{Y_{t+i}\}_{i\geq 0}} E[\sum_{i=0}^{\infty} b^i (C_{t+i}+G_{t+i}+K_{t+i})|\Omega_t]$$

$$\text{s.t } I_{t+i} = I_{t+i-1} + Y_{t+i} - S_{t+i} - Z_{t+i}$$

This formally justifies the remark made in the preceding section that a division can be thought of as a firm deciding about production and inventory behavior and minimizing the expected present discounted value of cost given current and expected sales.

The Inventory Equation.

Equation (4) indicates that inventory behavior depends on five structural parameters, a, b, c, d, k. Equation (4) is however homogeneous of degree zero in c, d, k. Absolute convexities cannot be determined; only relative convexities can. We may therefore normalize, by choosing $k = 1$; the goal of the estimation will be to recover a, b, c, d.

To solve equation (4), $B_1(L)$ can be rewritten, using $k = 1$, as:

(5) $$b^2L^2[F^4 - b^{-1}(c+2(b+1))F^3 + b^{-2}(c+2b+1+b(c+b+2)+db)F^2 - b^{-2}(c+2(b+1))F + b^{-2}].$$

The polynomial in F is such that if λ is a root, then $\lambda^{-1}b^{-1}$ is also a root. Call λ_1 and λ_2 the two smallest absolute value roots; they are either real or complex conjugates. Define

(6) $\psi_t \equiv B_2(L)Z_t + B_3(L)S_t + \xi_t$ and consider equation (4) for i=0:

$$E[B_1(L)I_t = \psi_t|\Omega_t] \Rightarrow$$

$$E[b^2L^2(F-\lambda_1)(F-\lambda_2)(F-\lambda_1^{-1}b^{-1})(F-\lambda_2^{-1}b^{-1})I_t = \psi_t|\Omega_t] \Rightarrow$$

$$E[b^2(1-\lambda_1L)(1-\lambda_2L)(F-\lambda_1^{-1}b^{-1})(F-\lambda_2^{-1}b^{-1})I_t = \psi_t|\Omega_t] \Rightarrow$$

$$E[(1-\lambda_1L)(1-\lambda_2L)I_t = (\lambda_1\lambda_2(1-\lambda_1bF)^{-1}(1-\lambda_2bF)^{-1})\psi_t|\Omega_t] \Rightarrow$$

$$E[(1-\lambda_1L)(1-\lambda_2L)I_t = b^{-1}(\lambda_1-\lambda_2)^{-1}\lambda_1\lambda_2\sum_{i=0}^{\infty}[(\lambda_1b)^{i+1} - (\lambda_2b)^{i+1}]\psi_{t+i}|\Omega_t].$$

As I_t, I_{t-1}, $I_{t-2} \in \Omega_t$, this implies:

$$I_t = (\lambda_1+\lambda_2)I_{t-1} - \lambda_1\lambda_2 I_{t-2}$$
$$+ b^{-1}(\lambda_1\lambda_2)(\lambda_1-\lambda_2)^{-1}\sum_{i=0}^{\infty}[(\lambda_1b)^{i+1} - (\lambda_2b)^{i+1}]E(\psi_{t+i}|\Omega_t)$$

Define a vector R such that:

$$R' = [-b^2,\ b(c+b+2),\ -(c+2b+1),\ 1,\ -b^2,\ b(c+b+2) + adb,\ -(c+2b+1),\ 1]$$

and define

$$\overline{Z}_t \equiv \begin{bmatrix} Z_t \\ \vdots \\ Z_{t-3} \end{bmatrix};\ \overline{S}_t \equiv \begin{bmatrix} S_t \\ \vdots \\ S_{t-3} \end{bmatrix}$$

We can now rewrite the above equation as:

$$(7) \quad I_t = (\lambda_1+\lambda_2)I_{t-1} - \lambda_1\lambda_2 I_{t-2}$$
$$+b^{-1}\lambda_1\lambda_2(\lambda_1-\lambda_2)^{-1}\sum_{i=0}^{\infty}[(\lambda_1 b)^{i+1} - (\lambda_2 b)^{i+1}]R'E\left(\begin{bmatrix}\overline{Z}_{t+2+i} \\ \overline{S}_{t+2+i}\end{bmatrix}\middle|\Omega_t\right)$$
$$+b^{-1}\lambda_1\lambda_2(\lambda_1-\lambda_2)^{-1}\sum_{i=0}^{\infty}[(\lambda_1 b)^{i+1} - (\lambda_2 b)^{i+1}]E(\xi_{t+i}|\Omega_t).$$

This is the solution to the optimization problem of a division. It shows that inventory depends on three sets of variables. It depends on itself lagged once and twice; it depends on lagged, current and expected future sales; it depends finally on current and expected unobservable cost disturbances.

The coefficients λ_1, λ_2 and the coefficients in R all depend on structural parameters (a, b, c, d). Even when λ_1 and λ_2 are complex conjugates, $\lambda_1 + \lambda_2$ and $\lambda_1\lambda_2$ are clearly real. Also, the relation of λ_1 and λ_2 to the structural parameters b, c, d (a does not appear in the polynomial of which λ_1, λ_2 are roots) can be characterized by the following two relations:

$$(8) \quad \begin{aligned} c &= (\lambda_1 + \lambda_2)\ (b + (\lambda_1\lambda_2)^{-1}) - 2(b + 1) \\ d &= b^{-1}[(\lambda_1\lambda_2)^{-1}\ (1 + b(\lambda_1 + \lambda_2)^2) + (\lambda_1\lambda_2)b^2 - (c+2b+1+b(c+b+2))] \end{aligned}$$

Using these two relations, it is easily shown that as c, d increase compared to k, i.e. costs of moving production become relatively insignificant, λ_2 goes to zero and equation (7) simplifies accordingly (with some care in the derivation to avoid dividing by zero). This proves the statement made in the preceding section. Note also that given b, c and d can be determined easily from $(\lambda_1 + \lambda_2)$ and $(\lambda_1\lambda_2)$.

Estimation of the Inventory Equation.

Equation (7) entirely characterizes inventory behavior. Further assumptions must however be made in order to estimate it and to recover the structural parameters.

