A&A 553, A76 (2013)
DOI: 10.1051/0004-6361/201220447

Astronomy & Astrophysics

The $R_h = ct$ universe without inflation

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Received 26 September 2012 / Accepted 3 April 2013

ABSTRACT

Context. The horizon problem in the standard model of cosmology (ΛDCM) arises from the observed uniformity of the cosmic microwave background radiation, which has the same temperature everywhere (except for tiny, stochastic fluctuations), even in regions on opposite sides of the sky, which appear to lie outside of each other's causal horizon. Since no physical process propagating at or below lightspeed could have brought them into thermal equilibrium, it appears that the universe in its infancy required highly improbable initial conditions.
Aims. In this paper, we demonstrate that the horizon problem only emerges for a subset of Friedmann-Robertson-Walker (FRW) cosmologies, such as ΛCDM, that include an early phase of rapid deceleration.
Methods. The origin of the problem is examined by considering photon propagation through a FRW spacetime at a more fundamental level than has been attempted before.
Results. We show that the horizon problem is nonexistent for the recently introduced $R_h = ct$ universe, obviating the principal motivation for the inclusion of inflation. We demonstrate through direct calculation that, in this cosmology, even opposite sides of the cosmos have remained causally connected to us – and to each other – from the very first moments in the universe's expansion. Therefore, within the context of the $R_h = ct$ universe, the hypothesized inflationary epoch from $t = 10^{-35}$ s to 10^{-32} s was not needed to fix this particular "problem", though it may still provide benefits to cosmology for other reasons.

Key words. gravitation – cosmic background radiation – cosmological parameters – cosmology: theory – inflation – early Universe

1. Introduction

The horizon problem arises out of the standard model of cosmology[1], which posits that the universe began with a hot big bang, followed by an expansion calculable from the solution to two rather simple differential equations. In this picture, the cosmos we see today began its existence as a hot, dense plasma of thermalized matter and radiation, subjugated dynamically by its own self-gravity and the possible effects of an unspecified dark energy, whose principal role is to bring about the apparent acceleration we see today (Riess 1998; Perlmutter 1999). The initial hot state was followed by a gradual cooling and a drop in density and energy.

The observational evidence for a "beginning" appears to be overwhelming. Distant galaxies are receding from us at a speed proportional to their distance. A pervasive, relic microwave background radiation has been discovered with a Planck spectrum corresponding to a (highly diminished) temperature of 2.7 K. And the measured cosmic abundances are in agreement with the ratios predicted by big-bang nucleosynthesis, which produced elements with a composition of roughly 75% hydrogen by mass, and about 25% helium.

Couched in the language of general relativity, these observations confirm certain assumed symmetries implied by the Cosmological Principle, which holds that the universe is homogeneous and isotropic on large scales, certainly larger than 100 Mpc. The differential equations describing the universal expansion emerge directly from Einstein's theory when these symmetries are adopted. They allow several different kinds of behavior as the constituents vary, and account very well for the global characteristics seen from the moment of the big bang all the way to the present time t_0, some 13.7 billion years later.

The horizon problem is viewed as a major shortcoming of the standard model – not as a result of an observational or internal inconsistency but, rather, from the fact that the universe seems to have required special initial conditions that are highly improbable according to our current understanding of physics. In its most direct manifestation, the horizon problem arises from the observed uniformity of the microwave background radiation, which has the same temperature everywhere, save for fluctuations at the level of one part in 100 000 seen in WMAP's measured relic signal (Spergel 2003). Regions on opposite sides of the sky, the argument goes, lie beyond each other's horizon (loosely defined as the distance light could have traveled during a time t_0), yet their present temperature is identical, even though they could not possibly have ever been in thermal equilibrium, since no physical process propagating at or below the speed of light could have causally connected them. The horizon problem therefore emerges as the inability of the standard model to account for this homogeneity on scales greater than the distance light could have traveled since the big bang, requiring

★ John Woodruff Simpson Fellow.

