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The Quest for Unity

General Relativity and Unitary Field Theories

Peter G. Bergmann

A centenary jubilee is a good time to evaluate the intellectual contributions of an outstanding individual. Enough time has elapsed to free us of the fashions of the moment; but that time is sufficiently short so that some of us who have come under Albert Einstein's influence are still alive. Einstein, who earned the greatest fame of all physicists in his own lifetime, has come to serve as a focus for a vast range of endeavors, both humane and scientific, on the occasion of his one-hundredth birthday. Having had the privilege of association with Einstein in my youth, I am happy to have this occasion to express my deep gratitude for the instruction and stimulation that I have received from him. Albert Einstein's memory will last as long as there are human beings who strive for a better society and for a deeper comprehension of the physical universe.

Einstein's contributions to physics are many, and they are being discussed by the participants of this centenary celebration — as they are at similar gatherings throughout the world. I shall address myself but to one contribution, Einstein's quest for unity in science, which found expression in his formulation of the theory of relativity and in his research for a unitary field theory that would lead beyond it.

Let me begin with the special theory of relativity. At the turn of the century there was, among the many puzzles confronting physicists, one that touched the very foundations of all natural science; it related to the nature of space and time. Most of physics was then dominated by mechanics, which dealt with the interaction of physical bodies. The crowning achievement of mechanics had been the complete and quantitative explanation of the workings of the solar system, so that astronomers were able to predict with great accuracy and complete reliability such events as eclipses decades and centuries away. The laws of mechanics had been formulated by Isaac Newton. They concerned the *accelera-*

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tions of the interacting bodies, determined by the forces of interaction, which in turn depended only on the (instantaneous) configuration. If Newton's laws were valid, then it followed that in our universe there is no possibility of identifying a state of rest, or for that matter of absolute motion. As far as absolute properties of space and time were concerned, the laws of mechanics called for a set of states of nonrotational uniform rectilinear motion, all of equal stature, which are usually referred to as inertial frames of reference.

The then-new physics of the electromagnetic field, brilliantly formulated by Faraday, Maxwell, and Lorentz, differed from the laws of mechanics in that they introduced the notion of the pervasive field, which was to fill the space between the particles. The laws of the field, however, involved a *velocity*, the speed with which any electromagnetic disturbance would spread in empty space; today we call this the speed of light. The electromagnetic laws would seem to single out one state, the state of absolute rest — that state in which in the absence of matter the speed of propagation of electromagnetic waves is isotropic. As everybody knows, the search for that state, or frame, of absolute rest was unsuccessful: it appeared that the electromagnetic field is totally insensitive to the absolute motion of the earth through space. This experimental fact, confirmed in the meantime in all manner of ways, apparently presented an internal inconsistency, unless of course you assumed that the earth represented the state of absolute rest. And that would have been a regression to Ptolemaic ideas, unacceptable to nineteenth-century scientists.

In this situation, increasingly tortured proposals were considered by the outstanding theorists of that time. Einstein's contribution was revolutionary because it was formally simple yet deep at the conceptual level. He demonstrated not only that observers in different states of motion would have different scales of distance and of time but also that the simultaneity of distant events would be observer- or frame-dependent if one accepted the proposition that the speed of light cannot be exceeded by any signaling device. By an intricate argument (into which I shall not enter here—but which is intricate not because of abstruse mathematics but because of a very delicate analysis of experimental procedures), he showed that once the notion of absolute time marks is dropped, two moving observers can *both* perceive the other's clocks to be slow, and both can perceive the other's yardsticks to be contracted. The paradox was resolved by a profound modification of classical space and time concepts.

A very few years later Minkowski discovered the natural mathematical formulation of Einstein's new physics, the four-dimensional space-time model. The relationship between space and time measurements of two observers moving differently was analogous to a rotation in four dimensions, except for a few signs that differed from an ordinary rotation.

Through his revision of the space-time concepts, Einstein had succeeded in removing from physics the apparent contradiction between the (classical) principle of relativity of mechanics and the laws of electrodynamics. To this extent unity was restored,

but a new contradiction had been created. Newtonian mechanics involved at its foundations the notion of absolute simultaneity; the forces between distant bodies—for instance between the sun and the earth—depended on their instantaneous distance from each other, which in relativity would differ for different observers. If the new theory of space and time was to prevail, mechanics needed to be modified.

