Working Paper
Series

FINRISK
National Centre of Competence in Research
Financial Valuation and Risk Management

National Centre of Competence in Research
Financial Valuation and Risk Management

Working Paper No. 301

The Quality of Public Information and the Term Structure of Interest Rates

Frederik Lundtofte

First version: February 2006
Current version: March 2007

This research has been carried out within the NCCR FINRISK project on "New Methods in Theoretical and Empirical Asset Pricing"

The Quality of Public Information and The Term Structure of Interest Rates

Frederik Lundtofte
First version: February, 2006
This version: March, 2007

Swiss Institute of Banking and Finance — University of St. Gallen, Rosenbergstrasse 52, CH-9000 St. Gallen, Switzerland. Phone: +41 71 224 7005. Fax: +41 71 224 7088. E-mail: frederik.lundtofte@unisg.ch

Abstract

This paper analyzes the term structure of interest rates in an exchange-only Lucas (1978) economy where consumers learn about a stochastic growth rate through observations of the endowment process and an external public signal. We allow for deluded consumers, who exaggerate the degree of covariation between the external public signal and the growth rate. With such consumers, there can be a premium for noisy external public information in long-term bonds and the social value of more precise public information can be negative. In contrast to Feldman (1989), we find that nonstochastic interest rates are not necessary for the expectations hypothesis to hold.

Keywords: learning, overconfidence, information quality, incomplete information, term structure of interest rates
JEL Classification Codes: C13, G12

I thank David Feldman, Paul Söderlind, and Fabio Trojani for helpful discussions, and the participants at the meeting of the Northern Finance Association in Montréal 2006, the VIII Workshop on Quantitative Finance in Venice 2007 and the SSES meeting in St. Gallen 2007. Financial support from the Swiss National Science Foundation (NCCR FINRISK and grant 100012-105745/1) is gratefully acknowledged.

1 Introduction

It is evident that consumers base their expectations on the information that they receive about news events, macroeconomic indicators, and corporations' earnings reports. Central banks, corporations and government agencies on their part face a number of issues concerning how much information they should disclose, in what form, and how often. Since there is a trade-off between timely but noisy information and slow but more accurate information, they need to strike the right balance between timely and frequent information and the accuracy of that information and guard against the potential damage caused by noise.

In October 2002, Goldman Sachs and Deutsche Bank jointly launched an auction-based market for so-called economic derivatives, i.e., derivatives on scheduled macroeconomic announcements. Initially, they covered non-farm payrolls, the Institute of Supply Management manufacturing report, and US retail sales ex-autos, and later, contracts on other releases, such as the US gross domestic product, the US international trade balance and the US consumer price index were introduced. Beber and Brandt (2006) measure market participants' ex-ante uncertainty regarding news releases using price data on some of these derivatives[1] and they then relate this measure to changes in implied volatilities of stock and bond options around macroeconomic announcements. They find that, for bonds, the relation between macroeconomic uncertainty and changes in implied volatility is statistically and economically highly significant, whereas the results for the aggregate stock index are considerably weaker, largely due to non-cyclical stocks not responding to macroeconomic news.

In this paper, we analyze the impact of public-information quality on the term

[1] Beber and Brandt (2006) focus on derivatives based on the non-farm payroll.

structure of interest rates. For this purpose, we employ an exchange-only Lucas (1978) economy, in which consumers learn about a stochastic growth rate through realizations of the endowment process and an external public signal. Consumers may be rational or they may be "deluded" in the sense that they exaggerate the degree of covariation between the external public signal and the unknown growth rate.

We find that, with rational consumers, the steady-state term premium reacts positively to an increasing precision in the external public signal, whereas, with deluded consumers, there can be a premium for noisy external public information. Therefore, we relate our paper to the issue of whether or not more precise public information increases social welfare. Hirshleifer (1971) shows that the social value of public information may not be positive. More recently, and in a different setting, Morris and Shin (2002) find results that are similar in spirit to those of Hirshleifer. Morris and Shin's results have gained some attention by the media (Economist, 2004) and spurred academic debate (Svensson, forthcoming; and Morris, Shin and Tong, forthcoming). In our model, we find that, with parameter values in line with US data, and assuming rational consumers, the social value of more precise public information is indeed positive. However, with deluded consumers, the social value of more precise public information can be negative.

In contrast to Feldman (1989), where consumers only base their estimates on output realizations, we find that, in the presence of external public information, nonstochastic interest rates are not necessary for the expectations hypothesis to hold. In fact, with rational consumers, the short interest rate must be stochastic on all but the initial date.

Moreover, in the steady state, the instantaneous variance of the short interest rate need not be decreasing in the precision of public information. If there is a positive instantaneous correlation between the endowment process and the growth rate, then bonds will hedge against a low future consumption and therefore, the representative consumer will accept a negative term premium. A positive instantaneous correlation between the endowment process and the growth rate is also a sufficient condition for the term premium to be decreasing in the term to maturity.

We show that the term structure is bounded and that, in general, it depends on calendar time through the estimation error. A sufficient condition for the long-run yield to maturity to be lower than the long-run short interest rate is a positive instantaneous correlation between the endowment process and the unobserved growth rate.

This paper is related to the literature on learning in financial markets. Notable works within this field include Williams (1977), Dothan and Feldman (1986), Detemple (1986), Feldman (1989), Feldman (1992), Brennan (1998), Veronesi (2000), Veronesi and Yared (2000), Riedel (2000), Yan (2001), Brennan and Xia (2001a,b), Xia (2001), Scheinkman and Xiong (2003), Feldman (2003), Dumas, Kurshev and Uppal (2004) and Li (2005). However, none of these contributions specifically address the topic of this paper: How does the quality of external public information affect the term structure of interest rates?

Recently, affine multifactor setups have been the interest of several empirical studies of the term structure (notable works are Dai and Singleton, 2000; de Jong, 2000; and Duffee, 2002). A theoretical framework for such specifications, which includes the term structure of Cox, Ingersoll and Ross (1985b) as a special case, is given in Duffie and Kan (1996). The incomplete information literature on the term structure (Dothan and Feldman, 1986; Feldman, 1989) adds realism to the general equilibrium model of Cox, Ingersoll and Ross (1985b) by allowing for an unobservable growth rate. In Dothan and Feldman (1986) and Feldman (1989), agents only learn through realized outputs. This paper contributes to this literature by analyzing how the quality of external public information affects the term structure of interest rates in a general equilibrium model in which consumers may be rational or deluded. In our model, consumers learn both through realized outputs and through external public information. We model external public information as a signal about the unobservable growth rate as in Veronesi's (2000) regime-shifting model of the stock market. This signal is a proxy for the many sources of public information mentioned above. News events, corporations' earnings reports, and macroeconomic indicators are all observable and therefore, it should be possible to test our model empirically.

The paper is organized as follows. In section 2, we introduce the economy, and in section 3, we present the theoretical results. In section 4, we conclude the paper. All proofs are in the Appendix.

2 The Economy

We consider a Lucas (1978) exchange economy with an exogenous aggregate endowment process, D_t. The consumption good is perishable; therefore, in each period, the entire endowment is consumed. The flow of aggregate endowments follows the process

$$\frac{\mathrm{d}D_t}{D_t} = \mu_t \mathrm{d}t + \sigma_D \mathrm{d}Z_t^D, \tag{1}$$

where the endowment growth rate (μ_t) is stochastic and evolves according to a mean-reverting Ornstein-Uhlenbeck process,

$$\mathrm{d}\mu_t = \kappa(\overline{\mu} - \mu_t)\mathrm{d}t + \sigma_\mu \mathrm{d}Z_t^\mu, \tag{2}$$

with σ_D, κ, and σ_μ being positive constants, and where Z^D and Z^μ are standard Brownian motions defined on a complete probability space, $(\Omega, F, \wp)$. Z^D and Z^μ have a constant instantaneous coefficient of correlation of $\rho_{\mu D}$, where $-1 \leq \rho_{\mu D} \leq 1$. We denote the instantaneous covariance between Z^D and Z^μ by $\sigma_{\mu D} \equiv \rho_{\mu D}\sigma_\mu\sigma_D$. The consumers do not observe the growth rate of the endowment process (μ_t). Instead, they have to estimate it from the realized values of the endowment process and a public signal (s). The public signal that they receive follows a diffusion process,

$$\mathrm{d}s_t = \mu_t \mathrm{d}t + \sigma_s \mathrm{d}Z_t^s, \tag{3}$$

where σ_s is a positive constant, and Z^s is a standard Brownian motion defined on the complete probability space, $(\Omega, F, \wp)$. Z^s is independent of both Z^D and Z^μ. However, the consumers steadfastly believe that the instantaneous correlation between Z_t^s and Z_t^μ is ϕ.[2] That is, they believe that the instantaneous correlation between the public signal and the growth rate is ϕ. Hence, if $\phi \neq 0$, then the consumers are "deluded" in the sense that they exaggerate the degree of covariation between the external public signal and the growth rate.[3] The case in which $\phi = 0$ corresponds to rational consumers.

In order to convert their original non-Markovian problem into a Markovian problem, the consumers need to determine the conditional distribution of the growth rate (Feldman, forthcoming). However, the consumers think that the signal has an instantaneous correlation of ϕ with the growth rate, when in reality, this correlation is zero. Given this belief, they determine the conditional distribution of the growth rate, and use as policies the optimal policies from the Markovian problem.

All consumers maximize expected life-time utility of intermediate consumption through a CRRA utility function, subject to a wealth constraint. The instantaneous utility of intermediate consumption is of the form

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \tag{4}$$

where $\gamma > 0$.