[1] Today, the standard model of cosmology (ΛCDM) is taken to be a particular solution to Einstein's equations of general relativity, in which the overall energy density is comprised of three principal components: (luminous and dark) matter, radiation, and an as yet unidentified dark energy, which is usually assumed to be a cosmological constant Λ.

some highly tuned spatial distribution of temperature that has no evident natural cause.

So serious has this shortcoming become that the inflationary model of cosmology (Guth 1981, 1982) was invented in part to resolve this possible discrepancy. In this picture, an inflationary spurt occurred from approximately 10^{-35} s to 10^{-32} s following the big bang, forcing the universe to expand much more rapidly than would otherwise have been feasible solely under the influence of matter, radiation, and dark energy, carrying causally connected regions beyond the horizon each would have had in the absence of this temporary acceleration.

In this paper, we will analyze the horizon problem more carefully than has been attempted before, demonstrating that the conflict between the observed properties of the cosmic microwave background (CMB) and ΛCDM is not generic to all FRW cosmologies. In particular, we will show that in the $R_h = ct$ universe, even opposite sides of the cosmos have remained causally connected to us – and to each other – from the very first moments of the expansion. In this cosmology, therefore, the inflationary model may be attractive for other reasons, but there is actually no real horizon problem for it to resolve. We will argue that the inconsistency between the observed properties of the CMB and ΛCDM could be an indication that the standard model requires a new ingredient – inflation – to fix it, but could also just be evidence in support of the $R_h = ct$ universe.

2. The $R_h = ct$ universe

It will be helpful in what follows for us to briefly review what we know about the $R_h = ct$ universe up to this point. From a theoretical standpoint, the $R_h = ct$ condition is required when one adopts both the Cosmological principle and Weyl's postulate together (Melia & Shevchuk 2012). If it turns out that the universe is indeed not correctly described by the $R_h = ct$ cosmology, then either the Cosmological principle or (more likely) Weyl's postulate would be called into question. That too would be quite interesting and profoundly important, but beyond the scope of our present discussion. (A more pedagogical description of the $R_h = ct$ universe is given in Melia 2012.)

Over the past several decades, ΛCDM has developed into a comprehensive description of nature, in which a set of primary constituents (radiation, matter, and an unspecified dark energy) are assumed with a partitioning determined from fits to the available data. ΛCDM must therefore deal with the challenges of accounting for varied observations covering disparate properties of the universe, some early in its history (as seen in the relic CMB) and others more recently (such as the formation of large-scale structure).

It is therefore desirable to keep testing the basic premises of the theory, particularly those that have the most profound consequence, such as the possible role that inflation may have played in the early universe. In recent papers, we have demonstrated that the $R_h = ct$ universe may be able to provide a physical basis for at least some of the features now emerging from the observations. For example, whereas one must infer empirically the overall equation-of-state of the constituents in ΛCDM, the total pressure p in $R_h = ct$ must be given by the simple expression $p = -\rho/3$, in terms of the total energy density ρ. This leads to a great simplification of the observable quantities, such as the luminosity distance and the redshift-dependent Hubble constant, both of which appear to be consistent with the data (see, e.g., Melia & Maier 2013; Wei et al. 2013). A more comprehensive description of the $R_h = ct$ universe, and tests of its predictions against the observations, may be found elsewhere (see, e.g., Melia 2012, 2013; Melia & Maier 2013; Wei et al. 2013). In the context of this paper, the $R_h = ct$ cosmology provides a good counterpoint to ΛCDM because, unlike the latter, the former does not require a period of inflation.

