The Proton Spin Puzzle: A Status Report*

Hai-Yang Cheng
Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China

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Abstract

The proton spin puzzle inspired by the EMC experiment and its present status are closely examined. Recent experimental progress is reviewed. Various factorization schemes due to the ambiguity arising from the axial anomaly are discussed. Some misconceptions in the literature about the $\overline{\mathrm{MS}}$ factorization scheme are clarified. It is stressed that the polarized nucleon structure function $g_1(x)$ is independent of the factorization scheme chosen in defining the quark spin density. Consequently, the anomalous gluon and sea-quark interpretations for the deviation of the observed first moment of $g_1^p(x)$ from the Ellis-Jaffe sum rule are equivalent. While it is well known that the total quark spin in the chiral-invariant (CI) factorization scheme (e.g. the improved parton model) can be made to be close to the quark model expectation provided that the gluon spin is positive and large enough, it is much less known that, contrary to the gauge-invariant scheme (e.g. the $\overline{\mathrm{MS}}$ scheme), the quark orbital angular momentum in the CI scheme deviates even farther from the relativistic quark model prediction. Recent developments in the NLO analysis of polarized DIS data, orbital angular momentum and lattice calculations of the proton spin content are briefly sketched.

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1 Introduction

Experimentally, the polarized structure functions g_1 and g_2 are determined by measuring two asymmetries:

$$A_{\parallel} = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}}, \qquad A_{\perp} = \frac{d\sigma^{\downarrow\rightarrow} - d\sigma^{\uparrow\rightarrow}}{d\sigma^{\downarrow\rightarrow} + d\sigma^{\uparrow\rightarrow}}, \tag{1.1}$$

where $d\sigma^{\uparrow\uparrow}$ ($d\sigma^{\uparrow\downarrow}$) is the differential cross section for the longitudinal lepton spin parallel (antiparallel) to the longitudinal nucleon spin, and $d\sigma^{\downarrow\rightarrow}$ ($d\sigma^{\uparrow\rightarrow}$) is the differential cross section for the lepton spin antiparallel (parallel) to the lepton momentum and nucleon spin direction transverse to the lepton momentum and towards the direction of the scattered lepton. From the parton-model or from the OPE approach, the first moment of the polarized proton structure function

$$\Gamma_1^p(Q^2) \equiv \int_0^1 g_1^p(x, Q^2)dx, \tag{1.2}$$

can be related to the combinations of the quark spin components via

$$\Gamma_1^p = \frac{1}{2}\sum_q e_q^2 \Delta q(Q^2) = \frac{1}{2}\sum_q e_q^2 \langle p, s|\bar{q}\gamma_\mu\gamma_5 q|p, s\rangle s^\mu, \tag{1.3}$$

where Δq represents the net helicity of the quark flavor q along the direction of the proton spin in the infinite momentum frame:

$$\Delta q = \int_0^1 \Delta q(x)dx \equiv \int_0^1 \left[q^\uparrow(x) + \bar{q}^\uparrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x)\right] dx. \tag{1.4}$$

At energies $\langle Q^2\rangle \sim 10\,\mathrm{GeV}^2$ or smaller, only three light flavors are relevant:

$$\Gamma_1^p(Q^2) = \frac{1}{2}\left(\frac{4}{9}\Delta u(Q^2) + \frac{1}{9}\Delta d(Q^2) + \frac{1}{9}\Delta s(Q^2)\right). \tag{1.5}$$

Other information on the quark polarization is available from the low-energy nucleon axial coupling constants g_A^3 and g_A^8:

$$\begin{aligned} g_A^3(Q^2) &\equiv \langle p, s|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|p, s\rangle s^\mu = \Delta u(Q^2) - \Delta d(Q^2), \qquad (1.6)\\ g_A^8(Q^2) &\equiv \langle p, s|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s|p, s\rangle s^\mu = \Delta u(Q^2) + \Delta d(Q^2) - 2\Delta s(Q^2). \end{aligned}$$

Since there is no anomalous dimension associated with the axial-vector currents A_μ^3 and A_μ^8, the non-singlet couplings g_A^3 and g_A^8 do not evolve with Q^2 and hence can be determined at $q^2 = 0$ from low-energy neutron and hyperon beta decays. Under SU(3)-flavor symmetry, the non-singlet couplings are related to the SU(3) parameters F and D by

$$g_A^3 = F + D, \qquad g_A^8 = 3F - D. \tag{1.7}$$

We use the updated coupling $g_A^3 = 1.2670 \pm 0.0035$ [1] and the values [2]

$$F = 0.463 \pm 0.008\,, \qquad D = 0.804 \pm 0.008\,, \qquad F/D = 0.576 \pm 0.016 \tag{1.8}$$

to obtain $g_A^8 = 0.585 \pm 0.025$.

Prior to the EMC measurement of polarized structure functions, a prediction for Γ_1^p was made based on the assumption that the strange sea in the nucleon is unpolarized, i.e., $\Delta s = 0$. It follows from (1.5) and (1.6) that

$$\Gamma_1^p(Q^2) = \frac{1}{12} g_A^3 + \frac{5}{36} g_A^8. \tag{1.9}$$

This is the Ellis-Jaffe sum rule [3]: $\Gamma_1^p = 0.185 \pm 0.003$ in the absence of QCD corrections and equals to 0.171 ± 0.006 at $Q^2 = 10\,\mathrm{GeV}^2$ to leading-order corrections. The 1987 EMC experiment [4] then came to a surprise. The result published in 1988 and later indicated that $\Gamma_1^p = 0.126 \pm 0.018$, substantially lower than the expectation from the Ellis-Jaffe conjecture. From the EMC measurement of Γ_1^p, we obtain

$$\Delta u = 0.77 \pm 0.06\,, \quad \Delta d = -0.49 \pm 0.06\,, \quad \Delta s = -0.15 \pm 0.06\,, \tag{1.10}$$

and

$$\begin{aligned} g_A^0(Q^2) &\equiv \langle p, s|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|p, s\rangle s^\mu \\ &= \Delta u(Q^2) + \Delta d(Q^2) + \Delta s(Q^2) \equiv \Delta\Sigma(Q^2) = 0.14 \pm 0.18 \end{aligned} \tag{1.11}$$

at $Q^2 = 10.7\,\mathrm{GeV}^2$. The EMC results exhibit two surprising features: The strange-sea polarization is sizeable and negative, and the total contribution of quark helicities to the proton spin is small and consistent with zero. This is sometimes referred to as the "proton spin crisis".

The so-called "proton spin crisis" is not pertinent since the proton helicity content explored in the DIS experiment is, strictly speaking, defined in the infinite momentum frame in terms of QCD current quarks and gluons, whereas the spin structure of the proton in the proton rest frame is referred to the constituent quarks. That is, the quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange. What trigged by the EMC experiment is the "proton helicity decomposition puzzle" rather than the "proton spin crisis" (for a review, see [5, 6, 7, 8, 9, 10, 11]). The non-relativistic SU(6) constituent quark model predicts $\Delta\Sigma' \equiv \Delta U + \Delta D = 1$ (ΔU denoting the constituent up quark spin and likewise for ΔD), but its prediction for the axial-vector coupling constant $g_A^3 = \frac{5}{3}$ is too large compared to the measured value of 1.2670 ± 0.0035 [1]. In the relativistic quark model, the proton is no longer a low-lying S-wave state since the quark orbital angular momentum is nonvanishing due to the presence of quark transverse momentum in the lower component of the Dirac spinor. Realistic models e.g. the cloudy bag model [12], predict $\Delta\Sigma' \approx 0.60$; that is, about 40% of the proton spin is carried by the orbital angular momentum of the constituent quarks. In the parton picture, the naive expectation of $\Delta\Sigma$, which is equal to $g_A^8 = 0.59$ under the assumption of vanishing sea polarization, is very close to the relativistic quark model's prediction of $\Delta\Sigma'$. One of the main theoretical problems is that hard gluons cannot induce sea polarization perturbatively for massless quarks due to helicity conservation. Hence, it is difficult to accommodate a large strange-sea polarization in the naive parton model.

