The Quality of Information and Incentives for Effort*

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Abstract

We study the relationship between the precision of information about the performance of an agent in a market, and the incentives this agent has for exerting effort to produce high quality. We show that this relationship is not monotonic. There exists a threshold beyond which any further improvement in the precision of information weakens the agent's incentive to produce high quality. Accordingly, both very precise and very imprecise information about the agent's performance may destroy its incentive to exert effort. A few applications of this result are discussed.

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1. Introduction

This paper concerns the relationship between the precision of public information about the performance of an agent in a market, and the incentives this agent has for exerting effort to produce high quality. We show that this relationship is not monotonic. There exists a threshold beyond which any further improvement in the precision of information weakens the agent's incentive to produce high quality. Accordingly, both very imprecise but also very precise public information about the agent's performance may destroy its incentive to exert effort.

We consider a dynamic model of a market for a good whose quality is not observable to the consumer at the time of purchase and is not contractible. Such goods are referred to as experience or credence goods in the literature.[1] Examples range from food and wine to used cars and expert advice. Importantly, because consumers cannot contract on the quality of the good, the price they are willing to pay depends on their beliefs about the good's expected quality. These beliefs are identified in our model with the reputation of the good's producer. A producer's reputation, in any given period, is determined by consumers' beliefs regarding the producer's type, taking into account the producer's incentives to exert effort to produce high quality. The beliefs regarding each producer's type are updated based on publicly available information about the producer's past performance, and the producers' incentives to exert high effort are determined by the return to reputation which depends on expected future prices.

For any given prior beliefs about the producer's ability, the producer's incentive to exert costly effort in order to produce high quality is increasing with the probability that the true quality of the good would be revealed. Hence, for any given consumers' prior beliefs, an improvement in consumers' ability to detect high and low quality goods has an obvious positive effect on the producer's incentives to produce high quality. However, as we show here, such an improvement may have an overall negative effect on the incentives to produce high quality because, in a dynamic setting, the consumers' prior beliefs are affected by the precision of their information.

If prior beliefs regarding a producer's ability are very precise, then a contradictory signal about the producer's ability would be attributed to either sampling error or a random shock in the production process, and would only have a small effect on the consumers' posterior beliefs. It follows that if the prior probability that a producer is competent is sufficiently high, then this producer could avoid effort and 'rest on its laurels' without incurring a significant loss of reputation. Similarly, if the prior probability that a producer is competent is sufficiently low, then it is difficult for such a producer to significantly improve its reputation by the production of high quality, because, as before, a signal that is inconsistent with the producer's reputation would be heavily discounted. Hence, because precise or concentrated priors are

[1] An experience or a credence good is distinguished by the fact that its quality cannot be determined by consumers at the time of purchase. The true quality of an experience good is revealed later when consumers experience its consumption. The true quality of a credence good is never fully revealed to consumers.

hard to change, precise information might generate perverse incentives. Consequently, an equilibrium in which competent producers exert costly effort in order to produce high quality goods and maintain their reputation could unravel as consumers' information becomes more precise.

Consider for example the case of a restaurant that serves sushi. An important aspect of the quality of sushi, which consumers have no way of verifying at the spot, is whether it contains germs that cause food disease. Sushi that contains such germs often causes stomach upset but not always. And not every stomach upset is related to the consumption of contaminated sushi. Therefore, a consumer of sushi can never tell for sure whether the sushi she ate is of high or low quality.

Sushi may contain germs either because the sushi chef is incompetent and simply does not know how to pick fresh fish, or because the sushi chef saves money and effort and buys inferior fish or fails to handle the fish properly. Suppose that consumers are aware of whether or not other consumers experienced a stomach upset after they had ate at the restaurant during the previous couple of periods, and that furthermore, the price of sushi in the restaurant depends on this information. The results reported in this paper suggest that as the consumers' information about the restaurant becomes more precise, for example, if the pool of consumers who share information about the quality of the sushi increases in size, then the incentives of competent sushi masters to exert effort may become weaker. Intuitively, suppose that consumers believe that the restaurant chef is competent with a very high probability, as would be the case if consumers are familiar with a large number of other satisfied consumers. Under such circumstances, news about a few cases of stomach upset would most likely not be attributed to the poor quality of sushi but to some other reason, and this might give the restaurant an incentive to rest on its laurels for a while, and cut cost by buying fish of inferior quality.

The contribution of this paper is that it provides an intuitive account of the relationship between the precision of consumers' beliefs, and hence producers' reputation, and producers' incentives. We build on the familiar idea that a producer with a good reputation might 'rest on its laurels' and produce low quality to argue that, by strengthening the inference that a producer with a good reputation is indeed very likely to be truly competent, an improvement in the precision of information may have a perverse effect on incentives. We identify a threshold beyond which any further improvement in the precision of publicly available information would reduce the incentives to exert effort in order to produce high quality.[2]

[2] It is well known that more precise *private information* may sometimes lead to a less efficient outcome. Consider for example a standard "lemons" market in which sellers know the quality of what they sell but buyers do not. Suppose that the adverse selection in this market is so bad that no trade takes place in equilibrium. In this market, if sellers were uninformed about the quality of the goods sold, then the efficient outcome in which buyers and sellers trade at a price that is equal to the average value of the good would prevail. A number of papers (see, e.g., Sakai, 1985; Gal-Or, 1988; Mirman et al., 1994; Harrington, 1995; and Schlee, 1996) describe environments in which *public information* about quality may sometimes have a

The relevant literature can be divided into three parts as follows.

Career Concerns

Holmström (1999) considered a model in which an agent's future career concerns influence its incentives to exert effort. The output produced by the agent is not contractible, so it is impossible to directly reward or penalize the agent based on its past performance. Rather, in each period, the agent's wage is determined based on the belief about its ability and its expected future effort. Initially, an agent in Holmström's model may exert some effort, but with time, as information about the agent's true ability becomes more and more precise, the agent's incentive to exert effort weakens, and the agent's level of effort decreases to zero.[3] Holmström shows that if the agent's ability changes stochastically over time, then an incentive to exert effort can be sustained, because in every period, the agent still has an incentive to prove anew that it has a high ability.

There are two main differences between Holmström's model and ours. First, in our model producers privately know their own types. In contrast, in Holmström's model the information about the agent's true ability is symmetric. Namely, the agent and the market are equally well informed about the agent's true ability.[4] Second, in our model the agent's ability and effort are strategic complements. They are strategic substitutes in Holmström's model. Accordingly, Holmström's conclusions about the effect of a change in the precision of the information about the agent's performance is very different from ours. While in our model, more precise information may weaken the agent's incentives to exert effort as explained above, in Holmström's model it unambiguously leads to stronger incentives to exert effort.

Dewatripont, Jewitt, and Tirole (1999) build on Holmström's model to characterize information structures in terms of their effect on the agent's incentives.[5] They identify information structures where more precise information may weaken the agent's incentives. They describe a number of examples that are all based on the following insight. Consider the agent's incentives to exert effort when information about its performance is given by the more informative signal (y, z) compared to the less informative signal y. Conditional on the realization of the signal y, suppose that the market's expectation of the agent's talent is increased when higher values of statistic z are observed. If higher effort by the agent tends to increase z (which follows from the monotone likelihood ratio property), then having z in the

negative value. However, the reason that public information may have a negative value in these models can be attributed to "non-convexities" of some kind, which is very different from the reasons discussed in this paper.

[3] Gibbons and Murphy (1992) considered an extension of Holmström's model in which the agent's output is contractible. They showed that the optimal compensation contract in such a setting optimizes over the combination of the implicit incentives from career concerns and the explicit incentives from the compensation contract. As the agent approaches retirement, the explicit incentives induced by the optimal compensation contract become stronger to make up for the weaker career concerns of such an agent.