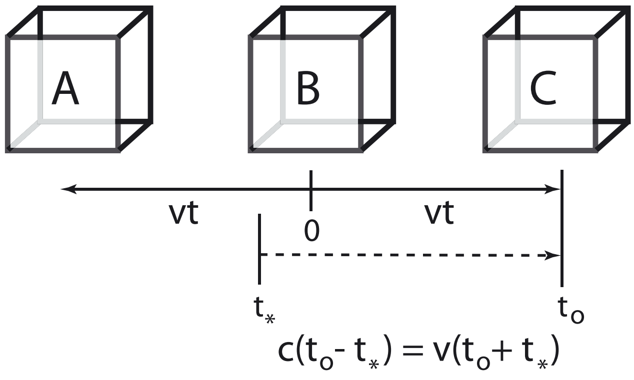

Fig. 1. Simple analogy in flat spacetime, in which 2 frames of reference (A and C) are moving away from us (situated in frame B), each at speed v, though in opposite directions. If v is close to c, after a time t in our frame, A and C are apparently beyond each other's horizon (here defined as ct), but A and C could still have communicated after $t = 0$ (and before t_0) if light were emitted by A at time t_*, such that $c(t_0 - t_*) = v(t_0 + t_*)$.

3. A simple analogy in flat spacetime

Let us now begin our discussion of the horizon problem by considering a much simpler analogy in flat spacetime, in which two frames of reference, A and C, coincident with ours (frame B) at time $t = 0$ (with t measured in our frame), are receding from us at speed v (see Fig. 1). If v is close to c, inevitably we reach the point where $2vt > ct$, and we would naively conclude that A and C are beyond each other's "horizon", here defined simply as ct. But there are actually several reasons why this statement is incorrect.

First, we cannot use the coordinates in our frame B to determine who is causally connected to A or C by simply adding velocities. To be specific, let's assume that $v = 0.9c$. Then, A's speed relative to C is not $2v = 1.8c$ but, by the relativistic addition of velocities, A is moving at speed $0.994475c$ relative to C, and is therefore a distance $0.994475ct'$ ($< ct'$) away from C as measured after a time t' in C's frame. In other words, there is no violation of causality when A's distance is measured by the observer in C using his coordinates, rather than those of another observer (in this case, ours) in frame B.

The second reason is that even though eventually $2vt > ct$, we would still conclude that A and C had communicated with each other after $t = 0$, as long as they initiated the process very soon after separation had begun. Suppose we see A sending a light signal to C at time t_*. We would conclude that the light signal had reached C by the time t_0, if

$$c(t_0 - t_*) = v(t_0 + t_*) \quad (1)$$

or, in other words, we would see that A and C had communicated with each other if their signals had been sent prior to the time

$$t_* \approx \frac{1}{2\gamma^2} t_0, \quad (2)$$

where $\gamma \equiv (1 - [v/c])^{-1/2}$ is the Lorentz factor. The emission time t_* is small, but not zero. So regardless of whether we argue from our perspective in B, or from that of the two other frames, all observers would agree that A and C had been in causal contact after $t = t' = 0$, as long as all three frames were coincident at the beginning. This is true in spite of the fact that $2vt_0 > ct_0$ in our frame. As we shall see shortly, this example is quite instructive because an effect similar to this emerges when we talk about proper distances between us and the various patches of the CMB on opposite sides of the sky (see Fig. 2 below).

4. Null geodesics in the FRW spacetime

The Friedmann-Robertson-Walker (FRW) metric for a spatially homogeneous and isotropic three-dimensional space may be written in terms of the cosmic (or proper) time t measured by a co-moving observer, and the corresponding radial (r) and angular (θ and ϕ) coordinates in the co-moving frame, for which the interval ds is

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{(1-kr^2)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \quad (3)$$

The constant k is +1 for a closed universe, 0 for a flat, open universe, or −1 for an open universe. The spatial coordinates in this frame remain "fixed" for all particles in the cosmos, while their physical separation varies by the scale factor $a(t)$. And to provide a framework for the analysis that follows, let us point out that, for radial motion, $a(t)dr/(1-kr^2)$ is the proper *incremental* distance realized when $dt = 0$, but only for each specific observer at his/her location. Nowadays, it is common for people to refer to the global quantity

$$R(t) \equiv a(t)\int \frac{dr}{\sqrt{1-kr^2}} \quad (4)$$

as the proper distance, but one must remember that $R(t)$ is calculated assuming the same common time t everywhere, which is *not* the physical time for that observer at locations other than at $r = 0$ (more on this below). Understanding this distinction will be critical for a full appreciation of why these co-moving coordinates often lead to confusion regarding what is – and what is not – causally connected.

