# The Quantum Theory of Atoms in Molecules

From Solid State to DNA and Drug Design

Edited by Chérif F. Matta and Russell J. Boyd



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