[4] Although the market is not able to directly observe the agent's effort in Holmström's model, it can infer it by solving the agent's optimization problem (Holmström, 1999, p. 171).

[5] See also Bar-Isaac and Ganuza (2005).

market information set enhances effort. However, if more effort on part of the agent tends to decrease z, then having z in the market information set would reduce the incentive for effort (pp. 193-4). Thus, the reason that better information may weaken incentives in Dewatripont et al.'s model is different from the reason that is described in this paper. Because, unlike in Holmström's model and in this paper, Dewatripont et al. only consider a two-period model, the informativeness of the signal has no effect on the market's prior beliefs at the beginning of each period as in our model. Furthermore, unlike the results obtained here, they show that under a number of "regularity" conditions, more precise information about the agent's performance unambiguously improves the agent's incentives to exert effort.

Reputation as Separation from Less Competent Types

In our model, as well as in all similar models, competent producers exert effort to produce high quality in order to maintain a 'reputation for competence.' The existence of incompetent producers is thus crucial for our results. For if all producers were known to be equally competent, then producers would not be capable of distinguishing themselves as 'more' competent than others, and would thus lose the incentive to exert costly effort. See for example, Mailath and Samuelson (2001), and in the different context of the enforcement of cooperation in repeated community prisoner's dilemma like games, Ghosh and Ray (1996). For obvious reasons, the mere existence of incompetent producers, by itself, is insufficient to provide competent producers with sufficient incentives. It must be that consumers assign a sufficiently high probability that any producer is incompetent to provide this particular producer with the incentive to exert the costly effort associated with the production of high quality so as to distinguish itself from less competent producers. Thus, another intuitive explanation for our main result is that as information becomes more precise, a competent producer finds it easier to distinguish itself from less competent types. The fact that separation becomes easier might imply that the incentive to exert costly effort in order to distinguish oneself is weakened.

The Market for Names

The relationship between reputation and incentives has also been explored in the context of the "market for names" where names serve as repositories for reputations (see Mailath and Samuelson, 2001; Tadelis 1999, 2002, and 2003; and the references therein). These authors studied the market for names that develops when producers of a certain good occasionally exit the market and sell their reputations to new entrants to the market. They have showed that such a 'market for names' provides an incentive to exert effort to produce high quality so as to build a 'name' or a reputation that can later be sold.

The rest of the paper proceeds as follows. In the next section, we describe the model. In Section 3, we show the existence of a threshold beyond which any further improvement in the precision of information would weaken the incentives to produce high quality. In section 4, we discuss a few extensions of the basic model. We conclude in Section 5 with a discussion of the possible implications of our analysis. All proofs are relegated to the appendix.

2. Model

We describe a simple model in which we can elucidate our main argument. A few extensions of the basic model are presented in Section 4. We consider a dynamic model of a market for an experience good. Time is discrete, and periods are indexed by $t \in \mathbb{I} \equiv \{..., 0, 1, ...\}$. There is a continuum of measure one of producers. There are two types of producers, competent and incompetent. The measure of incompetent producers is given by $\eta \in (0, 1)$. Producers discount future payoffs at the rate $\delta < 1$.

In every period, each competent producer may either exert a costly effort, at cost $c \in (0, 1)$, to produce one unit of a high quality good, or it may costlessly produce one unit of a low quality good. Incompetent producers are incapable of producing high quality goods. However, they may each costlessly produce one unit of the low quality good in every period.[6]

High and low quality goods cannot be distinguished by consumers at the time of purchase. A high quality good has value 1 and a low quality good has value 0 for consumers. Every period, produced goods are subject to inspection. We assume that high and low quality goods pass the inspection with probabilities π^H, and π^L, respectively, where $0 < \pi^L < \pi^H \leq 1$. Whether or not each producer passes or fails inspection is public information.[7] This public information is forgotten after $n \geq 1$ periods.

It follows that in every period, producers are sorted into 2^n submarkets, depending on whether they passed or failed inspection in the previous n periods. Let h_n denote an n-dimensional vector of passes and fails, and let H_n denote the set of all such n-dimensional vectors. In any period t, all the producers who have the same realized profile of passes and fails in the last n periods are sorted into the same submarket at t. We can thus identify every submarket with some n-dimensional profile of passes and fails $h_n \in H_n$.

We assume that in every period, demand in each submarket is infinitely elastic at the expected value of the good to consumers in that submarket. For every period $t \in \mathbb{I}$, let $p_t^{h_n}$ denote the price in submarket $h_n \in H_n$ at t. Our notion of market-equilibrium is defined as follows.

Definition. *A sequence of prices* $\left\{ \left(p_t^{h_n}\right)_{h_n \in H_n} \right\}_{t \in \mathbb{I}}$ *is a market-equilibrium if:*

1. *In every period* $t \in \mathbb{I}$, *producers produce the quality that maximizes the discounted value of their expected profits given the sequence of prices* $\left\{ \left(p_t^{h_n}\right)_{h_n \in H_n} \right\}_{t \in \mathbb{I}}$. *We assume, for simplicity, that in case of indifference, producers produce high quality.*

2. *In every period* $t \in \mathbb{I}$, *the prices in each submarket is equal to the expected quality of the good for consumers in that submarket.*

[6] Assuming instead that the produced quality is stochastic so that a competent producer who incurs the cost of producing high quality sometimes produces low quality, and vice-versa, does not change our results.

[7] The conclusions of the model remain qualitatively unchanged if a producer who has failed inspection is subject to a fine, provided, of course, that this fine is not so large so as to cause producers to stay out of the market.

Remark 1. Existence of a Market-Equilibrium

The model admits the existence of at least one market-equilibrium. In particular, the sequence of prices $\left\{\left(p_t^{h_n}\right)_{h_n \in H_n}\right\}_{t \in \mathbb{I}}$ where $p_t^{h_n} = 0$ for every $h_n \in H_n$ and $t \in \mathbb{I}$ and where no producer ever produces high quality is a market-equilibrium. To see this, observe that if prices are zero in every period, then no producer has any incentive to incur the cost required to produce high quality. Under this equilibrium, any passing of inspection would be attributed to inspection error.

Remark 2. Interpretation as a Model of Career Concerns

The description above has emphasized an interpretation of the model as that of a market for an experience good. However, the same assumptions also admit an interpretation of the model as that of an agent whose future career concerns influence its incentives to exert effort as in the "career concern" literature mentioned in the introduction. Under this alternative interpretation, instead of a continuum of producers, there is only one agent, who is initially believed to be competent with probability $1 - \eta$. The output of the agent is assumed to be non contractible, so that in every period, the agent is paid a wage that is based on the belief about its competence and its expected effort in that period. Under this alternative interpretation, the 2^n different submarkets may be thought of as the 2^n reputations that an agent might have in an environment where the market only obtains noisy signals about the agent's performance in the last n periods.

3. The Precision of Information

Ceteris paribus, the higher the cost of producing a high quality good, c, the weaker the incentive to produce it. The strength of incentives can therefore be measured by how high is the threshold cost above which competent producers may sometimes choose not to produce high quality. The higher this threshold, the stronger is the incentive to produce high quality.

This threshold obviously depends on the particular market equilibrium that is considered. For example, in the equilibrium in which no producer ever produces high quality, this threshold is zero. We are interested in the magnitude of this threshold in the efficient market equilibrium (where competent producers always produce high quality regardless of the particular submarket to which they have access in any given period). In other words, the efficient market equilibrium can be supported if the cost of producing high quality is zero, and by continuity, it can also be supported if the cost of producing high quality is sufficiently low. We define the "strength of incentives" to be equal to the maximal cost of producing high quality c under which the efficient market equilibrium can be sustained.