2 Experimental Progress

Before 1993 it took 5 years on the average to carry out a new polarized DIS experiment (see Table I). This situation was dramatically changed after 1993. Many new experiments measuring the nucleon and deuteron spin-dependent structure functions became available. The experimental progress is certainly quite remarkable in the past years.

Since experimental measurements only cover a limited kinematic range, an extrapolation to unmeasured $x \to 0$ and $x \to 1$ regions is necessary. At small x, a Regge behavior $g_1(x) \propto x^{\alpha(0)}$ is conventionally assumed in earlier experimental analyses. The improvement by the new measurements is two-folds: First, the small x region has been pushed down to the order 10^{-3} in SMC experiments (see Table I). Second, an extrapolation to the unmeasured small x region is done by performing a NLO QCD fit rather than by assuming Regge behaviour. The uncertainties of Γ_1 which mostly arise from the small x extrapolation are substantially reduced. From Table I it is also clear that the EMC experiment has been confirmed by all successive polarized DIS measurements.

Table I. Experiments on the polarized structure functions $g_1^p(x,Q^2)$, $g_1^n(x,Q^2)$ and $g_1^d(x,Q^2)$.

Experiment	Target	$Q^2(\mathrm{GeV}^2)$ range	x range	$\Gamma_1^{\mathrm{target}}(Q^2)$
E80/E130 [13, 14]	p	$1 < Q^2 < 10$	$0.1 < x < 0.7$	$\Gamma_1^p(10) = 0.17 \pm 0.05^*$
E142 [15]	n	$1 < Q^2 < 10$	$0.03 < x < 0.6$	$\Gamma_1^n(2) = -0.031 \pm 0.006 \pm 0.009$
E143 [16]	p,d	$1 < Q^2 < 10$	$0.03 < x < 0.8$	$\Gamma_1^p(3) = 0.132 \pm 0.003 \pm 0.009$
				$\Gamma_1^d(3) = 0.047 \pm 0.003 \pm 0.006$
E154 [17]	n	$1 < Q^2 < 17$	$0.014 < x < 0.7$	$\Gamma_1^n(5) = -0.041 \pm 0.004 \pm 0.006$
E155 [18]	p,d	$1 < Q^2 < 17$	$0.01 < x < 0.9$	$\Gamma_1^d(5) = 0.0266 \pm 0.0025 \pm 0.0071$
EMC [4]	p	$1 < Q^2 < 200$	$0.01 < x < 0.7$	$\Gamma_1^p(10.7) = 0.126 \pm 0.010 \pm 0.015^{**}$
SMC [19]	p,d	$1 < Q^2 < 60$	$0.003 < x < 0.7$	$\Gamma_1^p(10) = 0.120 \pm 0.005 \pm 0.006 \pm 0.014$
				$\Gamma_1^n(10) = -0.078 \pm 0.013 \pm 0.008 \pm 0.014$
				$\Gamma_1^d(10) = 0.019 \pm 0.006 \pm 0.003 \pm 0.013$
HERMES [20]	p,n,d	$1 < Q^2 < 10$	$0.023 < x < 0.6$	$\Gamma_1^n(3) = -0.037 \pm 0.013 \pm 0.008^\dagger$

* Obtained by assuming a Regge behavior $A_1 \propto x^{1.14}$ for small x.

** Combined result of E80, E130 and EMC data. The EMC data alone give $\Gamma_1^p = 0.123 \pm 0.013 \pm 0.019$.

† $\int_{0.021}^{0.85} g_1^p(x)dx = 0.122 \pm 0.003 \pm 0.010$ is obtained by HERMES for the proton target.

Comparing to the original EMC measurement, the statistic and systematic errors of the combined world average for Γ_1^p are substantially reduced. The result is $\Gamma_1^p = (0.12 \sim 0.13) \pm 0.007$ at $Q^2 = (5 \sim 10)\,\mathrm{GeV}^2$. Consequently, $\Delta\Sigma = (0.20 \sim 0.30) \pm 0.04$. For example, $\Delta\Sigma = 0.25 \pm 0.04$ leads to

$$\Delta u = 0.81 \pm 0.01\,, \quad \Delta d = -0.45 \pm 0.01\,, \quad \Delta s = -0.11 \pm 0.01\,. \tag{2.1}$$

We will employ (2.1) as the benchmarked values for Δq in ensuing discussions.

The Bjorken sum rule evaluated up to α_s^3 for three light flavors is [21]

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6}\frac{g_A}{g_V}\left[1 - \frac{\alpha_s(Q^2)}{\pi} - \frac{43}{12}\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.22\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]. \tag{2.2}$$

A serious test of the Bjorken sum rule, which is a rigorous consequence of QCD, became possible since 1993. The current experimental results are

$$\begin{aligned} \text{E143 [16]}: &\quad \Gamma_1^p - \Gamma_1^n = 0.164 \pm 0.021\,, \\ \text{SMC [19]}: &\quad \Gamma_1^p - \Gamma_1^n = 0.174^{+0.024}_{-0.012}\,, \end{aligned} \tag{2.3}$$

at $Q^2 = 5\,\mathrm{GeV}^2$, to be compared with the prediction

$$\Gamma_1^p - \Gamma_1^n = 0.181 \pm 0.003 \tag{2.4}$$

at the same energies. Therefore, the Bjorken sum rule has been confirmed by data to an accuracy of 10% level.

The quark polarization Δq is usually determined from the inclusive data of g_1 by assuming SU(3) flavor symmetry. Moreover, inclusive DIS determines the sum of polarized quark and antiquark distributions, but not the valence and sea quark spin distributions. Semi-inclusive polarized experiments in principle allow the determination of Δq for each flavor and disentangle valence and sea polarizations separately [22]. Hence, SU(3) flavor symmetry can be tested by comparing the measured first moments of the flavor distributions to the SU(3) predictions [23]. Semi-inclusive data are available from SMC [24] and HERMES [23] (see Table II). In order to present the experimental results, we digress for a moment to adopt a different definition for quark spin densities here: $\Delta q_s(x) = q_s^\uparrow(x) - q_s^\downarrow(x)$ and $\Delta\bar{q}(x) = \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)$, where $\Delta q_s(x) = \Delta q(x) - \Delta q_v(x)$ is the sea spin distribution for the quark flavor q. The SMC analysis is based on the assumption of SU(3) flavor symmetric sea: $\Delta\bar{u}(x) = \Delta\bar{d}(x) = \Delta\bar{s}(x) = \Delta u_s(x) = \Delta d_s(x) = \Delta s(x)$, while the HERMES results shown in Table II rely on the assumption of flavor independent polarization:

$$\frac{\Delta u_s(x)}{u_s(x)} = \frac{\Delta d_s(x)}{d_s(x)} = \frac{\Delta s(x)}{s(x)} = \frac{\Delta\bar{u}(x)}{\bar{u}(x)} = \frac{\Delta\bar{d}(x)}{\bar{d}(x)} = \frac{\Delta\bar{s}(x)}{\bar{s}(x)}. \tag{2.5}$$

Note that the ansatz[1] of $\Delta\bar{q}(x) = \Delta q_s(x)$ has to be made in both experiments in order to extract the valence quark polarization Δq_v from the measurement of Δq and $\Delta\bar{q}$, i.e., $\Delta q_v(x) = \Delta q(x) - \Delta\bar{q}(x)$. However, one caveat has to be mentioned: *A priori* the antiqaurk spin $\Delta\bar{q}$ can be different from the sea polarization Δq_s if they are not produced from gluons. For example, antiquarks are not polarized in the model of [25]. Under the assumption of SU(3) symmetric sea polarization, we are led to the predictions $g_A^3 = \Delta u_v - \Delta d_v$, $g_A^8 = \Delta u_v + \Delta d_v$ and hence $\Delta u_v = 2F = 0.93 \pm 0.02$ and $\Delta d_v = F - D = -0.34 \pm 0.02$. Note that the valence polarization Δq_v should be scale independent.