The first allows the parameters in equation (7) to be estimated. It assumes that S_t and ξ_t - and Z_t and ξ_t - are uncorrelated at all lags. This insures that the second set of terms in (7), lagged, current and expected sales is uncorrelated at all lags with the third set of terms involving unobservable disturbances. This shows in particular why the assumption that prices are uncorrelated with cost disturbances at all lags is required. If prices were affected by cost disturbances, they would affect current and possibly expected sales: the second set of terms would not be uncorrelated with the disturbance term.

The second specifies the processes generating ε_t, u_t and η_t. Estimation is substantially simpler when these are white noise; using Occam's razor, this is the assumption made here. The third set of terms in (7) reduces to $-\lambda_1\lambda_2(dbu_t + c\varepsilon_t + \eta_t)$.

The third specifies how expectations are formed, i.e. what is included in Ω_t. There are two possible assumptions. The first is to assume that we know which time series belong to Ω_t, the second is to assume that Ω_t includes some time series unobservable to the econometrician. Both assumptions lead to tractable estimation problems although the second one is substantially more costly to implement. Most of the estimation, reported in Section IV, proceeds under the first assumption; Section V reports results under the second assumption.

Under the first assumption, a simple first step is to assume that Ω_t includes only, in addition to current and lagged I's, current and lagged S's and Z's. The next section will specifically assume that (Z_t, S_t) has a fourth-order bivariate autoregressive representation. (Although the estimation will allow for seasonals in S, Z and the disturbances, they are in the derivation which follows put equal to zero but only for notational simplicity.) This representation can directly be written in quasi first-order form:

$$
(9) \quad \begin{bmatrix} Z_t \\ \vdots \\ Z_{t-3} \\ S_t \\ \vdots \\ S_{t-3} \end{bmatrix}
=
\begin{bmatrix}
a_{111} \cdots a_{114} & a_{121} \cdots a_{124} \\
\begin{matrix} 1 & & 0 & 0 \\ 0 & \ddots & & \\ 0 & 0 & 1 & 0 \end{matrix} & 0 \\
a_{211} \cdots a_{214} & a_{221} \cdots a_{224} \\
0 & \begin{matrix} 1 & & 0 & 0 \\ 0 & \ddots & & \\ 0 & 0 & 1 & 0 \end{matrix}
\end{bmatrix}
\begin{bmatrix} Z_{t-1} \\ \vdots \\ Z_{t-4} \\ S_{t-1} \\ \vdots \\ S_{t-4} \end{bmatrix}
+
\begin{bmatrix} \theta_{Zt} \\ 0 \\ \vdots \\ 0 \\ \theta_{St} \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

$$
\text{with } E(\theta_{Zt}\theta_{St})'(\theta_{Zt-i}\theta_{St-i}) \begin{array}{l} = 0 \text{ if } i \neq 0 \\ = \Theta \text{ if } i = 0 \end{array}
$$

Using the more concise notation developed above, this can be written as:

$$
\begin{bmatrix} \bar{Z}_t \\ \bar{S}_t \end{bmatrix} = A \begin{bmatrix} \bar{Z}_{t-1} \\ \bar{S}_{t-1} \end{bmatrix} + \theta_t \quad , \quad \text{so that } E\left(\begin{bmatrix} \bar{Z}_{t+i} \\ \bar{S}_{t+i} \end{bmatrix} \middle| \Omega_t\right) = A^i \begin{bmatrix} \bar{Z}_t \\ \bar{S}_t \end{bmatrix}
$$

We can then rewrite equation (7) as

$$(10)\quad I_t = (\lambda_1+\lambda_2)I_{t-1} - \lambda_1\lambda_2\, I_{t-2} + \lambda_1\lambda_2\, R'(I - (\lambda_1+\lambda_2)bA + \lambda_1\lambda_2\, b^2A^2)^{-1}\, A^2 \begin{bmatrix}\bar{Z}_t\\ \bar{S}_t\end{bmatrix} - \lambda_1\lambda_2(dbu_t + c\varepsilon_t + \eta_t)$$

This is the equation to be estimated in the next section. If we had carried the three seasonal components of ε, u, η and the seasonal component of the (S, Z) process, there would be an additional set of seasonals with unconstrained coefficients; this set will be included in the estimation. Note that in equation (10), all coefficients are real. λ_1, λ_2 and R depend on the structural parameters a, b, c, d. The matrix A characterizes the process generating S and Z. The disturbance term is white and unconstrained. I now turn to the joint estimation of (9) and (10).

Section IV. Estimation Results

This section proceeds in three steps.

The first is simply to run unconstrained regressions of equation (10). What can be learned from such regressions? Consistent estimates of $(\lambda_1 + \lambda_2)$ and $(\lambda_1\lambda_2)$ can be obtained. From equation (8), this implies that if b was known, consistent estimates of c, d could be obtained. The coefficient b is not known but it is reasonable to assume that b, being a monthly discount rate is between 1 and .98 (.98 monthly implies .79 annually), and to solve for both values. Note that this estimation gives us no information about a, the marginal inventory-sales desired ratio.

Why go through this rather pedestrian step? The reason is that the estimates of c, d obtained in this way will be consistent, conditional on b, even if divisions did not have rational expectations. More precisely they will be consistent if the divisions formed their forecasts of sales S and Z using past values of S or Z; this includes in particular adaptive, static, adaptive regressive expectation formation mechanisms. The necessary regressions have in fact already been run and reported in Section II, Table 3. Point estimates and asymptotic variances of c, d given b can be obtained using equation (8). They are reported in Table 5. The interpretation and the economic implications of the results will be given in Section VI. Note however that $\hat{c}$, $\hat{d}$ are of the expected sign, namely positive. c, which should be interpreted as the ratio of c to k as k is normalized to be unity, varies between .02 and 4.96 (equivalently the ratio of estimated k to c varies between 41.6 and .20). It is never significant at 95%. The coefficient $\hat{d}$, or again the ratio of d to k, varies between 0.00 and .35 and is either significant or close to significant. Varying b from 1 to .98 does not substantially affect the estimates.