The question we are interested in is what is the effect of a change in the precision of information on the strength of incentives as defined above.

Denote the threshold cost above which a competent producer may sometimes choose not to produce high quality when all other competent producers do produce high quality by $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$. The next theorem establishes the existence of a threshold on the precision of information, beyond which any further improvement in the precision of information would weaken the incentives of competent producers to produce high quality.

Theorem 1. *For every fixed values of $n \geq 3$, $\eta \in (0,1)$ and $\delta < 1$, there exists threshold values of π^L and π^H such that any further improvement in the precision of information, namely either an increase in the value of π^H or a decrease in the value of π^L results in a lower value of $c_n^* \left(\pi^H, \pi^L, \eta, \delta\right)$.*

The intuition for the theorem is the following: In the efficient market equilibrium, competent producers always produce high quality and incompetent producers always produce low quality. As π^L decreases to zero, it becomes more and more difficult for incompetent producers to pass inspection. Consequently, as π^L decreases to zero, the proportion of incompetent producers who have one or more passes decreases to zero, which in turn implies that the prices in all submarkets except the one that has n failures converge to one. But then a competent producer who has a large number of passes can rest on its laurels and produce low quality, knowing that it could still obtain a price that is close to 1, and that furthermore, it can always rebuild its reputation in the next period by the production of high quality. Such a producer would therefore produce low quality, unless the cost of producing high quality c is sufficiently low to discourage him from doing so.

On the other hand, as π^H increases to one, it becomes more and more difficult for competent producers to fail inspection which implies that even one failure out of n inspections is sufficient to convince consumers that a producer is incompetent. In this case, prices in all submarkets except the one that has n successes converge to zero. This implies that a competent producer who happened to fail inspection becomes discouraged and stops the production of high quality, because it has to pass inspection for n straight periods before it can fetch a positive price for the good it produces. Again, this implies that the cost of producing high quality c has to be low in order for the efficient market equilibrium to be sustained.

When π^L decreases to zero and π^H increases to one simultaneously, then the set of histories or submarkets can be partitioned into two subsets, one with a large number of passes, and one with a low number of passes.[8] Prices in the former set of submarkets converge to one, while prices in the latter set of submarkets converge to zero. Competent producers who have passed many inspections face weaker incentives to produce high quality because they can afford to rest on their laurels, while competent producers who have failed many inspections become discouraged.

To better understand the reason the theorem holds, consider a simple example where the length of memory, n, is equal to 2. For this case, it is possible to explicitly solve for the value of the threshold cost $c_2^* \left(\pi^H, \pi^L, \eta, \delta\right)$ beyond which any further improvement in the precision of information undermines incentives.

[8] The number of passes required to qualify for inclusion in the former set depends on the relative speed of convergence of π^L to zero and π^H to one, respectively.

Given a market equilibrium $\left\{\left(p_t^{h_n}\right)_{h_n\in H_n}\right\}_{t\in\mathbb{I}}$, let $U_t^{h_n}$ denote the expected discounted payoff of a competent producer with a history $h_n \in H_n$ in period t who proceeds to behave optimally in period t and onwards.

In the efficient market equilibrium, in every period t,

$$\begin{aligned}
U_t^{PP} &= p_t^{PP} - c + \delta\left(\pi^H U_{t+1}^{PP} + \left(1-\pi^H\right)U_{t+1}^{PF}\right) \\
U_t^{PF} &= p_t^{PF} - c + \delta\left(\pi^H U_{t+1}^{FP} + \left(1-\pi^H\right)U_{t+1}^{FF}\right) \\
U_t^{FP} &= p_t^{FP} - c + \delta\left(\pi^H U_{t+1}^{PP} + \left(1-\pi^H\right)U_{t+1}^{PF}\right) \\
U_t^{FF} &= p_t^{FF} - c + \delta\left(\pi^H U_{t+1}^{FP} + \left(1-\pi^H\right)U_{t+1}^{FF}\right)
\end{aligned} \tag{1}$$

where PP denotes the set of histories in which producers passed the last two inspections, PF denotes the set of histories in which producers failed the last inspection but passed the penultimate inspection, and so on.

The next lemma characterizes the behavior of competent producers in a market equilibrium.

Lemma 1. *Fix a market-equilibrium $\left\{\left(p_t^{h_n}\right)_{h_n\in H_n}\right\}_{t\in\mathbb{I}}$. In every period $t \in \mathbb{I}$, a competent producer with a history h_n produces high quality if and only if*

$$c \leq \delta\left(\pi^H - \pi^L\right)\left(U_{t+1}^{h_nP} - U_{t+1}^{h_nF}\right) \tag{2}$$

where $h_nP, h_nF \in H_n$ denote vectors whose $n-1$ first coordinates coincide with the last $n-1$ coordinates of h_n and that have a pass and fail, respectively, in the n-th coordinate.

The fact that in any stationary market equilibrium, and in particular in the efficient market equilibrium, $U_t^{ij} = U_{t+1}^{ij} \equiv U^{ij}$ and $p_t^{ij} = p_{t+1}^{ij} \equiv p^{ij}$ for every $ij \in \{PP, PF, FP, FF\}$ and period t, implies that it is possible to solve for $\left(U^{PP}, U^{PF}, U^{FP}, U^{FF}\right)$ in the four linear equations in (3.1) in terms of the market prices $\left(p^{PP}, p^{PF}, p^{FP}, p^{FF}\right)$ (the solution is long, and for that reason is not presented here).

In the efficient market equilibrium, the measure of competent and incompetent producers in any period in submarket PP is given by $(1-\eta)\left(\pi^H\right)^2$ and $\eta\left(\pi^L\right)^2$, respectively. The price in submarket PP is therefore given by

$$p^{PP} = \frac{(1-\eta)\left(\pi^H\right)^2}{(1-\eta)\left(\pi^H\right)^2 + \eta\left(\pi^L\right)^2} \tag{3}$$

in every period $t \in \mathbb{I}$. Similarly, the prices in submarkets PF, FP, and FF, are given by

$$p^{PF} = p^{FP} = \frac{(1-\eta)\,\pi^H\left(1-\pi^H\right)}{(1-\eta)\,\pi^H\left(1-\pi^H\right) + \eta\pi^L\left(1-\pi^L\right)}, \tag{4}$$

and

$$p^{FF} = \frac{(1-\eta)\left(1-\pi^H\right)^2}{(1-\eta)\left(1-\pi^H\right)^2 + \eta\left(1-\pi^L\right)^2}, \tag{5}$$

respectively, in every period $t \in \mathbb{I}$.

By plugging (3)-(5) into the solution of $\left(U^{PP}, U^{PF}, U^{FP}, U^{FF}\right)$ in terms of $\left(p^{PP}, p^{PF}, p^{FP}, p^{FF}\right)$ and then into (2) it is possible to explicitly solve for $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ as a function of π^H, π^L, η, and δ. It is then possible to differentiate $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ with respect to π^L and π^H and to show directly that it is increasing in π^L and decreasing in π^H in the neighborhood of $\pi^L = 0$ and $\pi^H = 1$. As an illustration, in Figure 1 below we plot $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ as a function of π^L holding the values of π^H, η, and δ constant, and in Figure 2 below we plot $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ as a function of π^H holding the values of π^L, η, and δ constant.