The measurement of the gluon spin ΔG by all possible means is very important both theoretically and experimentally (see [8] for various processes sensitive to the gluon spin distributions). A global fit to the present inclusive DIS data of $g_1(x)$ cannot even fix the sign of ΔG decisively (see Sec. 3.3), not mentioning its magnitude. One way of measuring $\Delta G(x)$ directly is via the photon gluon fusion process occurred in the semi-inclusive DIS reaction. A

[1] This ansatz is generally not fulfilled by the assumption of flavor independent polarization made by HERMES [23].

Table II. The SMC [24] and HERMES [23] results for the first moments of valence and sea spin distributions.

	SMC ($Q^2 = 10\,\mathrm{GeV}^2$)	HERMES ($Q^2 = 2.5\,\mathrm{GeV}^2$)
Δu_v	$0.77 \pm 0.10 \pm 0.08$	$0.57 \pm 0.05 \pm 0.08$
Δd_v	$-0.52 \pm 0.14 \pm 0.09$	$-0.22 \pm 0.11 \pm 0.13$
$\Delta \bar{u}$	$0.01 \pm 0.04 \pm 0.03$	$-0.01 \pm 0.02 \pm 0.03$
$\Delta \bar{d}$	$0.01 \pm 0.04 \pm 0.03$	$-0.01 \pm 0.03 \pm 0.04$
$x\Delta u_v$	$0.155 \pm 0.017 \pm 0.010$	$0.13 \pm 0.01 \pm 0.01$
$x\Delta d_v$	$-0.056 \pm 0.026 \pm 0.011$	$-0.02 \pm 0.02 \pm 0.02$

recent HERMES [26] measurement of the longitudinal spin asymmetry in photoproduction of pairs of hadrons with high transverse momentum indicates that $\langle \Delta G(x)/G(x) \rangle = 0.41 \pm 0.18 \pm 0.03$ at $\langle x \rangle = 0.17$ to LO QCD. Hence $\Delta G(x)$ is found to be positive in the intermediate x region.

3 Theoretical Progress

In my opinion there are four important progresses in theory:

- The role played by the gluon to the first moment of g_1 is clarified. There are two extreme factorization schemes of interest.
- First-principles calculations of the quark spin and orbital angular momentum by lattice QCD became available.
- A complete and consistent NLO analysis of g_1 data became possible.
- Evoluation and gauge dependence of the quark orbital angular momentum are explored.

3.1 Anomalous gluon and sea quark interpretations

3.1.1 Anomalous gluon interpretation

We see from Sec. II that the polarized DIS data indicate that the fraction of the proton spin carried by the light quarks inside the proton is $\Delta\Sigma = (0.20 \sim 0.30)$ and the strange-quark polarization is $\Delta s \approx -0.10$ at $Q^2 = (5 \sim 10)\,\mathrm{GeV}^2$. The question is what kind of mechanism can generate a sizeable and negative sea polarization. It is difficult, if not impossible, to accommodate a large Δs in the naive parton model because massless quarks and antiquarks created from gluons have opposite helicities owing to helicity conservation. This implies that sea polarization for massless quarks cannot be induced perturbatively from hard gluons, irrespective of gluon polarization. It is also unlikely that the observed Δs comes solely from nonperturbative effects or from chiral-symmetry breaking due to nonvanishing quark masses.

As an attempt to understand the polarized DIS data, we consider QCD corrections to the polarized proton structure function $g_1^p(x)$. To the next-to-leading order (NLO) of α_s, the expression for $g_1^p(x)$ is

$$\begin{aligned} g_1^p(x,Q^2) &= \frac{1}{2}\sum e_q^2\Big[\Delta C_q(x,\alpha_s)\otimes\Delta q(x,Q^2)+\Delta C_G(x,\alpha_s)\otimes\Delta G(x,Q^2)\Big] \\ &= \frac{1}{2}\sum e_q^2\Big[\Delta q^{(0)}(x,Q^2)+\Delta q^{(1)}(x,Q^2)+\Delta q_s^G(x,Q^2) \\ &\quad +\Delta C_q^{(1)}(x,\alpha_s)\otimes\Delta q^{(0)}(x,Q^2)+\Delta C_G^{(1)}(x,\alpha_s)\otimes\Delta G(x,Q^2)+\cdots\Big], \end{aligned} \tag{3.1}$$

where uses of $\Delta C_q^{(0)}(x)=\delta(1-x)$ and $\Delta C_G^{(0)}(x)=0$ have been made, $\otimes$ denotes the convolution

$$f(x)\otimes g(x)=\int_x^1\frac{dy}{y}\,f\left(\frac{x}{y}\right)g(y), \tag{3.2}$$

and ΔC_q, ΔC_G are short-distance quark and gluon coefficient functions, respectively. More specifically, $\Delta C_G^{(1)}$ arises from the hard part of the polarized photon-gluon cross section, while $\Delta C_q^{(1)}$ from the short-distance part of the photon-quark cross section. Contrary to the coefficient functions, $\Delta q_s^G(x)$ and $\Delta q^{(1)}(x)$ come from the soft part of polarized photon-gluon and photon-quark scatterings, respectively. Explicitly, they are given by

$$\Delta q^{(1)}(x,Q^2)=\Delta\phi_{q/q}^{(1)}(x)\otimes\Delta q^{(0)}(x,Q^2),\qquad \Delta q_s^G(x,Q^2)=\Delta\phi_{q/G}^{(1)}(x)\otimes\Delta G(x,Q^2), \tag{3.3}$$

where $\Delta\phi_{j/i}(x)$ is the polarized distribution of parton j in parton i. Diagrammatically, $\Delta\phi_{q/q}^{(1)}$ and $\Delta\phi_{q/G}^{(1)}$ are depicted in Fig. 1.

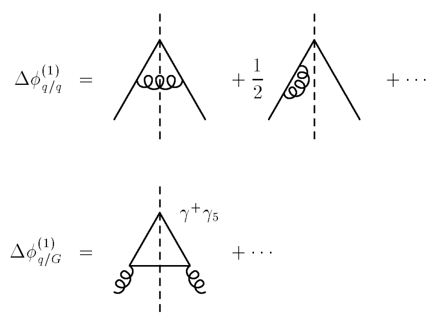

Figure 1: Diagrams for the quark spin distributions inside the parton: $\Delta\phi_{q/q}^{(1)}$ and $\Delta\phi_{q/G}^{(1)}$.

The photon-gluon scattering box diagram is ultraviolet finite but it depends on the choice of infrared and collinear regulators. However, the hard part of the box diagram should be soft-regulator independent. In the improved parton model, this is done by introducing a factorization scale $\mu_{\rm fact}$ so that the region $k_\perp^2\gtrsim\mu_{\rm fact}^2$ contributes to the hard photon-gluon cross section. The results are (see e.g. [8])

$$\begin{aligned} \Delta C_G^{(1)}(x,Q^2,\mu^2_{\rm fact})_{\rm CI} &= (2x-1)\left(\ln\frac{Q^2}{\mu^2_{\rm fact}}+\ln\frac{1-x}{x}-1\right), \\ \int_0^1 dx\Delta C_G^{(1)}(x)_{\rm CI} &= -\frac{\alpha_s}{2\pi}, \end{aligned} \tag{3.4}$$

where the reason for introducing the subscript "CI" will become clear below. Hence,

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x,Q^2) = \frac{1}{2}\left(1-\frac{\alpha_s}{\pi}\right)\sum_i \left[\Delta q_i(Q^2)_{\rm CI} - \frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2)\right]. \tag{3.5}$$

The $(1-\frac{\alpha_s}{\pi})$ term in Eq. (3.5) comes from the QCD loop correction, while the $\alpha_s\Delta G$ term arises from the box diagram of photon-gluon scattering. If the gluon polarization inside the proton is positive, a partial cancellation between $\Delta q_{\rm CI}$ and $\frac{\alpha_s}{2\pi}\Delta G$ will explain why the observed Γ_1^p is smaller than what naively expected from the Ellis-Jaffe sum rule. Note that unlike the usual QCD corrections, the QCD effect due to photon-gluon scattering is very special: The term $\alpha_s\Delta G$ is conserved to the leading-order QCD evolution; that is, ΔG grows with $\ln Q^2$, whereas α_s is inversely proportional to $\ln Q^2$.