Relativistic mechanics was designed to bridge the gap partially by making the mass velocity-dependent—hence the proportionality between mass and energy—and by modifying the force. These changes had, however, no effect on the dependence of the action at finite distances on absolute simultaneity. This could be accomplished only by replacing the Newtonian action by the intermediary of fields. Thus the need arose for a relativistic gravitational field.

It is possible to introduce relativistic field equations for a gravitational field with relatively little effort. Einstein was troubled, however, by two considerations. The first was that there were several ways of doing this, and very little grounds for choosing one way over the others. The second consideration was a peculiar property of gravitation, the universality of gravitational acceleration. In a gravitational field all bodies undergo the same acceleration—on the surface of the earth, for instance, 9.8 m/sec/sec.

In an electric field the force acting on a body depends on its electric charge; and the acceleration, on the ratio of its charge to its mass, e/m . No analogous parameter enters into the expression for acceleration caused by a gravitational field. This fact was already ascertained by Galileo and certainly recognized by Newton; but it remained a curiosity. It was Einstein who understood the implications. If gravitational acceleration is the same for all bodies, then it vanishes locally for an observer who himself undergoes the same acceleration. One is led naturally to the notion of a free-falling frame of reference rather than the inertial frame of reference. The difference between the two concepts is this: Whereas an inertial frame of reference presumably extends over the whole universe, a free-falling frame is defined only locally in a sufficiently small region. An astronaut or cosmonaut will perceive no gravitational field in his free-falling vehicle, but distant objects appear to be accelerated relative to himself. Thus the local uniformity of gravitational acceleration precludes the determination of inertial frames of reference by local means, replacing these frames by constructs that cannot be extended globally.

This line of reasoning leads to the general theory of relativity, Einstein's theory of the gravitational field. When the new theory was completed, some sixty years ago, it replaced the space-time of the special theory of relativity by a yet more general geometric concept, that of a Riemannian space-time; the latter locally has properties resembling those of the special theory but on a larger scale is much more involved, being a curved manifold.

I do not wish to give you the impression that the progress from the special to the general theory was straightforward or logically inescapable; far from it. If inertial frames cannot be determined by local observations, it might be possible to preserve the concept by

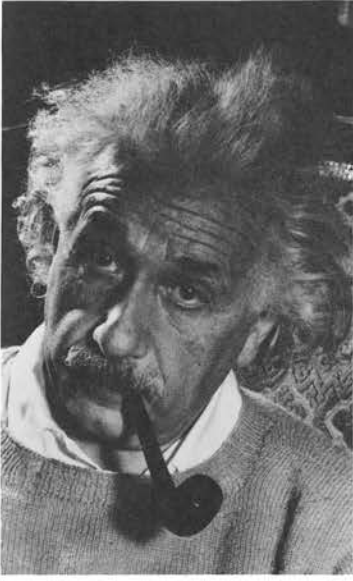
relying on observations of distant objects. This is in fact what astronomers do. But reliance *in principle* on distant objects runs counter to the spirit of a field theory, which relies on physical interaction of fields in the neighborhood of the particle, not at a distance; thus the embarrassment of instantaneous action at a distance is circumvented. From a logical point of view, the progress toward general relativity depended on a number of choices to be made; its eventual adoption, first by Einstein himself and later by the community of physicists, depended on the esthetic appeal of the finished theory and on its confirmation by experiment and observation.

As for experimental confirmations, the universality of gravitational acceleration has been confirmed to an accuracy beyond 10^{-11} . As for relativistic effects—that is, gravitational effects that deviate from the predictions of classical mechanics and of the special theory of relativity—all quantitative observations that can be performed with today's technology have confirmed Einstein's theory well within the bounds of instrumental error, including such cases where competing modern theories predicted different results. This is an ongoing enterprise.

The issue of the esthetic appeal of general relativity is closer to the principal theme of this discussion. Once more general relativity had restored a measure of unity to physics by modifying our ideas of space and time, which lie at the foundations of any dynamical conceptual construction. The new framework accommodated gravitation. Its essence was to be sought not in the properties of the single local free-falling frame of reference but in its relationship to free-falling frames in adjacent regions. These relations were subject to field laws that were chosen according to principles of formal simplicity and the requirement that for weak fields the classical results should agree in lowest approximation with those of the new theory.