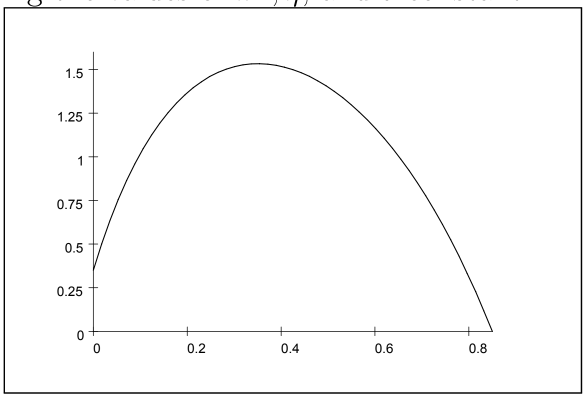

Figure 1. $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ as a function of π^L (for $\pi^H = .85$, $\eta = .2$, and $\delta = .9$).

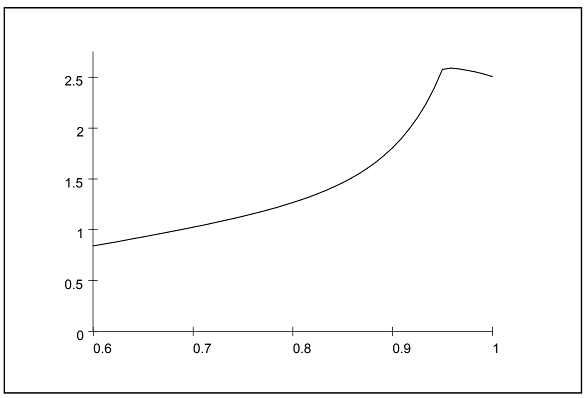

Figure 2. $c_2^*\left(\pi^H, \pi^L, \eta, \delta\right)$ as a function of π^H (for $\pi^L = .25$, $\eta = .2$, and $\delta = .9$).

Intuitively, what is going on is that as π^L decreases to zero, so few incompetent producers pass inspection that both p^{PF} and p^{PP} converge to 1. A competent producer who has passed inspection in the previous period (and is therefore either in submarket FP or PP) realizes that even if it fails inspection in the current period, it would still get a very good price, namely p^{PF}, in the next period, which decreases its incentive to produce high quality. Such a producer need not fear the stigma associated with having produced low quality because by producing high quality in the next period it can then pass inspection with a high probability and so gain access to submarket FP where the price p^{FP} is almost equal to the highest possible price it could get, p^{PP}, in the period after that. This implies that unless the cost of producing high quality c is very low, such a producer would indeed produce low quality. It follows that in order to sustain the efficient market equilibrium, the cost of producing high quality c has to decrease as π^L decreases to zero. More formally, the binding incentive constraint in this case is inequality (2) with $h_n \in \{PP, FP\}$, which implies that producers who have passed inspection in the previous period may want to rest on their laurels and produce low quality in the current period.

The intuition for what happens to the incentive to produce high quality as π^H increases to one is similar. In this case, it is inequality (2) with $h_n \in \{PF, FF\}$ that is binding, which implies that producers who have failed inspection in the previous period become discouraged and stop producing high quality. The reason for this is that when π^H is very close to 1, then a failure to pass inspection indicates that the producer is incompetent. Therefore, when π^H is very close to 1, the price in submarket p^{FP} is very close to zero, which undermines the incentives of producers who have failed inspection in the previous period to produce high quality. It follows that unless the cost of producing high quality c is very low, a competent producer who has failed the last inspection would produce low quality, which, in turn, implies that in order to to sustain the efficient market equilibrium, the cost of producing high quality c has to decrease as π^H increases to one.

If π^L decreases to zero and π^H increases to one simultaneously, then if π^L decreases to zero "faster" than π^H increases to one, then a pass becomes more informative than a fail, and the situation is similar to the situation in which π^L decreases to zero while π^H is held constant. If, on the other hand, π^H increases to one "faster" than π^L decreases to zero, then a fail becomes more informative than a pass, and the situation is similar to the situation in which π^H increases to one while π^L is held constant.[9]

Remark 3. Calculation of $c_n^* \left(\pi^H, \pi^L, \eta, \delta\right)$ for $n > 2$

In principle, it is possible to explicitly calculate $c_n^* \left(\pi^H, \pi^L, \eta, \delta\right)$ for every n in the same way it was calculated above for the case where $n = 2$. However, since the dimensionality of the calculation increases with n, such a calculation becomes rather tedious already with $n = 3$.

[9] If π^L decreases to zero and π^H increases to one at the "same speed" so that the probability that a producer is competent conditional on one pass and one failure converges to a constant that lies strictly between zero and one, then it may be that incentives improve as π^L decreases to zero and π^H increases to one. This is the reason we require that n be larger than or equal to 3 in Theorem 1. When $n \geq 3$, then even if the probability that a producer is competent conditional on some history converges to a constant that is stictly between zero and one, there are other histories to which a producer can fall back to and still be considered either competent or incompetent with a high probability.

4. Extensions

4.1. The Length of Memory

The precision of information and the length of memory are substitutes: both provide more precise information with respect to producers' competence. Therefore, in much the same way that too precise information can undermine the incentive to produce high quality as shown in the previous section, a longer memory can also undermine the incentive to produce high quality. We describe two results that illustrate this effect. We first show that as the length of memory is increased from one to two periods, the incentive to produce high quality becomes stronger when information is not very precise, but is undermined when it is more precise. And, second, we show that as the length of memory increases beyond a certain threshold, then the incentive to produce high quality is undermined, and as the length of memory tends to infinity, the incentive to produce high quality is completely eliminated.

4.1.1. $n = 1$ vs. $n = 2$

Consider a model that is identical to that described in Section 2, except that $n = 1$. That is, consumers only know if each producer has passed or failed inspection in the previous period. This implies that the market is composed of two submarkets: one for producers who passed inspection in the previous period, denoted P, and one for producers who failed, denoted F. In the efficient market equilibrium, the measures of competent and incompetent producers in submarket P are given by $(1-\eta)\,\pi^H$ and $\eta\pi^L$, respectively, in any period. The price in submarket P is therefore given by

$$p^P = \frac{(1-\eta)\,\pi^H}{(1-\eta)\,\pi^H + \eta\pi^L} \tag{6}$$

in every period $t \in \mathbb{I}$. Similarly, the price in submarket F is given by

$$p^F = \frac{(1-\eta)\left(1-\pi^H\right)}{(1-\eta)\,(1-\pi^H) + \eta\,(1-\pi^L)}. \tag{7}$$

An analog of Lemma 1 for the case where $n = 1$ implies that the efficient market equilibrium can be sustained as an equilibrium if and only if

$$c \leq \delta\left(\pi^H - \pi^L\right)\left(p^P - p^F\right), \tag{8}$$

and upon plugging (6) and (7) into (8), if and only if

$$c \leq \frac{\delta\eta\left(\pi^H - \pi^L\right)^2}{(\pi^H + \eta\pi^L)\,(1 + \eta - \pi^H - \eta\pi^L)}. \tag{9}$$

Examination of inequality (9) reveals that if inspection is already precise, that is, if π^H and π^L are close to 1 and 0, respectively, then the right-hand-side (RHS) of inequality (9) is close to δ, which implies, when δ is close to one, that the incentive to produce high quality is very strong. Hence, as π^H and π^L converge to 1 and 0, respectively, increasing the length of memory from one to two periods may weaken the incentive to produce high quality because if, say, $\pi^L = 1 - \pi^H$ so that π^L converges to zero at the same speed that π^H converges to one, then the RHS of inequality (2) for $h_n \in \{PP, FP\}$ and for $h_n \in \{PF, FF\}$, is bounded from above by $\delta\eta$ and $\delta(1-\eta)$, respectively.

On the other hand, if π^H and π^L are not close to 1 and 0, respectively, then it can be shown, numerically, that the RHS of inequality (9) is smaller than the RHS of inequality (2) for $h_n \in \{PP, FP\}$ and for $h_n \in \{PF, FF\}$, which implies that increasing the length of memory from one to two periods improves incentives in this case.