All consumers have identical preferences, information, and beliefs. Thus, the aggregation results from Rubinstein (1974) hold, and we can use a representative consumer framework, where this consumer has constant relative risk aversion and at time t, he maximizes expected utility of consumption conditional on the current

[2] Note that the instantaneous correlation matrix associated with the vector process (Z_t^D, Z_t^μ, Z_t^s) is valid (i.e., positive semidefinite) if and only if $\rho_{\mu D}^2 \leq 1$, $\phi^2 \leq 1$, and $\rho_{\mu D}^2 + \phi^2 \leq 1$. We assume that consumers only assign values to ϕ such that these conditions are fulfilled.

[3] This is similar to the way Scheinkman and Xiong (2003) and Dumas, Kurshev and Uppal (2004) model overconfidence.

values of the state variables,[4]

$$E_{W_t,D_t,m_t,t}\left\{\int\limits_{w=t}^{\tau} e^{-\beta(w-t)}\left(\frac{c_w^{1-\gamma}-1}{1-\gamma}\right)\mathrm{d}w\right\}. \tag{5}$$

Since this is a Lucas (1978) exchange economy with a perishable consumption good, aggregate consumption will equal the aggregate endowment in each period. That is, $c_w = D_w$ for all w.

In the early literature—for example, Detemple (1986), Dothan and Feldman (1986), and Feldman (1989)—the framework is often that of Cox, Ingersoll and Ross (1985a,b). We have chosen the Lucas (1978) framework, for its tractability. Primarily, the Lucas (1978) framework also enables us to solve for the term structure of interest rates under nonlogarithmic preferences. In contrast, the above-mentioned articles based on the Cox, Ingersoll and Ross (1985a,b) framework specialize to logarithmic preferences.

3 Theoretical Results

In this section, we present the theoretical results. First, we derive the evolution of the consumers' estimate of the unobserved growth rate, and thereafter, we analyze the interest rates' equilibrium term structures. Since we find that there can be a premium for noisy public information in bonds, we also analyze the social value of more precise public information.

[4]In the Markovian problem, the relevant state variables are: current wealth (W_t), the current value of the endowment (D_t), and the conditional mean of the growth rate according to his filtering (m_t).

3.1 Consumers' estimate of the growth rate

As follows from the preceding discussion, consumers will have to estimate the unobserved growth rate in the endowment growth equation, basing their estimates on the endowments' realized values and the public signal. The consumers need to determine the unknown growth rate's conditional distribution in order to convert their original optimization problem to a Markovian one. They condition on their observations on the endowments and the public signal, i.e., their information set is given by $G_t = \sigma((D_u, s_u); u \leq t)$. The deluded ($\phi \neq 0$) consumers perform their filtering under the impression that there is a nonzero correlation between the public signal and the growth rate, although the actual value is zero.

Assuming a Gaussian prior, finding the posterior distribution of the growth rate becomes a standard filtering problem, which fits into the Kalman-Bucy framework. Here, we will derive the evolution of the conditional mean of the unknown growth rate μ_t. Applying Theorem 12.7 in Liptser and Shiryaev (2001), we can find the stochastic differential equations (SDEs) of the conditional mean $m_t = E\left[\mu_t | G_t\right]$ and the conditional variance $v_t = E\left[(\mu_t - m_t)^2 \middle| G_t\right]$ of μ_t. v_t is sometimes called the "estimation error" or the "filtering error," since it measures the conditional mean squared error. The conditional mean—that is, the expected value of the unknown growth rate conditional on all available information—can be interpreted as the consumers' estimate of the growth rate.

Proposition 1 *If consumers' prior distribution over μ_0 is Gaussian with mean m_0 and variance v_0, and the instantaneous correlation between the external public signal* *and the growth rate is ϕ, then the conditional mean $m_t = E\left[\mu_t | G_t\right]$ satisfies*

$$\mathrm{d}m_t = \kappa(\overline{\mu} - m_t)\mathrm{d}t + \left(\frac{v_t + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D^2}, \frac{v_t + \phi\sigma_\mu\sigma_s}{\sigma_s^2}\right)\left(\begin{pmatrix} \frac{\mathrm{d}D_t}{D_t} \\ \mathrm{d}s_t \end{pmatrix} - m_t \begin{pmatrix} 1 \\ 1 \end{pmatrix}\mathrm{d}t\right), \tag{6}$$

and the conditional variance $v_t = E\left[(\mu_t - m_t)^2 \middle| G_t\right]$ of μ_t satisfies the Riccati equation

$$\frac{\mathrm{d}v_t}{\mathrm{d}t} = -2\kappa v_t + \sigma_\mu^2 - \frac{(v_t + \rho_{\mu D}\sigma_\mu\sigma_D)^2}{\sigma_D^2} - \frac{(v_t + \phi\sigma_\mu\sigma_s)^2}{\sigma_s^2}. \tag{7}$$

Furthermore, the posterior distribution of μ_t is also Gaussian, with $\mu_t | G_t \sim N(m_t, v_t)$.

Proof. See Theorem 12.7 in Liptser and Shiryaev (2001). ■

We can rewrite equations (1) and (3) as

$$\begin{pmatrix} \frac{\mathrm{d}D_t}{D_t} \\ \mathrm{d}s_t \end{pmatrix} = E\left[\mu_t | G_t\right]\begin{pmatrix} 1 \\ 1 \end{pmatrix}\mathrm{d}t + \begin{pmatrix} \sigma_D & 0 \\ 0 & \sigma_s \end{pmatrix}\begin{pmatrix} \mathrm{d}\widehat{Z}_t^D \\ \mathrm{d}\widehat{Z}_t^s \end{pmatrix}, \tag{8}$$

where

$$\begin{pmatrix} \mathrm{d}\widehat{Z}_t^D \\ \mathrm{d}\widehat{Z}_t^s \end{pmatrix} = \begin{pmatrix} \sigma_D & 0 \\ 0 & \sigma_s \end{pmatrix}^{-1}\left(\begin{pmatrix} \frac{\mathrm{d}D_t}{D_t} \\ \mathrm{d}s_t \end{pmatrix} - E\left[\mu_t | G_t\right]\begin{pmatrix} 1 \\ 1 \end{pmatrix}\mathrm{d}t\right).$$

It follows from standard filtering theory (Liptser and Shiryaev, 2001) that $\widehat{Z}_t^D$ and $\widehat{Z}_t^s$ are independent standard Brownian motions with respect to the consumers' filtration, $G_t = \sigma\left((D_u, s_u); u \leq t\right)$. Since the consumers believe that the correlation between the public signal and the growth rate is ϕ, they think that the dynamics of the conditional mean is given by

$$\mathrm{d}m_t = \kappa(\overline{\mu} - m_t)\mathrm{d}t + \left(\frac{v_t + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_t^D + \left(\frac{v_t + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_t^s, \tag{9}$$

where the estimation error, v_t, is the solution to the Riccati equation in (7). In the proposition below, we demonstrate some important characteristics of the estimation error:

Proposition 2 *The estimation error v_t proceeds monotonically from its initial value v_0 toward an asymptotic, nonnegative, stable, steady-state value, v_+, following the relation*

$$v_t = v_- \frac{\frac{v_+}{v_-} - \frac{v_0 - v_+}{v_0 - v_-} \exp\left\{-2\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\varphi t\right\}}{1 - \frac{v_0 - v_+}{v_0 - v_-} \exp\left\{-2\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\varphi t\right\}}, \tag{10}$$

where

$$v_+ = \frac{\sqrt{\left(\kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}\right)^2 + \sigma_\mu^2\left(1 - \phi^2 - \rho_{\mu D}^2\right)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)} - \left(\kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}\right)}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}}, \tag{11}$$

$$v_- = \frac{-\sqrt{\left(\kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}\right)^2 + \sigma_\mu^2\left(1 - \phi^2 - \rho_{\mu D}^2\right)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)} - \left(\kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}\right)}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}}, \tag{12}$$

and

$$\varphi = \frac{\sqrt{\left(\kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}\right)^2 + \sigma_\mu^2\left(1 - \phi^2 - \rho_{\mu D}^2\right)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)}}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}}. \tag{13}$$

Note that, according to the proposition above, the estimation error (v_t) is a deterministic function of time.

The following proposition states an important property of the stable steady-state value of the estimation error:

Proposition 3 *The stable steady-state value, v_+, of the estimation error is equal to zero if and only if the squared correlations sum to one ($\rho_{\mu D}^2 + \phi^2 = 1$) and $\kappa\sigma_s\sigma_D + \phi\sigma_\mu\sigma_D + \rho_{\mu D}\sigma_\mu\sigma_s \geq 0$.*

The proof is straightforward and is therefore omitted.

Further, we have the following:

Proposition 4 *If $\phi > 0$, then the steady-state estimation error is decreasing in the degree of delusion (ϕ),*

$$\frac{\partial v_+}{\partial \phi} \leq 0.$$

If consumers are deluded, then the steady-state estimation error, v_+, can be increasing with increasing precision of external public information.[5] However, if consumers are rational, then the steady-state estimation error is strictly decreasing in the precision of public information, except for the case when the endowment process is perfectly correlated with the unobserved growth rate and a specific technical condition applies, so that the steady-state estimation error is zero. Note also that the rate of convergence toward the stable steady-state value depends on the precision of external public information.

Proposition 5 *If $\phi = 0$, then the steady-state variance, v_+, is decreasing with increasing precision of external public information. That is,*

$$\frac{\partial v_+}{\partial \sigma_s} \geq 0,$$

with equality if and only if $\rho_{\mu D}^2 = 1$ and $\kappa \sigma_D^2 + \sigma_{\mu D} \geq 0$.