Utilizing the FRW metric in Einstein's field equations of general relativity, one obtains the corresponding FRW differential equations of motion alluded to in the introduction. These are the Friedmann equation,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho - \frac{kc^2}{a^2}, \quad (5)$$

and the "acceleration" equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3p)\cdot \quad (6)$$

An overdot denotes a derivative with respect to cosmic time t, and ρ and p represent the total energy density and total pressure, respectively. A further application of the FRW metric to the energy conservation equation in general relativity yields the final equation,

$$\dot{\rho} = -3H(\rho + p) \quad (7)$$

which, however, is not independent of Eqs. (5) and (6).

Using the definition of proper radius, we can now derive the differential equation for photon trajectories in a cosmology consistent with the FRW metric (Eq. (3)). We have

$$\dot{R} = \dot{a}r + a\dot{r}, \quad (8)$$

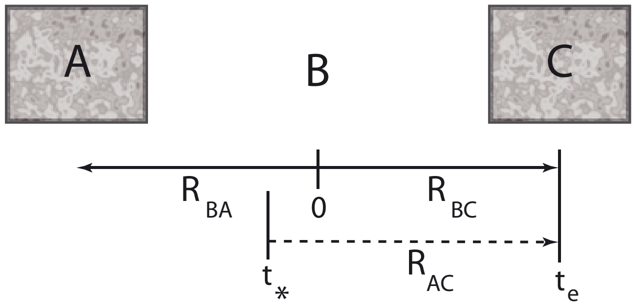

Fig. 2. An observer B is receiving light signals from two patches (A and C) of the CMB. We assume that the light was emitted by these sources at cosmic time t_e, when their proper distances from the observer were $R_{BA}(t_e)$ and $R_{BC}(t_e)$, respectively. In addition, we assume that A emitted an earlier light signal at time t_* that reached C, a proper distance $R_{AC}(t_e)$ away, at time t_e.

and the null condition applied to Eq. (3) yields

$$c\,dt = -a(t)\frac{dr}{\sqrt{1-kr^2}}, \quad (9)$$

where we have assumed propagation of the photon along a radius toward the origin. The best indications we have today are that the universe is flat so, for simplicity, we will assume $k = 0$ in all the calculations described below. Therefore, $\dot{r} = -c/a$ for a photon approaching the origin, and Eq. (8) becomes

$$\dot{R}_\gamma = c\left(\frac{R_\gamma}{R_h} - 1\right), \quad (10)$$

where we have added a subscript γ to emphasize the fact that this represents the proper radius of a photon propagating toward the observer. This expression makes use of the gravitational (or Hubble) radius (Melia 2007; Melia & Shevchuk 2012)

$$R_h = \frac{c}{H(t)}, \quad (11)$$

and we note that both R_γ and R_h are functions of cosmic time t. When the equation of state is written in the form $p = w\rho$ (for the total pressure p in terms of the total energy density ρ), it is not difficult to show from the Friedmann (5) and acceleration (6) equations that the gravitational radius satisfies the dynamical equation (Melia & Abdelqader 2009)

$$\dot{R}_h = \frac{3}{2}(1 + w)c. \quad (12)$$

Solving Eqs. (10) and (12) simultaneously yields the null geodesic $R_\gamma(t)$ linking a source of photons at proper distance $R_{src}(t_e) = R_\gamma(t_e)$ and cosmic time t_e, with the observer who receives them at time t_0, when $R_\gamma(t_0) = 0$.