The second step is the estimation of equation (9) and equation (10), subject to the full set of constraints. This is done first by using a two-step approximation to the full information maximum likelihood. Estimation of equation (9) is first performed, giving an estimate of A, $\hat{A}$. The estimate of $\hat{A}$ is then used for the estimation of equation (10) which gives estimates for (a, b, c, d). The information matrix is not block diagonal in A and (a, b, c, d): the method therefore is not as efficient asymptotically as the FIML method. The reason for using it is simply a reason of cost: the cost of maximizing the likelihood function with respect to 20 coefficients rather than 4 in the second step of the two-step method is very high. The next section gives results from FIML estimation for one particular division: they are substantially the same. The asymptotic standard errors and test statistics reported in this section are derived under the assumption that A is known, ignoring that $\hat{A}$ is substituted for A in the second step.

Table 5

Implied Values of c, d given b, from Reduced Form Regressions

	$\widehat{(\lambda_1+\lambda_2)}$	$\widehat{(\lambda_1\lambda_2)}$	b = 1.00 $\hat{c}$	b = 1.00 $\hat{d}$	b = .98 $\hat{c}$	b = .98 $\hat{d}$
CV	.998	.332	.004 (.00)	.336 (2.70)	.024 (.04)	.350 (2.73)
PT	1.235	.384	.954 (1.59)	.028 (1.48)	.970 (1.61)	.032 (1.60)
OD	1.242	.374	.562 (1.19)	.046 (1.97)	.578 (1.22)	.051 (2.04)
BK	1.281	.380	.652 (1.49)	.025 (1.74)	.666 (1.53)	.029 (1.81)
CD	1.119	.256	1.490 (1.39)	.073 (1.74)	1.508 (1.40)	.081 (1.80)
FD	1.233	.373	.538 (1.10)	.052 (1.92)	.554 (1.13)	.057 (2.03)
ML	1.349	.415	.599 (1.56)	.010 (1.33)	.613 (1.59)	.012 (1.50)
AM	1.063	.178	3.035 (1.39)	.074 (1.43)	3.054 (1.40)	.085 (1.49)
DG	1.154	.174	3.786 (1.36)	.002 (.39)	3.803 (1.37)	.004 (.50)
CP	1.008	.127	4.945 (1.02)	.111 (1.32)	4.964 (1.03)	.128 (1.35)

Asymptotic t statistics in parentheses.

$\widehat{(\lambda_1 + \lambda_2)}$ is the reported coefficient of I_{t-1} in Table 3.

$\widehat{(\lambda_1\lambda_2)}$ is the negative of the reported coefficient of I_{t-2} in Table 3.

The likelihood function is maximized using the Davidon Fletcher Powell algorithm until relative accuracy is reached and then the Newton-Raphson algorithm until tighter relative accuracy is again reached. As explained above, a set of twelve seasonal dummies with unconstrained coefficients is also included.

The results are given in Table 6. Table 6a gives the implied coefficients of the right hand side variables and can be compared to the unconstrained coefficients reported in Table 3 and repeated for convenience in Table 6a. It then gives the value of $(N-k)\left(\frac{SSR_c - SSR_u}{SSR_u}\right)$ where $N-k = 166-22$. Under the hypothesis that equation (10) is correct, this is distributed asymptotically $\chi^2_{(6)}$ as 6 restrictions are imposed on the unconstrained regression given in Table 3. Table 6b gives the implied values of the structural parameters a, b, c, d.

Table 6 contains bad news, namely that there is little hope of obtaining sensible coefficients for a, c, d if b is left free. The estimation does not determine b precisely and in some cases the point estimate is of the wrong sign. As c, d affect the results of estimation mainly through $(\lambda_1+\lambda_2)$ and $(\lambda_1\lambda_2)$, and as $(\lambda_1 + \lambda_2)$ and $(\lambda_1\lambda_2)$ are determined precisely, the implication of equation (8) is that nonsensible values of b imply nonsensible values of c, d. As a appears only in equation (10) in the product adb, nonsensible values of b and d give nonsensible values of a. What this does not explain is why the discount rate is badly estimated. The answer is probably that <u>given</u> $(\lambda_1 + \lambda_2)$, $\lambda_1\lambda_2$, i.e. letting c and d adjust so as to keep $\lambda_1 + \lambda_2$ and $\lambda_1\lambda_2$ unchanged, different values of b do not lead to very different behavior for a given division. This is supported both by the available t tests on b in Table 6b and the $\chi^2_{(1)}$ tests below in Table 7b which show that for most divisions except Ford and American Motors, the hypothesis that b = .98 cannot usually be rejected.

Table 6a. Implied Coefficients from Constrained Estimation; b Unconstrained

		I_{t-1}	I_{t-2}	S_t	$\sum_{i=0}^{3} S_{t-i}$	Z_t	$\sum_{i=0}^{3} Z_{t-i}$	SSR	$\chi^2_{(6)}$
CV	u	.99	-.33	-.26	.42	3.20	-2.00	6.93 E10	
	c	.94	-.27	-.27	.67	1.94	-2.10	8.34 E10	29.09
PT	u	1.23	-.38	-.43	.54	2.02	-1.35	1.33 E10	
	c	1.22	-.37	-.35	.60	.37	- .84	1.60 E10	29.03
OD	u	1.24	-.37	-.48	.57	2.82	-2.65	1.23 E10	
	c	1.15	-.28	-.37	.42	.68	+ .03	1.40 E10	18.60
BK	u	1.28	-.38	-.36	.45	2.03	- .63	9.28 E9	
	c	1.27	-.40	-.36	.50	1.33	.38	9.61 E9	5.08
CD	u	1.12	-.26	-.28	.17	6.00	- .16	1.97 E9	
	c	1.06	-.19	-.23	.10	4.74	-1.23	2.04 E9	5.08
FD	u	1.23	-.37	-.29	.26	4.55	-2.52	5.09 E10	
	c	1.10	-.22	-.13	.16	1.55	-1.50	6.76 E10	46.91
ML	u	1.35	-.41	-.38	.54	2.34	-2.35	5.63 E9	
	c	1.19	-.27	-.17	.51	-.39	- .59	8.26 E9	66.80
AM	u	1.06	-.17	-.44	.38	2.25	- .97	2.04 E9	
	c	1.11	-.19	-.45	.51	1.10	-1.46	2.77 E9	51.17
DG	u	1.15	-.17	-.44	.55	1.42	-1.18	7.30 E9	
	c	1.10	-.11	-.39	.45	.56	- .36	7.86 E9	10.96
CP	u	1.01	-.13	-.32	.42	1.64	- .53	2.03 E10	
	c	1.03	-.20	-.41	.52	-.41	- .05	2.31 E10	19.72

u: results from unconstrained regression, taken from Table 3.
c: results from constrained estimation.
Critical values of $\chi^2_{(6)}$: 18.54 at .005, 12.59 at .05, 10.64 at .10