3.2 Equilibrium

In this section, we will analyze the equilibrium term structure of interest rates. We assume that the financial market consists of i) a short-term bond (bank account) yielding a short-term interest rate of r_t; ii) a long-term bond; and iii) a claim to the

[5] One set of parameters that yields an estimation error that is increasing with increasing precision of external public information is the following: $\phi = -0.2$, $\kappa = 0.2$, $\rho_{\mu D} = 0.8$, $\sigma_\mu = 0.015$, $\sigma_D = 0.13$, $\sigma_s = 0.10$.

aggregate stream of endowments (a stock). Consumers solve the Markovian problem, i.e., they maximize the conditional expected utility in (5) by choosing optimal investment and consumption policies under the impression that the dynamics of the state variables is given by the following equations:

$$dW_t = W_t\alpha_t^S\left(\frac{\mathrm{d}S_t + D_t\mathrm{d}t}{S_t}\right) + W_t\alpha_t^B\frac{dP_t}{P_t} + W_t(1-\alpha_t^S-\alpha_t^B)r_t\mathrm{d}t - c_t\mathrm{d}t \qquad (14)$$

$$\frac{dD_t}{D_t} = m_t\mathrm{d}t + \sigma_D\mathrm{d}\widehat{Z}_t^D \qquad (15)$$

$$\mathrm{d}m_t = \kappa(\overline{\mu}-m_t)\mathrm{d}t + \left(\frac{v_t+\rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_t^D + \left(\frac{v_t+\phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_t^s, \qquad (16)$$

where W_t denotes wealth, α_t^S is the share of wealth invested in the stock, S_t is the endogenously determined price of a stock that pays out the aggregate endowments as dividends, α_t^B is the share of wealth invested in a long-term bond, P_t is the endogenously determined bond price, r_t is the endogenously determined interest rate, c_t denotes consumption, and $\widehat{Z}_t^D$ and $\widehat{Z}_t^s$ are independent Brownian motions. This is a standard Markovian problem. In order to determine the equilibrium term structure, we employ the stochastic discount factor approach (see the note in the Appendix).

Since, in each period, the representative consumer consumes the entire endowment ($c_w = D_w$), the stochastic discount factor is given by $\Lambda_t = e^{-\beta t}D_t^{-\gamma}$. Applying Ito's lemma, we can obtain the dynamics of the stochastic discount factor as

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = (-\beta - \gamma m_t + \frac{1}{2}\gamma(\gamma+1)\sigma_D^2)\mathrm{d}t - \gamma\sigma_D\mathrm{d}\widehat{Z}_t^D. \qquad (17)$$

From equation (58) in the Appendix, we can determine the interest rate endogenously as

$$r_t = \beta + \gamma m_t - \frac{1}{2}\gamma(\gamma+1)\sigma_D^2. \tag{18}$$

Hence, the short-term interest rate follows a Vasicek-type process,

$$\mathrm{d}r_t = \kappa(\overline{r} - r_t)\mathrm{d}t + \gamma\left(\frac{v_t + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_t^D + \gamma\left(\frac{v_t + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_t^s, \tag{19}$$

where $\overline{r} = \beta + \gamma\overline{\mu} - \frac{1}{2}\gamma(\gamma+1)\sigma_D^2$ is the long-run interest rate. The instantaneous variance of the short-term interest rate is

$$\sigma_r^2(v_t) = \gamma^2\left(\left(\frac{v_t + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)^2 + \left(\frac{v_t + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)^2\right). \tag{20}$$

In the proposition below, we summarize some of the properties of the instantaneous variance of the interest rate, $\sigma_r^2(v_t)$. Unlike Feldman (1989), who analyzes the term structure when consumers use only output information in their estimation, we find that the minimum variance of the interest rate can be different from zero because there are two sources behind the variation in interest rates: unanticipated shocks in the endowment process and unanticipated shocks in the public signal. In fact, we show that, with rational consumers, the instantaneous variance of the interest rate must be strictly positive for all $t > 0$.

Proposition 6 *The instantaneous variance of the interest rate, $\sigma_r^2(v_t)$, exhibits the following properties.*

(a) It is a quadratic convex function of the estimation error, v_t.

(b) It only depends on time through its dependence on the estimation error.

(c) It is a nondecreasing function of the estimation error if $\rho_{\mu D} \geq 0$ and $\phi \geq 0$.

(d) It is a decreasing–increasing function of the estimation error if $\rho_{\mu D} < 0$ and $\phi < 0$.

(e) If $\rho_{\mu D} < 0$ and $\phi < 0$, the instantaneous variance of the interest rate attains its minimum at $v_t = -\sigma_\mu \sigma_D \sigma_s (\rho_{\mu D}\sigma_s + \phi\sigma_D)/(\sigma_D^2 + \sigma_s^2)$, and the value of this minimum variance is

$$\sigma_r^2\left(-\frac{\sigma_\mu \sigma_D \sigma_s (\rho_{\mu D}\sigma_s + \phi\sigma_D)}{\sigma_D^2 + \sigma_s^2}\right) = \frac{\gamma^2 \sigma_\mu^2 (\rho_{\mu D}\sigma_D - \phi\sigma_s)^2}{\sigma_D^2 + \sigma_s^2} \geq 0. \tag{21}$$

(f) If $\phi = 0$, it is strictly positive for all $t > 0$.

Since the instantaneous variance of the interest rate only depends on time through the estimation error, the time path of this variance follows from the fact that the estimation error evolves monotonically in time from its initial value v_0 toward its stable steady-state value, v_+, (Proposition 2).

The steady-state variance of the interest rate need not be decreasing in the precision of public information. To see this, note that the partial derivative of $\sigma_r^2(v_+)$ w.r.t. the volatility of public information, σ_s, is given by

$$\frac{\partial \sigma_r^2(v_+)}{\partial \sigma_s} = 2\gamma^2 \left[\left(\frac{v_+ + \rho_{\mu D}\sigma_\mu \sigma_D}{\sigma_D^2} + \frac{v_+ + \phi\sigma_\mu\sigma_s}{\sigma_s^2}\right)\frac{\partial v_+}{\partial \sigma_s} + \frac{(v_+ + \phi\sigma_\mu\sigma_s)\,\phi\sigma_\mu}{\sigma_s^2} \right.$$
$$\left. - \frac{(v_+ + \phi\sigma_\mu\sigma_s)^2}{\sigma_s^3}\right]. \tag{22}$$

Thus, even in the case of rational consumers, this variance need not be decreasing in the precision of public information.[6]

Due to the tractability of our model, we can obtain a closed-form solution to the price of a long-term bond for all levels of risk aversion (cf. Cox, Ingersoll and Ross, 1985b, and Feldman, 1989, who specialize their analyses to logarithmic preferences):

[6] Note that the following two parameter sets yield a variance of the interest rate that is increasing in the precision of public information: i) $\phi = 0$, $\gamma = 3$, $\kappa = 0.2$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, $\rho_{\mu D} = -0.8$; and ii) $\phi = -0.2$, $\gamma = 3$, $\kappa = 0.2$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, $\rho_{\mu D} = -0.8$.

Proposition 7 *The price of a long-term bond maturing at time $T \leq \tau$ is given by*

$$P(m_t, t, T) = \exp\left\{-\left(\beta - \frac{\gamma(\gamma+1)\sigma_D^2}{2}\right)(T-t) - \gamma\left(\frac{1-e^{-\kappa(T-t)}}{\kappa}\right)m_t\right.$$
$$-\gamma\overline{\mu}\int_{u=t}^{T}(1-e^{-\kappa(T-u)})\mathrm{d}u + \gamma^2\int_{u=t}^{T}(v_u + \sigma_{\mu D})\left(\frac{1-e^{-\kappa(T-u)}}{\kappa}\right)\mathrm{d}u$$
$$+\frac{\gamma^2}{2}\int_{u=t}^{T}\left(\frac{v_u + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)^2\left(\frac{1-e^{-\kappa(T-u)}}{\kappa}\right)^2\mathrm{d}u$$
$$\left.+\frac{\gamma^2}{2}\int_{u=t}^{T}\left(\frac{v_u + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)^2\left(\frac{1-e^{-\kappa(T-u)}}{\kappa}\right)^2\mathrm{d}u\right\}. \tag{23}$$

Notice that although Feldman (1989) employs a Cox, Ingersoll and Ross (1985a) economy and we consider a Lucas (1978) economy, the results in Feldman (1989)—with the specific mean-reverting structure of our model imposed—are obtained as we let $\phi = 0$, $\gamma = 1$ and $\sigma_s \to \infty$.