One major conceptual difficulty was removed from the new theory some twenty years after its inception: the interaction of the local field with a particle. Every mass serves as a source of the gravitational field, just as each charge is a source of the electromagnetic field. At the site of a particle the field becomes very large. If the particle is conceived of as a mass point, the field becomes infinite. But the force that affects the kinematic behavior of the particle is determined by the surrounding field. What if that field is finite? The first response, historically, was a holding operation. If the particle itself was small, if its mass was slight, then one could imagine the field as it would be if the particle under consideration did not exist. Einstein then postulated that such a small particle would travel on a so-called geodesic, a curve in space-time that corresponds to unaccelerated motion in special relativity, or in the local free-falling frame. This assumption was in fact the point of departure for the geometric interpretation of the fact of uniform gravitational acceleration.

But what if the particle was not so small? How would one deal with the problem of a double star, for instance, in which it could not be reasonably assumed that the field caused by one star was larger than the field caused by the other star? Eventually Einstein,



*Above, Albert Einstein.
Right, Einstein and Bergmann.*
Photos by L. Aigner.



Infeld, and Hoffmann developed an approach that was applicable in such cases. They found that the field laws outside a particle could not be satisfied unless the particle itself behaved "properly." Viewed from that angle, the behavior of sources of the gravitational field was determined by the field laws themselves. This was a property not shared by other field laws, and certainly not by those of electrodynamics. Thus general relativity turned out, after all, to be conceptually more nearly of one piece than any physical theory then known.

With the laws of motion of particles having been absorbed into the logical structure of the field laws, mechanics (once the dominant structure of theoretical physics) was all but eliminated from it. The quest for unity had apparently reached its objective. But there were several hairs in that ointment.

Atomic physics, we know, is not governed by classical laws but obeys quantum rules. General relativity, however, is nonquantum. It satisfies essentially strictly deterministic laws, whereas quantum laws are essentially statistical. Einstein could never bring himself to accept statistics as the definitive form of the laws of nature, even though as a young person he had made major contributions to quantum theory and to statistical mechanics. He always considered statistical approaches preliminary to a better understanding, which would be strictly causal.

The second drawback of general relativity was that it treated particles as singularities of the field, infinities, and failed to explain their structural properties such as masses or charges. Finally, as nature is not purely gravitational but allows for other forces as well, the gravitational and the nongravitational fields appear to be essentially different. From the point of view of general relativity, gravitation is needed in order to give space and time their geometric structure; all the other forces are gratuitous.

Unitary field theory was intended to remedy all these blemishes.

From the early twenties to the end of his life, Einstein developed ever-new approaches to unitary field theory. At the time

of his death he was working, together with Bruria Kaufman, on the so-called asymmetric theory.

Riemannian geometry in four dimensions is a well-defined and fairly rigid structure, which admits very little variation in the proposed dynamical laws. Somehow this mold must be broken if more physical fields than gravitation are to be accommodated within the geometric framework. Before I discuss a few of these attempts, permit me to address myself to a preliminary question: What is geometry?

I suspect that there is no answer to this question that will satisfy everybody. Basically, geometry can be considered any kind of mathematical structure that begins with the construction of a set of points satisfying those minimal properties of continuity that justify speaking of a space. A space may, but need not, involve such concepts as volume and distance; it may, but need not, involve the existence of vector fields and the possibility of defining when two vectors at distinct locations are to be considered parallel. These are but examples of properties that geometric spaces can possess. Many more have been investigated; in fact, many more have been used by physicists in their endeavors to understand nature. Depending on the properties ascribed to a new model for space-time, the structures of geometric spaces can lend themselves to interpretations that are reminiscent of fields known to physicists. How does such a "geometrization" contribute to unification? Einstein has stated repeatedly that he did not consider the geometrization of physics a foremost or even a meaningful objective, and I believe that his comments remain valid today. What counts is not a geometric formulation, or picturization, but a real fusing of the mathematical structures intended to represent physical fields.

How can we visualize such a fusing? One possibility, suggested by the history of relativity itself, is that the decomposition of fields into gravitational, electromagnetic, and "strong" and "weak" nuclear forces might depend on the frame used for their description; that, for instance, a field that appears to be purely gravitational in one frame is mixed gravitational and electromagnetic in another frame. This is possible if the variety of equivalent frames, or modes of description, is sufficiently large.

There are other possibilities. Some fields may require additional fields complementing them before any meaningful differential operations can be defined. This situation obtains, for instance, in Weyl's geometry, on which I shall comment in a few minutes.