4.1.2. Large n

We show that the threshold cost, $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$, beyond which the efficient market equilibrium cannot be sustained decreases to zero as n increases.

A version of the "one-stage-deviation principle" (Fudenberg and Tirole, 1991, pp. 108-110) implies that the efficient market equilibrium can be sustained if and only if no producer in any submarket can benefit from a single deviation in which it produces low quality once and then high quality thereafter.

If all competent producers always produce high quality regardless of the submarket in which they happen to find themselves, then the prices in all submarkets remain constant, and do not change over time. We can therefore denote the price in submarket h_n by p^{h_n}, independently of the period. If we let $h_n(P)$ denote the number of passes in the vector h_n, then Bayesian updating implies that

$$p^{h_n} = \frac{(1-\eta)\left(\pi^H\right)^{h_n(P)}\left(1-\pi^H\right)^{n-h_n(P)}}{(1-\eta)\left(\pi^H\right)^{h_n(P)}\left(1-\pi^H\right)^{n-h_n(P)} + \eta\left(\pi^L\right)^{h_n(P)}\left(1-\pi^L\right)^{n-h_n(P)}} \tag{10}$$

for every submarket $h_n \in H_n$.

In the efficient market equilibrium, if a competent producer were to produce low quality in some period t, and then to continue producing high quality thereafter, then the distribution of submarkets to which this producer would have access to in the following n periods, after which the effect of this single deviation would disappear, would put a relatively bigger weight on submarkets with a larger number of fails and a smaller number of passes. It therefore follows that in order for producers to always produce high quality regardless of the submarket in which they happen to find themselves to be an equilibrium, it must be that such deviations are not profitable, or that

$$\begin{aligned}
& \delta \max_{h_n \in H_n, k \in \{1,\ldots,n\}} \left\{p^{h_n} - p^{(h_n:k\to F)}\right\} + \cdots + \delta^n \max_{h_n \in H_n, k \in \{1,\ldots,n\}} \left\{p^{h_n} - p^{(h_n:k\to F)}\right\} \\
\leq\ & \frac{\delta}{1-\delta} \max_{h_n \in H_n, k \in \{1,\ldots,n\}} \left\{p^{h_n} - p^{(h_n:k\to F)}\right\} \\
<\ & c,
\end{aligned}$$

where $(h_n : k \to F)$ denotes a vector that is identical to h_n except that it has a fail in the k-th place.

The next theorem shows that such deviations become less and less attractive as n increases.

Theorem 2. *The maximum difference*

$$\max_{h_n \in H_n, k \in \{1,\ldots,n\}} \left\{ p^{h_n} - p^{(h_n:k\to F)} \right\}$$

converges to zero as n increases.

Intuitively, the reason that the efficient market equilibrium becomes impossible to sustain as the length of memory increases is that as n increases, it becomes clearer whether any producer is competent or not. Prices in submarkets with histories that suggest that the producers there are competent converge to one, and prices in submarkets with histories that suggest that the producers there are incompetent converge to zero. It follows that as n increases the market makes increasingly similar inferences about the competence of producers whose record differs by only one failure. This implies that competent producers with a good record of passes may rest on their laurels and produce low quality without seriously damaging their reputations. This weakens the incentive to produce high quality in every period and undermines the efficient market equilibrium.

Remark 4. Speed of Convergence

Inspection of the proof of Theorem 2 reveals that both p^{h_n} and $p^{(h_n:k\to F)}$ converge to their limits exponentially fast in n. This implies that c_n^* decreases to zero exponentially fast with n.

Remark 5. Other Equilibria

The same intuition suggests that any equilibrium in which high quality is produced often would also be destabilized as the length of memory increases. However, we have not been able to formally establish such a result, and the question of what is the highest possible quality that can be sustained in a market equilibrium as n tends to infinity remains an open problem.

4.2. Endogenizing the Length of Memory, n

Many share the intuition that whatever happened in the distant past is of little relevance in the present. In the context of the model presented in Section 2, such an intuition implies that there is no need to consider the case where n is very large. In the context of this paper, an explanation of why such an intuition may be justified may proceed along the following lines: producers' abilities are subject to random shocks. It therefore follows that there is little reason to believe that there is any relationship between the types of a producer in periods t and t' if t and t' are very far from each other.

More formally, consider a model that is identical to the one presented in Section 2, except that each producer draws a new type (competent with probability $1-\eta$ and incompetent η) every k periods, where k is uniformly distributed over the set $\{1,...,K\}$ for some $K \geq 2$, independently across different producers and over each producer's personal history. Such an assumption implies:

1. that whatever happened more than K periods earlier is of no relevance for a producer's reputation, and

2. that what has happened k periods earlier is more relevant for a producer's reputation than what has happened $k+1$ periods earlier, for every $k \in \{1,...,K-1\}$. This observation is a consequence of the fact that whatever happened in the previous period is relevant for the reputation of all the producers who did not draw a new type in the current period; that whatever happened two periods earlier is relevant for the reputation of all the producers who did not draw a new type either in the current period or in the previous period; and so on.

The effect of this assumption on the prices in the different submarkets is that prices would put relatively more weight on what has happened in the recent past relative to what has happened in the more distant past. However, the properties of these prices, and the way they would respond to changes in the values of π^H and π^L remains qualitatively unchanged.[10] This implies, in particular, that our result about the negative effect of more precise information on incentives reported in Theorem 1 would holds in this case as well.

5. Discussion

Recently, testing and the general dissemination of the results of such testing have become very popular for students, teachers, caregivers, doctors, schools, nursing homes, and for other professions and for other institutions. The results reported in this study suggest that increased reliance on testing to improve incentives may fall short of expectations, and may even weaken incentives.

There are very few empirical studies of the benefits of testing. Jin and Leslie (2003) showed that a Los Angeles county requirement that restaurants post hygiene quality grade cards on their windows led to an increase in restaurants health inspection scores and to a decrease in the number of foodborne illness hospitalizations, which suggests that food quality has improved. Dranove et al. (2001) showed that doctors who are required to post their health care report cards tend to decline to treat more difficult, severely ill, patients. Consequently, health report cards may lead to a decrease in healthcare quality. Chipty

[10] Observe that the properties of prices that were used to prove Thereom 1 involve the convergence of prices to 1 and 0 as π^L tends to zero and as π^H tends to one, respectively, and the limit values of the derivatives of prices with respect to π^L and π^H, respectively.

(1995) exploited the cross-state variation in the choice of day-care regulations to identify the effect of regulation on the performance of the day-care market. She found that an increase in mandated annual inspections decreased equilibrium quality (as measured by staff/child ratios) for family day-care. Rosenthal (2004) has examined the effect of school inspections on the observed exam performance of the state secondary schools in the UK and concluded that inspection had a small but well-determined adverse effect on inspected schools. Finally, Clark and Tomlinson (2001) reported that the extent of monitoring does not seem to affect workers' effort levels based on employees' self-reported effort levels from the 1992 Survey of Employment in Britain. It thus appears that the evidence is consistent with the notion that the effect of improved inspection on outcomes is ambiguous.

We conclude with the following anecdotal evidence about the effect of safety regulation. In the U.S. the Occupational Safety and Health Administration (OSHA) requires employers to comply with a large number of regulations whose purpose is to ensure the safety and health of employees. OSHA routinely monitors violations of its regulations through surprise inspections, and fines those employers that are found to be violating its regulation. In order to relate our theory to the data provided by OSHA, suppose that an employer who has been found to violate OSHA regulation is perceived as riskier by employees, and holding every thing else fixed, it has to pay higher wages to its employees. That is, we assume that two employers with the same safety record would pay their employees different wages depending on whether they have been fined by OSHA or not.