From the note in the Appendix describing the stochastic discount factor approach, we know that the term premium can be obtained from the following equation:

$$E_t\left[\frac{\mathrm{d}P}{P}\right] - r_t\mathrm{d}t = -E_t\left[\frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}P}{P}\right]. \tag{24}$$

By Ito's lemma, we have

$$\mathrm{d}P = P_t\mathrm{d}t + P_m\mathrm{d}m + \frac{1}{2}P_{mm}(\mathrm{d}m)^2. \tag{25}$$

Therefore, we can rewrite our expression for the term premium as

$$E_t\left[\frac{\mathrm{d}P}{P}\right] - r_t\mathrm{d}t = -\frac{P_m}{P}E_t\left[\frac{\mathrm{d}\Lambda}{\Lambda}\mathrm{d}m\right] = -\gamma^2\left(\frac{1-e^{-\kappa(T-t)}}{\kappa}\right)(v_t + \sigma_{\mu D})\,\mathrm{d}t, \tag{26}$$

where the last equality follows from Proposition 7 and equations (9) and (17). Therefore, the instantaneous term premium at time t for the term to maturity $\tau_M \equiv T - t$

is given by

$$TP(t,\tau_M) = -\gamma^2 \left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)(v_t+\sigma_{\mu D})\,. \tag{27}$$

It follows from equations (25), (26) and (9) that the SDE of the bond price is

$$\frac{dP}{P} = \left(r_t - \gamma^2\left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)(v_t+\sigma_{\mu D})\right)dt - \gamma\left(\frac{v_t+\rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)\mathrm{d}\widehat{Z}_t^D$$
$$-\gamma\left(\frac{v_t+\phi\sigma_\mu\sigma_s}{\sigma_s}\right)\left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)\mathrm{d}\widehat{Z}_t^s. \tag{28}$$

From equation (27), we see that the sign of the term premium depends on the sign of $(v_t+\sigma_{\mu D})$: if the sign of $(v_t+\sigma_{\mu D})$ is positive, then the term premium is negative, and vice versa. This reverse relation occurs because if the sign of $(v_t+\sigma_{\mu D})$ is positive, then the instantaneous covariance between bond returns and aggregate consumption is negative and, therefore, the bond acts as a hedge against a low aggregate consumption and the representative consumer accepts a negative term premium; if instead the sign of $(v_t+\sigma_{\mu D})$ is negative, then the instantaneous covariance between bond returns and aggregate consumption is positive, and the representative consumer demands a positive term premium. Therefore, provided a nonzero term to maturity, a sufficient condition for a negative term premium is a positive correlation between the endowment process and the unobserved growth rate.[7]

We also find that, depending on the sign of $(v_t+\sigma_{\mu D})$, the instantaneous term premium can be increasing, constant, or decreasing in the term to maturity. Further, we find that, if consumers are rational, it is increasing in the precision of public information for all levels of risk aversion. That is, with rational consumers, one effect of less precise information is a decreasing instantaneous term premium: there

[7]Note, however, that, by virtue of Proposition 2, the estimation error v_t is in itself a function of the covariance $\sigma_{\mu D}$.

is in fact an instantaneous discount for imprecise public information in long-term bonds for all levels of risk aversion. The reason behind this result is that, as public information becomes more precise, the bond becomes a poorer hedge against a low future consumption, and thus demands a higher expected return. This result is in line with the result in Veronesi (2000) that there is a discount for imprecise public information in equities if the coefficient of relative risk aversion is greater than unity. However, with deluded consumers, the instantaneous term premium can be decreasing in the precision of public information, i.e., with deluded consumers, there can be a premium for imprecise public information. Provided that $\phi > 0$, the covariance between the endowment process and the bond return is increasing in the degree of delusion (ϕ). Therefore, the term premium is increasing in the degree of delusion (ϕ) if $\phi > 0$.

Moreover, the term premium is unbounded in the coefficient of relative risk aversion (see equation (27)). This stands in contrast to the result in Veronesi (2000) that the equity premium is bounded in the coefficient of relative risk aversion. In the following two propositions, we summarize our most important findings regarding the term premium.

Proposition 8 *The instantaneous term premium, $TP(t, \tau_M)$, is a(n) i) increasing, ii) constant, or iii) decreasing function of the term to maturity, τ_M, if and only if i) $v_t < -\sigma_{\mu D}$, ii) $v_t = -\sigma_{\mu D}$, or iii) $v_t > -\sigma_{\mu D}$.*

Proposition 9 *Provided that the term to maturity is positive, the instantaneous term premium, $TP(t, \tau_M)$, also has the following characteristics:*

(a) Its sign depends on the sign of $v_t + \sigma_{\mu D}$. That is, $\operatorname{sign}[TP(t, \tau_M)] = \operatorname{sign}[-(v_t + \sigma_{\mu D})]$.

(b) In the steady state, it is increasing in the precision of public information if consumers are rational. That is, if $\phi = 0$, then $\frac{\partial TP(t,\tau_M)}{\partial \sigma_s} \leq 0$, with equality if and only if $\rho_{\mu D}^2 = 1$ and $\kappa \sigma_D^2 + \sigma_{\mu D} \geq 0$.

(c) In the steady state, it is increasing in the degree of delusion (ϕ) if $\phi > 0$. That is, $\frac{\partial TP(t,\tau_M)}{\partial \phi} \geq 0$ if $\phi > 0$.

It follows from equation (28) that the instantaneous variance of the bond return is given by

$$\sigma_P^2(t, \tau_M) = \left(\frac{1 - e^{-\kappa \tau_M}}{\kappa} \right)^2 \sigma_r^2(v_t) = \gamma^2 \left(\frac{1 - e^{-\kappa \tau_M}}{\kappa} \right)^2 \left(\left(\frac{v_t + \sigma_{\mu D}}{\sigma_D} \right)^2 + \left(\frac{v_t + \phi \sigma_\mu \sigma_s}{\sigma_s} \right)^2 \right). \tag{29}$$

As the term to maturity increases, the bond return becomes increasingly volatile. Since the term $(1 - e^{-\kappa \tau_M})/\kappa$ does not depend on the precision of public information, the reaction to an increasing precision of public information follows directly from equation (22) and does not need to be stated explicitly.

The result that the bond return volatility is monotonically increasing and unbounded in the coefficient of relative risk aversion stands in contrast to the results regarding the volatility of equity returns in Veronesi (2000). In Veronesi's (2000) model, the volatility of equity returns is U-shaped with respect to the coefficient of relative risk aversion.

From Proposition 7, we can determine the term-structure function, the yield to maturity:

Proposition 10 *The yield to maturity is given by*

$$R(m_t, t, \tau_M) = -\frac{\ln P(m_t, t, \tau_M)}{\tau_M} = \beta - \frac{\gamma(\gamma + 1)\sigma_D^2}{2} + \gamma \left(\frac{1 - e^{-\kappa \tau_M}}{\kappa} \right) \frac{m_t}{\tau_M}$$

$$+\frac{1}{\tau_M}\gamma\overline{\mu}\int_{z=0}^{\tau_M}(1-e^{-\kappa(\tau_M-z)})\mathrm{d}z-\frac{\gamma^2}{\tau_M}\int_{z=0}^{\tau_M}(v_{t+z}+\sigma_{\mu D})\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)\mathrm{d}z$$

$$-\frac{\gamma^2}{2\tau_M}\int_{z=0}^{\tau_M}\left(\frac{v_{t+z}+\sigma_{\mu D}}{\sigma_D}\right)^2\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)^2\mathrm{d}z$$

$$-\frac{\gamma^2}{2\tau_M}\int_{z=0}^{\tau_M}\left(\frac{v_{t+z}+\phi\sigma_\mu\sigma_s}{\sigma_s}\right)^2\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)^2\mathrm{d}z. \tag{30}$$

The result follows directly from Proposition 7.

Note that, unlike in the case of complete information, the term-structure function also depends on calendar time through the estimation error. As in Feldman (1989), the yield to maturity can be split into three parts,

$$R(m_t,t,\tau_M)=AER(m_t,\tau_M)+MRPB(t,\tau_M)+JEB(t,\tau_M), \tag{31}$$

where

$$AER(m_t,\tau_M)\equiv\beta-\frac{\gamma(\gamma+1)\sigma_D^2}{2}+\gamma\left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)\frac{m_t}{\tau_M}+\frac{1}{\tau_M}\gamma\overline{\mu}\int_{z=0}^{\tau_M}(1-e^{-\kappa(\tau_M-z)})\mathrm{d}z \tag{32}$$

is the expected average interest rate, $AER(m_t,\tau_M)=E_t\left[\frac{1}{\tau_M}\int_{z=0}^{\tau_M}r_{t+z}\mathrm{d}z\right]$;

$$MRPB(t,\tau_M)\equiv-\frac{\gamma^2}{\tau_M}\int_{z=0}^{\tau_M}(v_{t+z}+\sigma_{\mu D})\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)\mathrm{d}z \tag{33}$$

is the market risk premium bias; and

$$\begin{aligned}JEB(t,\tau_M)\ \equiv\ &-\frac{\gamma^2}{2\tau_M}\int_{z=0}^{\tau_M}\left(\frac{v_{t+z}+\sigma_{\mu D}}{\sigma_D}\right)^2\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)^2\mathrm{d}z\\&-\frac{\gamma^2}{2\tau_M}\int_{z=0}^{\tau_M}\left(\frac{v_{t+z}+\phi\sigma_\mu\sigma_s}{\sigma_s}\right)^2\left(\frac{1-e^{-\kappa(\tau_M-z)}}{\kappa}\right)^2\mathrm{d}z\end{aligned} \tag{34}$$

is the Jensen's inequality bias. Compared to Feldman (1989), this part now contains an additional term related to the external public signal.

We find that the interest rates' term structure is bounded. In particular, as the term to maturity goes to zero, the expected average interest rate approaches the short-term interest rate:[8]

$$\lim_{\tau_M \to 0} AER(m_t,\tau_M) = r_t. \tag{35}$$

Moreover, the long-run expected average interest rate is given by the long-run interest rate,

$$\lim_{\tau_M \to \infty} AER(m_t,\tau_M) = \overline{r}. \tag{36}$$

The market risk premium bias goes to zero as the term to maturity goes to zero,

$$\lim_{\tau_M \to 0} MRPB(t,\tau_M) = 0. \tag{37}$$

In the long run, the market risk premium bias goes to a constant,

$$\lim_{\tau_M \to \infty} MRPB(t,\tau_M) = -\frac{\gamma^2(v_+ + \sigma_{\mu D})}{\kappa}. \tag{38}$$

Thus, a sufficient condition for a negative long-run market risk premium bias is a positive correlation between the endowment process and the unobserved growth rate.