As a consequence, Eq. (2.1) is modified to

$$\begin{aligned}
\Delta u_{\rm CI} - \frac{\alpha_s}{2\pi}\Delta G &= 0.81\pm 0.01\,, \\
\Delta d_{\rm CI} - \frac{\alpha_s}{2\pi}\Delta G &= -0.45\pm 0.01\,, \\
\Delta s_{\rm CI} - \frac{\alpha_s}{2\pi}\Delta G &= -0.11\pm 0.01\,,
\end{aligned} \tag{3.6}$$

and

$$g_A^0 = (\Delta u + \Delta d + \Delta s)_{\rm CI} - \frac{3\alpha_s}{2\pi}\Delta G = 0.25\pm 0.04 \tag{3.7}$$

at $Q^2 = 5 \sim 10\,{\rm GeV}^2$. Eqs. (3.6) and (3.7) imply that in the presence of anomalous gluon contributions, $\Delta\Sigma_{\rm CI}$ is not necessarily small while $\Delta s_{\rm CI}$ is not necessarily large. In the absence of sea polarization and in the framework of perturbative QCD, it is easily seen that $\Delta G \sim \Delta s(2\pi/\alpha_s) \sim 2.5$ at $Q^2 = 10\,{\rm GeV}^2$ and $\Delta\Sigma_{\rm CI} \sim 0.58$. It thus provides a nice and simple solution to the proton spin puzzle: This improved parton picture is reconciled, to a large degree, with the constituent quark model and yet explains the suppression of Γ_1^p, provided that ΔG is positive and large enough. This anomalous gluon interpretation of the observed Γ_1^p, as first proposed in [27, 28, 29] (see also [30]), looks appealing and has become a popular explanation since 1988.

Some historical remarks are in order:

- Long before the EMC experiment, there already existed three theoretical calculations of the photon-gluon box diagram. Kodaira [31] was the first one to compute the moments of the structure functions $g_{1,2}$. Since he worked in the OPE framework, there is no decomposition of g_A^0 in terms of quark and gluon spin components. The anomalous gluonic contribution to Γ_1^p was first put forward by Lam and Li [32] in 1982. A calculation of the gluonic coefficient function using the dimensional regularization was first made by Ratcliffe [33].

- The original results for the photon-gluon scattering cross section obtained by [27, 28, 29] are not perturbative QCD reliable as they depend on the choice of soft regulators. The first moment of $\Delta C_G^{(1)}(x)$ is equal to $-\alpha_s/(2\pi)$ in [29] but vanishes in the work of [28, 33]. After the soft part below the factorization scale $\mu_{\rm fact}$ is removed, the gluon coefficient function is given by Eq. (3.4) which is soft-cutoff independent.

3.1.2 Sea quark interpretation

According to the operator product expansion (OPE), the moments of structure functions can be expressed in terms of hard coefficients which are calculable by perturbative QCD and forward matrix elements of local gauge-invariant operators which are nonperturbative in nature:

$$\int_0^1 dx\, x^{n-1} g(x) = \sum_i C_i^n(Q^2)\langle N|O_i^n(0)|N\rangle. \tag{3.8}$$

It turns out that there is no gauge-invariant twist-2, spin-1 local gluonic operator for the first moment of $g_1(x)$ (see e.g. [34]). Here we face a dilemma here: On the one hand, the anomalous gluon interpretation sounds more attractive and is able to reconcile to a large degree with the conventional quark model; on the other hand, the sea-quark interpretation of Γ_1^p relies on a more solid theory of the OPE. In fact, these two popular explanations for the g_1^p data have been under hot debate over many years before 1995. Though the OPE approach is model independent, it faces the questions of what is the deep reason for the absence of gluonic contributions to Γ_1^p and how are we going to understand a large and negative strange-quark polarization ?

3.1.3 Factorization scheme dependence

In spite of much controversy on the aforementioned issue, this dispute was actually resolved almost a decade ago [35, 36]. The key point is that a different interpretation for Γ_1^p corresponds to a different factorization definition for the quark spin density and the hard photon-gluon cross section. The choice of the "ultraviolet" cutoff for soft contributions specifies the factorization convention.[2] More specifically, since $\Delta\phi^{(1)}$ in Eq. (3.1) is ultraviolet divergent, it is clear that, just like the case of unpolarized deep inelastic scattering, the coefficient functions ΔC_q and ΔC_G depend on how the parton spin distributions $\Delta\phi_{j/i}^{(1)}$ are defined, or how the ultraviolet regulator is specified on $\Delta\phi^{(1)}$. That is, the ambiguities in defining $\Delta\phi_{q/q}^{(1)}$ and $\Delta\phi_{q/G}^{(1)}$ are reflected on the ambiguities in extracting $\Delta C_q^{(1)}$ and $\Delta C_G^{(1)}$. Consequently, the decomposition of the photon-gluon and photon-quark cross sections into the hard and soft parts depends on the choice of the factorization scheme and the factorization scale μ. Of course, the physical quantity $g_1^p(x)$ is independent of the factorization prescription (for a review on the issue of factorization, see [8]).

However, the situation for the polarized DIS case is more complicated: In addition to all the ambiguities that spin-averaged parton distributions have, the parton spin densities are subject to two extra ambiguities, namely, the axial anomaly and the definition of γ_5 in n dimension. It is well known that the polarized triangle diagram for $\Delta\phi_{q/G}^{(1)}$ (see Fig. 1) has an axial anomaly. There are two extreme ultraviolet regulators of interest. One of them, which

[2]It is misleading to identify the regularization scheme for soft divergences, e.g. the off-shell scheme, with the factorization scheme; the former is merely employed to get rid of infrared and collinear divergences appearing in the calculation of partonic cross sections. However, the hard gluon coefficient functions are soft-cutoff independent, and they depend on the ultraviolet regulator on the triangle diagram for defining the polarized quark distribution inside the gluon. In other words, it is the choice of ultraviolet regulator rather than the soft cutoff that specifies the factorization scheme.

we refer to as the chiral-invariant (CI) factorization scheme (sometimes called as the "jet scheme" [37], the "parton model scheme" [11] or the "$k_\perp$ cut-off scheme"), respects chiral symmetry and gauge invariance but not the axial anomaly. This corresponds to a direct brute-force cutoff $\sim \mu$ on the $k_\perp$ integration in the triangle diagram (i.e. $k_\perp^2 \lesssim \mu^2$) with $k_\perp$ being the quark transverse momentum perpendicular to the virtual photon direction. Since the gluonic anomaly is manifested at $k_\perp^2 \to \infty$, it is evident that the spin-dependent quark distribution [i.e. $\Delta q^{(1)}(x)$] in the CI factorization scheme is anomaly-free. Note that this is the $k_\perp$-factorization scheme employed in the usual improved parton model [27, 28, 29].