The geodesic Eq. (10) can sometimes be solved analytically, for example, in ΛCDM early in the universe's history when radiation dominated ρ and p (for which $w = +1/3$). To see how this works in practice, let us consider a specific scenario (not unlike the simple thought experiment of Sect. 2), in which observer B is exchanging signals with patches A and C in the CMB, as shown in Fig. 2. The photons emitted by A and C at time t_e reach the observer B at a later time t_0 (and by symmetry, photons emitted by B at t_e also reach A and C at t_0). Earlier, A emitted a signal at time t_* ($< t_e$) in order to communicate with C at or before t_e. Thus, A and C will have been causally connected by the time their light was emitted at t_e toward B.

Now, by definition, the proper distance between A and C at time t_* is

$$R_{AC}(t_*) = a(t_*) \int_{t_*}^{t_e} c \frac{dt'}{a(t')}. \tag{13}$$

We'll think of t_e as the time of recombination, when matter and radiation separated. Therefore the universe was radiation dominated up to t_e, and

$$a(t) = (2H_0 t)^{1/2}, \tag{14}$$

where H_0 is the current value of the Hubble constant (Melia & Abdelqader 2009). With this expansion factor, Eq. (13) is easy to evaluate, yielding the result

$$R_{AC}(t_*) = 2ct_* \left[\left(\frac{t_e}{t_*}\right)^{1/2} - 1\right]. \tag{15}$$

But in order for the signal emitted by A at t_* to reach C at t_e, we must also have $R_{\gamma e}(t_*) = R_{AC}(t_*)$, and so for any arbitrary time t ($<t_e$),

$$R_{\gamma e}(t) = 2ct \left[\left(\frac{t_e}{t}\right)^{1/2} - 1\right]. \tag{16}$$

We have added an "e" to the subscript to emphasize that this is the particular geodesic reaching the observer (in this instance C) at time t_e. It is straightforward to show that Eq. (16) is the solution to Eq. (10), since in this case $R_h(t) = 2ct$. The properties of this geodesic trajectory are representative of all such null paths in FRW spacetime (Bikwa et al. 2012). Specifically, $R_{\gamma e}(t) \to 0$ as $t \to 0$, $R_{\gamma e}(t) \to 0$ as (in this particular case) $t \to t_e$, and $R_{\gamma e}(t)$ has a maximum at $t = t_e/4$, where $R_{\gamma e}(t_e/4) = ct_e/2$. All of these features are evident in the photon geodesics shown schematically in Figs. 3–5.

5. The horizon problem in ΛCDM

Figure 3 explains why the observed properties of the CMB are in conflict with ΛCDM – a situation known as the "horizon problem" in cosmology. In the following discussion, we will find it easier to consider geodesics drawn from the perspective of someone in patch C. This should not matter as far as calculated proper distances are concerned, because the cosmic time t is the same everywhere and if we determine that C has received a light signal from B during a certain time interval, then it stands to reason that B will likewise have received a signal from C during that time.

Drawn from the perspective of patch C, Fig. 3 shows the photon geodesics (solid curves) reaching the observer at two specific times. The first of these is the present (t_0 in his frame), and some of the light propagating along this path was emitted by B at time t_e, a proper distance $R_{BC}(t_e) = R_{\gamma 0}(t_e)$ away. The emission point is labeled δ in the figure. At time t_0, C is also receiving light from patch A, radiated at time t_h ($< t_e$), from a proper distance $R_{AC}(t_h) = R_{\gamma 0}(t_h)$ away, at the point labeled β. We're calling this time t_h because no light emitted by A at $t > t_h$ could have reached C by t_0. In other words, patch A lies beyond C's current photon horizon for $t > t_h$. (However, light emitted by A after t_h will become visible to C after t_0, and similarly for light emitted by B after t_e).