Table 6b

Values of the Structural Parameters; b Unconstrained

	a	b	c	d
CV	2.84 (3.73)	1.16 (2.41)	.22 (.35)	.29 (1.07)
PT	6.26 (3.79)	- .47 (-1.74)	1.69 (3.07)	-1.37 (2.68)
OD	5.92 (*)	.60 (*)	1.64 (*)	.32 (*)
BK	5.49 (*)	1.67 (*)	.06 (*)	.00 (*)
CD	9.18 (*)	.76 (*)	2.68 (*)	.27 (*)
FD	8.22 (3.05)	-1.35 (-7.50)	4.15 (3.87)	-1.10 (3.43)
ML	13.07 (3.45)	- .25 (-1.92)	2.55 (2.57)	-1.53) (-2.12)
AM	11.29 (2.33)	- 1.05 (- 7.5)	4.62 (2.61)	- .93 (-2.51)
DG	64.40 (*)	15.34 (*)	-5.68 (*)	.06 (*)
CP	-6.68 (*)	1.23 (*)	1.98 (1.70)	.025 (*)

t statistics in parentheses

*: not available. (As the information matrix is nearly singular, the N-R-D method inverts the sum of the information matrix and a diagonal matrix. Reported standard deviations overstate correct ones.)

Table 7 therefore gives the results of fully constrained estimation, with the additional restriction that $b = .98$ (Values of b between 1 and .95 do not affect the estimates of a, c, d in any significant way). Table 7 contains the main results of the paper. It is again composed of two subtables: Table 7a gives the implied coefficients of the right hand side variables, together again with the unconstrained coefficients repeated from Table 3. The last column gives the value of $(N-k)\left(\frac{SSR_{c1}-SSR_{c2}}{SSR_{c2}}\right)$ where SSR_{c1} is the sum of squared residuals obtained from fully constrained estimation with $b = .98$ and SSR_{c2} is the sum of squared residuals obtained from fully constrained estimation with b free. Under the hypothesis that $b = .98$, it is distributed asymptotically $\chi^2_{(1)}$. Table 7b gives the implied values of the structural coefficients a, c, d.

Consider first Table 7a. The coefficients of the constrained regression are in general close both to the coefficients of the constrained regression with b left free and to the unconstrained coefficients. This is particularly true of the coefficients of I_{t-1}, I_{t-2} and S_t which are very similar in the constrained and unconstrained case. The main discrepancy is between the unconstrained and constrained coefficients on Z_t. The unconstrained coefficient of Z_t is usually positive and large, implying that during the sample an increase in Z_t of 1 was associated with an increase in production of 4.2 for the average division. As no inventories are held against Z, a potential explanation is that increases in Z imply large expected increases in S or Z in the future and thus a current increase in production. The coefficient on lagged Z in the bivariate regressions of Z and S (not reported here) is indeed high. The effect of Z lagged once on S is for example 4.92 for Cadillac and of 1.76 for Buick. This explains why the constrained regression also leads to a positive sign on Z for 6 divisions. For 4 divisions however, the constrained regression does poorly for Z, suggesting that that part of the model is probably misspecified.

Table 7a. Implied Coefficients from Constrained Estimations; b = .98

		I_{t-1}	I_{t-2}	S_t	$\sum_0^3 S_{t-i}$	Z_t	$\sum_0^3 Z_{t-i}$	SSR	$\chi^2_{(1)}$
CV	u	.99	-.33	-.26	.42	3.20	-2.00	6.93 E10	
	c	.95	-.29	-.28	.68	1.93	-2.46	8.35 E10	.17
PT	u	1.23	-.38	-.43	.54	2.02	-1.35	1.33 E10	
	c	1.27	-.47	-.45	.70	.06	- .31	1.73 E10	11.61
OD	u	1.24	-.37	-.48	.57	2.82	-2.65	1.23 E10	
	c	1.14	-.28	-.40	.41	.79	.41	1.40 E10	.05
BK	u	1.28	-.38	-.36	.45	2.03	- .63	9.28 E9	
	c	1.24	-.34	-.29	.36	1.15	- .25	9.61 E9	.13
CD	u	1.12	-.26	-.28	.17	6.00	- .16	1.97 E9	
	c	1.06	-.22	-.28	.01	5.06	-2.58	2.07 E9	2.10
FD	u	1.23	-.37	-.29	.26	4.55	-2.52	5.09 E10	
	c	1.16	-.39	-.41	.52	-.51	- .55	9.91 E10	66.63
ML	u	1.35	-.41	-.38	.54	2.34	-2.35	5.63 E9	
	c	1.36	-.47	-.38	.73	1.04	- .17	10.26 E9	34.62
AM	u	1.06	-.17	-.44	.38	2.25	- .97	2.04 E9	
	c	1.06	-.23	-.44	.54	-.49	- .37	3.44 E9	34.58
DG	u	1.15	-.17	-.44	.55	1.42	-1.18	7.30 E9	
	c	1.12	-.18	-.32	.54	-.82	- .50	8.63 E9	14.00
CP	u	1.01	-.13	-.32	.42	1.64	- .53	2.03 E10	
	c	1.02	-.18	-.40	.51	-.45	- .14	2.32 E10	.62