The other ultraviolet cutoff on the triangle diagram of Fig. 1, as employed in the approach of the OPE, is chosen to satisfy gauge symmetry and the gluonic anomaly. As a result, chiral symmetry is broken in this gauge-invariant (GI) factorization scheme and the sea polarization is perturbatively induced from hard gluons via the anomaly. This perturbative mechanism for sea quark polarization is independent of the light quark masses. A straightforward calculation gives [38, 39]

$$\Delta\phi^{(1)}_{q/G}(x)_{\rm GI} = \Delta\phi^{(1)}_{q/G}(x)_{\rm CI} - \frac{\alpha_s}{\pi}(1-x), \tag{3.9}$$

where the term $\frac{\alpha_s}{\pi}(1-x)$ originates from the QCD anomaly arising from the region where $k_\perp^2 \to \infty$. Two remarks are in order. First, this term has been erroneously claimed[3] in some literature [40, 41, 9, 37] to be a soft term coming from $k_\perp^2 \sim m_q^2$. Second, although the quark spin distribution inside the gluon $\Delta\phi^{(1)}_{q/G}(x)$ cannot be reliably calculated by perturbative QCD, its difference in GI and CI schemes is trustworthy in QCD. Since the polarized valence quark distributions are $k_\perp$-factorization scheme independent, the total quark spin distributions in GI and CI schemes are related via Eqs. (3.3) and (3.9) to be [43]

$$\Delta q(x,Q^2)_{\rm GI} = \Delta q(x,Q^2)_{\rm CI} - \frac{\alpha_s(Q^2)}{\pi}\,(1-x)\otimes\Delta G(x,Q^2). \tag{3.10}$$

For a derivation of this important result based on a different approach, namely, the nonlocal light-ray operator technique, see Müller and Teryaev [44].

The axial anomaly in the box diagram for polarized photon-gluon scattering also occurs at $k_\perp^2 \to \infty$, more precisely, at $k_\perp^2 = [(1-x)/4x]Q^2$ with $x \to 0$. It is natural to expect that the axial anomaly resides in the gluon coefficient function $\Delta C_G^{(1)}$ in the CI scheme, whereas

[3]Some misconceptions in the literature about the $\overline{\rm MS}$ scheme have to be clarified. It has been argued [41] that the GI scheme is pathologic and inappropriate since $\Delta C_G^{(1)}(x)_{\rm GI}$, which is "hard" by definition, contains an unwanted "soft" term proportional to $(1-x)$ [see Eq. (3.12)]. The cross section of the photon-gluon box diagram contains a term proportional to $(1-x)$ if the infrared and collinear divergences are regulated by the quark mass or by the dimensional regulator. If the ultraviolet regulator for the triangle diagram is chirality-preserving, the $(1-x)$ term, which arises from the region where $k_T^2 \sim m_q^2$, does not contribute to the hard gluon coefficient, as it should be. However, if the ultraviolet regulator preserves the axial anomaly and gauge invariance, for example, the $\overline{\rm MS}$ regulator, chirality will be broken and the axial anomaly is absorbed in the quark spin density. It turns out that the effect of the axial anomaly, which is manifested at $k_T^2 \sim \mu^2_{\rm fact}$ (the factorization scale), has the x dependence of the $(1-x)$ form. This explains why ΔC_G in the $\overline{\rm MS}$ scheme has a term proportional to $(1-x)$ because the axial-anomaly effect must be subtracted from the previous gluon coefficient function computed in the chiral-invariant scheme. As pointed out in [39, 8] and again in [42], the $(1-x)$ term in the gluonic coefficient function in the $\overline{\rm MS}$ scheme is purely "hard", contrary to what has been claimed previously.

its effect in the GI scheme is shifted to the quark spin density. Since $\Delta C_G^{(1)}(x)$ is the hard part of the polarized photon-gluon cross section, which is sometimes denoted by $g_1^G(x)$, the polarized structure function of the gluon target, we have

$$\Delta C_G^{(1)}(x) = g_1^G(x) - \Delta\phi_{q/G}^{(1)}(x). \tag{3.11}$$

It follows from Eqs. (3.9) and (3.10) that

$$\Delta C_G^{(1)}(x)_{\rm GI} = \Delta C_G^{(1)}(x)_{\rm CI} + \frac{\alpha_s}{\pi}(1-x). \tag{3.12}$$

The first moments of $\Delta C_G(x)$, $\sum_q \Delta q(x)$ and $g_1^p(x)$ are given by

$$\begin{aligned} &\int_0^1 dx \Delta C_G^{(1)}(x)_{\rm GI} = 0, \qquad \int_0^1 dx \Delta C_G^{(1)}(x)_{\rm CI} = -\frac{\alpha_s}{2\pi}, \\ &\Delta\Sigma_{\rm GI}(Q^2) = \Delta\Sigma_{\rm CI}(Q^2) - \frac{n_f \alpha_s(Q^2)}{2\pi}\Delta G(Q^2), \end{aligned} \tag{3.13}$$

and

$$\begin{aligned} \Gamma_1^p &= \frac{1}{2}\sum e_q^2\left(\Delta q_{\rm CI}(Q^2) - \frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2)\right) \\ &= \frac{1}{2}\sum e_q^2 \Delta q_{\rm GI}(Q^2), \end{aligned} \tag{3.14}$$

where we have neglected contributions to g_1^p from $\Delta\phi_{q/q}^{(1)}$ and $\Delta C_q^{(1)}$. Note that $\Delta\Sigma_{\rm GI}(Q^2)$ is equivalent to the singlet axial charge $\langle p,s|J_\mu^5|p,s\rangle$. The well-known results (3.10-3.14) indicate that Γ_1^p receives anomalous gluon contributions in the CI factorization scheme (e.g. the improved parton model), whereas hard gluons do not play any role in Γ_1^p in the GI scheme such as the OPE approach. From (3.14) it is evident that the sea quark or anomalous gluon interpretation for the suppression of Γ_1^p observed experimentally is simply a matter of convention [35].

The $\overline{\rm MS}$ scheme is the most common one chosen in the GI factorization convention. The so-called Adler-Bardeen (AB) factorization scheme often adopted in the literature is obtained from the GI scheme by adding the x-independent term $-\alpha_s/(2\pi)$ to $\Delta C_G^{\rm GI}$ via

$$\Delta C_G(x)_{\rm AB} = \Delta C_G(x)_{\rm GI} - \frac{\alpha_s}{2\pi}, \tag{3.15}$$

while

$$\Delta q(x,Q^2)_{\rm AB} = \Delta q(x,Q^2)_{\rm GI} + \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\Delta G(y,Q^2). \tag{3.16}$$

In general, one can define a family of schemes labelled by the parameter a [53, 37]:

$$\Delta q(x)_a = \Delta q_{\rm GI}(x) + \frac{\alpha_s}{2\pi}\left[(2x-1)(a-1) + 2(1-x)\right]\otimes\Delta G(x), \tag{3.17}$$

which satisfy the relation

$$\Delta\Sigma_a = \Delta\Sigma_{\rm GI} + \frac{3\alpha_s}{2\pi}\Delta G, \tag{3.18}$$

to the first moment, but differ in their higher moments. The AB scheme corresponds to $a = 2$ and the CI scheme to $a = 1$. Since the x dependence of the axial-anomaly contribution in the quark sector is fixed to be $(1-x)$, it is obvious that all the schemes in this family including the AB scheme cannot consistently put all hard anomaly effects into gluonic coefficient functions unless $a = 1$, contrary to the original claim made in [41].