Suppose that the probability of detection of a safety violation is increasing in the number of annual inspections, and that "average safety" can be measured by the average number of violations per inspection. We are interested in relationship between the number of inspections and the number of average violations per inspection over time. Is it the case that as the number of inspections increases, the number of average violations per inspection increases too, as would be the case if increasing the number of inspections weakens the incentives to maintain safety?

This question can be answered using the data provided by OSHA on its homepage. OSHA's executive summary of its 20th century enforcement data reports that the history of OSHA regulation can be roughly divided into five period.

> "The first period, from the formation of OSHA in 1971 to 1976, was characterized by rapid growth in staff, inspections, violations, and penalties. During the second period, from 1977 to 1980, the agency revised its enforcement program to focus on inspection quality rather than quantity, instituted a new complaint policy and revised the penalty assessment methodology. Total inspections dropped dramatically during this period. During the 1981-1985/1986 period OSHA focused its efforts on 'cooperation rather than confrontation.' The agency reduced its enforcement staff, implemented 'records-review-only' inspections, and shifted towards more construction industry and small establishment inspections. As a result, inspection numbers increased while violations and penalties decreased.

During the 1985/1986-1991 period, the agency instituted the egregious case policy, eliminated the 'records-review-only' inspections, and reemphasized quality inspections. These changes resulted in a large decline in inspections but a sharp increase in violations and penalties."

Because for the first four periods, from 1971 to 1991, the nature of OSHA inspections changed from one period to the next, the resulting changes in the numbers of violations per inspection cannot be interpreted as indicative of changes in average safety. However, the nature of inspections remained more or less the same over the 1990s. OSHA's report concludes by mentioning that "For the year 1992 through 2000, the number of inspections conducted by OSHA declined by about 14 percent and the number violations dropped by about 48 percent (compared with the year 1991)." Thus over the 1990s, both the number of inspections and the number of violations per inspection, which is inversely related to average safety, have decreased. Although the number of inspections has gone down, average safety seems to have improved, as would be the case if the precision of information about employers' safety records was already past the threshold beyond which any further improvement would hurt the incentives to improve safety.

Appendix

Proof of Lemma 1. In every period t, a competent producer with a history h_n produces high quality if and only if,

$$\delta\left(\pi^H U_{t+1}^{h_n P} + \left(1-\pi^H\right) U_{t+1}^{h_n F}\right) - c \geq \delta\left(\pi^L U_{t+1}^{h_n P} + \left(1-\pi^L\right) U_{t+1}^{h_n F}\right)$$

if and only if (2). ■

Proof of Theorem 1. Consider the profile of strategies where competent producers produce high quality after every history. By definition of $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$, this profile of strategies is an efficient market equilibrium provided the cost c is smaller than or equal to $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$. A version of the "one-stage-deviation principle" (Fudenberg and Tirole, 1991, pp. 108-110) implies that for $c > c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$ there exists a submarket where a competent producer would strictly prefer to produce low quality and continue to produce high quality thereafter. We rely on this fact to show that $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$ must be increasing in π^L and decreasing in π^H in the neighborhood of $\pi^L = 0$ and $\pi^H = 1$.

Let $F_{t+k}\left(h_n\right)\left[h_n'\right]$ denote the probability that a competent producer who always produces high quality and has history h_n at t (recall that h_n describes a producer's inspection record in periods $t-n$ to $t-1$) has access to submarket h_n' at $t+k$. Let $G_{t+k}\left(h_n\right)\left[h_n'\right]$ denote the probability that a competent producer who produces low quality at t but who produces high quality in every other period and has history h_n at t has access to submarket h_n' at $t+k$.

The fact that information about passing and failing inspection is forgotten after n periods implies that for every $h_n \in H_n$,

$$G_{t+k}\left(h_n\right)\left[h_n'\right] = F_{t+k}\left(h_n\right)\left[h_n'\right]$$

for every $k \geq n+1$ and $h_n' \in H_n$.

It therefore follows that in the efficient market equilibrium, if a competent producer were to produce low quality in some period t and then continue to produce high quality thereafter, then the only cost of doing so would be that in the next n periods, the likelihood of gaining access to "good" submarkets that require a large number of passes would be a little lower, and the likelihood of gaining access to relatively "bad" submarkets would be a little higher.

Suppose that a competent producer has access to submarket h_n in period t. The benefit from producing low quality at t and continuing to produce high quality thereafter is c. The cost is equal to

$$\sum_{k \in \{1,\dots,n\}} \sum_{h_n' \in H_n} \left(F_{t+k}(h_n)[h_n'] - G_{t+k}(h_n)[h_n']\right) \delta^k p^{h_n'}. \tag{$\bigstar$}$$

We show that the cost of deviation decreases as π^L tends to zero and as π^H tends to one. This implies that $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$ is decreasing in the neighborhood of $\pi^L = 0$ and $\pi^H = 1$.

If all competent producers always produce high quality regardless of the submarket in which they happen to find themselves in any given period, then the prices in every submarket remain constant, and do not change over time. We can therefore denote the price in submarket h_n in period t by p^{h_n}, independently of the period. If we let $h_n(P)$ denote the number of passes in the vector h_n, then Bayesian updating implies that

$$p^{h_n} = \frac{(1-\eta)\left(\pi^H\right)^{h_n(P)}\left(1-\pi^H\right)^{n-h_n(P)}}{(1-\eta)\left(\pi^H\right)^{h_n(P)}\left(1-\pi^H\right)^{n-h_n(P)} + \eta\left(\pi^L\right)^{h_n(P)}\left(1-\pi^L\right)^{n-h_n(P)}} \tag{A1}$$

for every submarket $h_n \in H_n$.

As π^L tends to zero, prices in all the submarkets tend to 1, except the price in the submarket $h_n = (F, F, \dots, F)$ (where $h_n(P) = 0$) which tends to $\frac{(1-\eta)\left(1-\pi^H\right)^n}{(1-\eta)\left(1-\pi^H\right)^n + \eta} < 1$. The rates of convergence in the limit as π^L tends to zero are as follows: for $h_n \in H_n$ that is such that $h_n(P) \geq 2$

$$\lim_{\pi^L \searrow 0} \frac{dp^{h_n}}{d\pi^L} = 0; \tag{A2}$$

for $h_n \in H_n$ that is such that $h_n(P) = 1$

$$\lim_{\pi^L \searrow 0} \frac{dp^{h_n}}{d\pi^L} = -\frac{\eta}{(1-\eta)\pi^H\left(1-\pi^H\right)^{n-1}}; \tag{A3}$$

and for $h_n = (F, F, \dots, F)$ (that is such that $h_n(P) = 0$)

$$\lim_{\pi^L \searrow 0} \frac{dp^{h_n}}{d\pi^L} = \frac{(1-\eta)\left(1-\pi^H\right)^n \eta n}{\left((1-\eta)\left(1-\pi^H\right)^n + \eta\right)^2} > 0. \tag{A4}$$

We show that in every submarket $h_n \in H_n$, the cost of a one-time deviation, ($\bigstar$), is decreasing as π^L decreases to zero, provided that π^H is sufficiently close to one. The argument that shows that the cost of a one-time deviation is decreasing as π^H increases to one, provided that π^L is sufficiently close to zero is similar, and for this reason, is not given here. Because if no competent producer prefers to produce low quality once and high quality thereafter regardless of the submarket to which it has access then the profile in which all competent producers always produce high quality is an efficient market equilibrium, the "one-stage-deviation principle" implies that $c_n^*\left(\pi^H,\pi^L,\eta,\delta\right)$ is equal to the lowest cost of deviation among all submarkets.