As the term to maturity goes to zero, so does the Jensen's inequality bias:

$$\lim_{\tau_M \to 0} JEB(t,\tau_M) = 0. \tag{39}$$

Further, as the term to maturity goes to infinity, the Jensen's inequality bias approaches a non-positive constant,

$$\lim_{\tau_M \to \infty} JEB(t,\tau_M) = -\frac{\gamma^2}{2}\left(\frac{v_+ + \sigma_{\mu D}}{\kappa\sigma_D}\right)^2 - \frac{\gamma^2}{2}\left(\frac{v_+ + \phi\sigma_\mu\sigma_s}{\kappa\sigma_s}\right)^2. \tag{40}$$

[8] We find the limits by applying l'Hôpital's rule.

Hence, in total, the long-run yield to maturity is given by

$$\overline{R} = \overline{r} - \frac{\gamma^2(v_+ + \sigma_{\mu D})}{\kappa} - \frac{\gamma^2}{2}\left(\frac{v_+ + \sigma_{\mu D}}{\kappa\sigma_D}\right)^2 - \frac{\gamma^2}{2}\left(\frac{v_+ + \phi\sigma_\mu\sigma_s}{\kappa\sigma_s}\right)^2. \qquad (41)$$

Therefore, a sufficient condition for the long-run yield to maturity to be lower than the long-run short interest rate is a positive correlation between the endowment process and the unobserved growth rate. In this case, the instantaneous covariance between the aggregate consumption process and instantaneous bond returns will be negative.

In order to investigate the long-term rewards to imprecise information, we study the yield in excess of the expected average interest rate:

$$LREY(t, \tau_M) \equiv R(m_t, t, \tau_M) - AER(m_t, \tau_M),$$

and its response to less precise public information, $\partial LREY(t, \tau_M)/\partial\sigma_s$ in steady state.

Using parameter values that are in line with US data (Brennan and Xia, 2001b), we find that, assuming rational consumers, there is in fact a discount for imprecise public information also in long-term bonds (see Figure 1). However, using the same parameter set, and instead assuming deluded consumers, there is a premium for imprecise public information (see Figure 2).

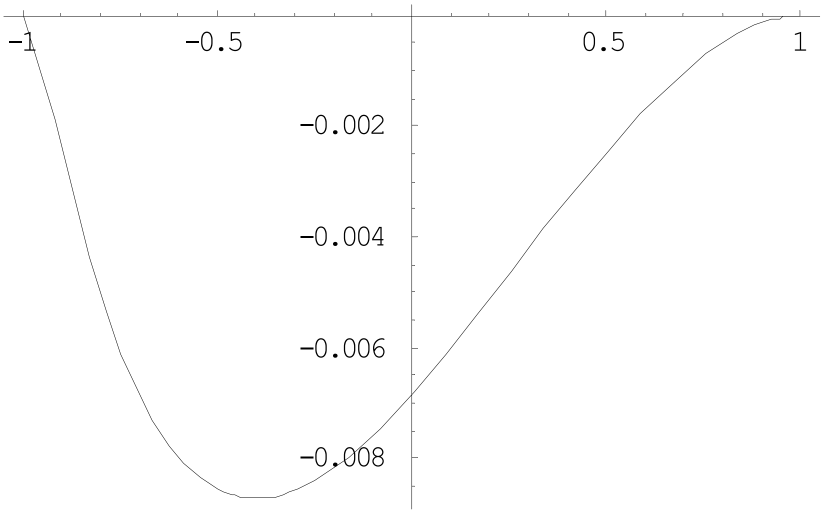

Figure 1: $\partial LREY(t, \tau_M)/\partial \sigma_s$ as a function of $\rho_{\mu D}$ in steady state, assuming rational ($\phi = 0$) consumers. Other parameter values are: $\gamma = 3$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, $\kappa = 0.2$, and $\tau_M = 10$.

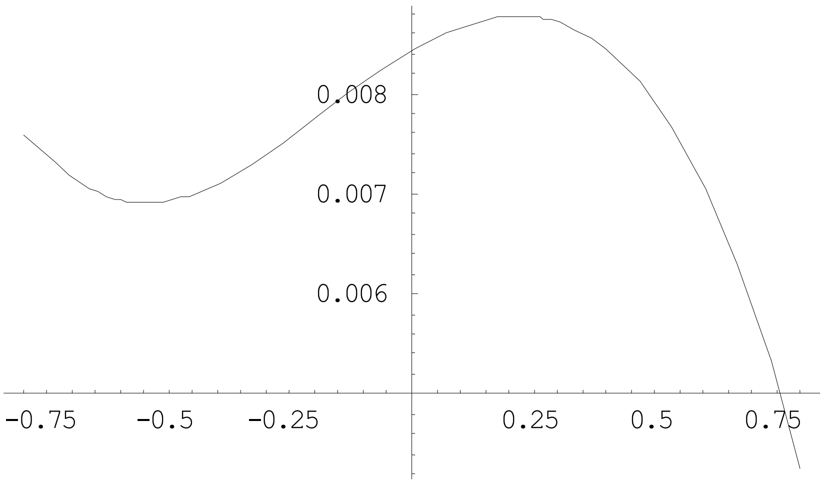

Figure 2: $\partial LREY(t, \tau_M)/\partial \sigma_s$ as a function of $\rho_{\mu D}$ in steady state, assuming deluded ($\phi = -0.2$) consumers. Other parameter values are: $\gamma = 3$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, $\kappa = 0.2$, and $\tau_M = 10$.

3.3 The Social Value of Public Information

Given the above result that imprecise public information can be rewarded in bonds, and the result in Yan (2001) that estimation uncertainty can be rewarded in long-run equity returns raises the question of the social value of more precise public information: if more precise public information has a negative effect on bond or equity returns, one would think that the social value of more precise public information may not always be positive in our setting. In other settings, Hirshleifer (1971) and, more recently, Morris and Shin (2002) have demonstrated that the social value of more precise public information may not be positive.[9] Let us now investigate the social value of public information in our framework.

For this purpose, let us study the value function of the representative individual:

Proposition 11 *In equilibrium, the value function of the representative individual is given by*

$$J(D_t, m_t, t) = \frac{D_t^{1-\gamma}}{1-\gamma} \int_{w=t}^{\tau} \exp\Bigg\{ -\left(\beta + (1-\gamma)\gamma\frac{\sigma_D^2}{2}\right)(w-t) + (1-\gamma)\left(\frac{1-e^{-\kappa(w-t)}}{\kappa}\right) m_t$$
$$+ (1-\gamma)\overline{\mu} \int_{u=t}^{w} (1-e^{-\kappa(w-u)})\mathrm{d}u + (1-\gamma)^2 \int_{u=t}^{w} (v_u + \sigma_{\mu D}) \left(\frac{1-e^{-\kappa(w-u)}}{\kappa}\right) \mathrm{d}u$$
$$+ \frac{(1-\gamma)^2}{2} \int_{u=t}^{w} \left(\frac{v_u + \sigma_{\mu D}}{\sigma_D}\right)^2 \left(\frac{1-e^{-\kappa(w-u)}}{\kappa}\right)^2 \mathrm{d}u$$
$$+ \frac{(1-\gamma)^2}{2} \int_{u=t}^{w} \left(\frac{v_u + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)^2 \left(\frac{1-e^{-\kappa(w-u)}}{\kappa}\right)^2 \mathrm{d}u \Bigg\} \mathrm{d}w$$
$$- \frac{1}{1-\gamma}\left(\frac{1-e^{-\beta(\tau-t)}}{\beta}\right). \tag{42}$$

In line with empirical evidence, we focus on conservative ($\gamma > 1$) consumers. With rational consumers and parameter values in line with US data (Brennan and Xia, 2001b), the social value of more precise information is positive (see Figure 3). However, as Figure 4 demonstrates, with deluded consumers, the social value of more precise information can be a negative.

[9] See also Svensson (forthcoming) and Morris, Shin, and Tong (forthcoming).

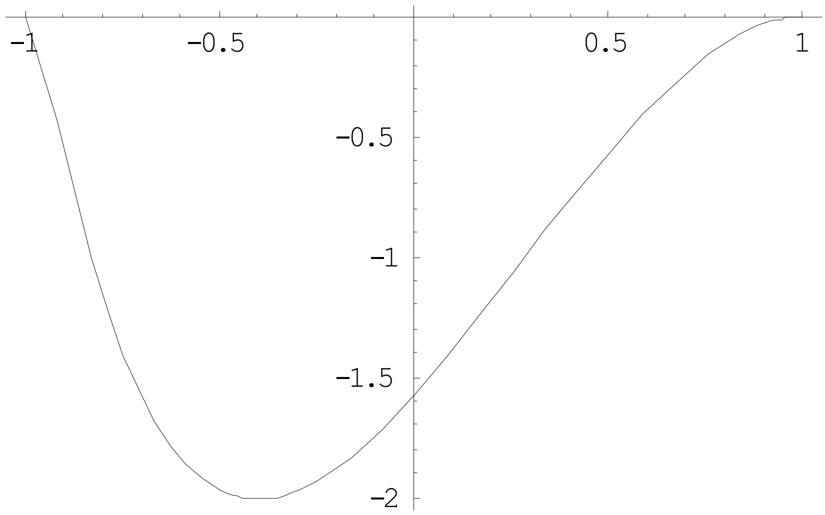

Figure 3: The steady-state value of $\partial J/\partial\sigma_s$ as a function of $\rho_{\mu D}$ at time $t = 0$, assuming rational ($\phi = 0$) consumers. Other parameter values are: $\beta = 0.05$, $\gamma = 3$, $\tau = 85$, $m_0 = 2\%$, $\overline{\mu} = 1.5\%$, $D_0 = 1$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, and $\kappa = 0.2$.