To develop and to survey such possibilities, a geometric formulation often is a real help. Essentially mathematicians and physicists proceed intuitively when they endeavor to create new concepts and relations. Geometry often helps to think in images. Thus, geometry may serve as a heuristic device. That might not exhaust the role of geometry, but it is a major part of it.

I cannot give you a complete listing of all attempts, by Einstein and by many others, to create generalizations of the four-dimensional Riemannian model of space-time. Though I have worked on unitary field theories myself, I cannot claim any com-

prehensive knowledge. One whole class of attempts may be characterized as maintaining the four-dimensionality of space-time but modifying or enriching the Riemannian structure. In this class belongs, for instance, Weyl's geometry. Weyl weakened Riemann's idea of an invariant distance at the infinitesimal level; he replaced it by the notion of relative distance. Only the ratio of two distances would have any invariant (frame-dependent) meaning. With this weakening of the metric concept, one cannot form differential structures without introducing a pseudovector field that looks like the potentials of the electromagnetic field.

Another enrichment, suggested originally by Cartan, generalizes the notion of parallel transport of vectors. In Riemannian geometry, if you introduce a free-falling frame of reference, then a vector is parallel to a vector if, in that frame, the components are the same. In Cartan geometry they may be rotated.

Finally, in Einstein's asymmetric theory the dot product of two vectors (at the same point) is not symmetric in the two vectors, $a \cdot b \neq b \cdot a$. In all three of these examples the minimal geometric structures are richer than in Riemannian geometry, so they are capable of accommodating a greater variety of physical fields.

In Weyl's geometry the gravitational and the electromagnetic structures are distinct in that they are not being converted into each other under changes of frame, but they are both required to produce a harmonic whole. In Einstein's asymmetric theory there exists one type of change of frame that mixes the Riemannian with the other parts of the geometry; in the Cartan geometry I do not see that kind of fusing, but from a somewhat different point of view, Cartan's geometry also hangs all together.

How can any enlargement of the geometry lead to an understanding of the properties of particles? That is a very difficult question to answer. The occurrence of singularities in a field theory represents a sort of breakdown of that theory: The field equations admit of solutions that go out of the control of those equations and ruin the causal character of the field laws. There seems little doubt that general relativity as we know it today leads to singularities under a variety of circumstances. There are no solutions that might be interpreted as particles which are everywhere finite. Once you are dealing with different field equations, you can hope that such solutions exist. I might add that the theorems concerning the unavoidability of singularities in the standard theory were all discovered long after Einstein's death, mostly by R. Penrose and S. Hawking. I have not seen their methods of proof extended to any of the unitary theories, but this may well be possible.

If nonsingular solutions do exist, then one can investigate whether they are in some manner related to the properties of particles that occur in nature. There is a way to relate the ratio of charge to mass of an elementary particle to a pure number, of the order of 10^{20} , depending on the kind of particle. A theory of elementary particles should yield, at the very least, numbers like this one.

Einstein hoped to obtain quantum rules in a similar fashion. If particles interact with each other, it is not likely that singularities

can be avoided in the course of time unless the initial conditions are just right. I suspect that few practitioners of unitary field theory today share these hopes; many of them feel, I believe, that it is worth their efforts to achieve success in other respects, even if quantum theory continues to flourish in its present form.

There are other kinds of unitary field theories, including some that today claim a great deal of interest. These theories utilize, in some way or another, an increase in the number of dimensions of space-time. One famous example is Kaluza's proposal. He increased the number of dimensions to five without changing the Riemannian character of the model. He was thus able to increase the number of components of the metric so as to accommodate the electromagnetic field as well. He set one extra component equal to a constant because he had no use for it. To account for the observed four-dimensionality of space-time, he assumed that no field depended on the fifth coordinate.

Strangely enough, Kaluza's field — though conceived of as a single structure, the metric — separated quite naturally into the gravitational and the electromagnetic fields in a manner that did not at all depend on the frame used. To this extent Kaluza's fusion of fields failed. But his idea continued to intrigue others, and several variants were tried in the course of the years. One, by Einstein, V. Bargmann, and myself, replaced Kaluza's assumption of strict independence from the fifth coordinate by a weaker assumption: that the universe is closed in the fifth dimension, that it looks a bit like a tube, and that the dependence on the fifth coordinate — limited as it must be if the circumference of the tube is sufficiently small — has something to do with quantum phenomena. Alas, the idea did not work out.