The cost of deviation ($\bigstar$) can be written as the following sum

$$
\begin{aligned}
(\bigstar) \quad = \quad & \sum_{k\in\{1,...,n\}} \sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} \left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right)\delta^k p^{h_n'} \\
& + \sum_{k\in\{1,...,n\}} \sum_{\{h_n'\in H_n : h_n'(P)=1\}} \left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right)\delta^k p^{h_n'} \\
& + \sum_{k\in\{1,...,n\}} \sum_{\{h_n'\in H_n : h_n'(P)=0\}} \left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right)\delta^k p^{h_n'}.
\end{aligned}
$$

The next three lemmas show that the derivative of ($\bigstar$) with respect to π^L is positive and arbitrarily large as π^L tends to zero, provided π^H is sufficiently close to one.

Lemma 2. *For every $h_n\in H_n$ and $k\in\{1,...,n\}$,*

$$
\lim_{\pi^L\searrow 0}\frac{d}{d\pi^L}\left[\sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} \left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right) p^{h_n'}\right] = 0. \tag{A5}
$$

Proof. By the chain rule, the left-hand-side of (A5) is equal to

$$
\lim_{\pi^L\searrow 0} \sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} \left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right)\frac{dp^{h_n'}}{d\pi^L} \tag{A6}
$$

$$
+\lim_{\pi^L\searrow 0} \sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} p^{h_n'}\frac{d}{d\pi^L}\left[\left(F_{t+k}\left(h_n\right)\left[h_n'\right] - G_{t+k}\left(h_n\right)\left[h_n'\right]\right)\right]. \tag{A7}
$$

We first show that (A6) is equal to zero. By (A2), $\lim\limits_{\pi^L\searrow 0}\frac{dp^{h_n'}}{d\pi^L}=0$ for every h_n' with $h_n'(P)\geq 2$. It therefore follows that both $\lim\limits_{\pi^L\searrow 0}\sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} F_{t+k}\left(h_n\right)\left[h_n'\right]\frac{dp^{h_n'}}{d\pi^L}$ and $\lim\limits_{\pi^L\searrow 0}\sum_{\{h_n'\in H_n : h_n'(P)\geq 2\}} G_{t+k}\left(h_n\right)\left[h_n'\right]\frac{dp^{h_n'}}{d\pi^L}$ are equal to zero, which implies that (A6) is equal to zero.

We now show that (A7) is equal to zero. Fix a vector $h_n\in H_n$ and a number $k\in\{1,...,n\}$. Let H_n^P denote the set of vectors with at least two passes whose first $n-k$ coordinates coincide with the last $n-k$ coordinates of h_n and who have a pass in the $n-k$-th place, and let H_n^F denote the set of vectors with at least two passes whose first $n-k$ coordinates coincide with the last $n-k$ coordinates of h_n and who have a fail in the $n-k$-th place. Let $h_n(P:k)$ denote the number of passes in the last k coordinates of h_n. For

$h'_n \in H_n^P$

$$
\begin{aligned}
& p^{h'_n} \frac{d}{d\pi^L} \left[\left(F_{t+k}\left(h_n\right)\left[h'_n\right] - G_{t+k}\left(h_n\right)\left[h'_n\right] \right) \right] && \text{(A8)} \\
= \;& p^{h'_n} \frac{d}{d\pi^L} \left[\left(\pi^H\right)^{h'_n(P:k)} \left(1-\pi^H\right)^{k-h'_n(P:k)} - \pi^L \left(\pi^H\right)^{h'_n(P:k)-1} \left(1-\pi^H\right)^{k-h'_n(P:k)} \right] \\
= \;& -p^{h'_n} \left(\pi^H\right)^{h'_n(P:k)-1} \left(1-\pi^H\right)^{k+1-h'_n(P:k)};
\end{aligned}
$$

and for $h_n \in H_n^F$

$$
\begin{aligned}
& p^{h'_n} \frac{d}{d\pi^L} \left[\left(F_{t+k}\left(h_n\right)\left[h'_n\right] - G_{t+k}\left(h_n\right)\left[h'_n\right] \right) \right] && \text{(A9)} \\
= \;& p^{h'_n} \frac{d}{d\pi^L} \left[\left(\pi^H\right)^{h'_n(P:k)} \left(1-\pi^H\right)^{k-h'_n(P:k)} - \left(1-\pi^L\right) \left(\pi^H\right)^{h'_n(P:k)} \left(1-\pi^H\right)^{k-1-h'_n(P:k)} \right] \\
= \;& p^{h'_n} \left(\pi^H\right)^{h'_n(P:k)} \left(1-\pi^H\right)^{k-1-h'_n(P:k)}.
\end{aligned}
$$

(A8) and (A9) imply that (A7) is equal to

$$
\begin{aligned}
& -\lim_{\pi^L \searrow 0} \sum_{h_n \in H_n^P} p^{h'_n} \left(\pi^H\right)^{h'_n(P:k)-1} \left(1-\pi^H\right)^{k-h'_n(P:k)} \\
& +\lim_{\pi^L \searrow 0} \sum_{h_n \in H_n^F} p^{h'_n} \left(\pi^H\right)^{h'_n(P:k)} \left(1-\pi^H\right)^{k-1-h'_n(P:k)} \\
= \;& -\sum_{l=1}^{k} \binom{k}{l-1} \left(\pi^H\right)^{l-1} \left(1-\pi^H\right)^{k-l} + \sum_{l=0}^{k-1} \binom{k}{l} \left(\pi^H\right)^{l} \left(1-\pi^H\right)^{k-1-l} \\
= \;& 0
\end{aligned}
$$

where the first equality follows from the fact that $\lim_{\pi^L \searrow 0} p^{h_n} = 1$ for every h_n with $h_n\left(P\right) \geq 2$, and the second equality follows from the fact that $\sum_{l=1}^{k} \binom{k}{l-1} \left(\pi^H\right)^{l-1} \left(1-\pi^H\right)^{k+1-l} = \sum_{l=0}^{k-1} \binom{k}{l} \left(\pi^H\right)^{l} \left(1-\pi^H\right)^{k-1-l}$. ■

Lemma 3. *For every $h_n \in H_n$ and $k \in \{1, ..., n\}$ for which $\sum_{\{h'_n \in H_n : h'_n(P)=1\}} \left(F_{t+k}\left(h_n\right)\left[h'_n\right] - G_{t+k}\left(h_n\right)\left[h'_n\right]\right) p^{h'_n}$ is positive,*

$$
\lim_{\pi^L \searrow 0} \frac{d}{d\pi^L} \left[\sum_{\{h'_n \in H_n : h'_n(P)=1\}} \left(F_{t+k}\left(h_n\right)\left[h'_n\right] - G_{t+k}\left(h_n\right)\left[h'_n\right]\right) p^{h'_n} \right] \tag{A10}
$$

tends to infinity as π^H increases to 1.