Finally, it should be stressed that the quark coefficient function $\Delta C_q^{(1)}(x)$ in the dimensional regularization scheme is subject to another ambiguity, namely, the definition of γ_5 in n dimension used to specify the ultraviolet cutoff on $\Delta\phi^{(1)}_{q/q}$ (see Fig. 1). For example, $\Delta C_q^{(1)}(x)$ calculated in the γ_5 prescription of 't Hooft and Veltman, Breitenlohner and Maison (HVBM) is different from that computed in the dimension reduction scheme. The result

$$\Delta C_q^{(1)}(x) = C_q^{(1)}(x) - \frac{2\alpha_s}{3}(1+x) \tag{3.19}$$

usually seen in the literature is obtained in the HVBM scheme, where $C_q(x)$ is the unpolarized quark coefficient function. Of course, the quantity $\Delta q^{(1)}(x) + \Delta C_q(x) \otimes \Delta q^{(0)}(x)$ and hence $g_1^p(x)$ is independent of the definition of γ_5 in dimensional regularization.

3.1.4 A brief summary

In addition to all the ambiguities that spin-averaged parton distributions have, the parton spin densities are subject to two extra ambiguities, namely, the axial anomaly and the definition of γ_5 in n dimension. There are two extreme cases of interest: Either the hard axial anomaly is manifested in the matrix elements of the quark current (GI scheme) or it is absorbed in the gluonic coefficient function so that the quark matrix element is anomaly-free (CI scheme). It should be stressed that in the so-called AB scheme, not all hard anomaly effects are lumped into ΔC_G and hence the corresponding quark matrix element is not anomaly-free. Of course, it appears that the CI scheme is close to the intuitive parton picture as the quark spin distribution which does not contain hard contributions from the anomaly is scale independent. (The price to be paid is that $\Delta q_{\rm CI}$ cannot be expressed as the matrix element of a local gauge-invariant operator and hence it is difficult to compute by lattice QCD. Also, *a priori* there is no reason that $\Delta\Sigma$ should have a simple quark interpretation [45].) Nevertheless, physically and mathematically GI and CI schemes are equivalent.

Two remarks are in order. (i) It is worth emphasizing that although the suppression of Γ_1^p can be accommodated by anomalous gluon and/or sea quark contributions, no quantitative prediction of Γ_1^p can be made. An attempt of explaining the smallness of g_A^0 has been made in the large-N_c approach [46]. (ii) So far we have focused on the perturbative part of the axial anomaly. The perturbative QCD result (3.10) indicates that the difference $\Delta q_s^{\rm GI} - \Delta q_s^{\rm CI}$ ($\Delta q = \Delta q_v + \Delta q_s$ with the valence polaization Δq_v being scheme independent) is induced perturbatively from hard gluons via the anomaly mechanism and its sign is predicted to be negative. By contrast, $\Delta q_s^{\rm CI}(x)$ can be regarded as an intrinsic sea-quark spin density produced nonperturbatively. The well-known solution to the $U_A(1)$ problem in QCD involves two important ingredients: the QCD anomaly and the QCD vacuum with a nontrivial topological structure, namely the θ-vacuum constructed from instantons which are nonperturbative gluon configurations. Since the instanton-induced interactions can flip quark helicity, in analog to the baryon-number nonconservation induced by the 't Hooft mechanism, the quark-antiquark pair created from the QCD vacuum via instantons can have a net helicity. It has been suggested that this mechanism of quark helicity nonconservation provides a natural and nonperturbative way of generating negative sea-quark polarization [47].

In retrospect, the dispute among the anomalous gluon and sea-quark explanations of the suppression of Γ_1 mostly before 1996 is considerably unfortunate and annoying since the fact that $g_1(x)$ is independent of the definition of the quark spin density and hence the choice of the factorization scheme due to the axial-anomaly ambiguity is presumably well known to all the practitioners in the field, especially to those QCD experts working in the area.

3.2 Lattice calculation of the proton spin content

The present lattice calculation is starting to shed light on the proton spin contents. An evaluation of $\Delta q_{\rm GI}$, the gauge-invariant quark spin component defined by $\Delta q_{\rm GI} = \langle p, s|\bar{q}\gamma^\mu\gamma_5 q|p, s\rangle s_\mu$, involves a disconnected insertion in addition to the connected insertion (see Fig. 2; the infinitely many possible gluon lines and additional quark loops are implicit). The sea-quark spin contribution comes from the disconnected insertion.

There are four lattice calculations done by Dong et al. [48], Fukugita et al. [49], Göckeler et al. [50, 51] in the quenched approximation and Güsken et al. [52] in full lattice QCD. Note that the disconnected contribution is not evaluated in [50, 51]. From Table III it is clear that the lattice results for g_A^0 and Δs are in agreement with experiment, while the full lattice QCD calculations for g_A^3, Δu and Δd are too small compared to experiment. In particular, there is a 30% discrepancy between the lattice QCD estimate of g_A^3 [52] and the experimental value. This points to the presence of sizeable higher order or even nonperturbative contributions to the renormalized factor $Z_A^{\rm NS}$ on the non-singlet current.

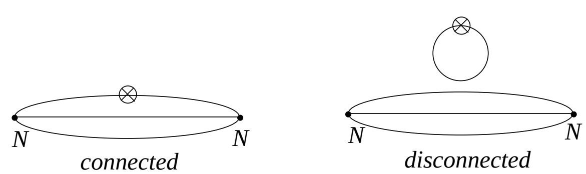

Figure 2: Connected and disconnected insertions arising from the flavor-singlet axial-vector current.

As for the chiral-invariant quantity $\Delta\Sigma_{\rm CI}$, it involves the matrix element of $\tilde{J}_5^+$ in light-front gauge where $\tilde{J}_{\mu 5}$ is an anomaly-free singlet axial vector current and hence sizeable gauge configurations are needed in lattice calculations for $\Delta\Sigma_{\rm CI}$. Nevertheless, it is conceivable to have lattice results for ΔG and $\Delta q_{\rm CI}$ soon in the near future. The lattice calculation of the quark total angular momentum was also available recently, see Sec. 3.4.

Table III. Quark spin contents of the proton from lattice calculations. Experimental results are taken from (2.1) for the first moments of quark spin distributions and Table II for the second moments† $\Delta^{(1)}q_v$ ($=\int_0^1 x\Delta q_v(x)dx$) of valence quark spin densities.

	Dong et al.	Fukugita et al.	Göckeler et al.*	Güsken et al.	Expt
g_A^0	0.25(12)	0.18(10)		0.20(12)	0.25(4)
g_A^3	1.20(10)	0.985(25)	1.24(10)	0.907(20)	1.2670(35)
g_A^8	0.61(13)			0.484(18)	0.585(25)
Δu	0.79(11)	0.638(54)	0.84(5)	0.62(7)	0.81(1)
Δd	-0.42(11)	-0.347(46)	-0.24(2)	-0.29(6)	-0.45(1)
Δs	-0.12(1)	-0.109(30)		-0.12(7)	-0.11(1)
$\Delta^{(1)}u_v$			0.198(8)		0.169(22) [24]
					0.13(1) [23]
$\Delta^{(1)}d_v$			-0.0477(33)		-0.055(29) [24]
					-0.02(3) [23]

*Since the disconnected contributions are not calculated in [50, 51], Δq receives contributions only from the valence component. The lattice results for $\Delta^{(1)}q_v$, which are presumably scale independent, are obtained in [51] at the scale $\mu^2 = 4\,\mathrm{GeV}^2$.

3.3 NLO analysis of polarization data

The experimental data of $g_1(x, Q^2)$ taken at different x-bin correspond to different ranges of Q^2; that is, Q^2 of the data is x-bin dependent. Hence, it is desirable to evolve the data to a common value of Q^2 in order to determine the moments of g_1 and test the Bjorken sum rule. Because of the availability of the two-loop polarized splitting functions $\Delta P_{ij}^{(1)}(x)$ [53, 54], it became possible to embark on a full next-to-leading order (NLO) analysis of the experimental data of polarized structure functions by taking into account the measured x dependence of Q^2 at each x bin. The NLO analyses have been performed in the $\overline{\mathrm{MS}}$ scheme (a family of the GI scheme) [55, 56, 57, 17, 58, 59, 60, 19, 61, 62, 63, 2, 64], the Adler-Bardeen (AB) scheme [41, 65, 17, 60, 62, 19] and the CI scheme [37, 62]. Of course, physical quantities such as the polarized structure function $g_1(x)$ are independent of choice of the factorization convention. Physically, the spin-dependent valence quark and gluon distributions should be the same in all factorization schemes.