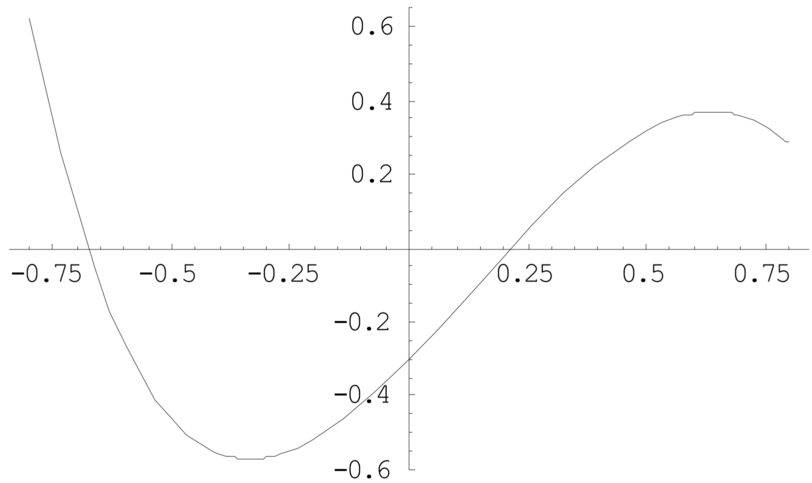

Figure 4: The steady-state value of $\partial J/\partial\sigma_s$ as a function of $\rho_{\mu D}$ at time $t = 0$, assuming deluded ($\phi = -0.3$) consumers. Other parameter values are: $\beta = 0.05$, $\gamma = 3$, $\tau = 85$, $m_0 = 2\%$, $\overline{\mu} = 1.5\%$, $D_0 = 1$, $\sigma_D = 0.13$, $\sigma_s = 0.10$, $\sigma_\mu = 0.015$, and $\kappa = 0.2$.

3.4 The Expectations Hypothesis

Cox, Ingersoll and Ross (1981) show that, when interest rates are stochastic, the only version of the expectations hypothesis that is consistent with general rational expectations equilibrium in continuous time is the so-called local expectations hypothesis:

$$E_t\left[\frac{\mathrm{d}P}{P}\right] = r_t\mathrm{d}t \text{ for all } \tau_M \text{ and } t. \tag{43}$$

That is, in continuous time, the expectations hypothesis says that, for all times to maturity and in all points in time, the instantaneous return on the long-term bond is equal to the short-term interest rate. Therefore, for the expectations hypothesis to hold, it is both necessary and sufficient that the term premium be zero. By equation (27), we see that the expectations hypothesis holds if and only if the estimation error is in the steady state and its steady-state value is $v_+ = -\sigma_{\mu D}$.

Feldman (1989), who investigates the term structure when rational consumers use only output information in their estimation, shows that for the expectations hypothesis to hold, interest rates have to be nonstochastic. In our model, nonstochastic interest rates are clearly not necessary for the expectations hypothesis to hold. Even as the endowment process becomes ineffective in conveying information about unanticipated changes in the investment opportunity set, external public information can still be effective—if there is a nonzero correlation between the consumers' estimate and the public signal. As seen in equation (19), there are two sources behind the variation in the interest rate—unanticipated shocks to the endowment process and unanticipated shocks to the public signal. In the steady state, the responsiveness of the interest rate to these two sources cannot be simultaneously eliminated (in general).

4 Conclusions

In this paper, we consider the term structure in an exchange-only Lucas (1978) economy in which the stochastic growth rate of the endowment process is unobservable and there is external public information. In addition, consumers may be rational or they may be deluded in the sense that they exaggerate the degree of covariation between the external public signal and the unobservable growth rate.

With rational consumers, the steady-state term premium is increasing in the precision of public information, i.e., with rational consumers, there is in fact an instantaneous discount for noisy external public information in long-term bonds. However, with deluded consumers, there can instead be an instantaneous premium for noisy external public information. These results can be compared to Veronesi's (2000) result that, for reasonable levels of risk aversion ($\gamma > 1$), there is no such premium in equities.

Moreover, we find that the instantaneous variance of the interest rate need not be decreasing in the precision of public information. Further, a positive correlation between the endowment process and the unobserved growth rate is a sufficient condition for the term premium to be negative. This is also a sufficient condition for the term premium to be decreasing in the term to maturity.

We derive a closed-form solution to the price of a long-term bond. Hence, we obtain the term-structure function (the yield to maturity). We show that the term structure is bounded and depends on calendar time through the estimation error. Here, too, a positive correlation between the endowment process and the unobservable growth rate is a sufficient condition for the long-run yield to maturity to be lower than the long-run short interest rate.

With parameter values in line with US data, we find that, with rational consumers, the social value of more precise public information is positive. However, with deluded consumers, this value can in fact be negative.

In contrast with Feldman (1989), where consumers learn about the growth rate only through output information, we find that, in the presence of external public information, nonstochastic interest rates are not necessary for the expectations hypothesis to hold.

Appendix

The Stochastic Discount Factor Approach

In this note, we provide a derivation of the basic pricing equations and discuss how they can be applied in the Lucas (1978) economy. The note is based on Cochrane (2001, pp. 28-33).

Consider the expected life-time utility of the representative consumer:

$$E_t \left[\int_t^\tau e^{-\beta(w-t)} u(c_w) \mathrm{d}w \right]. \tag{44}$$

Suppose that the representative consumer follows an optimal consumption path, in which the consumption streams are c_t^* from time t to time $t + \Delta t$ (where Δt is "very small") and c_w^* for $w \in [t + \Delta t, \tau]$. Consider a deviation from this optimal consumption path, in which the representative consumer buys ξ units of an arbitrary asset in the time-period from time t to time $t + \Delta t$. The asset has a price of V_t and pays a stream of cash-flows, X_w for $w \in [t + \Delta t, \tau]$. The representative consumer's consumption stream is thus given by

$$c_t = c_t^* - \xi V_t / \Delta t \text{ from } t \text{ to } t + \Delta t \tag{45}$$

$$c_w = c_w^* + \xi X_w \text{ for } w \in [t + \Delta t, \tau]. \tag{46}$$

His expected life-time utility is

$$\begin{aligned} J(\xi) &= u(c_t^* - \xi V_t/\Delta t) \int_t^{t+\Delta t} e^{-\beta(w-t)} dw + E_t \left[\int_{t+\Delta t}^{\tau} e^{-\beta(w-t)} u(c_w^* + \xi X_w) \mathrm{d}w \right] = \\ &= u(c_t^* - \xi V_t/\Delta t) \left(\frac{1 - e^{-\beta \Delta t}}{\beta} \right) + E_t \left[\int_{t+\Delta t}^{\tau} e^{-\beta(w-t)} u(c_w^* + \xi X_w) \mathrm{d}w \right]. \end{aligned} \tag{47}$$

Differentiating the above expression with respect to ξ yields

$$J'(\xi) = -u'(c_t^* - \xi V_t/\Delta t)V_t \left(\frac{1 - e^{-\beta \Delta t}}{\beta \Delta t} \right) + E_t \left[\int\limits_{t+\Delta t}^{\tau} e^{-\beta(w-t)} u'(c_w^* + \xi X_w) X_w \mathrm{d}w \right]. \tag{48}$$

Since c_t^* and c_w^* are optimal, the representative consumer's expected life-time utility reaches a maximum at $\xi = 0$, so $J'(0) = 0$. Hence,

$$u'(c_t^*)V_t \left(\frac{1 - e^{-\beta \Delta t}}{\beta \Delta t} \right) = E_t \left[\int\limits_{t+\Delta t}^{\tau} e^{-\beta(w-t)} u'(c_w^*) X_w \mathrm{d}w \right]. \tag{49}$$

Letting $\Delta t \longrightarrow 0$, we have

$$u'(c_t^*)V_t = E_t \left[\int\limits_{t}^{\tau} e^{-\beta(w-t)} u'(c_w^*) X_w \mathrm{d}w \right], \tag{50}$$

or, in Cochrane's (2001) notation

$$\Lambda_t V_t = E_t \left[\int\limits_{t}^{\tau} \Lambda_w X_w \mathrm{d}w \right], \tag{51}$$

where $\Lambda_u = e^{-\beta u} u'(c_u^*)$ is the "stochastic discount factor." In the Lucas (1978) economy considered in this paper, we have that, in equilibrium, the representative consumer's optimal consumption is equal to the aggregate endowment in each period, $c_u^* = D_u$ for $u \in [t, \tau]$.