Critical values of $\chi^2_{(1)}$: 7.87 at .005, 3.84 at .05, 2.70 at .10

Table 7b

Values of the Structural Parameters; b = .98

	a	b	c	d
CV	2.84 (3.59)	.98	.29 (.50)	.40 (3.33)
PT	1.96 (3.01)	.98	- .04 (- .22)	.09 (4.50)
OD	3.74 (2.12)	.98	1.21 (1.59)	.07 (2.33)
BK	12.78 (2.44)	.98	.88 (1.87)	.04 (2.50)
CD	6.56 (5.55)	.98	1.85 (2.01)	.13 (2.60)
FD	1.60 (1.77)	.98	.11 (.35)	.14 (3.50)
ML	4.44 (3.67)	.98	.24 (1.04)	.03 (2.50)
AM	1.38 (1.92)	.98	1.66 (1.59)	.13 (2.16)
DG	20.60 (2.19)	.98	3.29 (1.51)	.03 (3.00)
CP	3.36 (2.00)	.98	2.69 (1.36)	.15 (1.87)

t statistics in parentheses

Going from the unconstrained regression to the fully constrained regression with b free (Table 6a) increases the sum of squared residuals by 19% on average. Because of the large number of observations, the χ^2 statistic is large. The model is rejected at the .005 level for 5 divisions and rejected at the .10 level for 8 divisions. The model fits better the divisions of GM than those of Ford or Chrysler. It fits particularly well Buick and Cadillac. Going from the fully constrained regression with b free to the fully constrained regression with b = .98, the sum of squared residuals remains practically constant for 4 divisions. The χ^2 statistic is however large for 3 other divisions, the 2 divisions of Ford, Ford and Mercury Lincoln, and American Motors. In general, the assumed model fits observed behavior well, except for the reaction of production to Z_t for some divisions, especially Ford and American Motors.

Turning now to Table 7b, all the coefficients, except an insignificant estimate of c, have the correct sign. Some of the results hold for all divisions: The first is that a, d are both statistically significant: there is a significant cost of being away from target inventory; target inventory is a function of sales. The marginal desired inventory to sales ratio, a, is quite high, higher than the average inventory to sales ratio for the sample period for seven out of ten divisions. Although the coefficient d appears small, with an average value of .11, magnitudes are misleading: as Section V will show, its effect is substantial. The point estimate of c varies across divisions from 0.0 to 3.29 but is never significant; this suggests that the convexity of the cost function, relative to the cost of moving production — as k is normalized to be one — is not a main determinant of production and inventory behavior.

The next section considers some econometric extensions. The reader interested mainly in the economic implications of the results may go directly to Section VI.

Section V. Estimation Results. Extensions

This first subsection compares two-step versus efficient estimates; the second derives estimates of the technology under alternative assumptions about the information set.

Two-step versus Efficient Estimates.

The two-step method was used for reasons of cost but is not asymptotically efficient. Table 8 reports the results of full information maximum likelihood, which estimates jointly equations (9) and (10), for a particular division. The division chosen was Cadillac, for which the model seems to fit well. The results obtained using both methods are very similar, suggesting that the two-step method is quite good.

Alternative Assumptions about the Information Set.

Estimation in the previous section was performed under the joint hypothesis that prices (and therefore sales) were uncorrelated with cost disturbances and that the information set available to a division included only current and lagged sales, S and Z. This joint hypothesis implies that S and Z are statistically exogenous with respect to production and is testable by standard exogeneity tests. We therefore regress S and Z on lagged values of S, Z and Y and test the significance of lagged Y (it would be equivalent to regress S and Z on lagged values of S, Z and I).

These exogeneity tests, using a lag length of 4 for each variable, cannot reject exogeneity of Z but reject exogeneity of S at the 5% level for nine of the ten divisions. The source of the rejection is the same for all divisions, the coefficient on production lagged once is significant; it is however relatively small, from .18 for American Motors to .40 for Chevrolet.

Table 8. Comparison of Results Obtained with the Two-Step and the FIML Methods. Cadillac

Structural Parameters:	Two-Step	FIML
a	9.18	9.10
b	.76	.72
c	2.68	2.47
d	.27	.30
Implied Coefficients on:		
I_{t-1}	1.06	1.07
I_{t-2}	-.20	-.21
Z_t	4.74	5.32
Z_{t-1}	-1.68	-1.98
Z_{t-2}	1.59	1.07
Z_{t-3}	1.32	1.29
S_t	-.23	-.25
S_{t-1}	.10	.09
S_{t-2}	-.06	-.00
S_{t-3}	.05	.05

The two-step estimates are taken from Tables 6a, b.

The rejection implies rejection of at least one of the two hypotheses of the above joint hypothesis. It may mean that high production last period implies higher desired production this period, and thus lower prices and higher sales. It may alternatively mean that the information set available to a division is larger than we have assumed: production would then depend on more information about future sales than current and past sales, and would help predict future sales in the above regressions. As explained in Section III, the first hypothesis is maintained throughout this paper and I now relax the second.

Because some cars are ordered rather than bought from existing inventories, dealers may know next month's sales quite accurately. I consider therefore the assumption that next month's sales, S_{t+1} and Z_{t+1}, belong to this month's information set Ω_t. Two-step estimation is performed under this alternative assumption and the results reported in Table 9. The results are close to the results reported in Table 7b; the main difference is the decrease in the estimated value of a. This is not surprising as $E(S_{t+1}|\Omega_t)$ has now been replaced by S_{t+1} which has larger variance. The implied reduced forms are not reported. As before, they replicate closely the coefficients on lagged inventory and on sales S.

The second way to relax the initial assumption about information is to assume that we only observe a subset of the true information set Ω_t. The model, summarized by equation (7), still imposes restrictions on the multivariate process generating the variables I_t, S_t, Z_t:

Let $w_t = \{I_{t-1}, \cdots, S_{t-1}, \cdots, Z_{t-1}, \cdots\}$ and $w_t \subset \Omega_t$. Equation (7) can then be rewritten as:

Table 9

Values of the Structural Parameters; b = .98

$$\Omega_t = \{S_{t+1}, S_t, \cdots, Z_{t+1}, Z_t, \cdots\}$$

	a	b	c	d
CV	.92 (3.72)	.98	-.52 (2.47)	.30 (3.95)
PT	1.54 (2.48)	.98	-.07 (.50)	.06 (3.21)
OD	2.19 (2.02)	.98	.32 (1.33)	.04 (2.10)
BK	6.96 (2.01)	.98	.19 (1.26)	.02 (2.46)
CD	4.98 (3.63)	.98	.40 (1.73)	.04 (2.92)
FD	1.34 (1.81)	.98	.18 (.60)	.13 (3.09)
ML	4.88 (3.08)	.98	.29 (1.45)	.02 (2.44)
AM	.94 (1.65)	.98	1.21 (1.65)	.12 (2.40)
DG	16.14 (2.49)	.98	2.40 (1.95)	.02 (1.76)
CP	3.15 (1.94)	.98	2.22 (1.68)	.11 (1.92)

t statistics in parentheses

$$(11)\quad I_t = (\lambda_1+\lambda_2)I_{t-1} - \lambda_1\lambda_2 I_{t-2} + b^{-1}\lambda_1\lambda_2(\lambda_1-\lambda_2)^{-1}\sum_{i=0}^{\infty}[(\lambda_1 b)^{i+1} - (\lambda_2 b)^{i+1}]R'E\left(\begin{bmatrix}\bar{Z}_{t+2+i}\\ \bar{S}_{t+2+i}\end{bmatrix}\Big| w_t\right) + \sigma_t$$

where

$$\sigma_t \equiv b^{-1}\lambda_1\lambda_2(\lambda_1-\lambda_2)^{-1}\sum_{i=0}^{\infty}[(\lambda_1 b)^{i+1} - (\lambda_2 b)^{i+1}] \times \left(R'\left(E\left(\begin{bmatrix}\bar{Z}_{t+2+i}\\ \bar{S}_{t+2+i}\end{bmatrix}\Big|\Omega_t\right) - E\left(\begin{bmatrix}\bar{Z}_{t+2+i}\\ \bar{S}_{t+2+i}\end{bmatrix}\Big| w_t\right)\right) + E(\xi_{t+i}|\Omega_t)\right)$$

Under the maintained assumptions that technological disturbances are white, uncorrelated at all lags with sales, and that $w_t \subset \Omega_t$, σ_t is white and uncorrelated with the right hand side variables in (11).

Assume that I_t, S_t and Z_t have a fourth-order trivariate autoregressive representation, which is written directly in quasi first-order form:

$$\begin{bmatrix}\bar{Z}_t\\ \bar{S}_t\\ \bar{I}_t\end{bmatrix} = \underset{(12\times 12)}{B}\begin{bmatrix}\bar{Z}_{t-1}\\ \bar{S}_{t-1}\\ \bar{I}_{t-1}\end{bmatrix} + \varepsilon_t$$

Equation (11) can then be rewritten as:

$$(12)\quad I_t = (\lambda_1+\lambda_2)I_{t-1} - \lambda_1\lambda_2 I_{t-2} + \lambda_1\lambda_2\tilde{R}'(I - (\lambda_1+\lambda_2)bB + \lambda_1\lambda_2 b^2B^2)^{-1}B^3\begin{bmatrix}\bar{Z}_{t-1}\\ \bar{S}_{t-1}\\ \bar{I}_{t-1}\end{bmatrix} + \sigma_t$$

with $\tilde{R}' \equiv [R' \vdots\ 0\ \ 0\ \ 0\ \ 0]$.

Let e_i denote a (12×1) vector with 1 in the i^{th} line and 0 otherwise. The constraint on B imposed by the model is:

$$e_9'B = (\lambda_1+\lambda_2)e_9' - \lambda_1\lambda_2 e_{10}' + \lambda_1\lambda_2\tilde{R}'(I - (\lambda_1+\lambda_2)bB + \lambda_1\lambda_2 b^2B^2)^{-1}B^3$$

Table 10

Values of the Structural Parameters; b = .98

True information set unknown

	a	b	c	d
CV	2.00 (3.77)	.98	-.12 (.54)	.24 (4.00)
PT	1.55 (2.31)	.98	-.09 (.07)	.06 (6.10)
OD	55.23 (.31)	.98	1.00 (.81)	.002 (.25)
BK	6.17 (7.71)	.98	.90 (2.43)	.07 (7.20)
CD	4.61 (12.80)	.98	1.73 (2.66)	.23 (3.83)
FD	*	*	*	*
ML	*	*	*	*
AM	7.85 (9.12)	.98	2.82 (1.88)	.09 (2.25)
DG	1.69 (3.31)	.98	.90 (1.52)	.11 (2.75)
CP	3.54 (1.32)	.98	1.44 (1.77)	.06 (3.00)

*: Convergence not achieved

t statistics in parentheses

Maximum likelihood estimation estimates B subject to this constraint: the parameters are the 36 nontrivial elements of B and the structural parameters a, b, c, d. Until now, our attempts to use maximum likelihood estimation have not been successful: we have been unable to achieve convergence to anything resembling a global maximum.[1]

Consistent estimates of a, b, c, d can however be obtained as follows: the matrix B is first estimated by ordinary least squares and B is replaced by $\hat{B}$ in (12). Maximization of (12) over a, b, c, d is then performed as in the previous section. These estimates are given in Table 10. The method has some difficulty to estimate a and d separately; the results are roughly in line with the previous results in Tables 6b, 7b and 9.

Section VI. Implications

The findings of the previous two sections are twofold: Production smoothing exists but is more likely due to a cost of changing production than to a convex cost function. There is however a cost in being away from target inventory; this target is a function of sales.

How do these findings relate to the central macroeconomic issue about production - inventory behavior, namely whether inventory behavior is stabilizing or destabilizing, amplifies or dampens demand shocks? The framework used here makes clear that the answer depends on the sales process; for example for the sales process observed in the sample, Section II already gave the answer: an increase in sales led to a contemporaneous decrease in inventories, i.e. a less than complete increase in production; it however led to a movement of production over time such as to imply a larger variance for production than for sales. (If this was the only sales process we were interested in, there would be little justification for this paper). We can however address this question by characterizing the effects of simpler sales processes. I shall consider the class of first-order autoregressive processes for sales S_t, assuming constancy of both sales to non-U.S. dealers Z_t and cost disturbances. Specifically, I characterize the effects of an unexpected increase in sales of 1 in period 0, decaying at rate ρ over time, $\rho \varepsilon [0, 1)$. The tables below can also be interpreted as giving the weights of the moving average representation of production and inventory for a given sales process.