The Q^2 dependence of parton spin densities is determined by the spin-dependent DGLAP equations:

$$\begin{aligned}
\frac{d}{dt}\Delta q_{\rm NS}(x,t) &= \frac{\alpha_s(t)}{2\pi}\Delta P_{qq}^{\rm NS}(x)\otimes\Delta q_{\rm NS}(x,t),\\
\frac{d}{dt}\begin{pmatrix}\Delta q_{\rm S}(x,t)\\ \Delta G(x,t)\end{pmatrix} &= \frac{\alpha_s(t)}{2\pi}\begin{pmatrix}\Delta P_{qq}^{\rm S}(x) & 2n_f\Delta P_{qG}(x)\\ \Delta P_{Gq}(x) & \Delta P_{GG}(x)\end{pmatrix}\otimes\begin{pmatrix}\Delta q_{\rm S}(x,t)\\ \Delta G(x,t)\end{pmatrix},
\end{aligned}\tag{3.20}$$

with $t=\ln(Q^2/\Lambda^2_{\rm QCD})$,

$$\Delta q_{\rm NS}(x)=\Delta q_i(x)-\Delta q_j(x),\qquad \Delta q_{\rm S}(x)=\sum_i\Delta q_i(x),\tag{3.21}$$

and

$$\Delta P_{ij}(x) = \Delta P_{ij}^{(0)}(x) + \frac{\alpha_s}{2\pi}\Delta P_{ij}^{(1)}(x) + \cdots . \tag{3.22}$$

The spin-dependent anomalous dimensions are defined as

$$\Delta\gamma_{ij}^{n} = \int_0^1 \Delta P_{ij}(x)x^{n-1}dx = \Delta\gamma_{ij}^{(0),n} + \frac{\alpha_s}{2\pi}\Delta\gamma_{ij}^{(1),n} + \cdots . \tag{3.23}$$

To the NLO, $\Delta P_{qq}^{(1)}$ and $\Delta P_{qG}^{(1)}$ were calculated in the $\overline{\rm MS}$ scheme by Zijlstra and van Neerven [53]. However, the other two polarized splitting functions $\Delta P_{Gq}^{(1)}$ and $\Delta P_{GG}^{(1)}$ were not available until 1995 [54]. The recent analysis [2] shows that the NLO χ^2 is significantly smaller than that of LO, indicating the necessity of NLO fit of data in practice.

Since the unpolarized parton densities are mostly parameterized in the $\overline{\rm MS}$ scheme and the two-loop splitting functions are available in the same scheme, it is quite natural to perform the NLO analysis in the $\overline{\rm MS}$ scheme. In principle one can also work in any other factorization scheme. The splitting functions $\Delta P_{ij}^{(1)}$ in the CI scheme, for example, is obtained by applying Eq. (3.10) to the spin-dependent DGLAP equation, see [44, 42]. It is worth accentuating again that though it is perfectly all right to analyze the data in the AB scheme, one has to keep in mind that not all hard effects are absorbed in the gluonic coefficient functions in such a scheme.

The sea-quark and gluon spin distributions cannot be separately determined from current experimental data. In other words, while the shapes of the spin-dependent valence quark distributions are fairly constrained by the data, sea-quark and gluon spin densities are almost completely undetermined. In particular, ΔG is rather weakly constrained by the data. This is understandable because the gluon polarization contributes to g_1 only at the NLO level through the convolution $\Delta C_G \otimes \Delta G$. In principle, measurements of scaling violation in $g_1(x,Q^2)$ via, for example, the derivative of $g_1(x,Q^2)$ with respect to Q^2, in next-generation experiments will allow an estimate of the gluon spin density and the overall size of gluon polarization. Of course, the data should be sufficiently accurate in order to study the gluon spin density. Meanwhile, it is even more important to probe $\Delta G(x)$ independently in those hadron-hadron collision processes where gluons play a dominant role.

Though the polarized structure function is factorization scheme independent, it is important to perform NLO analyses in different schemes to test the reliability and consistency of the theory. It is found in [19, 58, 62] that the polarized parton distributions obtained in $\overline{\rm MS}$, AB and CI schemes agree well for the non-singlet spin densities, while the first moment of $\Delta G(x)$ obtained in the AB or CI scheme is different from that in the $\overline{\rm MS}$ scheme, reflecting that the present data can hardly constrain the gluon spin distribution. Typically, the extracted value of the gluon spin at $Q^2 = 1\,{\rm GeV}^2$ lies in the range $0 \lesssim \Delta G \lesssim 2$. However, the recent analysis [64] shows that the LO fit cannot decide on the sign of ΔG, while the NLO analysis yields a negative first moment of the gluon density.[4] This illustrates again that it is difficult to pin down the gluon spin distribution from present polarized DIS data. Note that

[4]It has been found by Jaffe [66] that the gluon spin component is negative $\Delta G \sim -0.4$ in the MIT bag model and even more negative in the non-relativistic quark model. However, it was explained in [67] that the negative ΔG obtained by Jaffe is a consequence of neglecting some self-interaction effects.

a recent NLO analysis in [62] shows that the polarized strange quark density is significantly different from zero independently of the factorization schemes used in the analysis:

$$\Delta s_{\overline{\mathrm{MS}}} = -0.102 \pm 0.012, \quad \Delta s_{\mathrm{CI}} = -0.064 \pm 0.010, \quad \Delta s_{\mathrm{AB}} = -0.058 \pm 0.012\,, \tag{3.24}$$

at $Q^2 = 1\,\mathrm{GeV}^2$. Note that the sea polarization Δs is scheme dependent:

$$\Delta s_{\mathrm{CI,AB}} = \Delta s_{\mathrm{GI}} + \frac{\alpha_s}{2\pi}\Delta G. \tag{3.25}$$

The presence of the sea polarization in the CI or AB scheme implies that the gluon polarization is not at its maximal value given by $\Delta G \sim -(2\pi/\alpha_s)\Delta s_{\mathrm{GI}}$.

3.4 Orbital angular momentum

The orbital angular momentum plays a role in the proton spin structure. For example, the growth of large ΔG with Q^2 is balanced by the large negative orbital angular momentum of the quark-gluon pair. It is also known that the reduction of the total spin component $\Delta\Sigma$ due to the presence of the quark transverse momentum in the lower component of the Dirac spinor is traded with the quark orbital angular momentum.

In the proton spin sum rule:

$$\frac{1}{2} = J_q + J_G, \tag{3.26}$$

the total angular momenta J_q and J_G of quarks and gluons respectively are gauge invariant. However, the decomposition into spin and orbital components, $J_q = \frac{1}{2}\Delta\Sigma + L_q$ and $J_G = \Delta G + L_G$, is gauge dependent. This leads to difficulties in defining the partonic spin densities: There exist no local gauge invariant operators that could represent the densities of gluon spin and the orbital angular momenta of quarks and gluons. It is known that the spin and orbital angular momenta of quarks and gluons appearing in the decomposition [34]

$$\begin{aligned} J^z &= J_q^z + J_G^z = S_q^z + L_q^z + S_G^z + L_G^z \\ &= \int d^3x \left[\frac{1}{2}\bar{\psi}\gamma^3\gamma^5\psi + \psi^\dagger(\vec{x}\times i\vec{\partial})^3\psi + (\vec{E}\times\vec{A})^3 - E^k(\vec{x}\times\vec{\partial})^3 A^k\right], \end{aligned} \tag{3.27}$$

obtained in the the light-cone gauge $A^+ = 0$ and in the infinite momentum frame, are separately gauge variant except for the quark spin operator S_q^z. However, the gluon spin and its distribution, for example, are physical, gauge invariant quantities and can be measured experimentally. The point is that ΔG can be expressed as the matrix element of a string-like nonlocal gauge-invariant operator O_G [34, 68]. As stressed in [69], what one measures experimentally is the matrix element of the gauge-invariant operator O_G. But in the light-cone gauge, the above operator reduces to the gluon spin operator $S_G^z = \int d^3x(\vec{E}\times\vec{A})^3$; that is, the gauge-invariant extension of the gluon spin operator in the light-cone gauge is measurable. Likewise, a gauge-invariant operator that reduces to the quark orbital angular momentum in the light-cone gauge has been discussed in [70] (see however a different discussion in [71]).