Since a zero-coupon bond can be seen as an asset that pays a stream of payoffs $1/\mathrm{d}t$ over a time interval of infinitesimal length $\mathrm{d}t$, we see from the basic pricing equation (51) that the price of such a bond maturing at time $T \in [t, \tau]$ is given by

$$V_t = E_t \left[\frac{\Lambda_T}{\Lambda_t} \right]. \tag{52}$$

We will now derive an expression for the risk premium of an asset. First, we study the difference between $\Lambda_{t+\Delta t} V_{t+\Delta t}$ and $\Lambda_t V_t$, using the basic pricing equation (51). Taking time-t expectations, we obtain

$$\Lambda_t V_t = E_t \left[\int\limits_{w=0}^{\Delta t} \Lambda_{t+w} X_{t+w} \mathrm{d}w \right] + E_t \left[\Lambda_{t+\Delta t} V_{t+\Delta t} \right]. \tag{53}$$

For small Δt, we thus have that

$$\Lambda_t V_t \approx \Lambda_t X_t \Delta t + E_t\left[\Lambda_{t+\Delta t} V_{t+\Delta t}\right], \tag{54}$$

or, rearranging,

$$0 \approx \Lambda_t X_t \Delta t + E_t\left[\Lambda_{t+\Delta t} V_{t+\Delta t} - \Lambda_t V_t\right]. \tag{55}$$

Letting $\Delta t \longrightarrow 0$ in equation (55), we obtain

$$0 = \Lambda X \mathrm{d}t + E_t\left[\mathrm{d}(\Lambda V)\right], \tag{56}$$

where we have dropped the t subscript for convenience. Using Ito's lemma on $\mathrm{d}(\Lambda V)$ in equation (56) and dividing by ΛV yield

$$0 = \frac{X}{V}\mathrm{d}t + E_t\left[\frac{\mathrm{d}\Lambda}{\Lambda} + \frac{\mathrm{d}V}{V} + \frac{\mathrm{d}\Lambda}{\Lambda}\frac{\mathrm{d}V}{V}\right]. \tag{57}$$

We can either view a risk free asset as an asset that pays zero dividends ($X = 0$) and climbs deterministically at a rate of r_t ($\frac{\mathrm{d}V_t}{V_t} = r_t \mathrm{d}t$), or, we can view it as an asset that pays a relative dividend of r_t ($\frac{X_t}{V_t} = r_t$) and has a constant price ($\mathrm{d}V = 0$). In any case, using equation (57), we find that for a risk free asset,

$$r_t \mathrm{d}t = -E_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\right]. \tag{58}$$

By substituting equation (58) into (57), we obtain an expression for the risk premium of an arbitrary asset,

$$E_t\left[\frac{\mathrm{d}V_t}{V_t}\right] + \frac{X_t}{V_t}\mathrm{d}t - r_t \mathrm{d}t = -E_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\frac{\mathrm{d}V_t}{V_t}\right]. \tag{59}$$

For a zero-coupon bond, $X_t = 0$ and, therefore, with P_t being the price of the bond, the term premium can be determined from the following relation:

$$E_t\left[\frac{\mathrm{d}P_t}{P_t}\right] - r_t \mathrm{d}t = -E_t\left[\frac{\mathrm{d}\Lambda_t}{\Lambda_t}\frac{\mathrm{d}P_t}{P_t}\right]. \tag{60}$$

Proof of Proposition 2

First consider the matrix of instantaneous correlations between Z_t^D, Z_t^μ and Z_t^s:

$$\boldsymbol{\varrho} \equiv \begin{pmatrix} 1 & \rho_{\mu D} & 0 \\ \rho_{\mu D} & 1 & \phi \\ 0 & \phi & 1 \end{pmatrix}.$$

Since $\boldsymbol{\varrho}$ has to be positive semidefinite, its determinant must be nonnegative:

$$\det \boldsymbol{\varrho} = 1 - \rho_{\mu D}^2 - \phi^2 \geq 0.$$

It follows that v_+ is always nonnegative.

Notice that the Riccati equation in (7) can be rewritten as

$$\frac{\mathrm{d}v_t}{\mathrm{d}t} = -\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)(v_t - v_+)(v_t - v_-). \tag{61}$$

The monotonicity of v_t and the stability of v_+ follows from the fact that

$$sign\left[\frac{\mathrm{d}v_t}{\mathrm{d}t}\right] = sign\left[-(v_t - v_+)(v_t - v_-)\right]. \tag{62}$$

The solution to equation (61) is given in Feldman (1989).

Proof of Proposition 4

Let $g(\phi) = \kappa + \phi\frac{\sigma_\mu}{\sigma_s} + \rho_{\mu D}\frac{\sigma_\mu}{\sigma_D}$ and $h(\phi) = \sigma_\mu^2\left(1 - \phi^2 - \rho_{\mu D}^2\right)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)$. Then, we have that

$$v_+ = \frac{\sqrt{g^2 + h} - g}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}}. \tag{63}$$

Therefore, for $\phi > 0$

$$\frac{\partial v_+}{\partial \phi} = \frac{\left(\frac{g}{\sqrt{g^2+h}} - 1\right)\frac{\partial g}{\partial \phi} + \frac{1}{2\sqrt{g^2+h}}\frac{\partial h}{\partial \phi}}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}} \leq 0. \tag{64}$$

Proof of Proposition 5

Letting $\phi = 0$ and taking the derivative of v_+ w.r.t. $1/\sigma_s^2$ in equation (11), we have

$$
\begin{aligned}
\frac{\partial v_+}{\partial\left(\frac{1}{\sigma_s^2}\right)} &= \frac{\kappa + (\sigma_{\mu D}/\sigma_D^2)}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)^2} \\
&\quad - \frac{1}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)^2}\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)} \\
&\quad + \frac{\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right) 2\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}} \\
&= \frac{2\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)^2 2\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}} \\
&\quad - \frac{2\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)^2 2\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}} \\
&= \frac{-\left(\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right) - \sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}\right)^2}{\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)^2 2\sqrt{\left(\kappa + (\sigma_{\mu D}/\sigma_D^2)\right)^2 + \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\left(\sigma_\mu^2 - \frac{\sigma_{\mu D}^2}{\sigma_D^2}\right)}} \leq 0.
\end{aligned}
\tag{65}
$$

By the chain rule,

$$
\frac{\partial v_+}{\partial \sigma_s} = \frac{\partial\left(\frac{1}{\sigma_s^2}\right)}{\partial \sigma_s}\frac{\partial v_+}{\partial\left(\frac{1}{\sigma_s^2}\right)} = -2\sigma_s^{-3}\frac{\partial v_+}{\partial\left(\frac{1}{\sigma_s^2}\right)} \geq 0. \tag{66}
$$

From (65), we see that $\frac{\partial v_+}{\partial\left(\frac{1}{\sigma_s^2}\right)} = 0$ if and only if $\rho_{\mu D}^2 = 1$ and $\kappa\sigma_D^2 + \sigma_{\mu D} \geq 0$.

Proof of Proposition 6

(a)–(e): Statements (a)–(e) are straightforward to show and their proofs are therefore omitted.

(f): It follows from Proposition 2 that the estimation error (v_t) evolves monotonically toward its stable steady state, v_+. Suppose now that $\sigma_r^2(v_t) = 0$ for some $t > 0$. Then, since $\phi = 0$, we must have $v_t = -\sigma_{\mu D}$ and $v_t = 0$. If $v_t = 0$ for some $t > 0$, it must be that $v_0 = 0$ and $v_+ = 0$. However, if $v_t = -\sigma_{\mu D}$ and $v_t = 0$ is to hold simultaneously, $\sigma_{\mu D}$ must be equal to zero. If $\sigma_{\mu D} = 0$, then the stable steady-state value, v_+, is greater than zero, which is a contradiction.

Proof of Proposition 7

It follows from equation (52) that the price of the bond is[10]

$$P(m_t, t, T) = E_t\left[\frac{\Lambda_T}{\Lambda_t}\right]. \tag{67}$$

Since $D_T = D_t \exp\left\{\int\limits_{u=t}^{T} (m_u - \sigma_D^2/2)\mathrm{d}u + \int\limits_{u=t}^{T} \sigma_D \mathrm{d}\widehat{Z}_u^D\right\}$, we have

$$\frac{\Lambda_T}{\Lambda_t} = e^{-\beta(T-t)}\frac{D_T^{-\gamma}}{D_t^{-\gamma}} = \exp\left\{-\beta(T-t) - \gamma\int\limits_{u=t}^{T}(m_u - \sigma_D^2/2)\mathrm{d}u - \gamma\int\limits_{u=t}^{T}\sigma_D \mathrm{d}\widehat{Z}_u^D\right\}. \tag{68}$$

By equation (9),

$$\int\limits_{u=t}^{T} m_u \mathrm{d}u = \int\limits_{u=t}^{T} \overline{\mu}\mathrm{d}u - \frac{1}{\kappa}m_T + \frac{1}{\kappa}m_t + \frac{1}{\kappa}\int\limits_{u=t}^{T}\left(\frac{v_u + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_u^D + \frac{1}{\kappa}\int\limits_{u=t}^{T}\left(\frac{v_u + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_u^s. \tag{69}$$

[10] We use $E_t[\circ]$ as a shorthand notation for "the conditional expected value at time t." It is calculated based on the current values of the state variables and the belief that the state variables evolve according to equations (14)–(16), i.e. the same information and beliefs that the representative consumer uses in his maximization of the Markovian problem.

Furthermore, the solution to equation (9) is

$$m_T = e^{-\kappa(T-t)}m_t + \int\limits_{u=t}^{T} e^{-\kappa(T-u)}\kappa\overline{\mu}\mathrm{d}u + \int\limits_{u=t}^{T} e^{-\kappa(T-u)}\left(\frac{v_u + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_u^D$$
$$+ \int\limits_{u=t}^{T} e^{-\kappa(T-u)}\left(\frac{v_u + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_u^s. \tag{70}$$

Inserting equation (70) into equation (69), we have

$$\int\limits_{u=t}^{T} m_u\mathrm{d}u = \int\limits_{u=t}^{T} \overline{\mu}(1-e^{-\kappa(T-u)})\mathrm{d}u + \left(\frac{1-e^{-\kappa(T-t)}}{\kappa}\right)m_t$$
$$+ \int\limits_{u=t}^{T}\left(\frac{1-e^{-\kappa(T-u)}}{\kappa}\right)\left(\frac{v_u + \rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_u^D + \int\limits_{u=t}^{T}\left(\frac{1-e^{-\kappa(T-u)}}{\kappa}\right)\left(\frac{v_u + \phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_u^s. \tag{71}$$

The result follows from inserting equation (71) into equation (68) and taking expectations.