As the focus is not on the intraindustry differences, I shall consider a "representative division," with the following values of the structural parameters a = 3, b = .98, c = 1, k = 1 and d = .1, which correspond roughly to the mean values obtained in Table 7b. There are two main results:

Production smoothing coming from cost of changing production tends to be stabilizing in the short run, but much less so in the medium run. This is different from production smoothing coming from a convex cost function. This is made clear in Table 11. This table reports the effects of a sales shock of 1, decaying at rate ρ = .9 in three different cases. In all three cases a = 3, b = .98 and d = .1. In the first case, production smoothing comes only from a convex cost function: c = 1, k = 0. In the second, it comes only from a cost of changing production: c = 0, k = 1. In the last, both costs are present: c = 1, k = 1.

In the first, the desire to keep the level of production approximately constant together with the desire to reach target inventory leads the firm to revise upwards its sequence of production. The largest increase occurs in the first month. In the second case, the desire to keep the change in production approximately constant has two effects: the first is to lead a smaller change in production than in sales in the first month; the second is to lead

Table 11. Production and Inventory Response. Different Values of c, k.

		No Cost of Changing Production: $k = 0$ $c = 1$		Linear Cost Function $k = 1$ $\varphi = 0$		Representative Division $k = 1$ $c = 1$	
Months:	Sales S	Inventory I	Production Y	Inventory I	Production Y	Inventory I	Production Y
0	1.00	.38	$\boxed{1.38}$	-.03	.97	-.14	.86
1	.90	.63	1.14	.39	$\boxed{1.32}$	.01	$\boxed{1.04}$
2	.81	.77	.95	.89	1.30	.20	1.00
3	.73	.85	.80	1.28	1.11	.36	.88
4	.66	$\boxed{.88}$	.68	1.50	.88	.47	.76
5	.59	.87	.58	$\boxed{1.57}$	.66	.53	.65
6	.53	.85	.50	1.53	.49	.56	.56
7	.48	.81	.43	1.42	.36	$\boxed{.57}$	.48
8	.43	.76	.38	1.28	.29	.56	.41
9	.38	.71	.33	1.14	.24	.53	.36

In all three cases: $a = 3$ $d = .1$ $b = .98$; Sales process $S_i = \rho^i S_0$, $S_0 = 1$.

to higher levels of production than in the first case for the next five months and to a substantially larger build up of inventories. Therefore, costs of moving have a "stabilizing" contemporaneous effect but allow over time for a larger "destabilizing" effect on production and build-up of inventories. In the last case both costs are present, leading to an initial decrease in inventories, followed by an increase until the seventh month.

The effect of target inventory varies very much with small changes in the sales process. It plays a significant role only if sales exhibit a high degree of persistence. This is shown more precisely in Table 12, which gives the effects of a sales shock with different degrees of persistence; $\rho = .9$, .8, .7 and .0. The structural parameters are those of a representative division, the same in all four cases.

Consider first the case of a purely temporary sales shock (ρ=.0). In this case there is no desired target inventory effect as next period's expected sales are always equal to zero (All variables are measured in terms of deviations from preshock values). Production increases contemporaneously by only 16% of the increase in sales and after their initial decrease, inventories return to zero over time. The dynamics do not change drastically for $\rho = .7$ or even $\rho = .8$: production adjusts only partially, reaching a peak after a month; it is "smoother" than sales. Although desired inventory is now positive, actual inventory remains negative all along the adjustment path.

The adjustment path changes drastically when $\rho = .9$. (The results for this case are repeated from Table 11.) Although production responds initially by slightly less than the increase in sales, the desired inventory effect becomes important, leading to a positive inventory after the first month and an increase in production larger than the increase in sales from the first

Table 12. Production and Inventory Response. Different Degrees of Persistence of Sales.

	$\rho = .0$			$\rho = .7$			$\rho = .8$			$\rho = .9$		
Months	S	Y	I	S	Y	I	S	Y	I	S	Y	I
0	1.00	.16	-.83	1.00	.53	-.46	1.00	.66	-.33	1.00	.86	-.13
1	.00	$\boxed{.18}$	-.64	.70	$\boxed{.60}$	-.56	.80	$\boxed{.78}$	-.35	.90	$\boxed{1.04}$	.01
2	.00	.16	-.48	.49	.52	-.52	.64	.71	-.28	.81	1.00	.20
3	.00	.12	-.36	.34	.42	-.44	.51	.59	-.20	.73	.88	.36
4	.00	.09	-.26	.24	.32	-.36	.41	.47	-.13	.66	.76	.47
5	.00	.07	-.19	.17	.24	-.28	.33	.37	-.08	.59	.65	.53
6	.00	.05	-.14	.12	.18	-.22	.26	.29	-.04	.53	.56	.56
7	.00	.03	-.10	.08	.13	-.17	.21	.23	-.02	.48	.48	$\boxed{.57}$
8	.00	.02	-.07	.06	.09	-.13	.17	.18	-.00	.43	.41	.56
9	.00	.02	-.05	.04	.07	-.10	.13	.14	.00	.38	.36	.53

In all four cases a = 3. b = .98 c = 1. d = .1 k = 1.

to the seventh month. This shows that whether inventory behavior is stabilizing or destabilizing--whatever criterion or definition is used--depends very much on the sales process.

Section VII. Relation to the Stock Adjustment Model

The most popular empirical model of inventory behavior is the stock adjustment model, introduced initially by Lovell. How does our model compare to it?

The underlying justification of inventory behavior is in many ways similar; in both models, costs of adjustment and target inventory play a central role. The stock adjustment model cannot however be formally derived as a special case of our model, with static expectations for example. It should therefore probably be considered as an alternative model. This suggests another way of comparing them, namely by considering the fit of the stock adjustment model compared to this model, for all divisions of the automobile industry. The stock adjustment model is usually written as:

$$I_t = I_{t-1} + \lambda(I_t^* - I_{t-1}) + \delta(E(S_t|\Omega_{t-1}) - S_t) \text{ and } I_t^* = \alpha_0 + \alpha S_t$$

The results of the regressions, ran under the assumption of static expectations $E(S_t|\Omega_{t-1}) = S_{t-1}$, are reported in Table 13. (This estimation parallels a similar estimation at the industry level by Irvine, 1981.)