For a given cosmology (in this case ΛCDM), the photon geodesic labeled $R_{\gamma 0}(t)$ is unique, and if we know the time t_e at which the light was emitted (say, because we know the redshift at which the CMB was produced), there is only one point δ

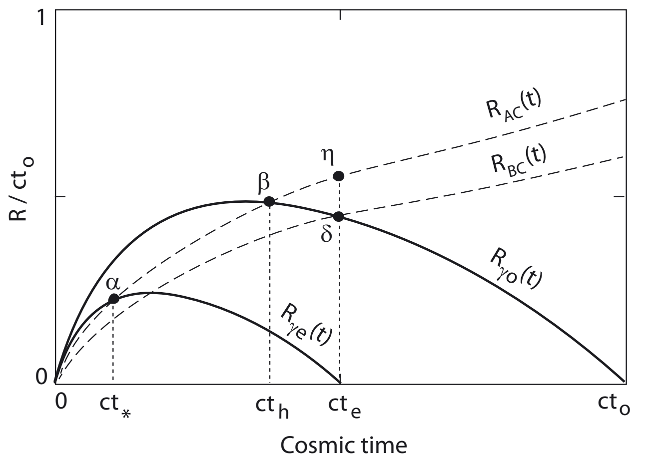

Fig. 3. Photon trajectories (solid curves) drawn from the perspective of an observer in patch C (see Fig. 2). The curve $R_{\gamma e}(t)$ is the null geodesic for light signals arriving at C at time t_e from any source at proper distance $R_{src}(t) = R_{\gamma e}(t)$ (for $t < t_e$), whereas $R_{\gamma 0}(t)$ is the corresponding null geodesic for photons arriving at C at time t_0 ($> t_e$), though from sources at $R_{src}(t) = R_{\gamma 0}(t)$ (at any $t < t_0$). The other curves and symbols are defined in the text.

that satisfies the necessary conditions for C to receive the signal at t_0. Our observations of the CMB allow us to see light emitted by patches A and C at $t_e \sim 380\,000$ years, apparently from equal proper distances on opposite sides of the sky, so that $R_{AC}(t_e) = 2R_{BC}(t_e)$ in this diagram. Is it possible within the purview of ΛCDM for us to find a time t_* to satisfy this proper-distance requirement, while A and C were still causally connected at time t_e?

The answer is yes, but not for a t_e as early as 380 000 years after the big bang. Writing

$$R_{AC}(t_e) = a(t_e) \int_{t_*}^{t_e} c \frac{dt'}{a(t')}, \tag{17}$$

we can trivially show for a radiation-dominated universe prior to t_e that

$$R_{AC}(t_e) = 2ct_e \left(1 - \left[\frac{t_*}{t_e}\right]^{1/2}\right). \tag{18}$$

For simplicity, and because the results don't depend on the details of the calculation, we will next calculate $R_{BC}(t_e)$ under the assumption that the universe was matter-dominated between t_e and t_0. (The addition of dark energy would change the proper distance by percentage points, but far from what is needed to change the answer.) So we will put

$$R_{BC}(t_e) = a(t_e) \int_{t_e}^{t_0} c \frac{dt'}{a(t')}, \tag{19}$$

where $a(t) = (3H_0 t/2)^{2/3}$ in Einstein-de-Sitter space. Thus

$$R_{BC}(t_e) = 3ct_e \left(\left[\frac{t_0}{t_e}\right]^{1/3} - 1\right). \tag{20}$$

Putting $R_{AC}(t_e) = 2R_{BC}(t_e)$ therefore gives

$$\left(\frac{t_0}{t_e}\right)^{1/3} = \frac{5}{3} - \frac{2}{3}\left(\frac{t_*}{t_e}\right)^{1/2}, \tag{21}$$

which shows that t_e is minimized when $t_* \to 0$ and has the value $t_e \approx 27t_0/125$, or roughly 1/5 of t_0. Thus, the early