In principle, the total angular momenta of quarks and gluons, J_q and J_G respectively, can be measured in deeply virtual Compton scattering in a special kinematic region where single quark scattering dominates [72]. It has also been suggested that the orbital angular momentum might be deduced from an azimuthal asymmetry in hadron production with a transversely polarized target.

The evolution of the quark and gluon orbital angular momenta was first discussed by Ratcliffe [73]. Ji, Tang and Hoodbhoy [74] have derived a complete leading-log evolution equation in the light-cone gauge:

$$\frac{d}{dt}\begin{pmatrix} L_q \\ L_G \end{pmatrix} = \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} -\frac{4}{3}C_F & \frac{n_f}{3} \\ \frac{4}{3}C_F & -\frac{n_f}{3} \end{pmatrix}\begin{pmatrix} L_q \\ L_G \end{pmatrix} + \frac{\alpha_s(t)}{2\pi}\begin{pmatrix} -\frac{2}{3}C_F & \frac{n_f}{3} \\ -\frac{5}{6}C_F & -\frac{11}{2} \end{pmatrix}\begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix}, \tag{3.28}$$

with the solutions

$$\begin{aligned} L_q(Q^2) &= -\frac{1}{2}\Delta\Sigma + \frac{1}{2}\frac{3n_f}{16+3n_f} + f(Q^2)\left(L_q(Q_0^2) + \frac{1}{2}\Delta\Sigma - \frac{1}{2}\frac{3n_f}{16+3n_f}\right), \\ L_G(Q^2) &= -\Delta G(Q^2) + \frac{1}{2}\frac{16}{16+3n_f} + f(Q^2)\left(L_G(Q_0^2) + \Delta G(Q_0^2) - \frac{1}{2}\frac{16}{16+3n_f}\right), \end{aligned} \tag{3.29}$$

where

$$f(Q^2) = \left(\frac{\ln Q_0^2/\Lambda^2_{\text{QCD}}}{\ln Q^2/\Lambda^2_{\text{QCD}}}\right)^{\frac{32+6n_f}{33-2n_f}} \tag{3.30}$$

and $\Delta\Sigma$ is Q^2 independent to the leading-log approximation. We see that the growth of ΔG with Q^2 is compensated by the gluon orbital angular momentum, which also increases like $\ln Q^2$ but with opposite sign. The solution (3.29) has an interesting implication in the asymptotic limit $Q^2 \to \infty$, namely

$$\begin{aligned} J_q(Q^2) &= \frac{1}{2}\Delta\Sigma + L_q(Q^2) \to \quad \frac{1}{2}\frac{3n_f}{16+3n_f}, \\ J_G(Q^2) &= \Delta G(Q^2) + L_G(Q^2) \to \quad \frac{1}{2}\frac{16}{16+3n_f}. \end{aligned} \tag{3.31}$$

Thus, history repeats herself: The partition of the nucleon spin between quarks and gluons follows the well-known partition of the nucleon momentum. Taking $n_f = 6$, we see that $J_q : J_G = 0.53 : 0.47$. If the evolution of J_q and J_G is very slow, which is empirically known to be true for the momentum sum rule that half of the proton's momentum is carried by gluons even at a moderate Q^2, then $\Delta\Sigma \sim 0.25$ at $Q^2 = 10\,\text{GeV}^2$ implies that $L_q \sim 0.13$ at the same Q^2, recalling that the quark orbital angular momentum is expected to be of order 0.20 in the relativistic quark model.

Recently the quark orbital angular momentum of the nucleon was calculated from lattice QCD by considering the form factor of the quark energy-momentum tensor $T_{\mu\nu}$ [75]. The total angular momentum of the quarks is found to be

$$J_q = 0.30 \pm 0.07\,. \tag{3.32}$$

That is about 60% of the proton spin is attributable to the quarks. Hence, the quark orbital angular momentum is $L_q = 0.17 \pm 0.06$ when combining with the previous quark spin content

$\frac{1}{2}\Delta\Sigma = 0.13 \pm 0.06$ [48]. Therefore, about 25% of the proton spin originates from the quark spin and about 35% comes from the quark orbital angular momentum.

It must be stressed that J_q should be factorization scheme independent. This means that a replacement of $\Delta\Sigma_{\rm GI}$ by $\Delta\Sigma_{\rm CI}$ in the spin sum rule (3.26) requires that the difference $\Delta\Sigma_{\rm CI} - \Delta\Sigma_{\rm GI} = (n_f\alpha_s/2\pi)\Delta G$ be compensated by a counterpart in the gluon orbital angular momentum:

$$L_q^{\rm CI} = L_q^{\rm GI} - \frac{n_f\alpha_s}{4\pi}\Delta G. \tag{3.33}$$

It is interesting to note that if ΔG is of order 2.5, one will have $\Delta\Sigma_{\rm CI} \sim 0.58$ and $L_q^{\rm CI} \sim 0$. In other words, while $\Delta\Sigma_{\rm CI}$ is close to the relativistic quark model value of $\Delta\Sigma$, $L_q^{\rm CI}$ deviates farther from the quark model expectation $L'_q \sim 0.20$ (see Sec. I).

4 Conclusions

The spin sum rule of the proton in the infinite momentum frame reads

$$\frac{1}{2} = J_q + J_G = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_G. \tag{4.1}$$

The quark spin can be inferred from polarized DIS measurements of $g_1(x)$ and its first moment. Due to the ambiguity arising from the axial anomaly, the definition of the sea polarization Δq_s and hence $\Delta q = \Delta q_v + \Delta q_s$ is $k_\perp$ factorization dependent, but J_q, $g_1(x)$ and Γ_1 are not. The only spin content which is for sure at present is the observed value $\Delta\Sigma \sim 0.20 - 0.30$ in the GI scheme (e.g. the $\overline{\rm MS}$ scheme). The recent lattice calculation yields $J_q = 0.30 \pm 0.07$. Therefore, we have $L_q^{\rm GI} \sim 0.10 \pm 0.06$ for $\Delta\Sigma_{\rm GI} \sim 0.25$, to be compared with the quark model prediction $L'_q \sim 0.20$. The values of $\Delta\Sigma$ and L_q in the CI scheme (e.g. the improved QCD parton model) or the AB scheme depend on the gluon spin. Since $L_q^{\rm CI} - L_q^{\rm GI} = -n_f\alpha_s\Delta G/(4\pi)$, it is clear that if the gluon polarization is positive and large enough, then $L_q^{\rm CI}$ will deviate even farther from the quark picture although $\Delta\Sigma_{\rm CI}$ can be made to be close to the constituent relativistic quark model. In the asymptotic limit, $J_q(\infty) = \frac{1}{2}\Delta\Sigma(\infty) + L_q(\infty) \sim \frac{1}{4}$ and $J_G(\infty) = \Delta G(\infty) + L_G(\infty) \sim \frac{1}{4}$. The recent lattice result $J_q = 0.30 \pm 0.07$ at a moderate Q^2 seems to suggest that the evolution of J_q and hence J_G is slow enough.

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