Consider first the fits, as measured by the sum of squared residuals, for the stock adjustment model and our model. A comparison of fits is appropriate as both models are regressed with the same number of observations and the same number of free parameters (a, b, c versus α, λ, δ). The result is a draw, each model dominating the other for five divisions. The estimated

Table 13. Stock Adjustment Model. Estimation

	λ	δ	α	D.W	SSR	Alternative Model SSR[1]
CV	.27 (5.40)	.12 (1.33)	.88	1.57	1.03 E11	.83 E11
PT	.12 (4.00)	.17 (1.82)	1.41	1.43	1.90 E10	1.73 E10
OD	.11 (3.40)	.24 (2.60)	.72	1.52	1.70 E10	1.40 E10
BK	.08 (2.80)	.25 (2.57)	2.00	1.37	1.23 E10	.96 E10
CD	.05 (1.50)	.09 (.99)	.60	1.76	2.58 E9	2.07 E9
FD	.12 (3.24)	-.21 (-2.10)	.50	1.60	8.13 E10	9.91 E10
ML	.08 (2.97)	-.01 (-.20)	2.62	1.45	8.43 E9	10.26 E9
AM	.10 (3.30)	-.03 (-.30)	.10	1.74	3.20 E9	3.44 E9
DG	.02 (1.10)	.18 (1.38)	2.50	1.81	8.53 E9	8.63 E9
CP	.12 (2.92)	.18 (1.57)	.66	1.83	2.28 E10	2.32 E10

Period of estimation 1966-4 to 1979-12; OLS; 12 monthly dummies.

(1). SSR repeated from Table 7a.

coefficients are usually of the right sign; all divisions however exhibit the characteristic emphasized by Feldstein and Auerbach for manufacturing: although the underlying justification would suggest that $\lambda = 1 - \delta$, this is strongly rejected for all divisions. There is also evidence of serial correlation, although not as strong as for aggregate manufacturing data.

Therefore, although our model does not dominate in terms of fit, it gives a more satisfying explanation for inventory behavior. Given the similarity of the results of Table 11 to the results obtained for manufacturing, this suggests that our model may also provide a more satisfying explanation for manufacturing as a whole. This is clearly only a conjecture.

Conclusion

This study has shown that the production behavior of the automobile industry is well explained by the assumption of intertemporal optimization with rational expectations. The underlying cost structure appears to have substantial costs of changing production as well as substantial costs of being away from target inventory, the latter being a function of current sales.

The last section suggests that this cost structure generates the type of time series behavior usually explained by the stock adjustment model and that the results of this study may be of relevance to more than the automobile industry.

Footnotes

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1. This sentence hides a lot of work and a superb program written by Angelo Melino and Danny Quah, based on work by Melino (1982). The program works with the eigenvector and eigenvalue matrices associated with B. In this case, the constraint simply allows to express 12 of the free elements of the eigenvector matrix as a function of the others and the eigenvalues and reduces the constrained maximization to an unconstrained one.

Data Appendix.

Production Data, U.S. and Canada:

Ward's Automotive Reports, weekly, 1965–1979

Sales Data, U.S.

Ward's Automotive Reports, weekly, 1965–1979

Factory Sales, U.S. and Canada, to U.S., Canada and other Exports:

Statistics Department. Motor Vehicle Manufacturers Association of the U.S., Inc. Detroit.

Dealers' Inventories, Computed by Perpetual Inventory Method, benchmarked December 31, 1979. Benchmark Source:

Days Supply, Ward's Automotive Reports, December 31, 1979.

Manufacturers' Inventories, Computed by Perpetual Inventory Method, benchmarked so that the minimum for each series is zero. Levels confirmed by private communication.

Strike Dummies (Used in Table 2 only).

GM:	September, October, November	1970
Ford:	September, October	1967
	September, October	1976
AM:	October, November	1969

References

Blanchard, Olivier J. "Identification in Dynamic Linear Models with Rational Expectations." Discussion paper 888, Harvard Institute of Economic Research, March 1982.

Blanchard, Olivier J. and Melino, Angelo. "Cyclical Behavior of Prices and Quantities in the Automobile Market." Mimeographed. Harvard, October 1981.

Blinder, Alan S. "Inventories and Sticky Prices: More on the Micro Foundations of Macroeconomics." American Economic Review, forthcoming, 1982.

Feldstein, Martin and Auerbach, Alan J. "Inventory Behavior in Durable Goods Manufacturing: the Target Adjustment Model." BPEA, 1976-2: 351-408.

Hansen, Lars P. and Sargent, Thomas J. "Formulating and Estimating Dynamic Linear Rational Expectations Models." J. Econ. Dynamics and Control 2, no. 1 (1980): 7-46.

Holt, Charles C.; Modigliani, Franco; Muth, John F.; and Simon, Herbert A. "Planning Production, Inventories and Work Force." Englewood Cliffs, N.J.: Prentice Hall, 1960.

Irvine, F. O.,Jr. "A Study of Automobile Inventory Investment." Econ. Inquiry 19 (July 1981): 353-379.

Lovell, Michael C. "Manufacturers' Inventories, Sales Expectations and the Acceleration Principle." Econometrica 29 (July 1961): 293-314.

Lucas, Robert E., Jr. "Econometric Policy Evaluation: A Critique." In "The Phillips Curve and Labor Markets," edited by Karl Brunner and Allan H. Meltzer. Carnegie-Rochester Conferences on Public Policy, vol. 1. Amsterdam: North Holland, 1976.

Melino, Angelo. "Essays in Macro Econometrics." Ph.D. dissertation, Harvard University, 1982.

Richardson, B. et al. "An Inventory of Selected Mathematical Models Relating to the Motor Vehicle Transportation System." Mimeographed. Policy Analysis Division, University of Michigan, September 1978.

Sargent, Thomas J. "Rational Expectations, Econometric Exogeneity and Consumption." JPE 86, no. 4 (August 1978): 673-700.

White, Lawrence J. "The Automobile Industry Since 1945." Cambridge, Mass.: Harvard University Press, 1971.