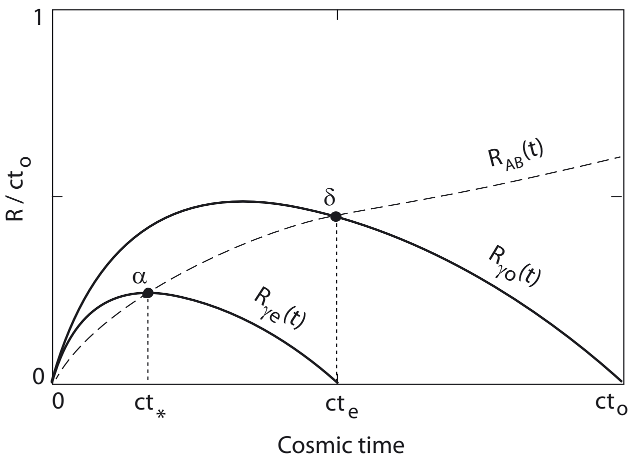

Fig. 4. Photon trajectories (solid curves) drawn from the perspective of an observer in patch B (see Fig. 2). By symmetry, the curve $R_{\gamma e}(t)$ is the null geodesic for light signals arriving at A or C at time t_e from B emitting from a proper distance $R_{AB}(t) = R_{\gamma e}(t)$ (for $t < t_e$), whereas $R_{\gamma 0}(t)$ is the corresponding null geodesic for photons arriving at B at time t_0 ($>t_e$). The other curves and symbols have the same meanings as in Fig. 3.

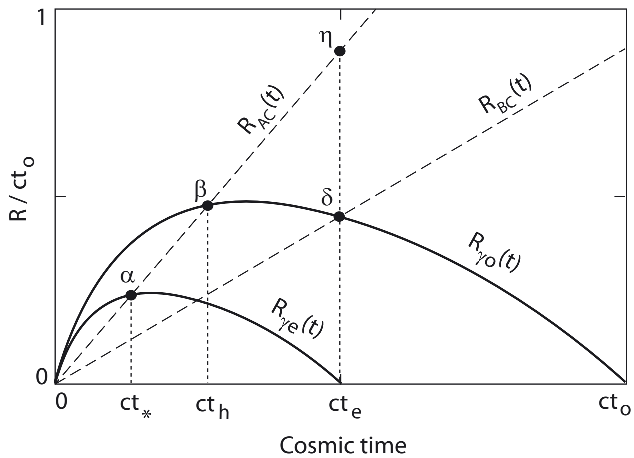

Fig. 5. Photon trajectories (solid curves) drawn from the perspective of an observer in patch C (see Fig. 2) for a universe in which $R_h = ct$. All the symbols have the same meanings as those of Fig. 3.

deceleration of the universe following the big bang would not permit patches A and C to recede from each other quickly enough for us to now see the light they emitted from opposite sides of the sky as early as $t_e = 380\,000$ years. We could, however, see their light if it were emitted within the past 10 billion years or so, *even though they are on opposite sides of the sky*.

This point needs to be emphasized because the situation is similar to that of our thought experiment in Sect. 2. Just because two patches are moving in opposite directions from us is not a sufficient reason for them to be out of causal contact. Nonetheless, the fact that we see the CMB so early in the universe's history means that either (1) there was some additional acceleration – possibly inflation – that increased $R_{AC}(t_e)$ to twice $R_{BC}(t_e)$ by t_e; or (2) the early deceleration was not as strong as that in ΛCDM, or was even completely absent. As we shall see in the next section, there is no such horizon problem in the $R_h = ct$ universe.

Before jumping to that case, however, let us consider one more variation on the scenario we have just explored, one in which the physical conditions in A and C are equilibrated by virtue of a "beacon" sending out signals from B that reach these patches before they emit at t_e. This situation is depicted in Fig. 4. Here, we need

$$R_{AB}(t_*) = R_{BC}(t_*) = 2ct_* \left(\left[\frac{t_e}{t_*} \right]^{1/2} - 1 \right), \tag{22}$$

but also

$$R_{AB}(t_e) = R_{BC}(t_e) = 3ct_e \left(\left[\frac{t_0}{t_e} \right]^{1/3} - 1 \right) \tag{23}$$

(under the same conditions as before). With

$$R_{AB}(t_e) = R_{BC}(t_e) = \frac{a(t_e)}{a(t_*)} R_{AB}(t_*), \tag{24}$$

it is straightforward to see that the result for t_e is the same as in Fig. 3, i.e., the earliest t_e that would have allowed A and C to be causally connected – and yet appear on opposite sides of the sky – is roughly 1/5 of t_0 within the context of ΛCDM. This is hardly surprizing, of course, given the high degree of symmetry implicit in the FRW metric.