Another idea, discovered and rediscovered several times over, was not to kill the supernumerary field component but to retain it and assign it such tasks as to serve as a cosmological parameter. Brans's and Dicke's so-called tensor-scalar theory is one of these attempts, though I believe that these authors were initially unaware of the preceding history of their idea.

There are other methods for increasing the dimensionality of space-time. One is to permit the coordinates of space-time to assume complex values. Penrose's twistor formalism is a case in point. Complexification is utilized by some authors as a mere technical device for discovering new solutions of Einstein's equations in the real domain; this is a productive approach, but it has little to do with unitary field theory. Others, and I believe Penrose is among them, take complex space-time seriously. They hope to break new ground. Formally, a complex number is a pair of real numbers. A complex four-dimensional space or space-time is in that sense equivalent to a real eight-dimensional manifold. But if the pairing into sets of complex coordinates (or dimensions) is taken seriously, then the rules of algebra and of analysis applied to complex numbers are equivalent to the introduction of an additional invariant structure — the so-called complex structure — which must be reproduced under all changes of frame. Thus the structure of a complex space differs significantly from that of a real space having twice as many dimensions. Penrose hopes that

by pursuing this line of inquiry he may succeed in understanding elementary particles and perhaps also the quantum character of nature.

If complex numbers are good, hypercomplex numbers may be better. Hypercomplex numbers are one way of looking at algebras that have at least some of the properties of the algebra of ordinary numbers. Whereas complex numbers are equivalent to pairs of real numbers, hypercomplex numbers involve larger multiplets. Their rules of arithmetic cannot be so simple as those involving real and complex numbers. They will involve noncommutative products ($ij \neq ji$). Most systems of hypercomplex numbers also contain null divisors, nonzero elements whose product with some other nonzero numbers equals zero.

One particular type of hypercomplex algebras is known as Grassmann algebras. The product of any two of the basic elements of a Grassmann algebra is anticommutative, $ij + ji = 0$. Interest in Grassmann algebras and in fields formed with their help originated with mathematicians and physicists impressed with the possibility of using them in elementary particle physics. It had been observed that there are collections of elementary particles that resemble each other even though some members of the set have integral spin and others half-odd spin. In quantum theory the state vectors (or wave functions) belonging to particles with integral spin are symmetric with respect to the permutation of particles; those belonging to particles with half-odd spin are antisymmetric. One type obeys Einstein-Bose statistics, the other Fermi-Dirac statistics. Some elementary particle physicists believe that there must be some changes in frame that change one kind of particle into the other field. Formally such a scheme can be set up, most conveniently with the help of Grassmann "numbers." These endeavors go under the name of supersymmetry. If they involve an attempt at unitary field theory, they are called supergravity.

There is some formal resemblance between complex field theories and supergravity. As for motivation, I am impressed with the seriousness of these novel attempts to draw inspiration from elementary particle physics, an area in which large numbers of people are obtaining new and exciting insights. Supergravity meets one objection that has been raised against the search for unitary field theory: that it has been purely speculative, without nurture from the findings of experimental physics. Supersymmetry and supergravity are speculative, to be sure; but they are influenced by high-energy physics, and that to me is a very attractive feature. There are also many unsolved problems in these attempts. I certainly do not wish to give you the impression that I am all sold on supergravity. Rather, I should say that these many years after Albert Einstein's death, a new generation of unitary field theorists is taking up the torch, and they are proceeding along novel lines. They have good contact with other frontier areas of physics; one can only wish them well.

In twentieth-century theoretical physics there have emerged a number of major areas, each dominated by a closely reasoned and closely linked set of laws. These areas have emerged in response

to the human quest for understanding, for comprehending the individual event as an instance of an overriding general principle. Albert Einstein created one such area, the theory of gravitation, and he did so by deepening our grasp of the nature of space and time, the scaffolding on which all of physical science takes place. He had hoped to expand and to strengthen this scaffolding so as to take in the physics of the atom and of the subatomic world as well, but this attempt did not succeed in his lifetime.

It behooves us to proceed, each of us, in the manner we judge best, whether or not it resembles closely Einstein's own way. All our endeavors are supported by what he had achieved, and our resolve is strengthened as we perceive not only Einstein's tenacity but also his creativity and flexibility. To most of us it is given to contribute but one small step or two toward man's understanding of nature. Let us be content with that. The quest for unity will never be sated. Each achievement will reveal new vistas and mysteries to be conquered.