Proof. By the chain rule, (A10) is equal to

$$
\lim_{\pi^L \searrow 0} \sum_{\{h'_n \in H_n : h'_n(P)=1\}} p^{h'_n} \frac{d}{d\pi^L} \left[\left(F_{t+k}\left(h_n\right)\left[h'_n\right] - G_{t+k}\left(h_n\right)\left[h'_n\right] \right) \right] \tag{A11}
$$

$$+\lim_{\pi^L \searrow 0} \sum_{\{h'_n \in H_n : h'_n(P)=1\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right) \frac{dp^{h'_n}}{d\pi^L}. \tag{A12}$$

We first show that (A11) is equal to zero. By (A1), $p^{h'_n}$ has the same value for every h'_n that is such that $h'_n(P) = 1$, and can be therefore placed before the summation sign in (A11). Observe that

$$\begin{aligned} \sum_{\{h'_n \in H_n : h'_n(P)=1\}} \left[\left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right)\right] &= \binom{k}{1} \pi^H \left(1-\pi^H\right)^{k-1} \\ &\quad - \pi^L \left(1-\pi^H\right)^{k-1} \\ &\quad - \binom{k-1}{1} \pi^H \left(1-\pi^L\right)\left(1-\pi^H\right)^{k-2} \\ &= \left(1-\pi^H\right)^{k-2}\left(\pi^H - \pi^L + k\pi^H\left(\pi^L - \pi^H\right)\right), \end{aligned} \tag{A13}$$

from which it follows that

$$\sum_{\{h'_n \in H_n : h'_n(P)=1\}} \frac{d}{d\pi^L}\left[\left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right)\right] = \left(k\pi^H - 1\right)\left(1-\pi^H\right)^{k-2}.$$

Finally, because $\lim_{\pi^L \searrow 0} p^{h'_n} = 1$ for h'_n with $h'_n(P) = 1$, it follows that (A11) tends to zero as π^H increases to 1.

We now show that (A12) tends to infinity as π^H increases to 1. Because, by (A3) $\lim_{\pi^L \searrow 0} \frac{dp^{h'_n}}{d\pi^L} = -\frac{\eta}{(1-\eta)\pi^H(1-\pi^H)^{n-1}}$ for every $h'_n \in H_n$ that is such that $h'_n(P) = 1$, (A12) is equal to

$$-\frac{\eta}{(1-\eta)\pi^H(1-\pi^H)^{n-1}} \lim_{\pi^L \searrow 0} \sum_{\{h'_n \in H_n : h'_n(P)=1\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right),$$

which, by (A13), is equal to

$$\frac{\eta\left(k\pi^H - 1\right)}{(1-\eta)\left(1-\pi^H\right)^{n+1-k}}.$$

The last term tends to infinity as π^H increases to 1. ■

Lemma 4. *For every $h_n \in H_n$ and $k \in \{1, ..., n\}$ for which $\sum_{\{h'_n \in H_n : h'_n(P)=0\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right) p^{h'_n}$ is positive,*

$$\lim_{\pi^L \searrow 0} \frac{d}{d\pi^L}\left[\sum_{\{h'_n \in H_n : h'_n(P)=0\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right) p^{h'_n}\right]$$

tends to zero as π^H increases to 1.

Proof. Observe that

$$\sum_{\{h'_n \in H_n : h'_n(P)=0\}} F_{t+k}(h_n)[h'_n] = \left(1-\pi^H\right)^k,$$

and

$$\sum_{\{h'_n \in H_n : h'_n(P)=0\}} G_{t+k}(h_n)[h'_n] = \left(1-\pi^L\right)\left(1-\pi^H\right)^{k-1}.$$

Consequently,

$$\sum_{\{h'_n \in H_n : h'_n(P)=0\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right) = \left(1-\pi^H\right)^{k-1}\left(\pi^L - \pi^H\right).$$

Because by (A1), $p^{(F,\ldots,F)} = \frac{(1-\eta)\left(1-\pi^H\right)^n}{(1-\eta)(1-\pi^H)^n + \eta(1-\pi^L)^n}$,

$$\begin{aligned} & \frac{d}{d\pi^L}\left[\sum_{\{h'_n \in H_n : h'_n(P)=0\}} \left(F_{t+k}(h_n)[h'_n] - G_{t+k}(h_n)[h'_n]\right) p^{h'_n}\right] \\ = {} & \frac{(1-\eta)\left(1-\pi^H\right)^{n+k-1}}{(1-\eta)\left(1-\pi^H\right)^n + \eta\left(1-\pi^L\right)^n} + \left(1-\pi^H\right)^{k-1}\left(\pi^L - \pi^H\right)\frac{dp^{(F,\ldots,F)}}{d\pi^L}. \qquad \text{(A14)} \end{aligned}$$

(A4) implies that the limit of (A14) as π^L tends to zero is equal to

$$\frac{(1-\eta)\left(1-\pi^H\right)^{n+k-1}}{(1-\eta)\left(1-\pi^H\right)^n + \eta} - \frac{(1-\eta)\eta n \pi^H \left(1-\pi^H\right)^{n+k-1}}{\left((1-\eta)\left(1-\pi^H\right)^n + \eta\right)^2},$$

which tends to zero as π^H increases to 1. ■

Finally, the fact that, as we have shown, $c_n^*\left(\pi^H, \pi^L, \eta, \delta\right)$ is increasing in π^L and decreasing in π^H in the neighborhood of $\pi^L = 0$ and $\pi^H = 1$ implies that there exists threshold values of π^L and π^H such that any improvement in the precision of information beyond this threshold would undermine incentives to produce high quality. This completes the proof of Theorem 1. ■

Proof of Theorem 2. Rewrite p^{h_n} from (10) as

$$p^{h_n} = \frac{1}{1 + \frac{\eta}{(1-\eta)}\left(\frac{\pi^L}{\pi^H}\right)^{h_n(P)}\left(\frac{1-\pi^L}{1-\pi^H}\right)^{n-h_n(P)}}.$$

Suppose that $h_n(P) \in \{1, \ldots, n\}$ is such that $\frac{h_n(P)}{n}$ converges to some constant $\kappa \in [0,1]$ as n tends to infinity. Observe that

$$\begin{aligned} \lim_{n \nearrow \infty}\left(\frac{\pi^L}{\pi^H}\right)^{h_n(P)}\left(\frac{1-\pi^L}{1-\pi^H}\right)^{n-h_n(P)} &= \lim_{n \nearrow \infty}\left(\left(\frac{\pi^L}{\pi^H}\right)^{\kappa}\left(\frac{1-\pi^L}{1-\pi^H}\right)^{1-\kappa}\right)^n \\ &= \begin{cases} \infty & \text{if } \kappa < \kappa^* \\ 1 & \text{if } \kappa = \kappa^* \\ 0 & \text{if } \kappa > \kappa^* \end{cases} \end{aligned}$$

where $\kappa^* \equiv \frac{\log\left(\frac{1-\pi^L}{1-\pi^H}\right)}{\log\left(\frac{1-\pi^L}{1-\pi^H}\right)-\log\left(\frac{\pi^L}{\pi^H}\right)}$ is such that $\left(\frac{\pi^L}{\pi^H}\right)^{\kappa^*}\left(\frac{1-\pi^L}{1-\pi^H}\right)^{1-\kappa^*}=1$. It therefore follows that

$$\lim_{n\nearrow\infty} p^{h_n} = \begin{cases} 0 & \text{if } \lim_{n\nearrow\infty} \frac{h_n(P)}{n} < \kappa^* \\ 1-\eta & \text{if } \lim_{n\nearrow\infty} \frac{h_n(P)}{n} = \kappa^* \\ 1 & \text{if } \lim_{n\nearrow\infty} \frac{h_n(P)}{n} > \kappa^* \end{cases}$$

Since the number of passes in the vector $(h_n : k \to F)$, denoted $(h_n : k \to F)(P)$, is between $h_n(P) - 1$ and $h_n(P)$, depending on whether h_n has a pass or fail in the k-th place, $\lim_{n\nearrow\infty} \frac{h_n(P)}{n} = \kappa$ if and only if $\lim_{n\nearrow\infty} \frac{(h_n:k\to F)(P)}{n} = \kappa$, and so

$$\lim_{n\nearrow\infty} p^{(h_n:k\to F)} = \lim_{n\nearrow\infty} p^{h_n}.$$

■

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