6. No horizon problem in the $R_h = ct$ universe

The null geodesics and proper distances for the $R_h = ct$ universe are shown in Fig. 5, again from the perspective of an observer in patch C (see Fig. 2). The principal difference between this cosmology and ΛCDM is that here $a(t) \propto t$ for all cosmic time t. Therefore, without necessarily having to go through the same detailed derivations as before, we can immediately write down

$$R_{AC}(t_e) = ct_e \,\ln(t_e/t_*), \tag{25}$$

and

$$R_{BC}(t_e) = ct_e \,\ln(t_0/t_e). \tag{26}$$

The question now is "under what conditions could A and C have been causally connected at t_e, yet with $R_{AC}(t_e) = 2R_{BC}(t_e)$ (which means that here on Earth we would be seeing light emitted by A and C at time t_e from opposite sides of the sky)? Clearly, this condition is met when

$$t_* = t_e \left(\frac{t_e}{t_0} \right)^2, \tag{27}$$

and just as was true for the thought experiment in Sect. 2, this time t_* may be small – but it is not zero.

7. Conclusions

By carefully tracing null geodesics through the FRW spacetime, we have affirmed the well-known "horizon" problem in ΛCDM, highlighting its inability to account for the observed properties of the CMB without some additional early acceleration that would have permitted light emitted at $t_e \sim 380\,000$ years after the big bang to now reach us from opposite sides of the sky. The inflationary model was invented largely to offset this flaw in the basic theory.

However, we have also demonstrated that the "horizon" problem is not generic to all FRW cosmologies. In particular, we have demonstrated that the recently introduced $R_h = ct$ universe can easily accommodate a situation in which patches of the CMB, seemingly well beyond each other's photon horizon on opposite sides of the sky, were nonetheless in causal contact with each other after the big bang and emitted the radiation we see today under identical physical conditions. To be clear, these patches were not really beyond each other's photon horizon, though this mistaken inference is often made because of the confusion over the meaning of proper distance $R = a(t)r$. The reason is that $R(t)$ does not represent the distance ct light has traveled during a time t. One can easily convince themselves of this by a quick inspection of Eq. (10). Therefore a simple comparison of the proper distance between two patches of CMB on opposite sides of the sky with the distance ct_0 light could have traveled since the big bang is not sufficient to demonstrate whether or not these patches are causally connected.

For various reasons, some of which have been discussed elsewhere (Melia 2007, 2013; Melia & Shevchuk 2012; Melia & Maier 2013; Wei et al. 2013), we believe that the $R_\mathrm{h} = ct$ universe may provide a theoretical context for understanding at least some of the features that have emerged via fits to the data using the standard model. In this paper, we have extended the argument in favor of the $R_\mathrm{h} = ct$ universe by demonstrating that it can also explain how apparently disconnected regions of the CMB on opposite sides of the sky were nonetheless in physical equilibrium when they produced the microwave radiation we see today.

Therefore, inflation was not needed to fix a "horizon" problem that does not exist in this cosmology. In the end, this may turn out to be the most important feature of the $R_\mathrm{h} = ct$ universe because the inflationary scenario is still not completely understood, and may not be able to overcome all of its inherent problems (e.g., the so-called monopole problem; Guth & Tye 1980; Einhorn 1980).

Acknowledgements. This research was partially supported by ONR grant N00014-09-C-0032 at the University of Arizona, and by a Miegunyah Fellowship at the University of Melbourne. I am particularly grateful to Amherst College for its support through a John Woodruff Simpson Lectureship.

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