Proof of Proposition 8

Taking the derivative w.r.t. τ_M in equation (27) we have

$$\frac{\partial TP(t,\tau_M)}{\partial \tau_M} = -\gamma^2 e^{-\kappa\tau_M}\left(v_t + \sigma_{\mu D}\right), \tag{72}$$

and the result follows.

Proof of Proposition 9

(a): The result follows directly from equation (27).

(b): The result follows from inserting $v_t = v_+$ into equation (27) and taking the derivative w.r.t. σ_s:

$$\frac{\partial TP(t,\tau_M)}{\partial \sigma_s} = -\gamma^2\left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)\frac{\partial v_+}{\partial \sigma_s}. \tag{73}$$

From Proposition 5, we know that if $\phi = 0$, then $\frac{\partial v_+}{\partial \sigma_s} \geq 0$, with equality if and only if $\rho_{\mu D}^2 = 1$ and $\kappa\sigma_D^2 + \sigma_{\mu D} \geq 0$.

(c): In the steady state, the derivative of the term premium w.r.t. ϕ is given by

$$\frac{\partial TP(t,\tau_M)}{\partial \phi} = -\gamma^2 \left(\frac{1-e^{-\kappa\tau_M}}{\kappa}\right)\frac{\partial v_+}{\partial \phi}.$$

From Proposition 4, we know that $\frac{\partial v_+}{\partial \phi} \leq 0$ if $\phi > 0$, and the result follows.

Proof of Proposition 11

Since, in equilibrium, $c_w = D_w$ for all w, the value function of the representative consumer is given by

$$J(D_t, m_t, t) = E_t\left[\int_{w=t}^{\tau} e^{-\beta(w-t)}\left(\frac{D_w^{1-\gamma}-1}{1-\gamma}\right)\mathrm{d}w\right]. \tag{74}$$

We have that

$$D_w = D_t \exp\left\{\int_{u=t}^{w}(m_u - \sigma_D^2/2)\mathrm{d}u + \int_{u=t}^{w}\sigma_D \mathrm{d}\widehat{Z}_u^D\right\}. \tag{75}$$

Therefore, it follows that

$$\int_{w=t}^{\tau} e^{-\beta(w-t)}\left(\frac{D_w^{1-\gamma}-1}{1-\gamma}\right)\mathrm{d}w =$$

$$= \frac{D_t^{1-\gamma}}{1-\gamma}\int_{w=t}^{\tau}\exp\left\{\left(-\beta-(1-\gamma)\sigma_D^2/2\right)(w-t) + (1-\gamma)\int_{u=t}^{w} m_u du + (1-\gamma)\int_{u=t}^{w}\sigma_D \mathrm{d}\widehat{Z}_u^D\right\}$$
$$-\frac{1}{1-\gamma}\left(\frac{1-e^{-\beta(\tau-t)}}{\beta}\right). \tag{76}$$

By an argument similar to the one we used in the proof of Proposition 7, we have that

$$\int_{u=t}^{w} m_u \mathrm{d}u = \int_{u=t}^{w}\overline{\mu}(1-e^{-\kappa(w-u)})\mathrm{d}u + \left(\frac{1-e^{-\kappa(w-t)}}{\kappa}\right)m_t$$
$$+\int_{u=t}^{w}\left(\frac{1-e^{-\kappa(w-u)}}{\kappa}\right)\left(\frac{v_u+\rho_{\mu D}\sigma_\mu\sigma_D}{\sigma_D}\right)\mathrm{d}\widehat{Z}_u^D + \int_{u=t}^{w}\left(\frac{1-e^{-\kappa(w-u)}}{\kappa}\right)\left(\frac{v_u+\phi\sigma_\mu\sigma_s}{\sigma_s}\right)\mathrm{d}\widehat{Z}_u^s. \tag{77}$$

The result follows from inserting equation (77) into equation (76) and taking expectations.

References

Beber, A. and M. W. Brandt (2006): "Resolving Macroeconomic Uncertainty in Stock and Bond Markets," Working paper, Fuqua School of Business, Duke University.

Brennan, M. J. (1998): "The Role of Learning in Dynamic Portfolio Decisions," *European Finance Review*, Vol. 1, pp. 295–306.

Brennan, M. J. and Y. Xia (2001a): "Assessing Asset Pricing Anomalies," *Review of Financial Studies*, Vol. 14, pp. 905–945.

Brennan, M. J. and Y. Xia (2001b): "Stock Return Volatility and Equity Premium," *Journal of Monetary Economics*, Vol. 47, pp. 249–283.

Cochrane, J. H. (2001): *Asset Pricing*, Princeton University Press, Princeton.

Cox, J. C., J. E. Ingersoll, and S. A. Ross (1981): "A Re-examination of Traditional Hypotheses About the Term Structure of Interest Rates," *Journal of Finance*, Vol. 36, pp. 769–799.

Cox, J. C., J. E. Ingersoll, and S. A. Ross (1985a): "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, Vol. 53, pp. 363–384.

Cox, J. C., J. E. Ingersoll, and S. A. Ross (1985b): "A Theory of the Term Structure of Interest Rates," *Econometrica*, Vol. 53, pp. 385–407.

Dai, Q. and K. Singleton (2000): "Specification Analysis of Affine Term Structure Models," *Journal of Finance*, Vol. 55, pp. 1943-1978.

de Jong, F. (2000): "Time Series and Cross-Section Information in Affine Term-Structure Models," *Journal of Business and Economic Statistics*, Vol. 18, pp. 300-314.

Detemple, J. B. (1986): "Asset Pricing in a Production Economy with Incomplete Information," *Journal of Finance*, Vol. 41, pp. 383–391.

Dothan, M. U. and D. Feldman (1986): "Equilibrium Interest Rates and Multiperiod Bonds in a Partially Observable Economy," *Journal of Finance*, Vol. 41, pp. 369–382.

Duffee, G. (2002): "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, Vol. 57, pp. 405-443.

Duffie, D. and R. Kan (1996): "A Yield-Factor Model of Interest Rates," *Mathematical Finance*, Vol. 6, pp. 379-406.

Duffie, D. and K. Singleton (1997): "An Econometric Model of the Term Structure of Interest-Rate Swap Yields," *Journal of Finance*, Vol. 52, pp. 1287-1321.

Dumas, B., A. Kurshev, and R. Uppal (2004): "What To Do About Excessive Volatility?" Working paper, London Business School.

Economist (2004): "It's Not Always Good to Talk," July 22, 2004, Issue no. 71.

Feldman, D. (1989): "The Term Structure of Interest Rates in a Partially Observable Economy," *Journal of Finance*, Vol. 44, pp. 789–812.

Feldman, D. (1992): "Logarithmic Preferences, Myopic Decisions, and Incomplete Information," *Journal of Financial and Quantitative Analysis*, Vol. 27, pp. 619–629.

Feldman, D. (2003): "The Term Structure of Interest Rates: Bounded or Falling?" *European Finance Review*, Vol. 7, pp. 103–113.

Feldman, D. (forthcoming): "Incomplete Information Equilibria: Separation Theorems and Other Myths," *Annals of Operations Research.*

Hansen, L. P. and T. J. Sargent (1982): "Instrumental Variables Procedures for Estimating Linear Rational Expectations Models," *Journal of Monetary Economics*, Vol. 9, pp. 263–296.

Hirshleifer, J. (1971): "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, Vol. 61, pp. 561-574.

Li, G. (2005): "Information Quality, Learning, and Stock Market Returns," *Journal of Financial and Quantitative Analysis*, Vol. 40, pp. 595–620.

Liptser, R. S. and A. N. Shiryaev (2001): *Statistics of Random Processes I & II.* Springer-Verlag, Berlin.

Lucas, R. E. Jr. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, Vol. 46, pp. 1429–1445.

Morris, S. and H. S. Shin (2002): "The Social Value of Public Information," *American Economic Review*, Vol. 92, pp. 1521-1534.

Morris, S., H. S. Shin and H. Tong (forthcoming): "Response to 'The Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con'," *American Economic Review.*

Riedel, F. (2000): "Decreasing Yield Curves in a Model with an Unknown Constant Growth Rate," *European Finance Review*, Vol. 4, pp. 51–67.

Rubinstein, M. (1974): "An Aggregation Theorem for Securities Markets," *Journal of Financial Economics*, Vol. 1, pp. 225–244.

Scheinkman, J. A. and W. Xiong (2003): "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, Vol. 111, pp. 1183-1219.

Svensson, L. E. O. (forthcoming): "The Social Value of Public Information: Morris and Shin (2002) is Actually Pro Transparency, Not Con," *American Economic Review.*

Veronesi, P. (2000): "How Does Information Quality Affect Stock Returns?" *Journal of Finance*, Vol. 55, pp. 807–837.

Veronesi, P. and F. Yared (2000): "Short and Long Horizon Term and Inflation Risk Premia in the US Term Structure: Evidence from an Integrated Model for Nominal and Real Bond Prices under Regime Shifts," Working paper, University of Chicago.

Williams, J. T. (1977): "Capital Asset Prices with Heterogeneous Beliefs," *Journal of Financial Economics*, Vol. 5, pp. 219–239.

Xia, Y. (2001): "Learning About Predictability: the Effect of Parameter Uncertainty on Dynamic Asset Allocation," *Journal of Finance*, Vol. 56, pp. 205–246.

Yan, H. (2001): "Estimation Uncertainty and the Equity Premium," Working paper, University of Texas at Austin.